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Chapter 1: Linear Equations in One Variable (4 weeks)

Utah Core Standard(s):

- Solve linear equations in one variable. (8.EE.7)
 - a) Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
 - b) Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Academic Vocabulary: linear expression, simplify, evaluate, linear equation, equivalent expression, solve, solution, inverse operations, like terms, distributive property, ratio, no solution, infinitely many solutions

Chapter Overview:

By the end of eighth grade, students should be able to solve any type of linear equation in one variable. This includes equations with rational number coefficients that require expanding expressions using the distributive property, collecting like terms, and equations with variables on both sides of the equal sign. The chapter utilizes algebra tiles as a tool students can use to create visual, concrete representations of equations. Students manipulate the tiles while simultaneously manipulating the abstract representation of the equation, thus gaining a better understanding of the algebraic processes involved in solving equations. The goal is that students transition from the concrete process of solving equations to the abstract process of solving equations. Students should understand and know the laws of algebra that allow them to simplify expressions and the properties of equality that allow them to transform a linear equation into its simplest form, thus revealing the solution if there is one. Applications are interwoven throughout the chapter in order that students realize the power of being able to create and solve linear equations to help them solve real-world problems. The ability to solve real-world problems by writing and solving linear equations gives purpose to the skills students are learning in this chapter.

Connections to Content:


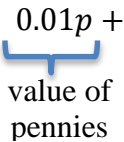

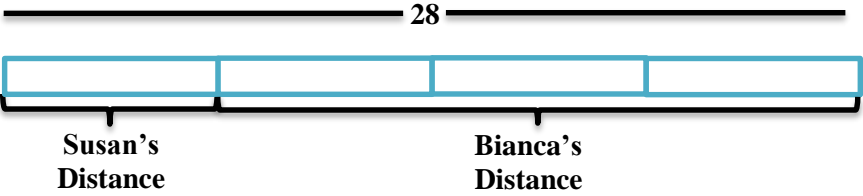
Prior Knowledge

In previous coursework, students used properties of arithmetic to generate equivalent expressions, including those that required expansion using the distributive property. Students solved one- and two-step equations. Students have also solved real-life mathematical problems by creating and solving numerical and algebraic expressions and equations.

Future Knowledge

Later in this book, students will analyze and solve pairs of simultaneous linear equations. They will also create and solve linear equations in two-variables to solve real-world problems. A student's understanding of how to solve a linear equation using inverse operations sets the foundation for understanding how to solve simple quadratic equations later in this course and additional types of equations in subsequent coursework. Additionally, in subsequent coursework, students will be creating equations that describe numbers or relationships for additional types of equations (exponential, quadratic, rational, etc.)

MATHEMATICAL PRACTICE STANDARDS

	<p>Make sense of problems and persevere in solving them.</p>	<p>At Discovery Preschool, parents who have two students enrolled get a discount on the second child. The second child’s tuition is 10 dollars less per day than the first child’s. If Tess has her two children enrolled for 5 days and her total bill for both children is \$200, how much does she pay each day for her second child to attend daycare?</p> <p>Uncle Hank has another riddle for his nephews. He tells them, “I have the same number of nickels and pennies. I have 4 times as many quarters as nickels. I have 3 more dimes than quarters. I have a total of \$6.14. Whoever can solve my riddle will get my coins.”</p> <p>Ben has started the equation for solving the riddle. Finish writing the equation that represents the riddle.</p> $0.01p +$  <p>How many of each type of coin does Uncle Hank have?</p> <p><i>Students may use a variety of strategies (diagrams, equations, tables, graphs) to solve these and other real-world problems presented throughout the chapter. Regardless of the strategy used, students must analyze givens, constraints, relationships, and goals and identify correspondences between the different approaches and representations used to solve the problems. Ultimately, students will see that these problems can be solved by creating linear equations that describe relationships between quantities. The ability to create equations to model real-world situations is a valuable tool for students going forward.</i></p>
	<p>Reason abstractly and quantitatively.</p>	<p>Bianca ran three times farther than Susan. Together they ran 28 miles. The following is a model that represents this situation.</p>  <ol style="list-style-type: none"> Write an equation that represents this situation. How far did each girl run? <p><i>A variety of models are used throughout this chapter to help students make sense of quantities and their relationship in a problem. The concrete bar model shown above helps students to abstract the situation and represent it symbolically. Once the equation is solved, students must interpret the solution in the context and attend to the meaning of the quantities.</i></p>



Construct viable arguments and critique the reasoning of others.

Two students, Arthur and Oliver, each have some chocolates. They know that they have the same number of chocolates. Arthur has two tubs of chocolates, one bag of chocolates, and twenty-five loose chocolates. Oliver has two tubs of chocolates, two bags of chocolates, and seven loose chocolates. Determine the number of chocolates in a tub. Determine the number of chocolates in a bag. Include any pictures, models, or equations you used to solve the problem and clearly explain the strategy you used.

Sam was asked to evaluate the expression $5x + 3x + 20$ for $x = 100$. Sam's work is shown below. What mistake did Sam make? Help Sam to answer the question correctly.

Sam's Work:

$$5x + 3x + 20 = 100$$

$$8x + 20 = 100$$

$$8x = 80$$

$$x = 10$$

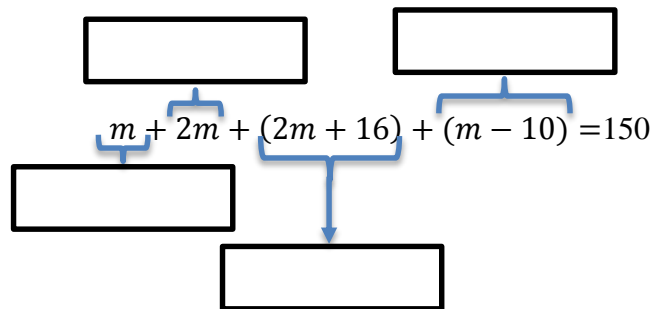
Students may use a variety of strategies to solve the chocolate problem above. They must be able to justify their conclusions, communicate them to others, and respond to the arguments made by others. In the second example, students are asked to examine a problem that has been solved incorrectly, explain what the error is, and solve the problem correctly. This type of error analysis requires students to possess a clear understanding of the mathematical concepts and skills being studied.




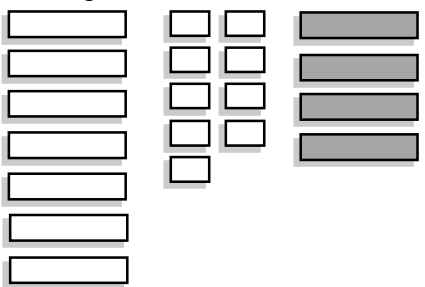
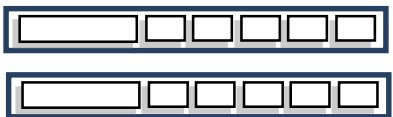
Model with mathematics.




A marble jar has twice as many blue marbles as red marbles, 16 more green marbles than blue marbles, and 10 fewer white marbles than red marbles. The jar has a total of 150 marbles. Use this information to answer the questions that follow.

The following equation represents this situation. Match each piece of the equation to the appropriate marble color. Write your answer in the boxes provided.



Determine how many marbles of each color are in the jar.

		<p>Josh works 40 hours a week as a nurse practitioner. He makes time and a half for every hour he works over 40 hours. Josh works 60 hours one week and earns \$2100. Part of an equation that represents this situation is shown below.</p> <div style="text-align: center;"> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">over-time pay rate</div> <div style="text-align: center; margin: 5px 0;">↓</div> $\text{---}(40) + 1.5p(\text{---}) = 2100$ <div style="text-align: center; margin: 5px 0;">↑</div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">hours worked at regular rate</div> </div> <p>Fill in the blanks in the equation above so that it matches the story. What is Josh's regular hourly rate? What is Josh's overtime hourly rate? <i>Throughout eighth grade, students will build linear models to represent real-world situations, moving fluently between the verbal representation, concrete models, and abstract or symbolic representation. These models map the relationships between quantities in a given situation, allow students to solve many real world problems, and help students to draw conclusions and make decisions in a given situation.</i></p>
	<p>Use appropriate tools strategically.</p>	<p>The following is a model of the equation $7x + 9 - 4x = 2(x + 5)$. Create this model with your tiles and solve the equation, showing your solving actions below.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <p><i>Concrete models are used throughout this chapter as a tool to assist students in becoming proficient in the abstract process of solving any type of linear equation. Once students have mastered how to solve a linear equation, this skill becomes a tool for accessing more advanced mathematical content.</i></p>

	<p>Attend to precision.</p>	<p>Find and Fix the Mistake: Ricardo solved the following equation incorrectly. Circle the mistake and describe the mistake in words. Then, solve the equation correctly.</p> $2x + x + 5x = 56$ $7x = 56$ $x = 8$ <p style="margin-left: 150px;">Combine like terms. Divide both sides by 7.</p> <p><i>When students analyze errors made by others, they must be clear in their understanding of the content and skills being learned. Students identify, explain, and correct common errors that are made when solving equations.</i></p>
	<p>Look for and make use of structure.</p>	<p>Use the following equations to answer the questions that follow.</p> $\frac{x + 3}{2} = 5 \qquad \frac{x}{2} + 3 = 5 \qquad \frac{1}{2}(x + 3) = 5 \qquad \frac{x}{2} + \frac{3}{2} = 5$ <p>Examine each of the equations above. Circle the equations that are equivalent. Think about the structure of the expressions on the left side of the equation. It may help to use your tiles and draw a model of each equation.</p> <p>Consider the expression $4a - 12$. Write 3 different expressions that if set equal to $4a - 12$ would result in the equation having infinite solutions.</p> <p><i>In order to solve equations, students must make sense of the structure of the expressions in an equation. They must be able to view expressions as a single object or as composed of several objects in order to determine the solving actions and in order to operate on the expression correctly. In order to determine whether two expressions are equivalent and generate equivalent expressions, students must compose and decompose expressions.</i></p>
	<p>Look for and express regularity in repeated reasoning.</p>	<p>Directions: Write the story that goes with the expressions and equation in each problem. Solve for the unknown information and interpret your answer.</p> <p>Number of weeks: w Sophie's Money: $300 - 40w$ Raphael's Money: $180 + 20w$ $300 - 40w = 180 + 20w$</p> <p><i>This chapter deals with linear expressions and equations. Linear functions grow at a constant rate. In the problem above, Sophie's money is decreasing at a rate of \$40 per week while Raphael's is increasing at a rate of \$20 per week. Students may realize that Raphael is closing the gap in the amount of money each child has by \$60 a week and use this reasoning to help determine how long it will take for both children to have the same amount of money.</i></p>

1.0 Anchor Problem: Chocolate



Directions: Consider the following situations. Then answer the questions below. Include any pictures, models, or equations you used to solve the problem and clearly explain the strategy you used.

Situation 1: Two students, Theo and Lance, each have some chocolates. They know that they have the same number of chocolates. Theo has four full bags of chocolates and five loose chocolates. Lance has two full bags of chocolates and twenty-nine loose chocolates.

Determine the number of chocolates in a bag. Determine the number of chocolates each child has.

Situation 2: Two students, Arthur and Oliver, each have some chocolates. They know that they have the same number of chocolates. Arthur has two tubs of chocolates, one bag of chocolates, and twenty-five loose chocolates. Oliver has two tubs of chocolates, two bags of chocolates, and seven loose chocolates.

Determine the number of chocolates in a tub. Determine the number of chocolates in a bag.

Situation 3: Two students, Abby and Amy have the same number of chocolates. Abby has one full tub of chocolates and 21 remaining chocolates. Amy has one full tub of chocolates and 17 remaining chocolates.

Determine the number of chocolates in a tub.

Section 1.1: Creating and Solving Multi-Step Linear Equations

Section Overview:

This section begins with a review of writing and simplifying algebraic expressions. Students review what an algebraic expression is and what it means to simplify an algebraic expression. They also evaluate algebraic expressions and create expressions to represent real-world situations. This work with expressions sets the foundation for the study of linear equations. Students learn what a linear equation is, what it means to solve a linear equation, and the different outcomes that may occur when solving an equation (one solution, no solution, and infinitely many solutions). Section one focuses on equations with one solution. Students solve linear equations whose solutions require collecting like terms. They then move to equations whose solution requires the use of the distributive property and collecting like terms. Applications that can be solved using the types of equations being studied in a lesson are interwoven throughout. Scaffolding has been provided in order to aide students in the process of creating equations to represent and solve real-world problems.

Concepts and Skills to Master:


By the end of this section, students should be able to:

1. Understand the meaning of linear expression and linear equation.
2. Solve multi-step linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
3. Write and simplify linear expressions and equations that model real-world problems.

1.1a Class Activity: Simplifying Linear Expressions


1. Emma is playing a popular video game and is determined to beat the high score. The game saves her place so that each time she plays it again, she picks up in the same place with the same number of points. Emma downloads the video game on Monday night and starts playing, scoring a bunch of points. On Tuesday, she scores an additional 500 points. On Wednesday she doubles her score from the previous day. On Thursday, she scores the same number of points that she scored on Monday.
 - a. Miguel's teacher asks him to write an expression that represents Emma's total score after she is done playing on Thursday. Miguel writes the following expression:

$$2(p + 500) + p$$

Miguel's teacher lets him know that his expression is correct. Write in words what each piece of Miguel's expression represents in the story problem. 

- b. Nevaeh writes the following expression to represent Emma's score on Thursday.

$$2p + 1000 + p$$

The teacher lets her know that she is also correct. How did Nevaeh represent the problem differently than Miguel? 

- c. Can you think of another expression to represent Emma's score on Thursday?
- d. If Emma scored 700 points on Monday, evaluate each of the three expressions above to determine how many points Emma has on Thursday.

In this lesson, we will study linear expressions. Miguel, Nevaeh, and you all wrote linear expressions to represent Emma's total number of points on Thursday. A **linear expression** is a mathematical phrase consisting of numbers, unknowns (symbols that represent numbers), and arithmetic operations. Linear expressions describe mathematical or real-world situations.

The following are all examples of linear expressions:

- $3x - 5$
- $2x - x - 17$
- $3x$
- $6(2x - 5) + 11$
- $3x + 7x - 3 + 2x$
- 25

Below are **equivalent linear expressions** that represent Emma's score from the video game example. Equivalent expressions have the same meaning. Two expressions are considered **equivalent** when a substitution of any number for the unknown p in each of the expressions produces the same numerical result. Substituting in a specific number for the unknown in an expression and calculating the resulting value is called **evaluating** the expression.

$$2(p + 500) + p$$

$$2p + 1000 + p$$

$$3p + 1000$$

For ease of communicating mathematical ideas, we will consider a linear expression in the form $Ax + B$ where A and B are numbers and x represents an unknown, the **simplified form of a linear expression**. In the example above, the simplified form of the expression is $3p + 1000$.

In this lesson, we will be using tiles to model and simplify linear expressions.

Key for Tiles:



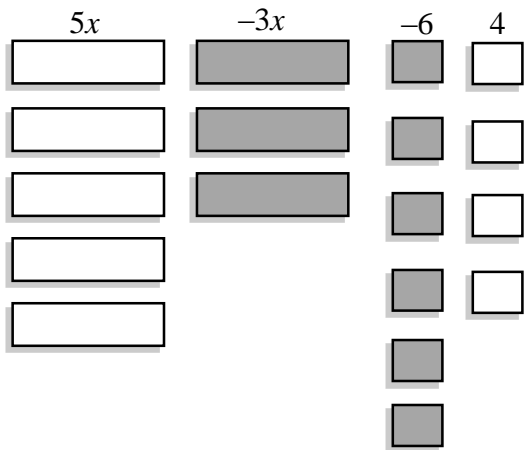




Remember that a positive tile and a negative tile can be combined to create a **zero pair** or add to zero.




2. The following is a model of the expression $5x + (-3x) - 6 + 4$

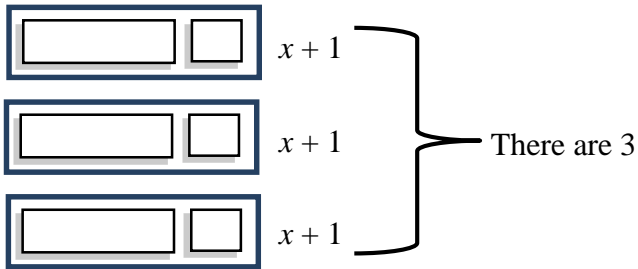


- a. Find zero pairs, and write the simplified form of this expression.
- b. Evaluate this expression when $x = 8$.

3. Using your tiles, model the expression $-4x + 3 + 5x + (-1)$.

- a. Find zero pairs and write the simplified form of this expression.
- b. Evaluate this expression when $x = -5$.

4. The following is a model of the expression $3(x + 1)$.



- a. Write the simplified form of this expression.
- b. Evaluate this expression when $x = -4$.

5. Using your tiles, model the expression $2(2x - 1)$.

- a. Write the simplified form of this expression.
- b. Evaluate this expression when $x = 0$.

Directions: Model and simplify each expression. 

6. $2(x + 2) - x$

7. $3 - 3x + 4(x - 3)$

Directions: Simplify each expression. Use the tiles if you need to.

8. $-4x + 3 + 5(2x - 1)$

9. $12 - (x - 2) + 4x$

10. Write three expressions that are equivalent to $6x + 12$. Use the tiles if you need to.



11. Write three expressions that are equivalent to $4x - 2$. Use the tiles if you need to.

12. A group of friends goes to a movie on Friday night. Each friend purchases a movie ticket that costs \$8, a small popcorn that costs \$3.50, and a medium drink that costs \$2.25.

- a. Circle the expression(s) below that represent the **total amount of money spent** by the group if f represents the number of friends that went to the movie. There may be more than one answer.



$8 + 3.50 + 2.25$

$f(8 + 3.50 + 2.25)$

$f + 8 + 3.50 + 2.25$

$f + 13.75$

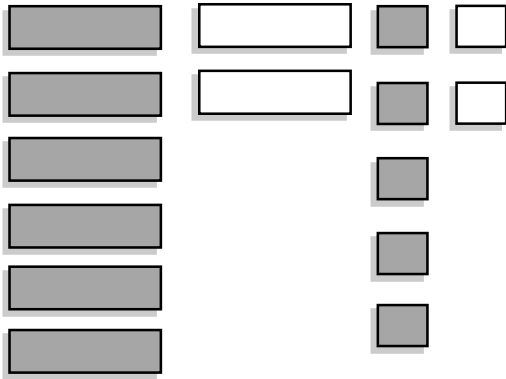
$8f + 3.5f + 2.25f$

$13.75f$

- b. If 5 friends go to the movie, how much money will **each person** spend? How much money will the **entire group** spend?

1.1a Homework: Simplifying Linear Expressions

1. The following is a model of the expression $-6x + 2x - 5 + 2$

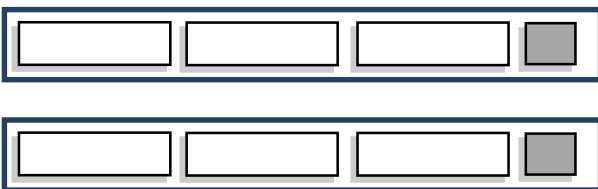


- Find zero pairs, and write the simplified form of this expression.
- Evaluate this expression for $x = 2$.

2. Model the expression $5x + 2 - x - 4$.

- Simplify the expression modeled above.
- Evaluate this expression for $x = -3$.

3. The following is a model of the expression $2(3x - 1)$.



- Write the simplified form of this expression.
- Evaluate this expression for $x = 4$.

Directions: Simplify each expression. Use the tiles if needed.

4. $2(x + 3) + 4x$	5. $4 + 2(x - 2) - 4$
6. $6(2x - 4) - 3x$	7. $4 - (2x + 3) + 5x$
8. $-6x - 4 + 5x + 7$	9. $\frac{1}{2}(4x + 12) + 2x$

10. Write three expressions that are equivalent to $3x + 15$.

11. Write three expressions that are equivalent to $2x + 4x - 15$.

12. Dan's basketball coaches have set up the following schedule for practice: 10 minutes warm-up and stretching; 15 minutes of defensive drills; 10 minutes of passing drills; 20 minutes of shooting practice, and x minutes of time to scrimmage.

- a. Circle the expression(s) below that represent the amount of time Dan will practice **each week** if they practice **3 days a week**. There may be more than one answer.

$$10 + 15 + 10 + 20 + x$$

$$3(10 + 15 + 10 + 20 + x)$$

$$3(55 + x)$$

$$165 + x$$

$$165 + 3x$$

$$168x$$

- b. If Dan's team scrimmages for 35 minutes each practice, how long is **each** practice? How long do they practice **each week** (again assuming they practice 3 days a week)?

Find and Fix the Mistake: In each of the following problems, a common error has been made when

simplifying the expressions. Identify the mistake, explain it, and simplify the expression correctly.



<p>13. $5x + 4x - x$ $9x$ Explain mistake:</p> <p>Correct answer:</p>	<p>14. $2(x + 5)$ $2x + 5$ Explain mistake:</p> <p>Correct answer:</p>
<p>15. $5 - (x - 2)$ $5 - x - 2$ $3 - x$ Explain mistake:</p> <p>Correct answer:</p>	<p>16. $-3(2x - 5)$ $-6x - 15$ Explain mistake:</p> <p>Correct answer:</p>
<p>17. $2x + 3 + 4x$ $9x$ Explain mistake:</p> <p>Correct answer:</p>	<p>18. $x + 3x + 6x$ $10x^2$ Explain mistake:</p> <p>Correct answer:</p>

19. Evaluate the expression $2x + 4$ for the following values of x .



a. $x = 1$	b. $x = -1$
c. $x = 0$	d. $x = \frac{1}{2}$

20. Evaluate the expression $-3x + 2$ for the following values of x .

a. $x = 1$	b. $x = -1$
c. $x = 0$	d. $x = \frac{1}{3}$

1.1b Class Activity: Writing Linear Expressions to Model Real World Situations

1. Aria and her friends are playing a game. The expressions below represent the amount of money each player has at the end of the game where m is the amount of money a player started with.

- a. Match each player to the correct expression.  

Expressions		
A. $2(3m - 100)$ Player:	B. $\frac{m}{2} + 100 - 25$ Player:	C. $2m - 100 - 25$ Player:
D. $\frac{(m+100)}{2} - 25$ Player:	E. $2(m - 100) - 25$ Player:	F. $2\left(\frac{m}{3} - 100\right)$ Player:

Player Scenarios	
Peta collects the following cards: <ul style="list-style-type: none"> You are doing a great job at work and just received a bonus. Double the amount of money you currently have. It's time to pay property taxes. Pay the bank \$100. It's the first day of school. Pay the store \$25 for school supplies. 	Miya collects the following cards: <ul style="list-style-type: none"> Congratulations! Your art submission won first place. Collect \$100. Your house was damaged in a flood. Pay half of your remaining money for repairs. You got a parking ticket. Pay \$25.
Aria collects the following cards: <ul style="list-style-type: none"> It's time to pay property taxes. Pay the bank \$100. You are doing a great job at work and just received a bonus. Double the amount of money you currently have. It's the first day of school. Pay the school \$25 for supplies. 	Hadley collects the following cards: <ul style="list-style-type: none"> You won the lottery! Triple the amount of money you currently have. It's time to pay property taxes. Pay the bank \$100. You are doing a great job at work and just received a bonus. Double the amount of money you currently have.
Lea collects the following cards: <ul style="list-style-type: none"> Your house was damaged in a flood. Pay half of your remaining money for repairs. Congratulations! Your art submission won first place. Collect \$100. You got a parking ticket. Pay \$25. 	Sierra collects the following cards: <ul style="list-style-type: none"> Your two sisters want to join the game. Divide the money you currently have between the three of you. It's time to pay property taxes. Pay the bank \$100. You are doing a great job at work and just received a bonus. Double the amount of money you currently have.

- b. If each player started the game with \$1,000, who won the game?

Directions: For #2 – 4, circle the expression(s) that correctly model each situation. There may be more than one

answer.  

2. Tim took his friends to the movies. He started with \$40 and bought 3 movie tickets that each cost x dollars. He also bought one tub of popcorn that cost \$5.75.

a. Which of the following expression(s) represent the amount of money Tim has left?

$$40 - x - x - x - 5.75$$

$$40 - 3x - 5.75$$

$$40 - 3 - 5.75$$

$$-3x + 34.25$$

$$34.25 - 3x$$

$$31.25x$$

b. If each movie ticket costs \$6, how much money does Tim have left?

3. Master Tickets charges \$35 for each concert ticket, plus an additional \$2 service fee for each ticket purchased. Kanye purchased x concert tickets.

a. Which of the following expression(s) represent the amount of money Kanye spent?

$$35x + 2$$

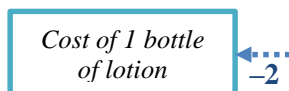
$$37x$$

$$35x + 2x$$

$$x(35 + 2)$$

b. If Kanye purchased 4 concert tickets, how much did he spend?

4. Sara bought 3 baby outfits that cost p dollars each and one bottle of baby lotion. The baby lotion costs 2 dollars less than an outfit. The following is a model of this situation.



a. Which of the following expression(s) represent the amount of money Sara spent?

$$p + p + p - 2$$

$$3p + (p - 2)$$

$$p + p + p + (p - 2)$$

$$4p - 2$$

$$3p - 2$$

$$2p$$

b. If each baby outfit costs \$5, how much did Sara spend?

5. Antony and his friends went to a fast food restaurant for lunch. They ordered and ate 3 Big Micks, 2 Mick-Chicken Sandwiches, and 4 large fries. A Mick-Chicken Sandwich has 200 fewer calories than a Big Mick. A large fry has 50 fewer calories than a Big Mick.
- a. Part of an expression that represents the total number of calories consumed by Antony and his friends is shown below. Fill in the remaining pieces of the expression on the lines provided and simplify the expression.

$$\begin{array}{c} \text{\# of chicken} \\ \text{sandwiches} \\ \downarrow \\ \text{---}(c) + 2(\text{---}) + \text{---}(c - 50) \\ \uparrow \qquad \qquad \qquad \uparrow \\ \text{calories in a} \qquad \qquad \text{calories in a} \\ \text{Big Mick} \qquad \qquad \text{large fry} \end{array}$$

- b. Write the simplified form of the expression.

- c. If a Big Mick has 550 calories in it, how many calories did Antony and his friends consume?

1.1b Homework: Writing Linear Expressions to Model Real World Situations

1. Mateo sends approximately twice as many text messages as his mom each month. His dad sends approximately 500 fewer text messages than Mateo each month. The following expression represents the total number of text messages sent by the family each month:

$$t + \frac{t}{2} + (t - 500)$$

- a. Write in words what each piece of the expression represents in the story.

t : _____

$\frac{t}{2}$: _____

$(t - 500)$: _____

- b. If Mateo sends approximately 3,000 texts per month, approximately how many texts does his entire family send each month (assuming he has no other family members)?

Directions: For #2 – 3, circle the expression(s) that correctly model the situation. There may be more than one answer.

2. Christina is purchasing one of each of the following for her nieces for Easter: a pack of sidewalk chalk that costs \$2.25, a bottle of bubbles that costs \$1, and a chocolate bunny that costs \$3.50.
- a. Which of the following expression(s) represent the amount of money Christina will spend if she has n nieces?

$6.75 + n$

$n(2.25 + 1 + 3.5)$

$6.75n$

$2.25n + n + 3.5n$

6.75

$n(2.25n + 1n + 3.5n)$

- b. If Christina has 5 nieces, how much money will she spend on gifts for Easter?

3. At Six East Shoe Store, a pair of boots is \$30 more than a pair of sandals. A pair of sandals costs s dollars each. Emily purchased 2 pairs of boots and 3 pairs of sandals.
- a. Which of the following expression(s) represent the amount of money Emily spent?

$2(s + 30) + 3s$

$(s + 30) + (s + 30) + (s + 30) + s + s$

$2s + 3(s + 30)$

$(s + 30) + (s + 30) + s + s + s$

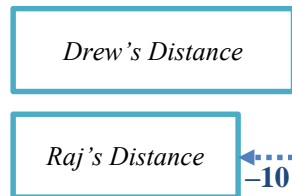
$5s + 90$

$5s + 60$

- b. If a pair of sandals is \$15, how much did Emily spend in all?

Directions: Write a linear expression **in simplified form** that represents each of the following situations. For these problems, simplified form is $Ax + B$ where A and B are numbers and x represents an unknown.

4. Drew and Raj are both training for a bike race. Raj bikes 10 miles less than Drew each day that they train. The following is a model of this situation.



- Write an expression **in simplified form** that represents the number of miles Drew bikes if Raj bikes m miles each day.
 - Write an expression **in simplified form** that represents the total number of miles Raj and Drew bike if they each train 5 days a week and m represents the number of miles Raj bikes each day.
 - If Raj bikes 15 miles each day, how many miles do Raj and Drew bike together each week (again assuming they train 5 days a week)?
5. Naja is paid p dollars per hour she works. For every hour she works over 40 hours, she is paid time and a half which means she is paid 1.5 times her normal hourly rate. She worked 50 hours last week. The following is a model of this situation.
- Irene tried to write an expression that represents the amount Naja earned last week but needs your help. Help Irene finish the expression by filling in the blanks.

$$40(\underline{\quad}) + \underline{\quad}(1.5p)$$

↑
hours worked
at regular rate

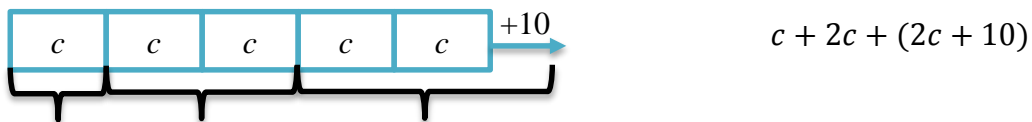
over-time
pay rate
↓

- Simplify the completed expression above.
- If Naja's regular hourly rate is \$30 per hour, how much did she earn last week?

1.1c Class Activity: Solving Multi-Step Linear Equations (combine like terms)

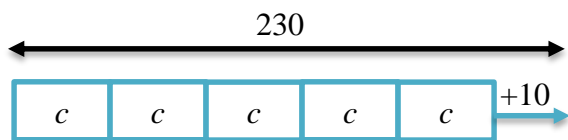
In the lessons up to this point, we have been working with linear expressions. We reviewed how to **simplify** a linear expression and how to **evaluate** a linear expression for a given value of x . We will now begin our work with linear equations. When we **solve** a linear equation, our task is to find the values of the unknown that make the equation true.

- Damion and his friends went trick-or-treating. The next day, they got together and counted their candy. Damion had twice as much candy as Nick. Bo had 10 more pieces than Damion. The following model and expression represent the amount of candy the boys have together:



- Show on the model and the expression which pieces represent the amount of candy each of the boys has.

What if we also knew that together the boys have 230 pieces of candy? Let's look at a model for this:





$c + 2c + (2c + 10)$ and 230 are both linear expressions that represent the amount of candy the boys have together. When we set two linear expressions equal to each other, we create a linear equation. A **linear equation** is an assertion or statement that two linear expressions are equal to each other. Using the candy example, we can create the following equation:

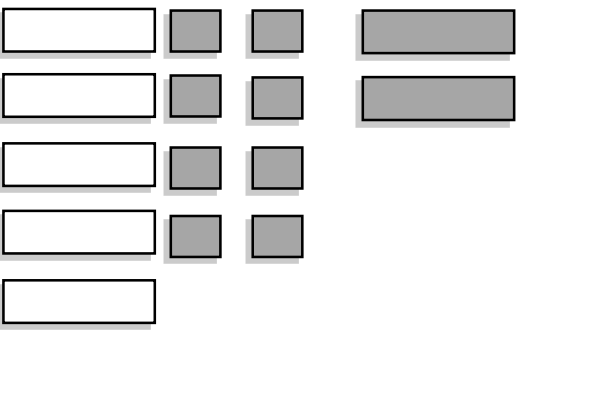
$$\begin{array}{c}
 \text{"is equal to"} \\
 \downarrow \\
 \underbrace{c + 2c + (2c + 10)}_{\text{Expression 1}} = \underbrace{230}_{\text{Expression 2}}
 \end{array}$$

- Model this equation with your tiles and solve for c .
- What does c represent in the context?
- How many pieces of candy do each of the boys have?

A **solution** to an equation is a number that makes the equation true when substituted for the unknown. In the example above, the solution is 44. Verify that when you substitute 44 in for c the equation is true.

It is important to note that when we create an equation, the two expressions on either side of the equal sign might be true for 1) **one value of x** (as we saw in the candy example above), 2) **no values of x** (there is not a number that can be substituted for the variable to make the equation true), or 3) **all values of x** (every number we substitute in for the variable will make the equation true). In the first section, we will study equations that have one solution.

2. The following is a model of the equation $5x - 8 - 2x = 4$. Create this model with your tiles and solve the equation, showing the solving actions (steps) below.  

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- a. Solving Actions (show each step below):

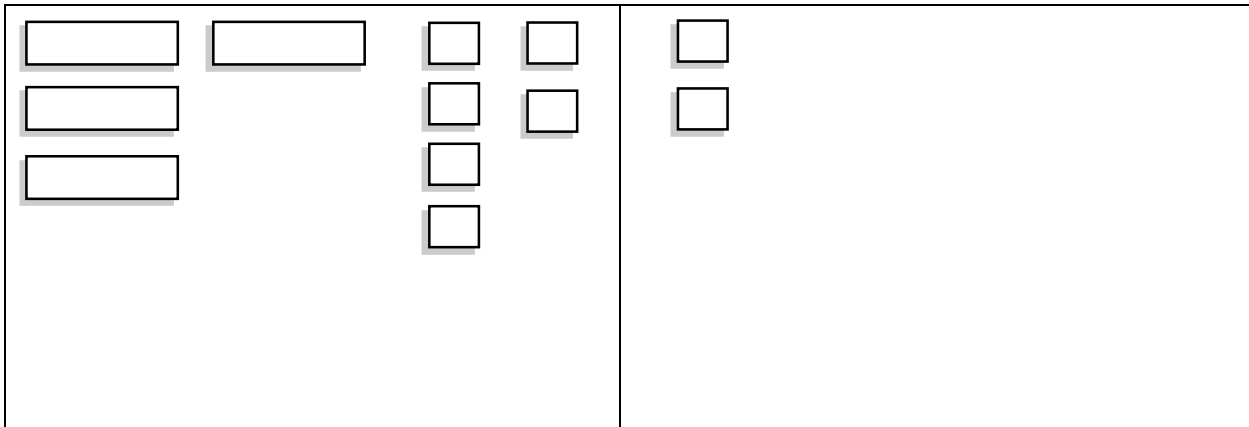
$$5x - 8 - 2x = 4$$

- b. Verify the solution in the space below.

Directions: Model and solve the following equations. Show the solving actions and verify your solution.

<p>3. $4x + 3x - 1 = 6$</p>	<p>4. $10 = -x + 3x + 4$</p>
<p>5. $2x - x + 4 = -8$</p>	<p>6. $10 = -2x - 3 + 4x + 5$</p>

7. The following is a model of an equation.



- a. Write the symbolic representation (equation) for this model.
- b. Solve the equation.

Directions: Solve the following equations.

8. $-7x + 5x + 3 = -9$	9. $17 = m + 5 - 3m$
10. $0.5b + 2b = -50$	11. $-\frac{2}{3} = -\frac{4}{3} + 6r$

12. **Find and Fix the Mistake:** Ricardo solved the following equation incorrectly. Circle the mistake and

describe the mistake in words. Then, solve the equation correctly.




$$2x + x + 5x = 56$$

$$7x = 56$$

Combine like terms.

$$x = 8$$

Divide both sides by 7.

13. Carson and his family drove to Disneyland. They started driving on Thursday and then stopped for the night. On Friday, they drove twice as many miles as they had on Thursday. On Saturday, they drove fifty miles more than they had on Friday. Carson's mom asked him to write an equation to determine how many miles they drove each day. 

Carson wrote the following equation: $m + 2m + (2m + 50) = 650$

- a. Match each expression with what it represents in the story.

m	The number of miles driven on Friday
$2m$	The number of miles driven on Saturday
$(2m + 50)$	The total number of miles driven.
$m + 2m + (2m + 50)$	The number of miles driven on Thursday.

- b. If Carson and his family live 650 miles from Disneyland, how many miles did Carson's family drive each day?

Thursday: _____ **Friday:** _____ **Saturday:** _____

14. George started writing a story that matches the expressions and equation shown on the left. Pieces of the story are missing. Help him finish the story, solve the equation, and determine each person's age.



Ages

Talen's age: t
 Peter's age: $8t + 3$
 $t + (8t + 3) = 39$

Story

I am trying to figure out Peter and Talen's ages. Peter tells me that he is three more than...

Together, Talen and Peter's ages...

How old are Talen and Peter?

Talen's age: _____

Peter's age: _____

1.1c Homework: Solving Multi-Step Linear Equations (combine like terms)

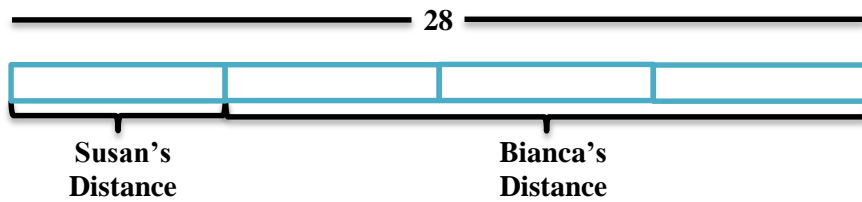
Directions: Solve the following equations. Verify your solutions.

1. $7x - 3x = 24$	2. $6a + 5a = -11$	3. $-4n - 2n = 6$
4. $5a + 8 + (-2a) = -7$	5. $18 = 3x + 4 + 2$	6. $-2x - 7 - 4x = 17$
7. $s + 3s = -24$	8. $-5n + 4n - 7 = 8$	9. $x - 4 + 3 = -6$
10. $6 = 4t - 3t - 2$	11. $2x - 9x + 17 = -4$	12. $x + 3x + 4x = 56$
13. $8x + 25 - 6x = 35$	14. $-32 = -4x - 2x + 4$	15. $1.3b - 0.7b = 12$
16. $0.4y + 0.1y = -2.5$	17. $\frac{2}{5}x - \frac{1}{5}x = 9$	18. $r - \frac{5}{4} + \frac{1}{2}r = \frac{13}{4}$
19. $3x + 5 + 6x - 7 = 25$	20. $7d - 12 + 3d = 15$	21. $22 = 5c + 3c - c + 8$

Find and Fix the Mistake: In #22 – 23, a common mistake has been made. Circle the mistake and describe the mistake in words. Then, solve the equation correctly.

<p>22. $2x + 4x - 2 = 20$</p> <p>$6x - 2 = 20$ Combine like terms ($2x$ and $4x$)</p> <p>$4x = 20$ Combine like terms ($6x$ and -2)</p> <p>$x = 5$ Divide by 4.</p> <p>Explanation of mistake:</p> <p>Solve correctly:</p>	<p>23. $3x - 8x - 5 = 10$</p> <p>$5x - 5 = 10$ Combine like terms ($3x$ and $-8x$).</p> <p>$5x = 15$ Add 5 to both sides.</p> <p>$x = 3$ Divide both sides by 5.</p> <p>Explanation of mistake:</p> <p>Solve correctly.</p>
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24. Bianca ran three times farther than Susan. Together they ran 28 miles. The following is a model that represents this situation.



- Write an equation that represents this situation. _____
- How far did each girl run?

Susan: _____ **Bianca:** _____

25. Write a story that matches the expressions and equation shown on the left. Then solve the equation and find each person's age.

Ages

Felipe's age: f

Felipe's sister's age: $f - 6$

Felipe's mom's age: $3f - 9$

$$f + (f - 6) + (3f - 9) = 60$$

Story

Felipe's age: _____ Felipe's sister's age: _____ Felipe's mom's age: _____

Hint: When writing stories like the one above, be sure to relate the unknown to its meaning in the story. Don't use the actual unknown in the story. For example, in the situation above, do not write "Felipe's sister is f minus 6 years old." Think about how Felipe's sister's age is related to Felipe's age which is represented by f in the expression. Also, remember that your story needs a question.

1.1d Class Activity: Equations with Fractions


1. Use the following equations to answer the questions that follow.

$$\frac{x + 3}{2} = 5$$

$$\frac{x}{2} + 3 = 5$$

$$\frac{1}{2}(x + 3) = 5$$

$$\frac{x}{2} + \frac{3}{2} = 5$$

- a. Examine each of the equations above. Circle the equations that are equivalent. Think about the structure of the expressions on the left side of the equation. It may help to use your tiles and draw a model of each equation. 
- b. Solve each of the equations in the space above. Did you find that some equations were easier to solve than others? Why or why not?

When faced with an equation with fractions, we can transform it into an equation that does not contain fractions. This is called **clearing of fractions**. In the problems above, in order to clear the fractions, we need to get rid of the 2 in the denominator of each equation.

- c. Can you think of a way to eliminate the 2 in each equation before you start to solve the equation? Test your method and re-solve each of the equations above.

Directions: Solve each equation by first clearing the fractions.

$$2. \frac{x-1}{4} = 6$$

$$3. \frac{x}{2} + \frac{1}{4} = \frac{7}{4}$$

$$4. \frac{1}{3}(2x - 4) = -6$$

$$5. \frac{2x}{3} + 4 = \frac{14}{3}$$

$$6. \frac{4}{5} = 3x + \frac{1}{5}$$

$$7. \frac{-2x-5}{3} = 3$$

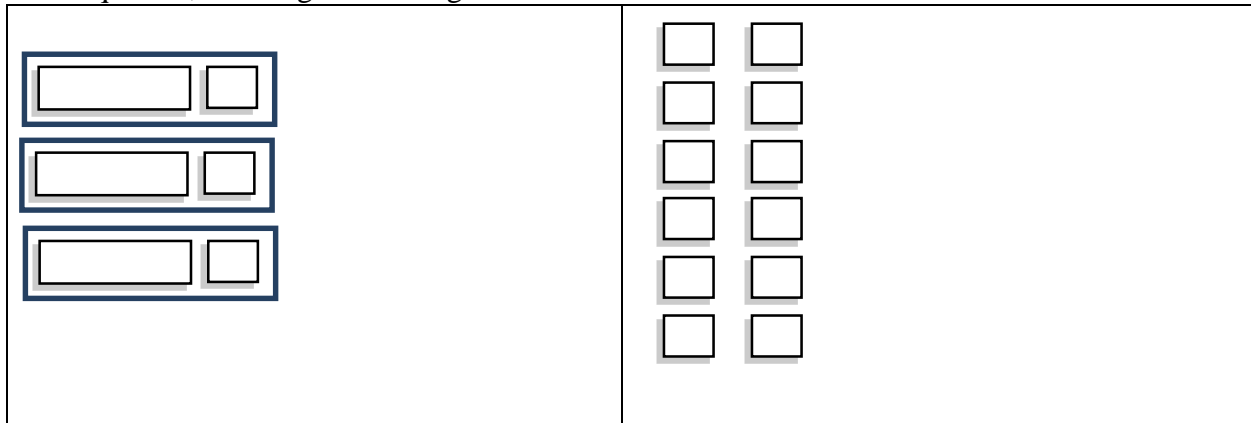
$$8. \frac{x}{12} + \frac{1}{3} = \frac{1}{4}$$

$$9. -6 = \frac{(3x-12)}{3}$$

$$10. \frac{1}{2}(4x + 12) = 2$$

1.1e Class Activity: Solving Multi-Step Linear Equations (distribute and combine like terms)

1. The following is a model of the equation $3(x + 1) = 12$. Create this model with your tiles and solve the equation, showing the solving actions below.



Solving Actions:

$$3(x + 1) = 12$$

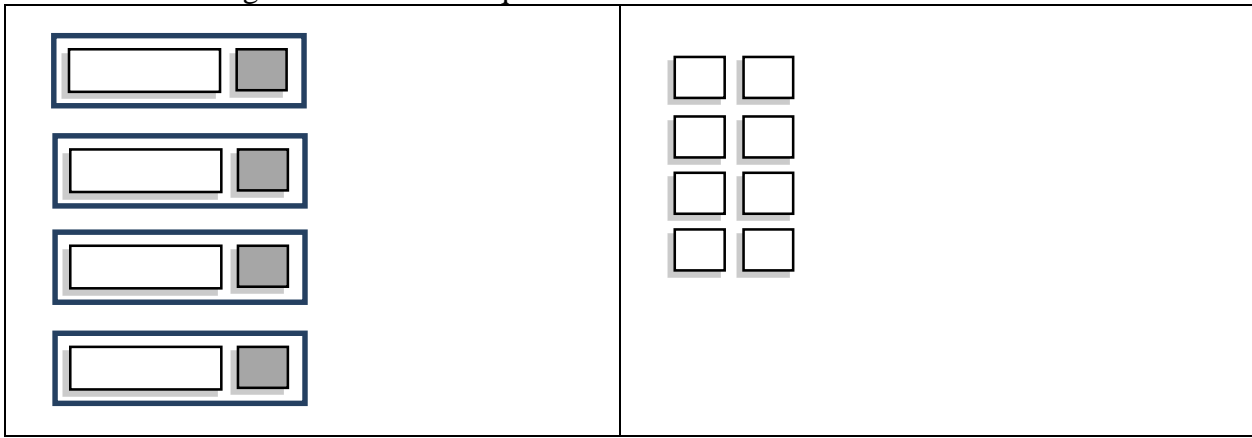
Directions: Model and solve the following equations.

2. $2(x + 5) = 14$

3. $2(3x + 1) - 2x = 10$

4. $-12 = 3(x - 2)$

5. The following is a model of an equation.



- a. Write the symbolic representation for this model.
- b. Solve the equation.

Directions: Solve the following equations without the use of the tiles.

6. $-2(x + 1) = 8$	7. $13 = -3(x - 4) - 8$
8. $5 + 2(3a - 1) = 15$	9. $\frac{1}{2}(2t + 4) = -8$

$$10. \frac{x}{3} + \frac{x-2}{5} = 6$$

$$11. 14 = 5 - 3(x - 2)$$

12. Part of a story that matches the expressions and equation shown on the left has been written for you. Finish the story, solve the equation, and determine how much time Theo spends training in each sport.

Triathlon Training Schedule

Minutes spent swimming: x

Minutes spent running: $2x$

Minutes spent biking: $2x + 30$

$3x + 4(2x) + 2(2x + 30) = 510$ min.

Story

Theo is training for a triathlon. He runs twice as long as he swims. He bikes...

He swims three times a week, runs four times a week, and bikes...

If he spends a total of 510 minutes per week training, how many minutes does he spend on each exercise at a time?

Minutes spent swimming: _____ **Minutes spent running:** _____ **Minutes spent biking:** _____

13. Write a story that matches the expressions and equation shown on the left. Then, solve the equation and determine how much each ride at the fair costs.

A Trip to the Fair

Cost of a pony ride: b

Cost to ride the Ferris wheel: $\frac{1}{2}b$

Cost to bungee jump: $2b + 5$

$3b + 4\left(\frac{1}{2}b\right) + (2b + 5) = \33

Story

Cost of a pony ride: _____ **Cost to ride the Ferris wheel:** _____ **Cost to bungee jump:** _____

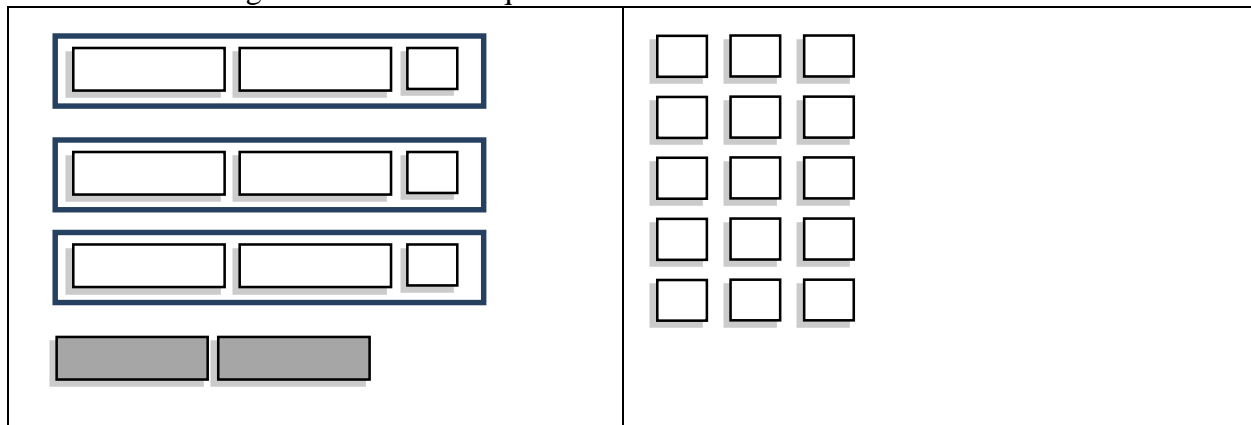
14. Solve this riddle. “Consider the numbers 3, 8, and 7. Find a fourth number so that the average of the numbers is 7.” The following equation represents this situation.

The diagram shows the equation $\frac{3 + 8 + 7 + x}{4} = 7$. Three empty rectangular boxes are connected to the equation by blue arrows. One box is on the left with an arrow pointing to the numerator. Another box is on the right with an arrow pointing to the denominator. A third box is centered below the equation with an arrow pointing to the denominator.

- Fill in the boxes above telling what each piece of the equation represents.
- Solve the equation and find the fourth number.

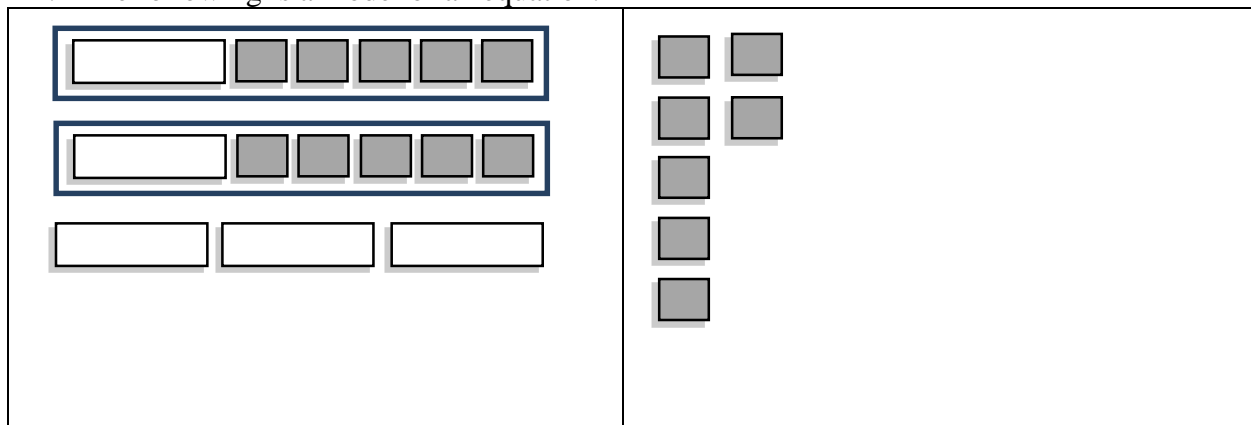
1.1e Homework: Solving Multi-Step Linear Equations (distribute and combine like terms)

1. The following is a model of an equation.



- Write the symbolic representation (equation) for this model.
- Solve the equation.


2. The following is a model of an equation.



- Write the symbolic representation for this model.
- Solve the equation.

Directions: Solve the following equations. Verify your solutions.

3. $3(4x - 2) = 30$	4. $-24 = 4(2 + 2x)$	5. $-16 = 2(4x + 8)$
6. $3(x + 10) + 5 = 11$	7. $3t - 2 + t - 5t = -1$	8. $-24 = 2(1 - 5x) + 4$
9. $-2(a + 3) + 4a = 18$	10. $28 = 5x + 3(x + 4)$	11. $4x - 3(x - 2) = 21$
12. $0 = -2(x + 5) + 3x$	13. $\frac{1}{3}(x + 6) = 1$	14. $5 - 4(2b - 5) + 3b = 15$
15. $10 = 3(x - 2) - 2(5x - 1)$	16. $0.2(10t - 4) - t = 1.2$	17. $-\frac{1}{7}(x - 7) + 22 = 26$
18. $-10 = \frac{3x}{4} + \frac{x}{2}$	19. $\frac{x-2}{3} = \frac{1}{2}$	20. $-(x - 5) + 2 - x = 3$

Find and Fix the Mistake: In the following problems, a common mistake has been made. Circle the mistake and describe the mistake in words. Then, solve the equation correctly. 

21. $7 + 2(3x + 4) = 3$

$7 + 6x + 4 = 3$ Distribute the 2.

$11 + 6x = 3$ Combine like terms (7 and 4).

$6x = -8$ Subtract 11 from both sides.

$x = -\frac{8}{6}$ Divide both sides by 6.

$x = -\frac{4}{3}$ Simplify the fraction.

Explanation of Mistake:

Solve Correctly:

22. $-2(y - 3) + 5 = 3$

$-2y - 6 + 5 = 3$ Distribute the -2.

$-2y - 1 = 3$ Combine like terms (-6 and 5)

$-2y = 4$ Add 1 to both sides.

$y = -2$ Divide both sides by -2.

Explanation of Mistake:

Solve Correctly:

23. $5 - (2x - 7) = 14$

$5 - 2x - 7 = 14$ Distribute the negative sign.

$-2 - 2x = 14$ Combine like terms (5 and -7).

$-2x = 16$ Add 2 to both sides.

$x = -8$ Divide both sides by -2.

Explanation of Mistake:

Solve Correctly:

24. $\frac{1}{3}(x + 15) = 11$

$\frac{1}{3}x + 5 = 11$ Distribute the $\frac{1}{3}$.

$\frac{1}{3}x = 6$ Subtract 5 from both sides.

$x = 2$ Divide both sides by 3.

Explanation of Mistake:

Solve Correctly:

25. The expressions below show the grams of fat in sandwiches at a popular fast food restaurant. Use these expressions and the equation to write a story and determine the number of grams of fat in each sandwich.

Fast Food Calories

Crispy Chicken: f

Single burger with cheese: $f + 11$

Double burger with cheese: $2f$

$3f + 2(f + 11) + 4(2f) = 204$ g.

Story

Fat grams in a Crispy Chicken: _____

Fat grams in a single burger with cheese: _____

Fat grams in a double burger with cheese: _____

26. Mia has taken two quizzes in math so far this quarter and scored a 75% on the first and an 82% on the second. What must she score on the third quiz in order to have an average of 80% on her three quizzes?

3. Use the story below about Farmer Ted and his animals to answer the questions that follow.

Farmer Ted's Animals

Weight of a cow: $\underline{\quad c \quad}$

Weight of a horse: $\underline{\hspace{2cm}}$

Weight of a sheep: $\underline{\hspace{2cm}}$

Weight of a pig: $\underline{\hspace{2cm}}$

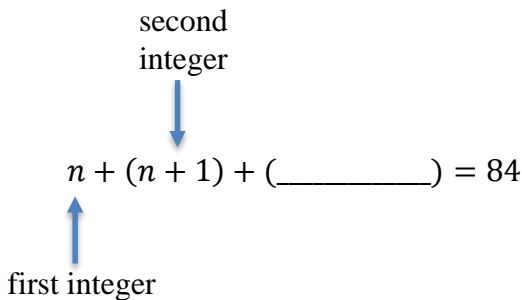
Story

Farmer Ted is weighing his animals. He knows that a pig weighs approximately $\frac{1}{4}$ as much as a cow. He also knows that a Clydesdale horse weighs about twice what a cow weighs. A sheep weighs approximately 100 pounds less than a pig.

- On the lines above to the left, write the expressions that match the weight of each of the animals if a cow weighs c pounds.
- Write an expression for the weight of one cow, one horse, one sheep, and one pig.
- If Farmer Ted puts 3 cows, 2 Clydesdale horses, 4 sheep and 1 pig on a giant scale used for weighing semi-trucks, the scale reads 7,850 pounds. Approximately how much does each animal weigh?

Cow: $\underline{\hspace{2cm}}$ **Clydesdale horse:** $\underline{\hspace{2cm}}$ **Sheep:** $\underline{\hspace{2cm}}$ **Pig:** $\underline{\hspace{2cm}}$

4. Miley is trying to solve the following riddle: “The sum of three consecutive integers is 84. What are the integers?” She writes part of an equation that can be used to solve this riddle.



- Help Miley complete the equation above by filling in the blank.
- Find the three integers.

5. Eli is making lemonade for a party. Expressions showing the ratio of water to sugar to lemon juice used to make lemonade are shown on the left.

Making Lemonade

Cups of water: c

Cups of sugar: $\frac{1}{4}c$

Cups of lemon juice: $\frac{1}{2}c$

$$c + \frac{1}{4}c + \frac{1}{2}c = 14 \text{ cups}$$

Story

- Write a story that matches the expressions and equation shown on the left.
- Solve the equation above. How many cups of each ingredient is Eli planning to use?

Cups of water: _____

Cups of sugar: _____

Cups of lemon juice: _____

6. Use the incomplete story and the expressions and equation below to answer the questions that follow.

Triangles

$m\angle A: x$

$m\angle B: 3x$

$m\angle C: x - 20$

$$x + 3x + (x - 20) = 180^\circ$$

Story

In $\triangle ABC$, the measure of $\angle B$ is three times larger than...

The measure of $\angle C$ is 20° less than...

The sum of the angles in a triangle is...

What is the measure of each angle in the triangle?

- Finish the story above so that it matches the expressions and equation shown on the left.
- What is the measure of each angle in the triangle?

$m\angle A =$ _____

$m\angle B =$ _____

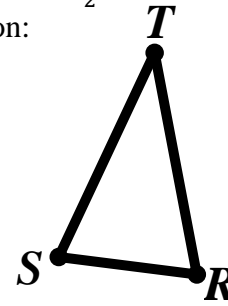
$m\angle C =$ _____

7. In $\triangle RST$, $\angle R$ and $\angle S$ have the same measure. The measure of $\angle T$ is $\frac{1}{2}$ the measure of $\angle R$ and $\angle S$. Marie drew the following model and picture to represent this situation:

$\angle R$

$\angle S$

$\angle T$



- a. Help Marie write an equation that represents the sum of the angles in $\triangle RST$. Remember the sum of the angles in a triangle is 180° .
- b. Solve the equation and find the measure of each angle.

$m\angle R$: _____

$m\angle S$: _____

$m\angle T$: _____

8. Use the expressions and equation below to answer the questions that follow.

Rectangles

Width of a rectangle: w

Length of a rectangle: $2w$

$2w + w + 2w + w = 42$ ft.

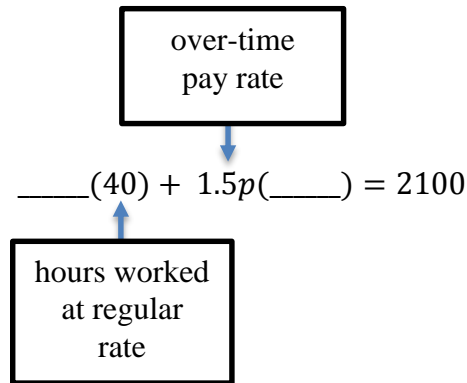
Story

- a. Draw a picture of the rectangle that accurately depicts the ratio of the length of the rectangle to its width.
- b. Write a story in the space provided that matches the expressions and equation.
- c. What is a different but equivalent way of writing the equation above?
- d. Solve the equation and find the length and width of the rectangle.

Length: _____

Width: _____

9. Josh works 40 hours a week as a nurse practitioner. He makes time and a half for every hour he works over 40 hours. Josh works 60 hours one week and earns \$2100. Part of an equation that represents this situation is shown below.



- a. Fill in the blanks in the equation above so that it matches the story.
- b. What is Josh's regular hourly rate? _____
- c. What is Josh's overtime hourly rate? _____
10. The ratio of girls to boys at The Gymnastics Preparation Center is 3:2. If there are 180 kids that train at The Gymnastics Preparation Center, how many of them are girls? How many of them are boys?
11. The average of three numbers is 14. The largest number is two more than twice the smallest. The second largest number is twice the smallest number. Find the three numbers.

3. During the summer, Victoria plays soccer and takes swim and piano lessons. Each swim lesson is 15 minutes shorter than a soccer practice. Each piano lesson is twice as long as a soccer practice. Use this information to answer the questions that follow.

a. The following expressions represent how long an activity is each time she goes. Write the name of the activity that matches each expression on the lines provided.

t : _____

$t - 15$: _____

$2t$: _____

b. Victoria has soccer three times a week, swimming four times a week, and piano twice a week. She spends a total of 435 minutes each week doing these three activities a week. Write an equation that represents this situation.

c. How long is one session of each activity?

Soccer: _____

Swimming: _____

Piano: _____

4. The ratio of freshmen to sophomores to juniors to seniors in band is 1:2:3:2. If there are a total of 240 students in the band, how many are in each grade level?

Freshmen: _____

Sophomores: _____

Juniors: _____

Seniors: _____

5. The art teacher is making salt dough for an upcoming project. The ratio of flour to salt to water used to make salt dough is shown below.

Making Salt Dough

Cups of flour: $2c$

Cups of salt: c

Cups of water: $\frac{3}{4}c$

$2c + c + \frac{3}{4}c = 60$ cups

Story

- a. Write a story that matches the expressions and equation shown on the left.
 b. Solve the equation. How many cups of each ingredient is the art teacher planning to use?

Cups of flour: _____

Cups of salt: _____

Cups of water: _____

6. Use the story below about a triangle to answer the questions that follow.

Angles in a Triangle

$m\angle A$: _____

$m\angle B$: _____

$m\angle C$: _____ x _____

Equation: _____

Story

In $\triangle ABC$, the measure of $\angle B$ is three times larger than the measure of $\angle C$. The measure of $\angle A$ is twice as large as the measure of $\angle C$. The sum of the angles in a triangle is 180° .

- a. Write the expressions and equation matching the story on the lines provided above.
 b. What is the measure of each angle in the triangle?

$m\angle A =$ _____

$m\angle B =$ _____

$m\angle C =$ _____

9. Use the expressions and equation below about the cost of clothes to answer the questions that follow.

The Cost of Clothes

Cost of a shirt: c

Cost of a pair of jeans: $c + 12$

$$3c + 2(c + 12) = \$164$$

Story

- a. Write a story that matches the expressions and equation in the space provided.
- b. Solve the equation to determine the cost of a shirt and the cost of a pair of jeans.

Cost of a shirt: _____ **Cost of a pair of jeans:** _____

Directions: Write and solve an equation to answer each of the following problems. Use pictures and models to help you. Refer back to similar problems you have already seen in the chapter to help you if you get stuck. Make sure your answers are displayed clearly with the appropriate units.

10. The width of a rectangle is five less than three times the length of the rectangle. If the perimeter of the rectangle is 70 inches what are the dimensions of the rectangle?

11. At Shoes for Less, a pair of shoes is \$15 less than a pair of boots. Cho purchased three pairs of shoes and two pairs of boots for \$120. How much does a pair of boots cost?

12. Central Lewis High School has five times as many desktop computers as laptops. The school has a total of 360 computers. How many laptops does Central Lewis High School have?
13. In $\triangle LMN$, the measure of $\angle L$ is equal to the measure of $\angle M$. The measure of $\angle N$ is twice the measure of $\angle M$. Find the measure of each angle in $\triangle LMN$.
14. Adam is trying to solve the following riddle: “The sum of three consecutive integers is -36 . What are the integers?” Solve Adam’s riddle.
15. Afua got a 90% on her first math exam, a 76% on her second math exam, and a 92% on her third math exam. What must she score on her fourth exam to have an average of 88% in the class?
16. At Discovery Preschool, parents who have two students enrolled get a discount on the second child. The second child’s tuition is 10 dollars less per day than the first child’s. If Tess has her two children enrolled for 5 days and her total bill for both children is \$200, how much does she pay each day for her second child to attend daycare?

1.1g Self-Assessment: Section 1.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Understand the meaning of linear expression and linear equation.	I am struggling with the difference between a linear equation and a linear expression.	If given a list of expressions and equations, I can tell which ones are expressions and which are equations.	I can define linear expression and equation in my own words and provide examples of each.	I can define linear expression and equation in my own words and provide examples of each. I know what it means to simplify an expression, to evaluate an expression, and to solve an equation.
2. Solve multi-step linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	I am struggling to solve most of the equations on the following page.	I can solve the equations in Set A on the following page.	I can solve all of the equations in Set A and all but one of the equations in Set B.	I can solve all of the equations in Sets A and B on the following page.
3. Write and solve linear expressions and equations that model real world problems.	When faced with a word problem similar to those in this chapter, I am having a difficult time seeing how the pieces of an equation relate to the story and I usually skip these problems.	When faced with a word problem similar to those in this chapter, I can match the different pieces of an equation that has been given to me to the story and solve the equation.	When faced with a word problem similar to those in this chapter, I can identify the important quantities in a practical situation, complete partial expressions and equations that have been given to me, solve the equation, and interpret the solution in the context.	When faced with a word problem similar to those in this chapter, I can identify the important quantities in a practical situation, write expressions and equations showing the relationship between the quantities, solve the equation, and interpret the solution in the context.

1. Sam was asked to evaluate the expression $5x + 3x + 20$ for $x = 100$. Sam's work is shown below.

Sam's Work:

$$5x + 3x + 20 = 100$$

$$8x + 20 = 100$$

$$8x = 80$$

$$x = 10$$

- a. What mistake did Sam make? Help Sam to answer the question correctly.

2. Solve the following equations.

Set A

1. $8x - 6x + 1 = 11$	2. $-4x - 6 + x = -12$	3. $2(x + 3) + 4x = 40$
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Set B

4. $-2(2x + 3) + 3x = -2$	5. $7k - (k + 6) + 2k = 22$	6. $\frac{1}{2}(4x - 16) + 8x = 12$
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3. Jesse and her brothers Nick and Owen are saving money over the summer. Each week, Jesse saves twice as much as Owen. Owen saves \$5 more than Nick. At the end of four weeks, the three of them have saved a total of \$220. How much money does each person save per week?

Section 1.2: Creating and Solving Multi-Step Linear Equations with Variables on Both Sides

Section Overview:


In this section students will solve equations with unknowns on both sides of the equal sign. From here, students will apply the skills learned so far in the chapter and solve a variety of linear equations with rational number coefficients. Up to this point, students have only encountered linear equations with a unique solution (one solution). In the latter part of this section, students will be introduced to linear equations in one variable with no solution or infinitely many solutions. Students will analyze what it is about the structure of an equation and the solving outcome ($x = a$, $a = b$, or $a = a$ where a and b are different numbers) that results in one solution, infinitely many solutions, or no solution (is true for a unique value, no value, or all values of the unknown).

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Solve multi-step linear equations that have one solution, infinitely many solutions, or no solution.
2. Understand what it is about the structure of a linear equation that results in equations with one solution, infinitely many solutions, or no solutions.
3. Identify and provide examples of equations that have one solution, infinitely many solutions, or no solution.

1.2a Class Activity: Solving Multi-Step Linear Equations (variables on both sides)

1. The following is a model of the equation $5x + 2 = 3x + 12$. Create this model with your tiles and solve the equation, showing your solving actions below. 

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- a. Solving Actions:
 $5x + 2 = 3x + 12$

- b. How can you verify your solution?

Directions: Model and solve the following equations.

2. $2x = x + 4$	3. $3x + 3 = 2x + 7$	4. $x + 10 = 2x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2 - 5x = -6x + 5$

Directions: Solve the following equations. You may use the tiles to help you.

8. $4x = 2x + 12$

9. $5x + 3 = x + 27$

10. $-4x + 6 = 3x - 36$

11. $0.7x - 0.6 = -0.2x - 0.42$

12. $8 - 4x = 4x$

13. $\frac{1}{3}x - 8 = 12 + \frac{4}{3}x$

14. $\frac{x+3}{2} = \frac{x-1}{4}$

15. $\frac{(x-3)}{3} = \frac{(2x+4)}{5}$

1.2a Homework: Solving Multi-Step Linear Equations (variables on both sides)

Directions: Solve the following equations. Verify your solutions.

1. $3x = 2x + 2$	2. $3x + 5 = 4x + 1$	3. $6x + 3 = 3x + 12$
4. $3x + 8 = 2x + 10$	5. $5x + 3 = 3x + 7$	6. $5a - 5 = 7 + 2a$
7. $3b - 6 = 8 - 4b$	8. $-3y + 12 = 3y - 12$	9. $3x = -3x + 1$
10. $-2x + 8 = -3x - 1$	11. $3p - 4 = 5p + 4$	12. $3 - 0.25x = -\frac{1}{2}x + 9$

Directions: In the following problems, a common mistake has been made. Circle the mistake and describe the mistake in words. Then, solve the equation correctly.

13. $4x - 8 = -2x + 20$

$2x - 8 = 20$ Combine like terms ($4x$ and $-2x$)

$2x = 28$ Add 8 to both sides.

$x = 14$ Divide both sides by 2.

Explanation of Mistake:

Solve Correctly:

14. $6x + 4 = -2x$

$8x = 4$ Add $2x$ to both sides.

$x = \frac{4}{8}$ Divide both sides by 8.

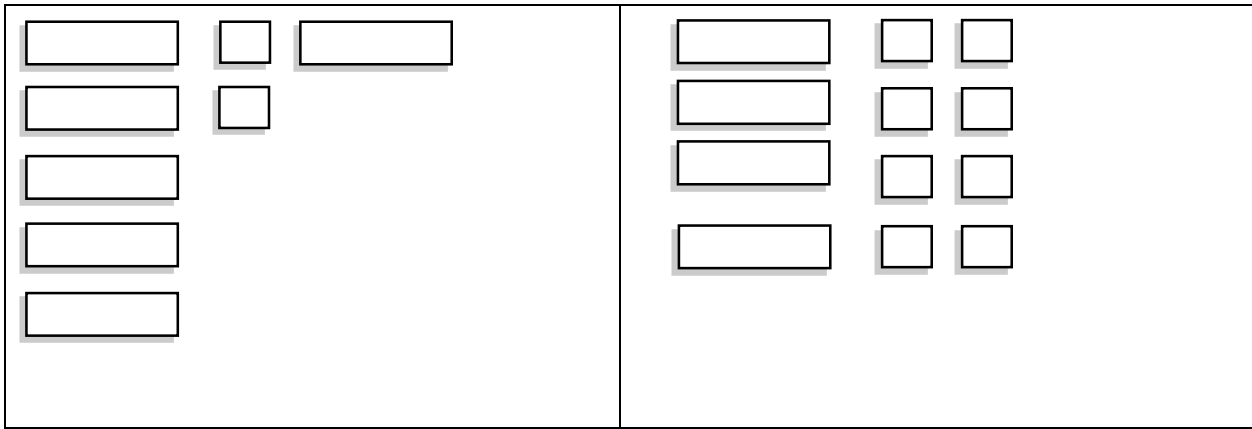
$x = \frac{1}{2}$ Simplify the fraction.

Explanation of Mistake:

Solve Correctly:

1.2b Class Activity: Solving Multi-Step Linear Equations (putting it all together)

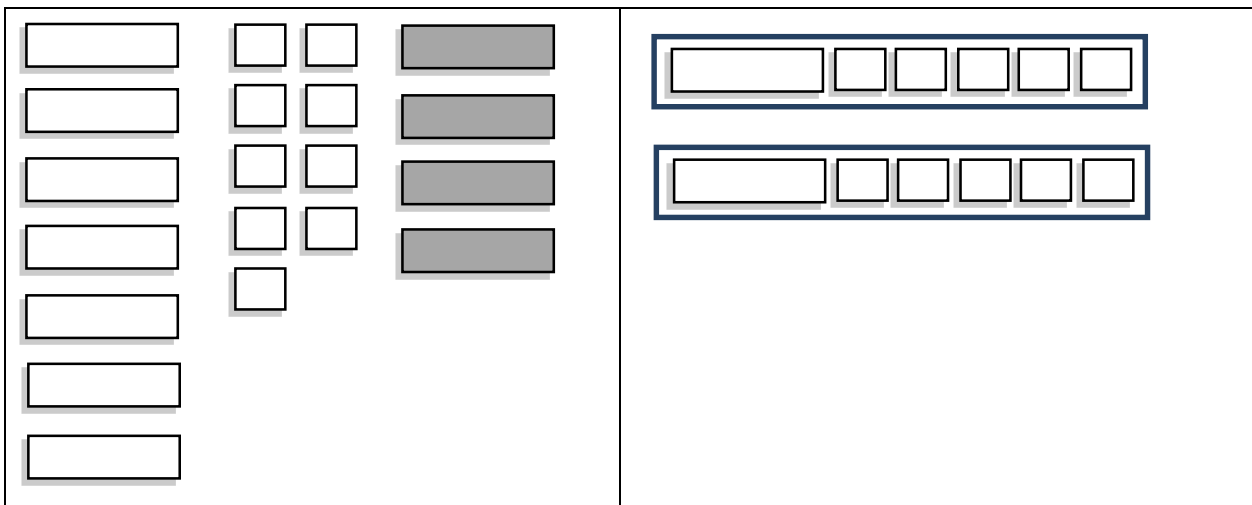
1. The following is a model of the equation $5x + 2 + x = 4x + 8$. Create this model with your tiles and solve the equation, showing your solving actions below.



- a. Solving Actions:
 $5x + 2 + x = 4x + 8$

- b. Verify your solution.

2. The following is a model of the equation $7x + 9 - 4x = 2(x + 5)$. Create this model with your tiles and solve the equation, showing your solving actions below.

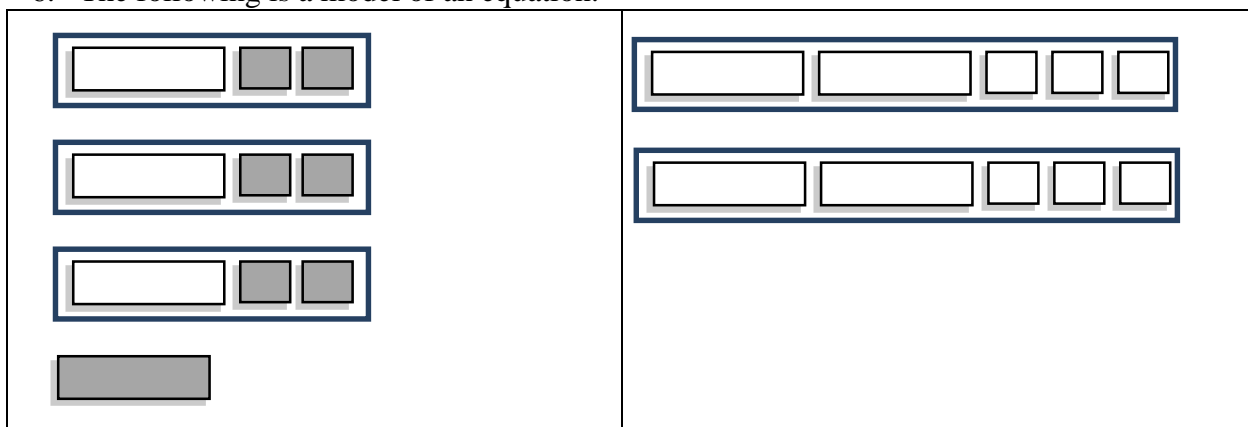


- a. Solving Actions:
 $7x + 9 - 4x = 2(x + 5)$

Directions: Model and solve the following equations.

<p>3. $2(x + 3) = 5x - 3$</p>	<p>4. $8x + 3 - 2x = 3x + 12$</p>	<p>5. $10x + 2 - 3x = 3(2x + 2)$</p>
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6. The following is a model of an equation.



a. Write the symbolic representation for this model.

b. Solve the equation.

Directions: Solve the following equations without the use of the tiles.

$$7. 9x - 4 = 7 - 2x$$

$$8. 2(a + 2) = 11 + a$$

$$9. 5x - 4x + 18 = 3x + 2$$

$$10. 2t + 21 = 3(t + 5)$$

$$11. 7x + 2 - 4x = 7 + 2x + 4$$

$$12. 4(1 - x) + 3x = -2(x + 1)$$

$$13. \frac{1}{4}(12x + 16) = 10 - 3(x - 2)$$

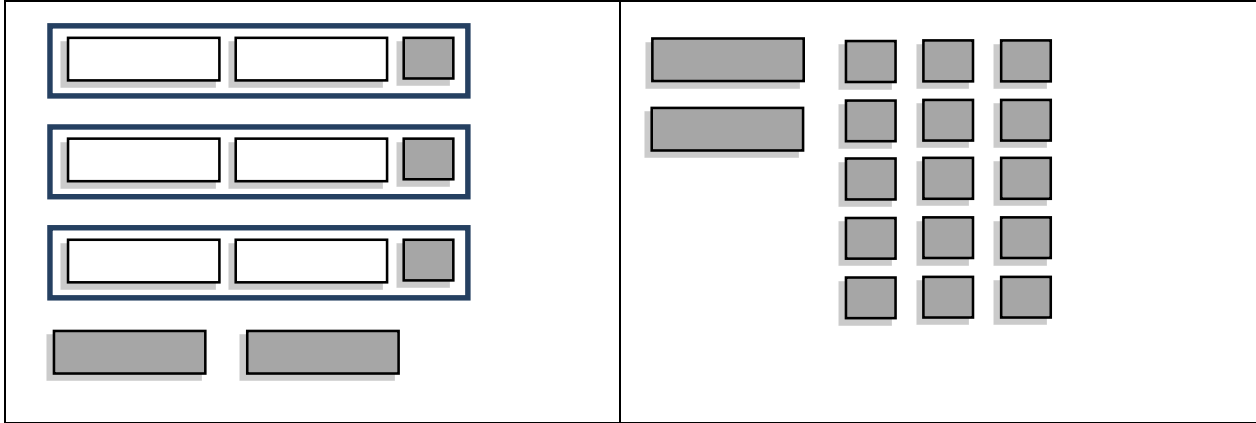
$$14. \frac{2x-9}{3} = 8 - 3x$$

$$15. \frac{y}{3} + 5 = \frac{y}{2} + 3$$

$$16. \frac{1}{2}(2n + 6) = 5n - 12 - n$$

1.2b Homework: Solving Multi-Step Linear Equations (putting it all together)

1. The following is a model of an equation.



- a. Write the symbolic representation of the equation for this model.

- b. Solve the equation.

Directions: Solve the following equations. Verify your solutions.

2. $x + 3x = 9 + x$	3. $4c + 4 = c + 10$	4. $3(4x - 1) = 2(5x - 7)$
5. $3x + 10 + 2x = 2(x + 8)$	6. $2(x + 8) = 2(2x + 1)$	7. $4(x + 3) = x + 26 + x$

8. $3a + 5(a - 2) = 6(a + 4)$	9. $13 - (2c + 2) = 2(c + 2) + 3c$	10. $2(4x + 1) - 2x = 9x - 1$
11. $2 - (2x + 2) = 2(x + 3) + x$	12. $3(x - 6) = 4(x + 2) - 21$	13. $3(y + 7) = 2(y + 9) - y$
14. $-4(x - 3) = 6(x + 5)$	15. $\frac{1}{2}(12 - 2x) - 4 = 5x + 2(x - 7)$	16. $\frac{1}{2}(12n - 4) = 14 - 10n$

1.2c Class Activity: Creating and Solving Linear Equations to Model Real World Problems Part II

Directions: Write a story that goes with the expressions and equation in each problem. Solve for the unknown information and interpret your answer.



1. Birthday Parties

Number of people at a birthday party: p

Cost of party at Boondocks: $8p + 60$

Cost of party at Raging Waters: $20p$

$$8p + 60 = 20p$$

Story

- Write a story that matches the expressions and equations.
- Solve the equation in the space above.
- Interpret your answer.

2. A Number Trick

Starting number: n

Lily's number: $3(n + 5)$

Kali's number: $(n - 5)$

$$3(n + 5) = (n - 5)$$

Story

- Write a story that matches the expressions and equations.
- Solve the equation in the space above.
- Interpret your answer.

3. Savings

Number of weeks: w

Sophie's Money: $300 - 40w$

Raphael's Money: $180 + 20w$

$$300 - 40w = 180 + 20w$$

Story

- Write a story that matches the expressions and equations.
- Solve the equation in the space above.
- Interpret your answer.

1.2c Homework: Creating and Solving Linear Equations to Model Real World Problems Part II

Directions: Write the story that goes with the expressions and equation in each problem. Solve for the unknown information and interpret your answer.

1. Fixing Your Car

Time (hours): h

Cost of Mike's Mechanics: $15h + 75$

Cost of Bubba's Body Shop: $25h$

$$15h + 75 = 25h$$

Story

- Write a story that matches the expressions and equations.
- Solve the equation in the space above.
- Interpret your answer.

2. World Languages

Number of years: t

of students in French: $160 - 9t$

of students in Spanish: $85 + 6t$

$$160 - 9t = 85 + 6t$$

Story

- Write a story that matches the expressions and equations.
- Solve the equation in the space above.
- Interpret your answer.

3. Downloading Music

of songs downloaded: s

Monthly cost at bTunes: $0.99s$

Monthly cost at iMusic: $10 + 0.79s$

$$0.99s = 10 + 0.79s$$

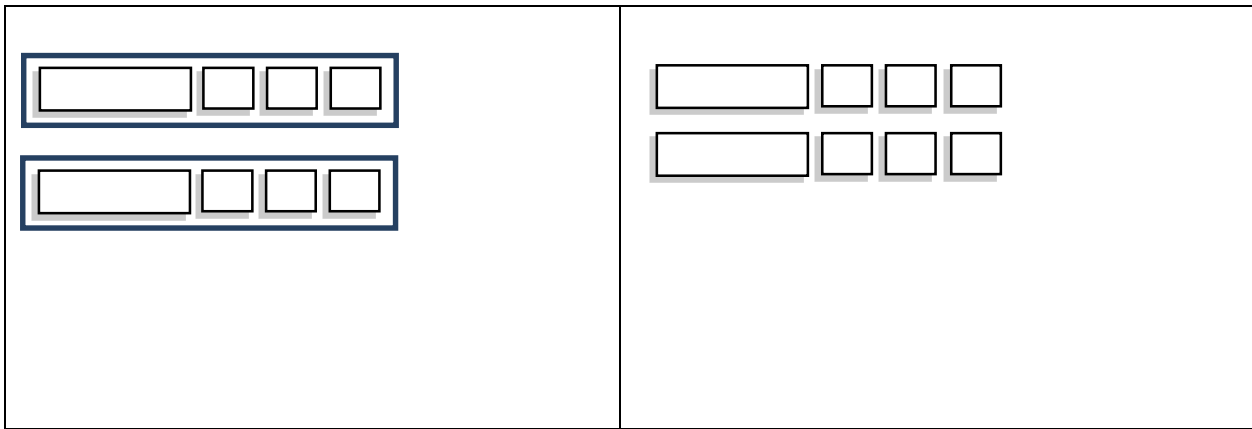
Story



- Write a story that matches the expressions and equations.
- Solve the equation in the space above.
- Interpret your answer.

1.2d Class Activity: Solving Multi-Step Linear Equations (the different solving outcomes)

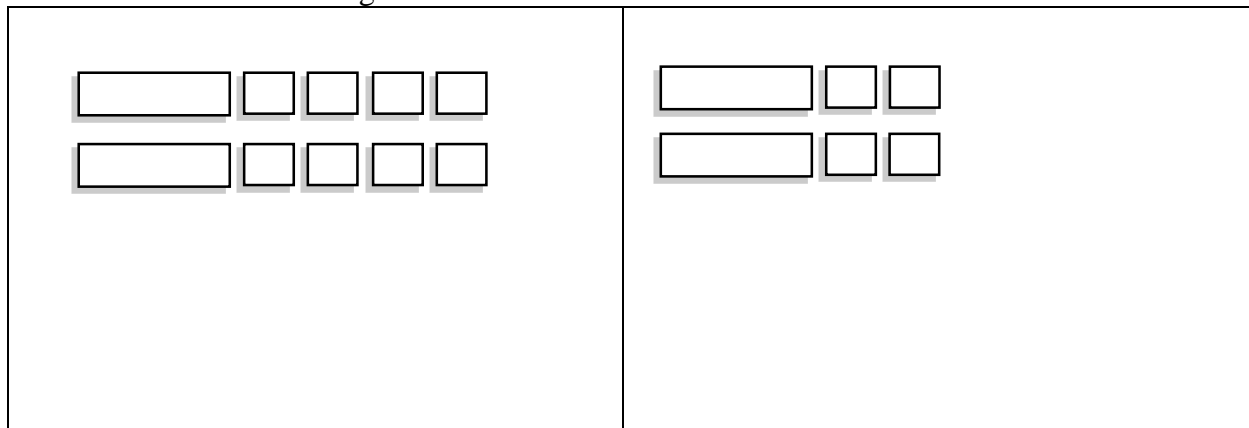
Up to this point, we have solved linear equations with a unique solution (**one solution**). In this lesson, we encounter equations that when solved have **infinitely many solutions** and **no solution**.



1. Consider the following model:



- a. Make some observations about the model above.
- b. Write the symbolic representation (equation) for this model and then solve the equation you wrote.
- c. What happened when you solved the equation? What is it about the structure of the equation that led to the solution?   n #
- d. Build or draw your own equation using your tiles that would result in the same solution as the one above.
- e. Solve the equation you built. What do you notice?


2. Consider the following model:




- Make some observations about the model above.
- Write the symbolic representation for this model and then solve the equation you wrote.
- What happened when you solved the equation? What is it about the structure of the equation that led to the solution?  
- Build or draw your own equation using your tiles that would result in the same solution as the one above.
- Solve the equation you built. What do you notice?

Directions: Solve the following equations. If there is one solution, state what the solution is. Otherwise, state if there are infinitely many solutions or no solution.

3. $x - 1 = x + 1$	4. $5x - 10 = 10 - 5x$
5. $4(m - 3) = 10m - 6(m + 2)$	6. $4(x - 4) = 4x - 16$
7. $2x - 5 = 2(x - 5)$	8. $3x = 3x - 4$
9. $3v + 5 + 2v = 5(2 + v)$	10. $5 - (4a + 8) = 5 - 4a - 8$
11. $\frac{2x+8}{2} = x + 4$	12. $\frac{1}{3}(x - 2) = \frac{x}{3} - \frac{2}{3}$

13. What is it about the structure of an expression that leads to one solution, infinitely many solutions, or no solution? Provide examples to support your claim. 

Directions: Without solving completely, determine the number of solutions by examining the structure of the equation. 

14. $6a - 3 = 3(2a - 1)$	15. $5x - 2 = 5x$	16. $8x - 2x + 4 = 6x - 1$
17. $5m + 2 = 3m - 8$	18. $2(3a - 12) = 3(2a - 8)$	19. $\frac{3x-12}{3} = x + 4$
20. $\frac{2x+2}{4} = \frac{x+1}{2}$	21. $x + \frac{1}{5} = \frac{x+1}{5}$	22. $\frac{x}{2} - 4 = \frac{1}{2}(x - 8)$

23. Consider the expression $4a - 12$. Write 3 different expressions that if set equal to $4a - 12$ would result in the equation having infinite solutions.
24. Consider the expression $x + 1$. Write 3 different expressions that if set equal to $x + 1$ would result in the equation having no solution.
25. Consider the expression $2x + 6$. Write 3 different expressions that if set equal to $2x + 6$ would result in the equation having one solution.
26. Determine whether the equation $7x = 5x$ has one solution, infinitely many solutions, or no solution. If it has one solution, determine what the solution is.

1.2d Homework: Solving Multi-Step Linear Equations (the different solving outcomes)



Directions: Without solving completely, determine the number of solutions of each of the equations.

1. $x - 211 = x$	2. $3(m - 3) = 3m - 9$	3. $5 - x = -x + 5$
4. $-4m + 12 = 4m + 12$	5. $-3(x + 2) = -3x + 6$	6. $\frac{x-3}{5} = \frac{x}{5} - \frac{3}{5}$

Directions: Solve the following equations. If there is one solution, state what the solution is. Otherwise, state if there are infinitely many solutions or no solution. Show all your work.

7. $3x + 1 - 3(x - 1) = 4$	8. $3(a + 6) - 2(a - 6) = 6$
9. $3(r - 4) = 3r - 4$	10. $2(x + 1) = 3x + 4$
11. $3 - (4b - 2) = 3 - 4b + 2$	12. $3 - (4b - 2) = 3 - 4b - 2$

13. $2y - 5y + 6 = -(3y - 6)$	14. $f + 1 = 7f + 12 - 11 - 6f$
15. $12 + 8a = 6a - 6$	16. $\frac{1}{2}(6m - 10) = 3m - 5$

Directions: Fill in the blanks of the following equations to meet the criteria given. In some cases, there may be more than one correct answer.  

17. An equation that yields one solution: $8x + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}x + 10$

18. An equation that yields no solution: $8x + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}x + 10$

19. An equation that yields infinitely many solutions: $8x + 24 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}}x + \underline{\hspace{1cm}})$

Directions: Create your own equations to meet the following criteria.

20. An equation that yields one solution of $x = 5$.

21. An equation that yields no solution.

22. An equation that yields infinitely many solutions.

23. **Challenge:** Can you think of an equation with two solutions?

4. Hugo and Maggie both solved the following equation for x .

$$3(x + 2) = 12$$

Hugo's Method: $3(x + 2) = 12$ $3x + 6 = 12$ Distribute the 3 $3x = 6$ Subtract 6 from both sides $x = 2$ Divide both sides by 3	Maggie's Method: $3(x + 2) = 12$ $x + 2 = 4$ Divide both sides by 3 $x = 2$ Subtract 2 from both sides
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a. Examine the solutions. Did both people solve the equation correctly?



b. The equations below are a mirror of the equations above; however the numbers 2, 3, and 12 in the original equation have been replaced with p , q , and r which represent any number. Solve the equations below for x , using both Hugo and Maggie's methods.

Hugo's Method: $p(x + q) = r$	Maggie's Method: $p(x + q) = r$
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5. Solve the following equations for x .

a. $2x + 3x = 15$

b. $ax + bx = 15$

c. Can you think of a different way of solving $ax + bx = 15$ for x ?

6. Solve the following equations for x if a , b , and c represent real numbers not equal to 0. If you get stuck, put actual numbers in for a , b , and c and think about how you would solve these equations and then apply that thinking to the equations below.

a. $ax = bx + c$

b. $\frac{ax+b}{c} = 5$

1.2e Homework: Abstracting the Solving Process

1. Solve the following equations for x . State the solving actions.

a. $x - 6 = 8$

b. $x - b = 8$ where b represents any number

c. $x - b = c$ where b and c represent any number

2. Solve the following equations for x . State the solving actions.

a. $\frac{x}{4} = 20$

b. $\frac{x}{d} = 20$ where d represents any number not equal to zero

c. $\frac{x}{d} = c$ where c and d represent any number not equal to zero

3. Solve each of the following equations for x . State the solving actions.

a. $\frac{x}{3} + 5 = -1$

b. $\frac{x}{a} + b = c$

4. Solve the following equations for x where $a, b, c, d, e,$ and f are real numbers not equal to 0. If you get stuck, put actual numbers in for $a, b,$ and c and think about how you would solve these equations and then apply that thinking to the equations below.

a. $bx + d = a$	b. $\frac{ax-c}{d} = b$
c. $\frac{x}{ab} = c$	d. $abx = c$
e. $ex + fx - c = d$	f. $ax - c = -bx + d$

1.2f Self-Assessment: Section 1.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Solve multi-step linear equations that have one solution, infinitely many solutions, or no solution.	I am struggling to solve most of the equations on the following page.	I can solve the equations in Set A on the following page.	I can solve all of the equations in Sets A and B on the following page.	I can solve all of the equations in Sets A, B, and C on the following page.
2. Understand what it is about the structure of a linear equation that results in equations with one solution, infinitely many solutions, or no solutions.	I don't understand what the different solving outcomes of a linear equation are.	I can tell whether an equation has one, no, or infinite solutions only after I have solved it but I sometimes get confused about the difference between no solution and infinite solutions.	I can tell whether an equation has one, no, or infinite solutions only after I have solved it.	I can look at an equation and without solving the equation entirely determine whether it will have one solution, no solution, or infinite solutions just by examining the structure of the equation.
3. Identify and provide examples of equations that have one solution, infinitely many solutions, or no solutions.	I don't know what it means for an equation to have one, no, or infinite solutions.	When given a list of equations, I can identify equations that have one, no, or infinite solutions but I have to draw a model or solve the equation in order to tell.	When given a list of equations, I can identify equations that have one, no, or infinite solutions without solving the equations.	When given a list of equations, I can determine whether the equations will have one, no, or infinite solutions. I can also generate my own equations that will result in one, no, or infinite solutions.

1. Solve the following equations.
2. Before solving each equation, examine the structure of the equation and determine whether it will have one, no, or infinite solutions.

Set A

1. $8g + 9 = 4g + 1$	2. $x + 1 = x - 2$	3. $-2x + 4x - 10 = -3x$
----------------------	--------------------	--------------------------

Set B

4. $2a - 12 = 3(a - 6)$	5. $-5x + 2(x - 1) = -12 + 2x$	6. $5 - (2a - 3) = 5 - 2a + 3$
-------------------------	--------------------------------	--------------------------------

Set C

7. $-2(x - 1) + 4x = 3(2x - 2)$	8. $\frac{1}{2}(6x - 8) + 2x = -x - 16$	9. $\frac{1}{4}(16x - 1) = 4x - \frac{1}{4}$
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Chapter 2: Exploring Linear Relations (4 weeks)

Utah Core Standard(s):

- Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has a greater speed.* (8.EE.5)
- Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b . (8.EE.6)
- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (8.F.4)

Academic Vocabulary: proportional relationship, proportional constant, unit rate, rate of change, linear relationship, slope(m), translation, dilation, y -intercept(b), linear, right triangle, origin, rise, run, graph, table, context, geometric model, constant difference, difference table, initial value, slope-intercept form.

Chapter Overview:


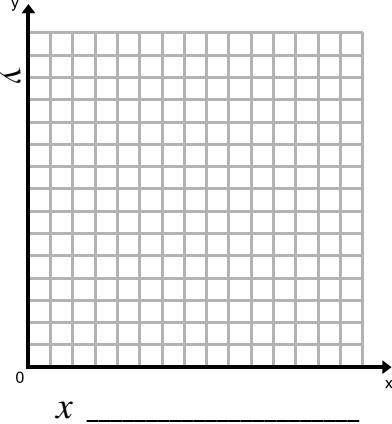

Students begin this chapter by reviewing proportional relationships from 6th and 7th grade, recognizing, representing, and comparing proportional relationships. In eighth grade, a shift takes place as students move from proportional linear relationships, a special case of linear relationships, to the study of linear relationships in general. Students explore the growth rate of a linear relationship using patterns and contexts that exhibit linear growth. During this work with linear patterns and contexts, students begin to surface ideas about the two parameters of a linear relationship: constant rate of change (slope) and initial value (y -intercept) and gain a conceptual understanding of the slope-intercept form of a linear equation. This work requires students to move fluently between the representations of a linear relationship and make connections between the representations. After exploring the rate of change of a linear relationship, students are introduced to the concept of slope and use the properties of dilations to show that the slope is the same between any two distinct points on a non-vertical line. Finally, students synthesize concepts learned and derive the equation of a line.




Connections to Content:

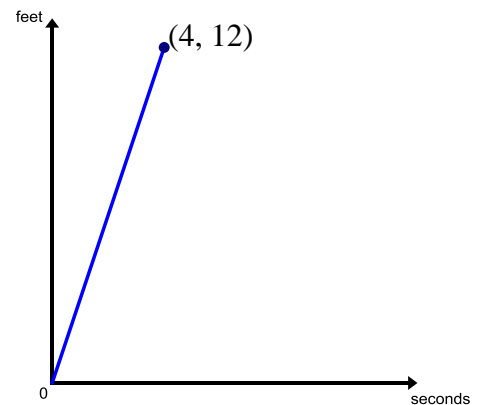
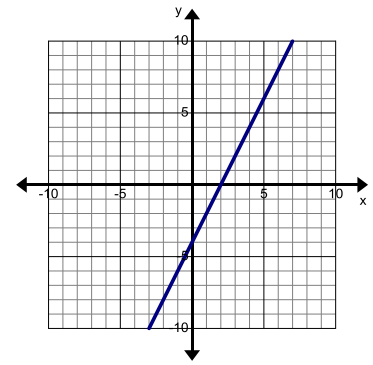
Prior Knowledge: This chapter relies heavily on a student's knowledge about ratios and proportional relationships from 6th and 7th grade. Students should come with an understanding of what a unit rate is and how to compute it. In addition they need to be able to recognize and represent proportional relationships from a story, graph, table, or equation. In addition they must identify the constant of proportionality or unit rate given different representations.




Future Knowledge: After this chapter students continue to work with linear relationships and begin work with functions. They will work more formally with slope-intercept form as they write and graph equations for lines. This will set the stage for students to be able to graph and write the equation of a line given any set of conditions. Students use their knowledge of slope and proportionality to represent and construct linear functions in a variety of ways. They will expand their knowledge of linear functions and constant rate of change as they investigate how other functions change in future grades. The work done in this chapter is the foundation of the study of how different types of functions grow and change.

MATHEMATICAL PRACTICE STANDARDS:

	<p>Make sense of problems and persevere in solving them.</p>	<p>Gourmet jellybeans cost \$9 for 2 pounds.</p> <p>a. Complete the table.</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 15%;">Pounds</td> <td style="width: 10%;">.5</td> <td style="width: 10%;"></td> <td style="width: 10%;">3</td> <td style="width: 10%;">4</td> <td style="width: 10%;"></td> <td style="width: 10%;">8</td> <td style="width: 10%;">10</td> <td style="width: 10%;"></td> <td style="width: 10%;">20</td> </tr> <tr> <td>Total Cost</td> <td></td> <td>\$9</td> <td></td> <td></td> <td>\$27</td> <td></td> <td></td> <td></td> <td></td> </tr> </table>	Pounds	.5		3	4		8	10		20	Total Cost		\$9			\$27				
		Pounds	.5		3	4		8	10		20											
Total Cost		\$9			\$27																	
		<p>b. Label the axes. Graph the relationship.</p> <div style="text-align: center;">  </div>	<p>c. What is the unit rate?</p> <p>d. Write a sentence with correct units to describe the rate of change.</p> <p>e. Write an equation to find the cost for any amount of jellybeans.</p> <p>f. Why is the data graphed only in the first quadrant?</p>																			
<p><i>As students approach this problem they are given some real world data and asked to graph and analyze it. They must make conjectures about the unit rate of the line and understand the correspondences between the table, graph, and equation. The final question asks students to conceptualize the problem by having them explain why only the first quadrant was used.</i></p>																						
	<p>Reason abstractly and quantitatively.</p>	<p>Graphing points can be time-consuming. Develop a procedure for calculating the slope without graphing each point. Explain your procedure below. Show that it works for problems 1-4 above.</p> <p>Discuss the methods for calculating slope without using right triangles on a graph. Write what you think about the methods.</p> <p>Now discuss this formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$ What does it mean? How does it work?</p> <p><i>By examining how the rise and run is found amongst a variety of points students begin understand that the rise is the difference of the y values and the run is the difference of the x values. They must abstract the given information and represent it symbolically as they develop and analyze the slope formula.</i></p>																				

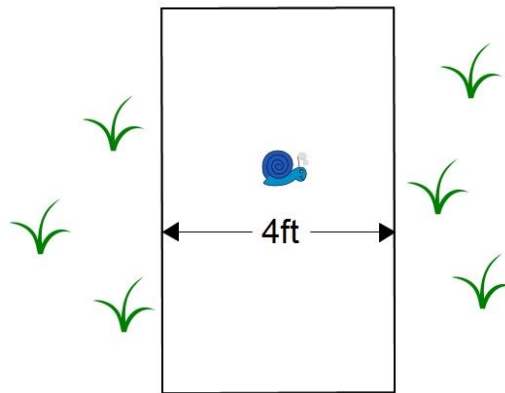
	<p>Construct viable arguments and critique the reasoning of others.</p>	<p>On the line to the right choose any two points that fall on the line. (To make your examination easier choose two points that fall on an intersection of the gridlines).</p> <p>From the two points create a right triangle, the line itself will be the hypotenuse and the legs will extend from the two points and meet at a right angle.</p> <p>Compare the points that you chose and your triangle with someone in your class. Discuss the following: Did you both choose the same points? How are your triangles the same? How are your triangles different? What relationship exists between your triangles?</p> <p><i>Upon comparing their triangle with a class member students begin to discover that any right triangle constructed on the line is related through a dilation. By talking with one another they can analyze different triangles and discuss the proportionality that exists between them. This also gives students the opportunity to help one another learn how to accurately construct the right triangle on the graph used to find slope. In addition, they begin to make conjectures about how slope can be found from any two points on the line.</i></p>
	<p>Model with mathematics.</p>	<p>a. Create your own story that shows a proportional relationship. b. Complete a table and graph to represent this relationship. Be sure to label the axes of your graph. c. Write an equation that represents your proportional relationship.</p> <p><i>This question asks students to not only create their own proportional relationship but to model it with a table, graph, and equation. Students show their understanding of how a proportional relationship is shown in several representations.</i></p>
	<p>Attend to precision.</p>	<p>The graph below shows the distance a cat is from his bowl of milk over time. Which sentence is a good match for the graph?</p> <p>A. The cat was 12 feet away from the milk and ran toward it reaching it after 4 seconds. B. The cat was 4 feet away from the milk and ran toward it reaching it after 12 seconds. C. The cat ran away from the milk at a rate of 3 feet per second. D. The cat ran away from the milk at a rate of 4 feet per second. E. The cat was 12 feet away from the milk and ran away from it at a rate of 4 feet per second.</p> <p><i>Upon examining the graph students must attend to precision as they discuss the ordered pair (4,12) and analyze exactly what it is telling us about the cat. Students often confuse the given information with rate of change and fail to recognize what quantity each number in the ordered pair represents. Also they must communicate what the direction of the line is telling us. Later on they are asked to make their own graphs and must use the correct units and labels to communicate their thinking.</i></p>



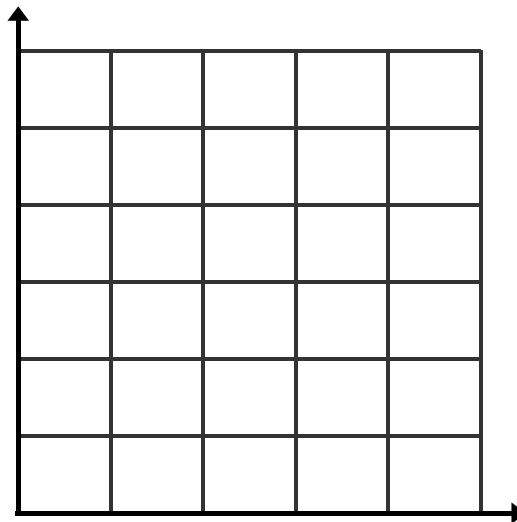
	<p style="text-align: center;">Use appropriate tools strategically.</p>	<p>Your group task is to build a set of stairs and a handicap ramp at the side. Both the stairs and the ramp will begin at the same place (at ground level) and end at the height of three feet. Answer the following questions as you develop your design.</p> <ul style="list-style-type: none"> • How many steps do you want or need? • How deep should each step be (we will call this the run)? Why do you want this run depth? • How tall will each step be (we will call this the rise)? Why do you want this rise height? • What is the total distance (total depth for all steps) you will need (at the base) for all of the stairs? This would be a measurement at ground level from stair/ramp start point to stair/ramp end point. <p>Sketch the ramp as viewed from the side on graph paper below. Label and sketch the base and height, for example: stair-base (in inches or feet) and height (in inches or feet).</p> <p><i>As students design a set of stairs and ramps they must use decide how they can use the tools (graph paper, ruler, pencil) provided them most efficiently. They will need to generate a graph that displays their designs and use the graph as a tool to analyze the slope of the stairs and ramp.</i></p>
	<p style="text-align: center;">Look for and make use of structure.</p>	<p>Examine the graphs and equations given above. Describe the general form of a linear equation. In other words, in general, how is a linear equation written? What are its different parts?</p> <p><i>As students examine many equations in slope-intercept form and interpret the slope and y-intercept they see the structure of the general form for a linear equation begin to emerge. Even if they write the initial value or y-intercept first they can step back and look at an overview of the general form of the equation and shift their perspective to see that the order in which you write your slope and initial value does not matter.</i></p>
	<p style="text-align: center;">Look for and express regularity in repeated reasoning.</p>	<p>In each graph below, how many right triangles do you see?</p> <ul style="list-style-type: none"> • Trace the triangles by color. • For each triangle write a ratio comparing the lengths of its legs or $\frac{\text{height}}{\text{base}}$. Then simplify the ratio $\frac{\text{height}}{\text{base}} = \frac{\quad}{\quad}$. <div style="text-align: right;"> </div> <p><i>In this series of problems students repeatedly find the height/base ratio of triangles that are dilations of one another and infer that slope can be calculated with the rise/run ratio by choosing any two points. A general method for finding slope as rise/run is discovered.</i></p>

2.0 Anchor Problem: Proportionality and Unit Rate

Toby the snail is crossing a four foot wide sidewalk at a constant rate. It takes him 1 minute and 36 seconds to scoot across **half** the width of the sidewalk, as pictured below.



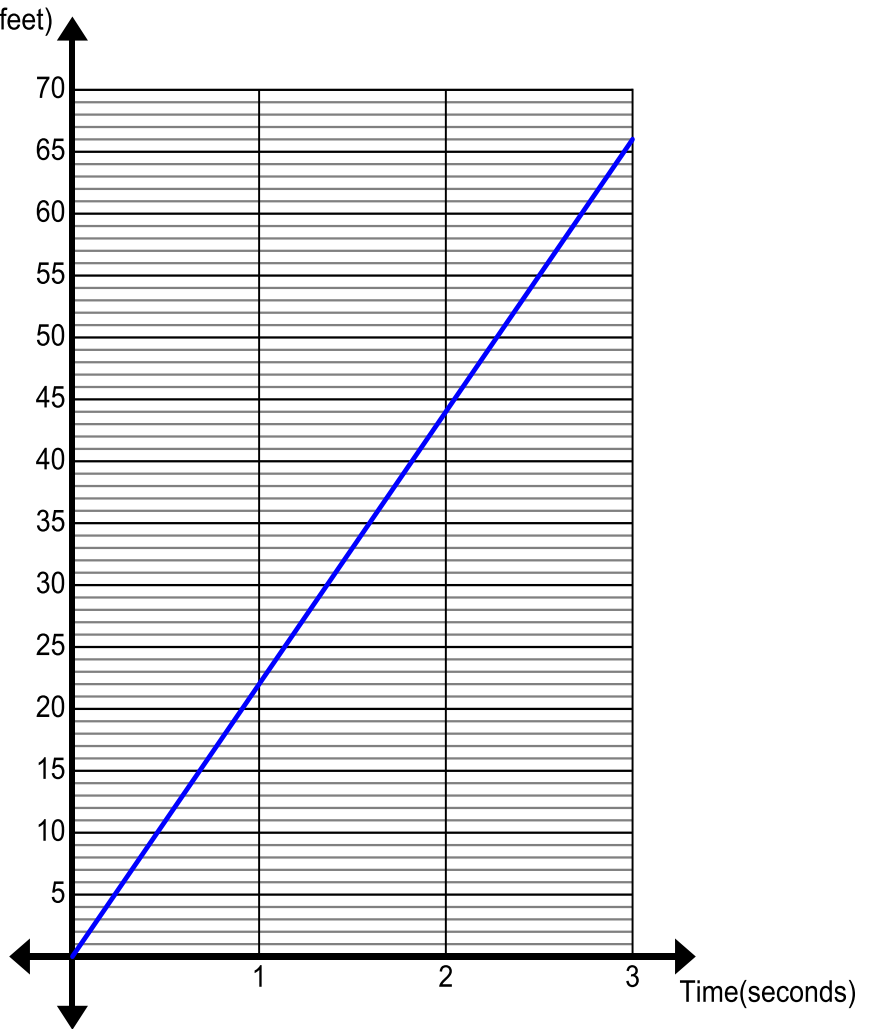
1. Find the unit rate for this proportional relationship. Be sure to explain what this unit rate means.
2. Write an equation that describes this proportional relationship if x is the amount of time it takes Toby to cross the sidewalk and y is the distance he has traveled. Use this equation to make a table of values to graph the first 5 seconds of Toby's journey.



3. How long will it take Toby to cross the sidewalk?

A large group of cyclists are on the sidewalk heading in Toby's direction. The graph below shows the rate at which they travel.

- Find and describe the unit rate for the group of cyclists. Highlight this unit rate on the graph.
- Describe how you can find the unit rate at a different location on the graph.
- The cyclists are 4300 feet away from Toby on the sidewalk. Will Toby cross the sidewalk before the cyclists reach him? Justify your answer.



Section 2.1: Analyze Proportional Relationships

Section Overview:

The section begins by reviewing proportional relationships that were studied in 6th and 7th grade. By investigating several contexts, students study the proportional constant or unit rate in tables, graphs, and equations. They recognize that a proportional relationship can be represented with a straight line that goes through the origin and compare proportional relationships represented in many ways. In the last lesson of the section a bridge from proportional relationships to linear relationships is achieved as students translate the graph of a proportional relation away from the origin and analyze that there is no effect on steepness of the line or rate at which it changes but that the relation is no longer proportional. They begin to see a proportional relationship as a special subset of a linear relationship where the rate of change is a proportional constant or unit rate and the graph of the relationship is a line that goes through the origin. Students also investigate the transition of the proportional constant or unit rate to rate of change, that is, if the input or x -coordinate changes by an amount A , the output or y -coordinate changes by the amount m times A .

Concepts and Skills to Master:

By the end of this section, students should be able to:

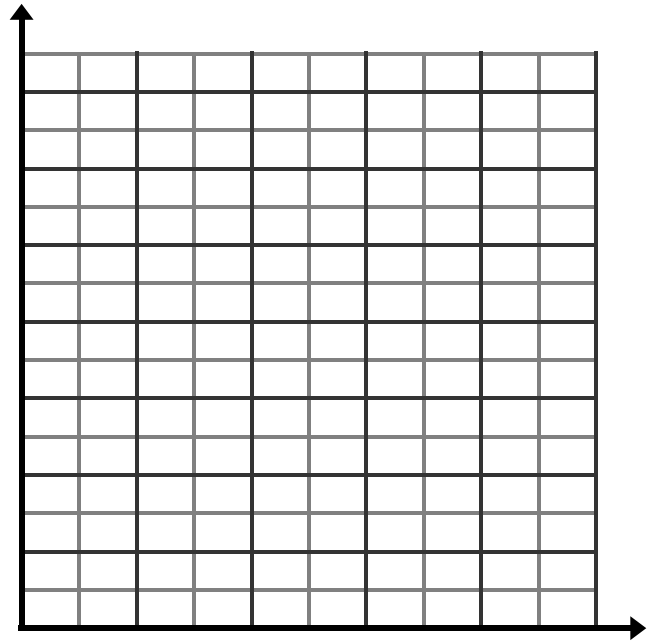
1. Graph and write equations for a proportional relationship and identify the proportional constant or unit rate given a table, graph, equation, or context.
2. Compare proportional relationships represented in different ways.
3. Know that the graph of a proportional relationship goes through the origin.

2.1a Class Activity: Proportional Relationships

In the previous chapter, you wrote equations with one variable to describe many situations mathematically. In this chapter you will learn how to write an equation that has two variables to represent a situation. In addition to writing equations you can represent real-life relationships in others ways. In 6th and 7th grade you studied **proportional relationships** and represented these relationships in various ways. The problems given below will help you to review how ratio and proportion can help relate and represent mathematical quantities from a given situation.

1. Julie is picking teammates for her flag football team. She picks three girls for every boy.
 - a. Complete the table below to show the relationship of boys to girls on Julie's team.
 - b. Graph the girl to boy relationship for Julie's team with boys on the x -axis and girls on the y -axis.

Boys	Girls
1	
2	
	9
4	



- c. Find the ratio of girls to boys for several different ordered pairs in the table.

$$\frac{\text{number of girls}}{\text{number of boys}} =$$

- d. Fill in the boxes to show the relationship between girls and boys on Julie's team.

$$\boxed{} = 3 \cdot \boxed{}$$

- e. Use the equation and graph to determine how many girls would be on the team if Julie chose 10 boys to be on the team.
- f. Use the equation and graph to determine how many boys are on the team if Julie chose 18 girls.

In the previous example the two quantities of interest are in a **proportional relation**.

Recall that when two quantities are proportionally related, the ratio of each y value to its corresponding x value is constant. This constant is called the **constant of proportionality** or **proportional constant**.

The ratio that related the number of boys to girls was 3. This is the proportional constant for this relationship.

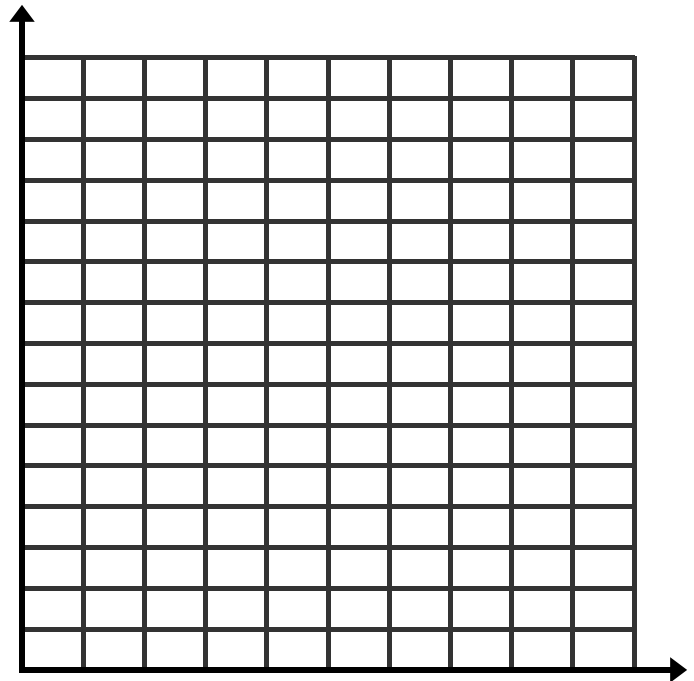
2. Carmen is making homemade root beer for an upcoming charity fundraiser. The number of pounds of dry ice to the ounces of root beer extract (flavoring) is proportionally related. If Carmen uses 12 pounds of dry ice she will need to use 8 ounces of root beer extract.
- Write a ratio/proportional constant that relates the number of pounds of dry ice to the number of ounces of root beer extract.
 - Write a ratio/proportional constant that relates the number of ounces of root beer extract to the number pounds of dry ice.

Notice that the proportional constant depends on how you define your items.

- c. Complete the table below to show the relationship between number ounces of root beer extract x and number of pounds of dry ice y needed to make homemade root beer.

Ounces of Root Beer Extract (x)	Pounds of Dry Ice (y)
0	
	$\frac{3}{2} = 1.5$
2	
3	
	6
8	

- d. Graph and label this relationship below.



d. What is the proportional constant for this relationship?

e. Write an equation that shows the relationship between the number of ounces of root beer extract (x) and the number of pounds of dry ice (y) needed to make homemade root beer.

Every ratio has an associated rate. **Unit rate** is another way of interpreting the ratio's proportional constant. The statement below describes how unit rate defines y and x in a proportional relationship.

If quantities y and x are in proportion then the **unit rate** of y with respect to x is the amount of y that corresponds to one unit of x . If we interchange the roles of y and x , we would speak of the unit rate of x with respect to y .

5. In the previous problem Carmen was making homemade root beer. Express the proportional constant as a unit rate.

6. What would the unit rate be if we interchanged the roles of x and y ?

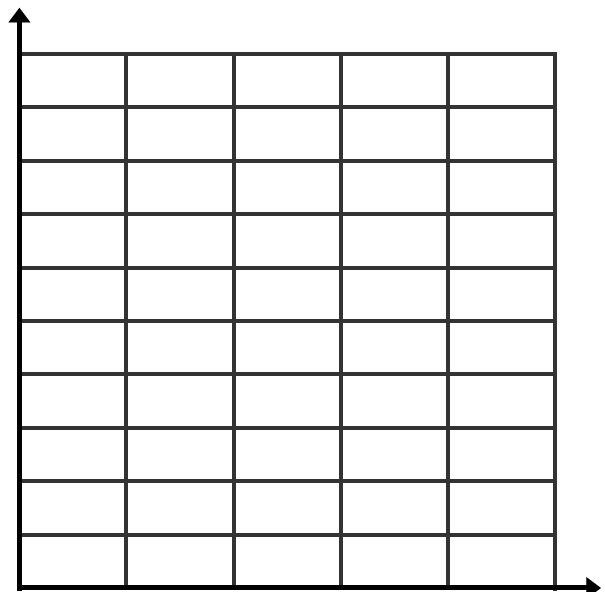
In the problem below use the properties of a proportional relationship to help you answer the question.

7. Doug is pouring cement for his backyard patio that is 100 square feet. The cement comes out of the truck at a constant rate. It is very important that he gets all the cement poured before 12:00 noon when it gets too hot for the cement to be mixed properly. It is currently 11:00 AM and he has poured 75 square feet of concrete in the last 3 hours. At this rate will he finish before noon?

a. Fill in the missing items in the table if x represents the number of hours that have passed since Doug began pouring concrete and y represents the amount of concrete poured

Time elapsed (hours) x	Amount of concrete poured (square feet) y
0	
1	
2	
3 (11:00AM)	75
4	

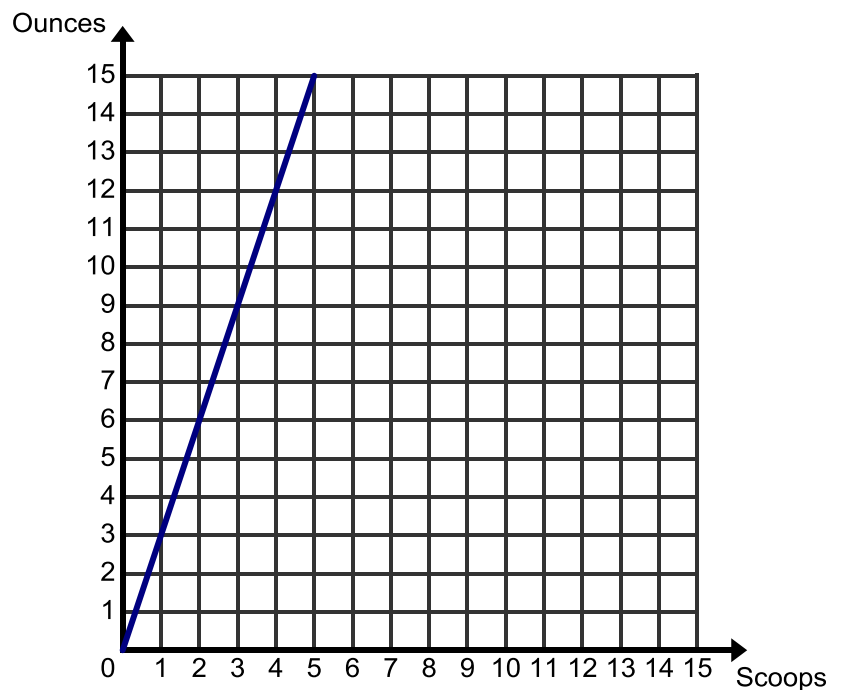
b. Graph the relationship below.



- c. What is the unit rate for this relationship? In other words, how many square feet of concrete can Doug pour in 1 hour.
- d. Which equation given below best describes this relationship?
 - a) $y=25x$
 - b) $y=75x$
 - c) $x=25y$
 - d) $y=11x$
- e. Will Doug finish the job in time? Justify your answer.

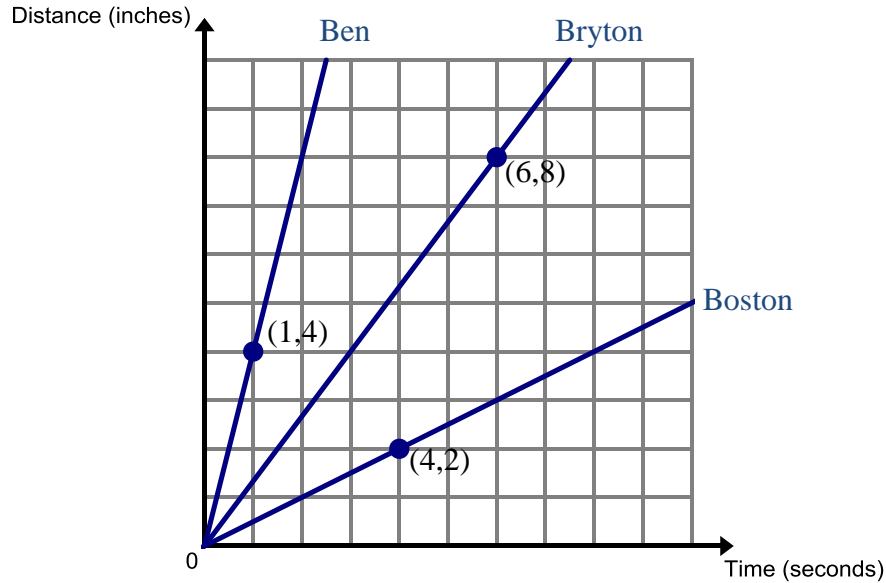
10. Vanessa is mixing formula for her baby. The graph given to the right describes the relationship between the ounces of water to the scoops of formula to make a properly mixed bottle.

- a. Does the graph describe a proportional relationship? Justify your answer.
- b. What is the unit rate for this relationship? Show on the graph how you can see the unit rate.
- c. At a different location on the graph show and explain how you can find the unit rate.



- d. Write an equation to relate the ounces of water to the scoops of formula.
- e. How many scoops of formula must Vanessa use to make 9 ounce bottle for her baby?

11. Ben, Boston, and Bryton have each designed a remote control monster truck. They lined them up to crush some mini cars in the driveway. The lines on the graph below show the distance in inches that each monster truck travels over time in seconds.



- Bryton states that each truck is traveling at a constant rate. Is his statement correct? Why or why not.
 - What do the values in the ordered pairs given on the graph represent?
 - Find the unit rate for each boy's truck.
Ben:

Bryton:

Boston:
 - Which boy's truck is moving the fastest?
12. Explain why the graph of a proportional relationship makes a straight line.
13. Summarize what you know about proportional relationships using bulleted list in the space below.

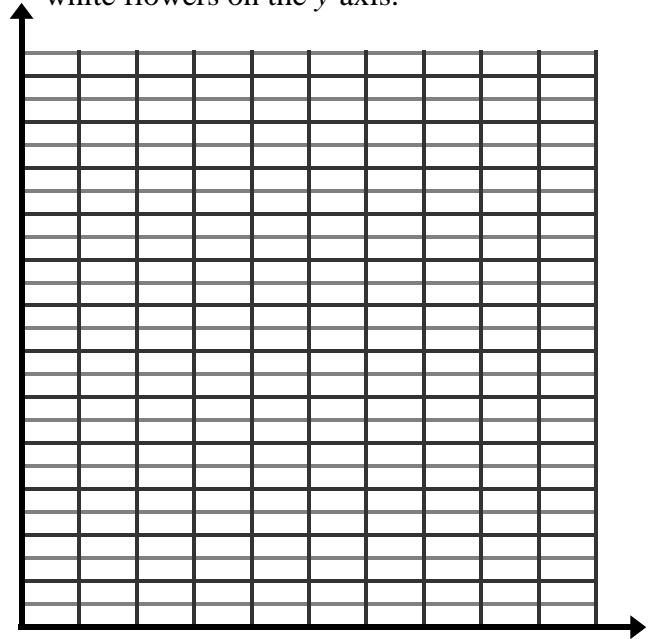
2.1a Homework: Proportional Relationships.

1. A florist is arranging flowers for a wedding. For every 2 pink flowers in a vase, he also includes 8 white flowers.

a. Complete the table below to show the relationship of white to pink flowers in each vase.

Pink	White
1	
	8
	12
6	

b. Graph the white flower to pink flower relationship for each vase with pink flowers on the x -axis and white flowers on the y -axis.



c. Find the ratio of white to pink for several different ordered pairs in the table.

$$\frac{\text{number of white flowers}}{\text{number of pink flowers}} =$$

d. Fill in the boxes to show the relationship between white flowers and pink flowers in a vase.

$$\boxed{} = 4 \cdot \boxed{}$$

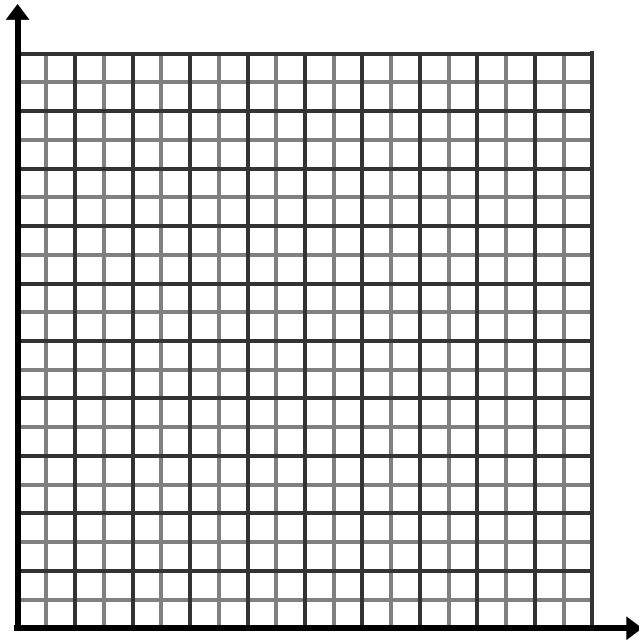
e. Use the equation and graph to determine how many white flowers there would be if the florist included 20 pink flowers.

2. You are going to Europe for vacation and must exchange your money. The exchange rate of Euros to Dollars is a proportional relationship. The table below shows the exchange rate for Euros y to Dollars x .

a. Complete the table.

Dollars(x)	Euros(y)
2	
4	3
6	
	6

b. Graph the Euro to Dollar relationship.



c. What is the unit rate for this relationship?

d. Write an equation that represents this relationship.

e. If you exchanged \$20, how many Euro would you get?

f. If you received 36 Euro, how many dollars did you exchange?

3. In the problem given above we found that the unit rate described the number of Euros in a Dollar. Now interchange the roles of y and x for this relationship. Find the exchange rate for Dollars (y) to Euros (x). Use the questions below to help you do this.

a. Make a table of values to represent the Dollar to Euro relationship.

Euros(x)	Dollars(y)

b. What is the unit rate for this Dollar to Euro relationship?

c. Write an equation that represents this relationship.

Use the equation to answer the following questions.

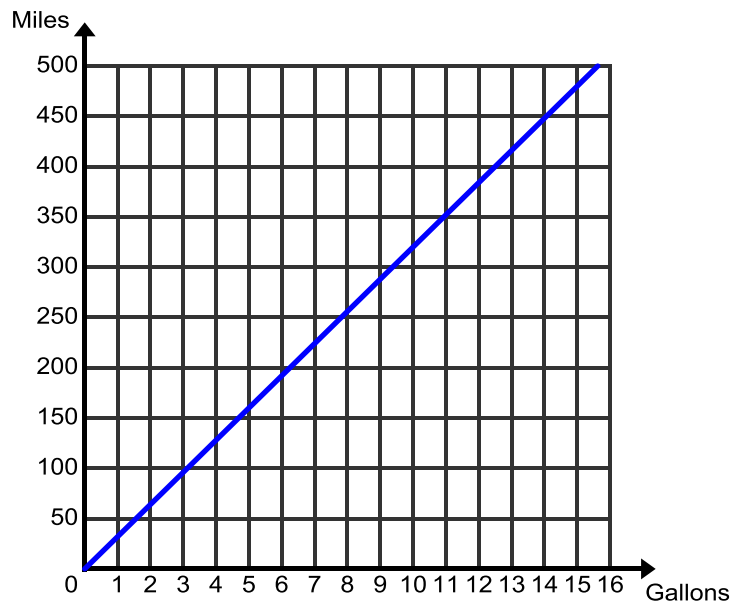
d. While in Europe you find a shirt that you want to buy that is marked at 25 Euros. You only have \$32 to exchange for the Euros. Do you have enough money? Explain.

e. Upon returning home from Europe you have 100 Euros left. How many Dollars can you get for 100 Euros?

4. The graph given below shows the gas mileage that Penny gets in her car. The ratio 192:6 describes the miles to gallons fuel rate for her car.

a. What is the unit rate for this relationship?

b. Use the graph to approximate how many miles Penny can go if she has a 15 gallon tank in her car.



5. A proportional constant of $\frac{1}{3}$ relates the number of inches a flower grows to the number of weeks since being planted.

a. Fill in the missing items in the table if x represents the number of weeks that have past and y represents the height of the flower.

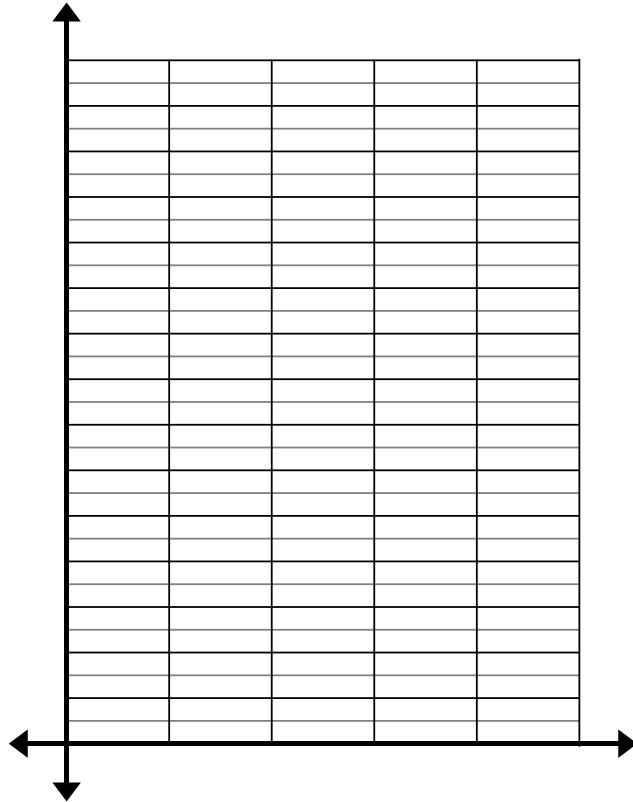
x (weeks)	1	3		9	30
y (height)			2		

- b. Write an equation that represents this relationship and use the equation to predict how tall the flower will be after 8 weeks.
- c. Is it probable for the flower to continue to grow in this manner forever?

6. Padma needs to buy 5 pounds of candy to throw at her city's annual 4th of July Parade; the picture below shows how much it costs to buy Salt Water Taffy at her local grocery store.

Salt Water Taffy
3 pounds for
\$15.00

- a. Find the unit rate for this proportional relationship. Be sure to describe what this unit rate means.
- b. Write an equation that describes this proportional relationship if x is the number of pounds and y is the cost. Use this equation to make a table of values to graph how much up to five pounds of taffy will cost.



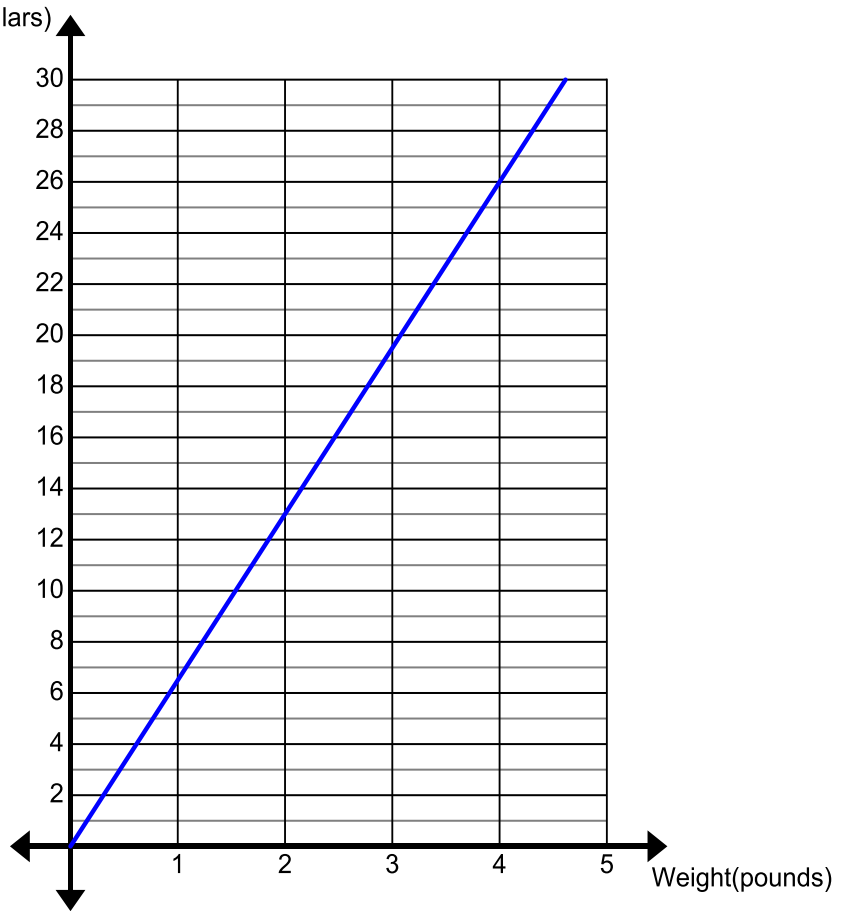
- c. How much will it cost Padma to buy the 5 pounds of candy that she needs?

7. Padma also sees that she can buy Tootsie Rolls at the grocery store. The graph below shows cost of tootsies rolls per pound purchased. (Note: When cost is involved unit rate is often referred to as *unit price*.)

a. Find and describe the unit rate for the Tootsie Rolls. Highlight this unit rate on the graph.

b. Describe how you can find the unit rate at a different location on the graph.

c. Graph the line for the Salt Water Taffy on the grid with the Tootsie Roll. Label each line with the candy it represents.

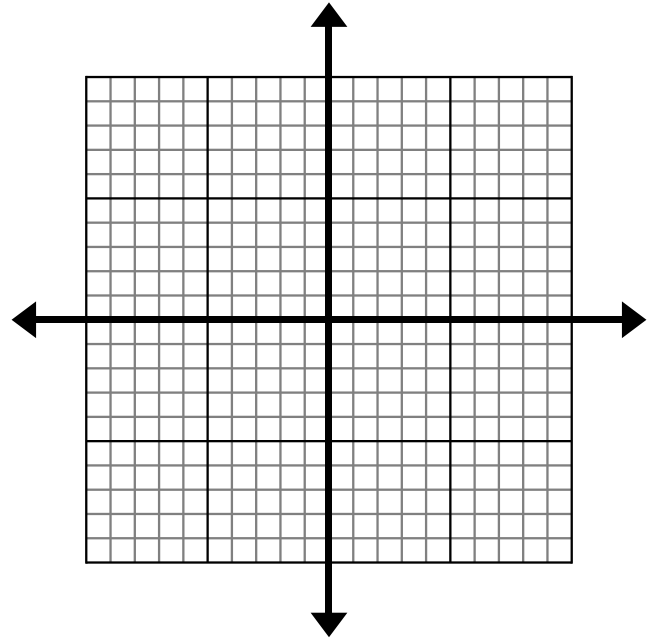


d. What is the better deal for Padma, should she buy the Salt Water Taffy or the Tootsie Rolls? Justify your answer.

8. Create your own relationship.

a. Create your own story that shows a proportional relationship.

b. Complete a table and graph to represent this situation. Be sure to label your graph and table.



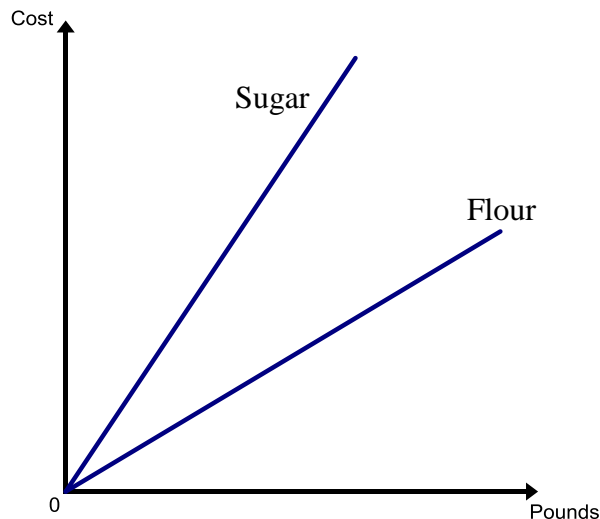
c. Write an equation that represents your proportional relationship.

2.1b Class Activity: Comparing Proportional Relationships



Proportional relationships can help us to compare and analyze quantities and to make useful decisions. Complete the tasks given below that compare proportional relationships.

Emma is putting together an order for sugar, flour, and salt for her restaurant pantry. The graph below shows the cost y to buy x pounds of sugar and flour. One line shows the cost of buying x pounds of flour and the other line shows the cost of buying x pounds of sugar.



1. From the graph which ingredient costs more to buy per pound? Justify your answer.
2. The cost to buy salt by the pound is less than sugar and flour. Draw a possible line that could represent the cost to buy x pounds of salt.

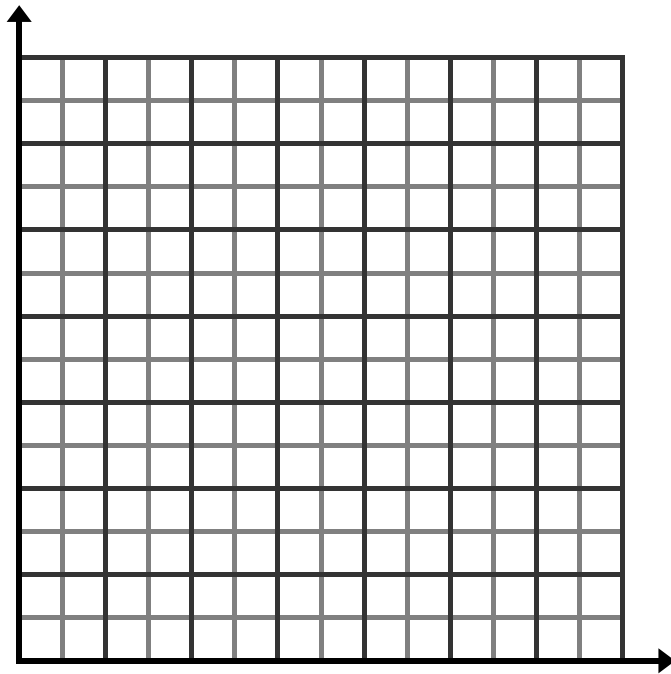
Don and Betsy are making super smoothies to re-energize them after a long workout. Betsy follows the recipe which calls for 2 cups of strawberries for every 3 bananas. Don wants twice as much as Betsy so he makes a smoothie with 4 cups of strawberries and 5 bananas.

Don tastes his smoothie and says, “This tastes too tart, there are too many strawberries!”



3. Explain why Don’s smoothie is too tart.
4. Find and describe the unit rate for Betsy’s smoothie.

5. Find and describe the unit rate for Don's smoothie.
6. Write an equation that relates the number of strawberries(x) to the number of bananas(y) for Besty's smoothie.
7. Write an equation that relates the number of strawberries(x) to the number of bananas(y) for Don's smoothie.
8. Use your equations to make tables to graph both of these lines on the same grid. Be sure to label which line belongs to which person.



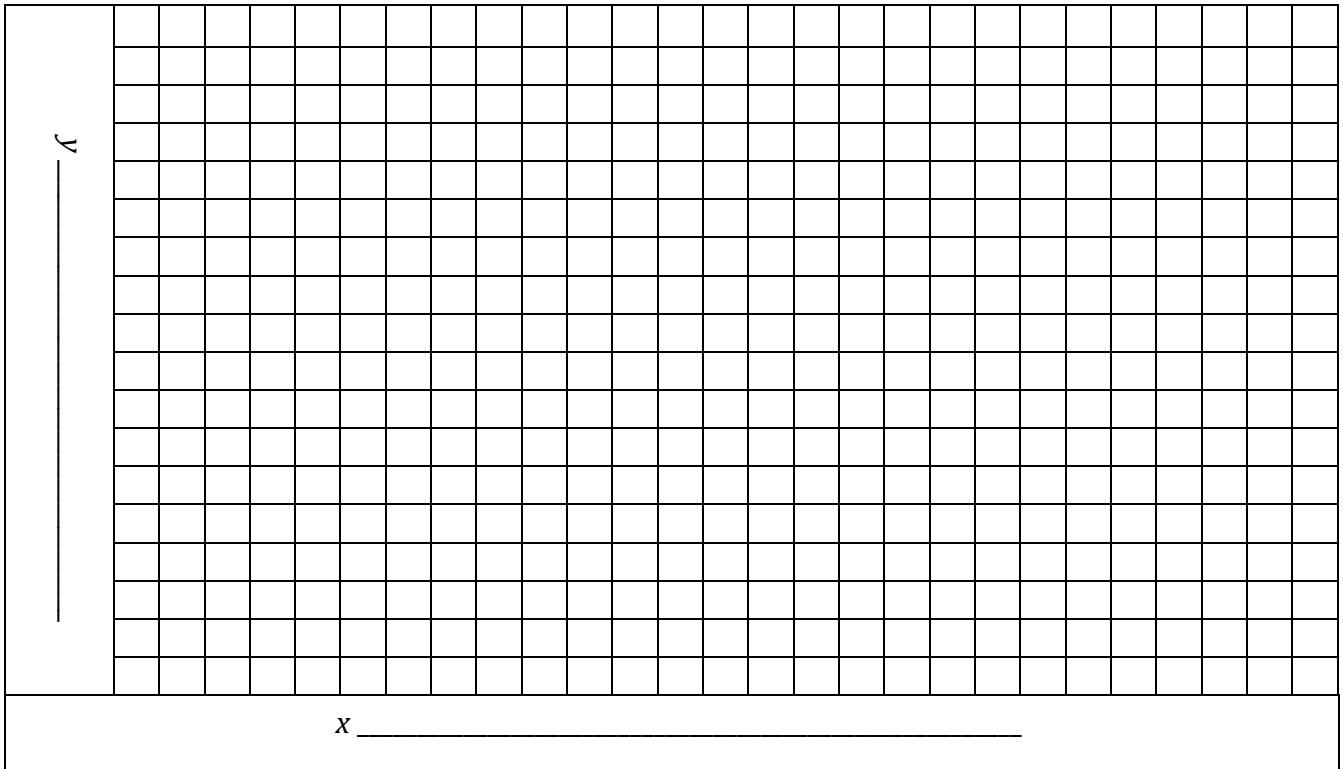
9. Explain how the steepness of the lines relates to the unit rate.

10. For the recreational activities below, compare the cost y per hour x by looking at graphs and equations.

- Fill in the missing representations. If the information is given in a table, fill in the story and equation. If the information is given in an equation, fill in the story and table, etc.
- Find the unit rate or slope for each situation.
- Graph all situations on the given graph on the next page. Remember to Label the axes. Label the lines with the situation names.

<p>a. Long Distance Phone Call: It costs \$10 per hour to talk on the phone long distance.</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 2px;">Hours(x)</th> <th style="padding: 2px;">Cost(y)</th> </tr> </thead> <tbody> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> </tbody> </table> <p>Equation: $y = 10x$</p> <p>Unit Rate:</p>	Hours(x)	Cost(y)											<p>b. Roller skating</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 2px;">Hours(x)</th> <th style="padding: 2px;">Cost(y)</th> </tr> </thead> <tbody> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="text-align: center;">2</td><td style="text-align: center;">5</td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> </tbody> </table> <p>Equation:</p> <p>Unit Rate:</p>	Hours(x)	Cost(y)			2	5							<p>c. Music Lessons The bill for private guitar lessons was \$75. The lesson lasted 3 hours.</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 2px;">Hours(x)</th> <th style="padding: 2px;">Cost(y)</th> </tr> </thead> <tbody> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> </tbody> </table> <p>Equation:</p> <p>Unit Rate:</p>	Hours(x)	Cost(y)										
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<p>d. Parks</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 2px;">Hours(x)</th> <th style="padding: 2px;">Cost(y)</th> </tr> </thead> <tbody> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="text-align: center;">24</td><td style="text-align: center;">6</td></tr> </tbody> </table> <p>Equation:</p> <p>Unit Rate:</p>	Hours(x)	Cost(y)									24	6	<p>e. Bungee Jumping It costs \$20 to bungee jump for 15 minutes.</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 2px;">Hours(x)</th> <th style="padding: 2px;">Cost(y)</th> </tr> </thead> <tbody> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> </tbody> </table> <p>Equation:</p> <p>Unit Rate:</p>	Hours(x)	Cost(y)											<p>f.</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 2px;">Hours(x)</th> <th style="padding: 2px;">Cost(y)</th> </tr> </thead> <tbody> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> <tr><td style="height: 15px;"> </td><td> </td></tr> </tbody> </table> <p>Equation:</p> <p>Unit Rate:</p>	Hours(x)	Cost(y)										
Hours(x)	Cost(y)																																					
24	6																																					
Hours(x)	Cost(y)																																					
Hours(x)	Cost(y)																																					

Graph



2.1b Homework: Comparing Proportional Relationships

Use the graph completed during the class activity to answer questions 1-10 below.

1. Order the five activities from highest cost to lowest cost per hour.
2. How do you compare the cost per hour by looking at the graph?
3. How do you compare the cost per hour by looking at the equations?
4. Create a sixth activity in column f on page 23. Think of a situation which would be less expensive than Bungee Jumping, but more expensive than the others. Fill in the table and make the graph.

Answer the questions below:

5. As the rate gets _____, the line gets _____.
6. Renting the pavilion at the park for 3 hours costs _____.
7. Talking on the phone for 2.5 hours costs _____.
8. Bungee jumping for _____ hours costs \$40.
9. For \$10 you can do each activity for approximately how much time?

a. Long Distance	
b. Roller Skating	
c. Music Lessons	
d. Park	
e. Bungee	
f.	

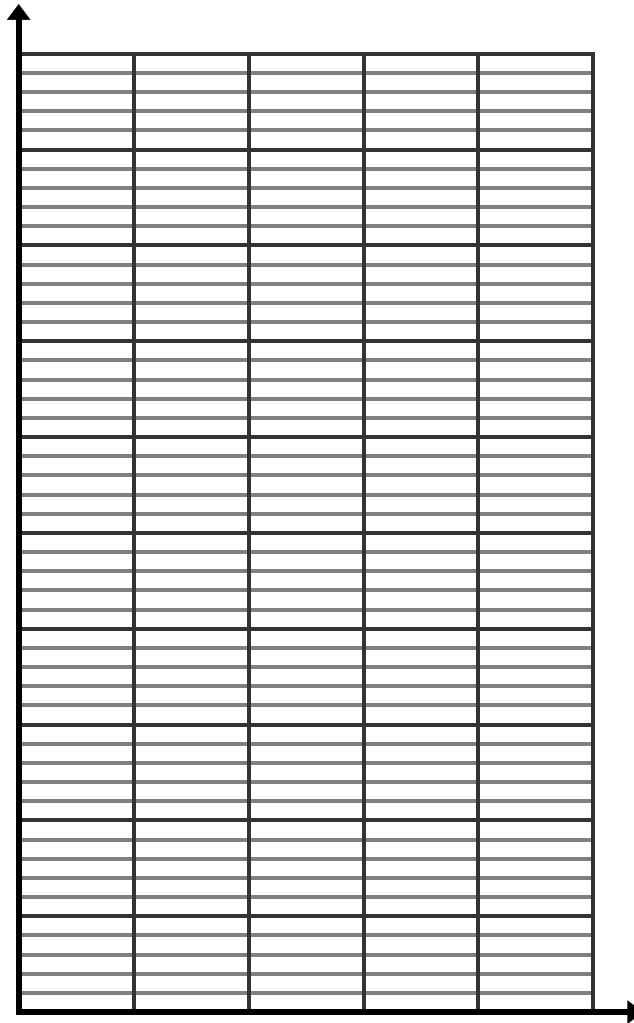
10. Did you use the tables, equations or graphs to answer questions 5-10? Why?

13. The table given below shows how much money Charlie earned every day that he worked last week. He gets paid the same rate every hour.

	Tuesday	Wednesday	Friday
Hours Worked	4	5	3.5
Money Earned	\$38.00	\$47.50	\$33.25

Sophia earns \$10.50 per hour at her job.

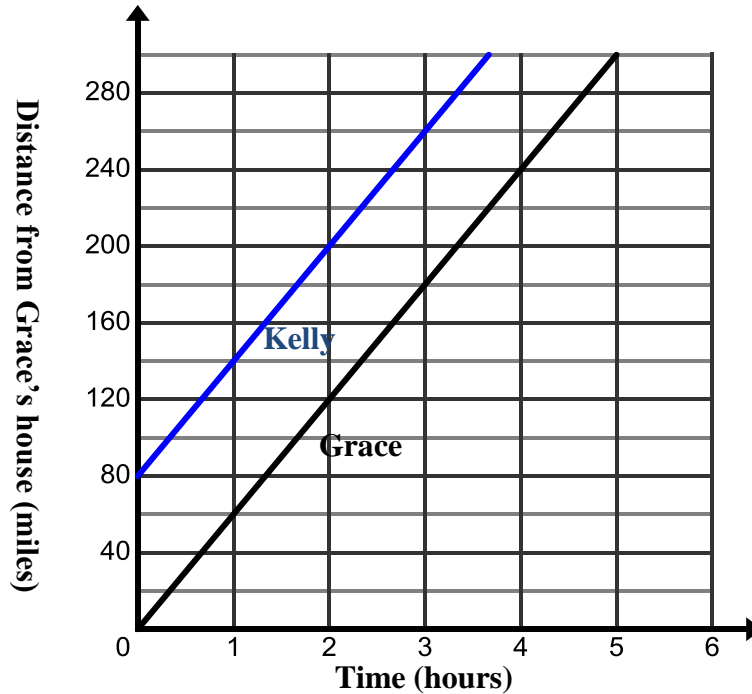
- a. Using the same coordinate plane, draw a line that represents Charlie's earnings if x represents the number of hours worked and y represents the amount of money earned. Also draw a line that represents how much Sophia earns. Label each line with the person's name.



- b. How can you use this graph to determine who makes more money?

2.1c Class Activity: Proportional Relationships as Linear Relationships

1. Two cousins, Grace and Kelly, are both headed to the same summer camp. They both leave from their own house for camp at the same time. The graph below represents the girls' trips to camp.



- a. Analyze the graph to determine which girl is traveling faster.
- b. Complete the table below for Grace and Kelly.

Grace		
<i>Time(x)</i>	<i>Distance(y)</i>	$\frac{y}{x}$
0		
1	60	
	120	
3		
	240	

Kelly		
<i>Time(x)</i>	<i>Distance(y)</i>	$\frac{y}{x}$
0		
	120	
2		
	200	
4		

- c. What do you notice about the ratio $\frac{y}{x}$ for Grace? What do you notice about the ratio $\frac{y}{x}$ for Kelly? What is this ratio describing?

- d. Describe why Kelly's driving relationship is not proportional?
- e. Is it possible to still describe the rate at which Kelly drives? If so, what is it?


Often the rate at which a relationship changes is shown by seeing that the changes from one measurement to another are proportional; that is, the quotient of the change in y values with respect to the x values is constant. This is called the **Rate of Change**.

Both of the relationships described above have a constant rate of change of 60 mph. This constancy defines them as **linear relationships**. (Their graphs produce straight lines).

- f. Use what you learned above to see if you can write an equation that represents each girl's distance y from Grace's house after x hours.

Grace: _____

Kelly: _____

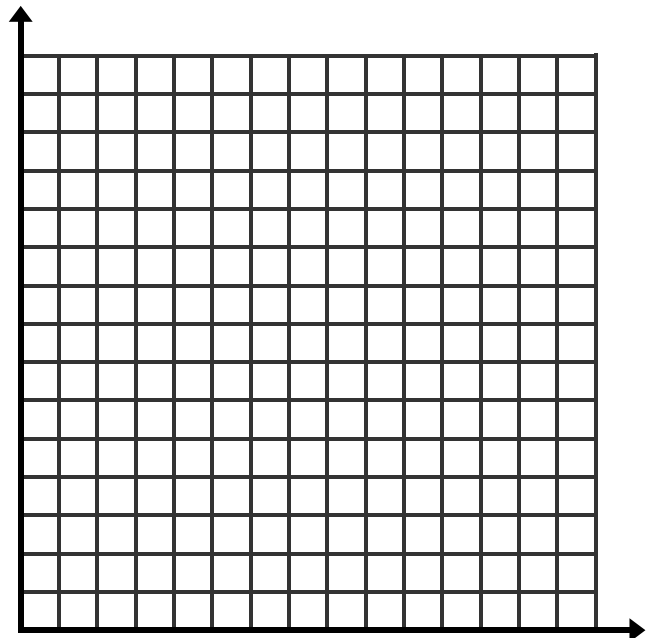
- 2. Agatha makes \$26 for selling 13 bags of popcorn at the Juab County Fair. 

- a. Find and describe the rate of change for this relationship.

- b. Complete the table that shows the amount of money Agatha makes for selling up to three bags of popcorn.

x	y

- c. Graph the dollars to bags of popcorn relationship.



- d. Highlight on the graph where you can see the rate of change.
- e. Write an equation that represents the relationship between the number of bags of popcorn that Agatha sells(x) and the amount of money she makes (y).
Equation:_____

3. At the Sanpete County Fair Fitz gets paid \$8 a day plus \$2 for every bag of popcorn that he sells.

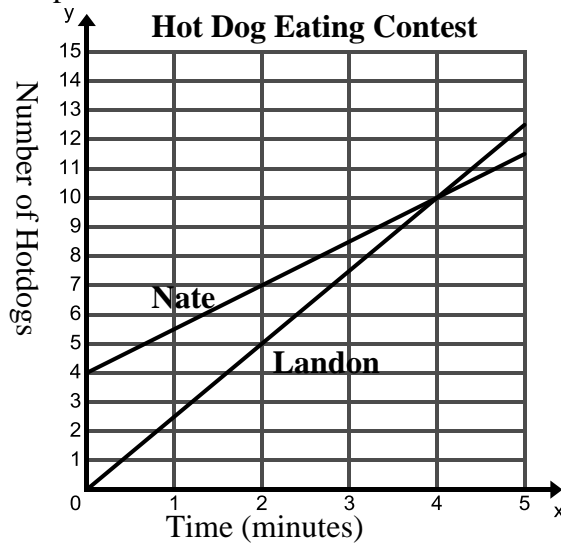
- a. Find and describe the rate of change for this relationship.
- b. Complete the table that shows the amount of money that Fitz makes for selling up to three bags of popcorn.

x	y

- c. Graph this relationship on the same coordinate plane as Agatha’s line on the previous page.
- d. Highlight on the graph where you can see the rate of change.
- e. Write an equation that represents the relationship between the number of bags of popcorn that Fitz sells(x) and the amount of money he makes(y).
Equation:_____
- f. Write at least two sentences that explain the similarities and differences between Agatha’s and Fitz’s relationship.

2.1c Homework: Proportional Relationships as Linear Relationships

1. Nate and Landon are competing in a 5 minute long Hot Dog eating contest. Nate has a special strategy to eat 4 hot dogs before the competition even begins to stretch out his stomach. The graph below represents what happened during the competition.



- a. Complete the tables where t = time in minutes and h = number of total hotdogs consumed.

Landon		
t	h	$\frac{h}{t}$
0	0	
1	2.5	
3	7.5	
	12.5	

Nate		
t	h	$\frac{h}{t}$
0		
1	5.5	
2		
	8.5	
5		

- b. Determine the rate of change (the number of hot dogs consumed per minute for each boy).
- c. Write an equation that represents the number of hotdogs h for each boy after t minutes.
 Landon: _____
 Nate: _____
- d. For which person, Landon or Nate, is the relationship between time and number of hot dogs eaten proportional?? Justify your answer.

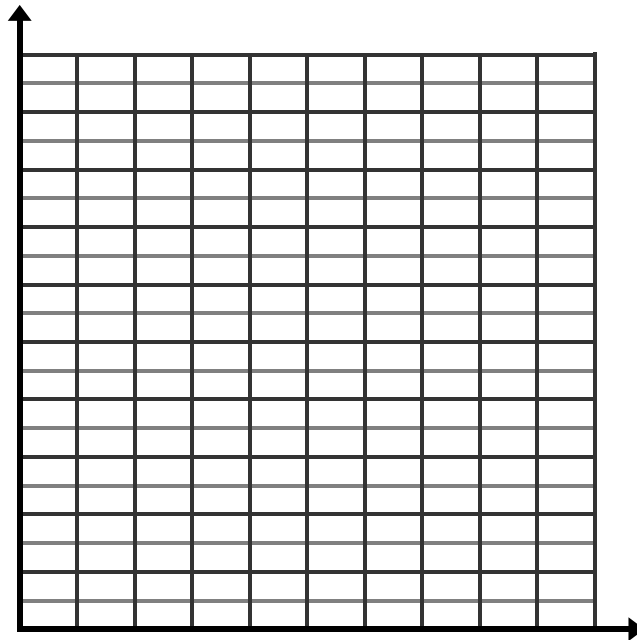
2. During her Tuesday shift at Sweater Barn, Fiona sells the same amount of sweaters per hour. Two hours into her shift Fiona has sold 8 sweaters.

a. Find and describe the rate of change for this relationship.

b. Complete the table given below where x is the number of hours worked and y is the total number of sweaters sold.

x	y

c. Graph the relationship on the grid below.



d. Write an equation that represents the relationship between the number of hours Fiona works(x) and the amount of sweaters she sells(y).

Equation: _____

e. Does this represent a proportional relationship? Explain how you know.

3. On Saturday Fiona gets to work 15 minutes early and sells three sweaters before her shift even begins. She then sells 4 sweaters every hour for the rest of her shift.

a. Find and describe the rate of change for this relationship.

b. Complete the table that represents this relationship.

x (hours)	y (sweaters sold)
0	3
1	
	11
3	

c. Graph this relationship on the same coordinate plane as Tuesday's information on the previous page.

d. Write an equation that represents the relationship between the number of hours Fiona works(x) and the amount of sweaters she sells(y).

Equation: _____

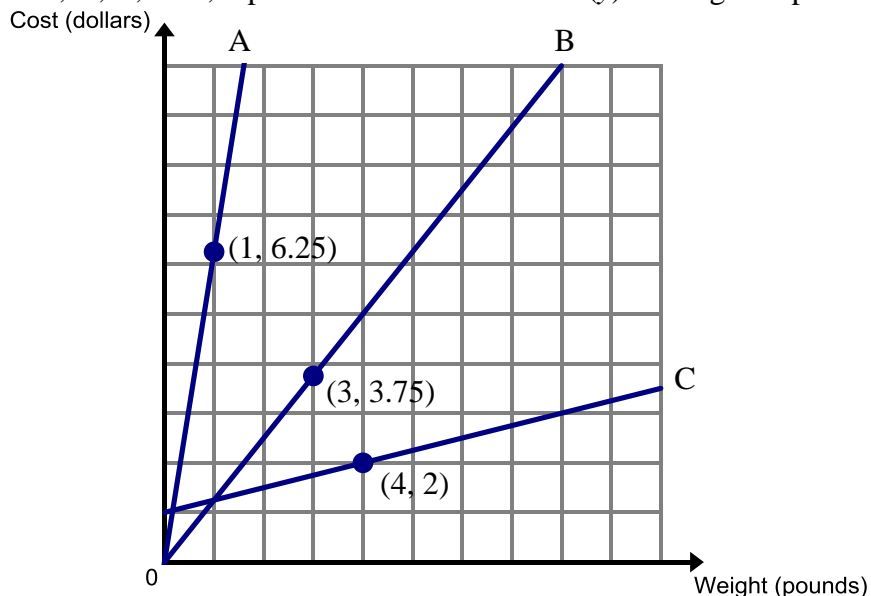
e. Does this represent a proportional relationship? Explain how you know.

f. Compare the rate of change of both of the lines on the previous page by highlighting the change on the graph. What do you notice?

4. Nayala bought 5 pounds of mangos for \$6.25.

a. What is the price per pound for the mangos that she bought?

b. Which line below, A, B, or C, represents the cost in dollars(y) to weight in pounds(x) relationship?



2.1d Self Assessment: Section 2.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample criteria are provided along with sample problems for each skill/concept on the following page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
<p>1. Graph and write equations for a proportional relationship and identify the proportional constant or unit rate given a table, equation, or contextual situation.</p> <p><i>See sample problem #1</i></p>	<p>I can correctly answer only 1 of the three parts of the question.</p>	<p>I can correctly answer 2 of the three parts of the question.</p>	<p>I can correctly answer all three parts of the question but cannot explain my answers.</p>	<p>I know how to find the unit rate for both Callie and Jeff and state what the unit rate is describing. I can accurately label the graphs and write an equation that represents the amount of money earned in relationship to the number of papers each person delivered.</p>
<p>2. Compare proportional relationships represented in different ways.</p> <p><i>See sample problem #2</i></p>	<p>I do not know how to compare these proportional relationships.</p>	<p>I can find the unit rate for only one relationship.</p>	<p>I can find the unit rate for each relationship represented and then compare the unit rates to determine who makes more money. I do not know how to make a third representation for someone who makes more money than Addy and Rachel.</p>	<p>I can find the unit rate for each relationship represented and then compare the unit rates to determine who makes more money. I can also create a third representation for someone who makes more money than Addy and Rachel.</p>

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
3. Know that the graph of a proportional relationship goes through the origin. <i>See sample problem #3</i>	I might be able to make a graph of Jeff's savings but I do not know how it relates to the other graph.	I can make a graph that show's Jeff's savings but I don't know how it relates to the other graph.	I can make a graph that shows Jeff's savings. I can discuss some of the similarities and differences between the two graphs.	I can make (and clearly label) a graph that shows Jeff's savings. I can discuss the similarities and differences between the two graphs including a discussion about proportionality and slope.

Sample Problem #1

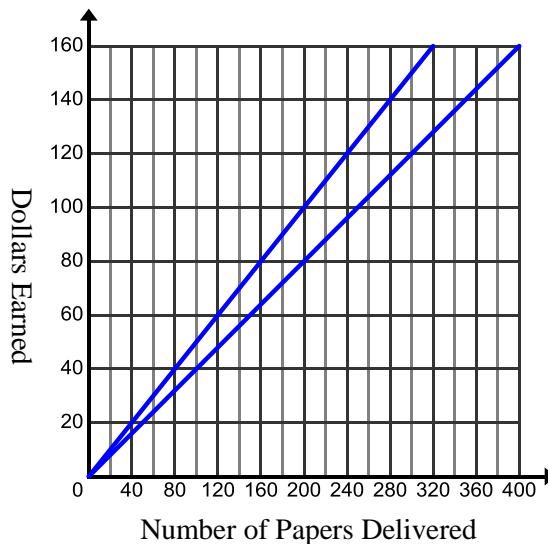
Callie and Jeff each have a job delivering newspapers. Jeff gets paid \$140 dollars for delivering 350 papers. Callie gets paid \$100 for delivering 200 papers.

- a. Find the unit rate for each person. Be sure to state what the unit rate is describing.

Callie:

Jeff:

- b. The graph given below shows the money earned to papers delivered relationship. Label which line represents each person.



- c. Write an equation that represents how much each person earns if x is the number of papers they deliver and y represents how much money they make.

Callie:

Jeff:

Sample Problem #2

Below is a table of how much money Rachel earns on her paper route. She gets paid the same amount of money per paper delivered.

Number of Papers Delivered	Money Earned
75	\$36
150	\$72
225	\$108

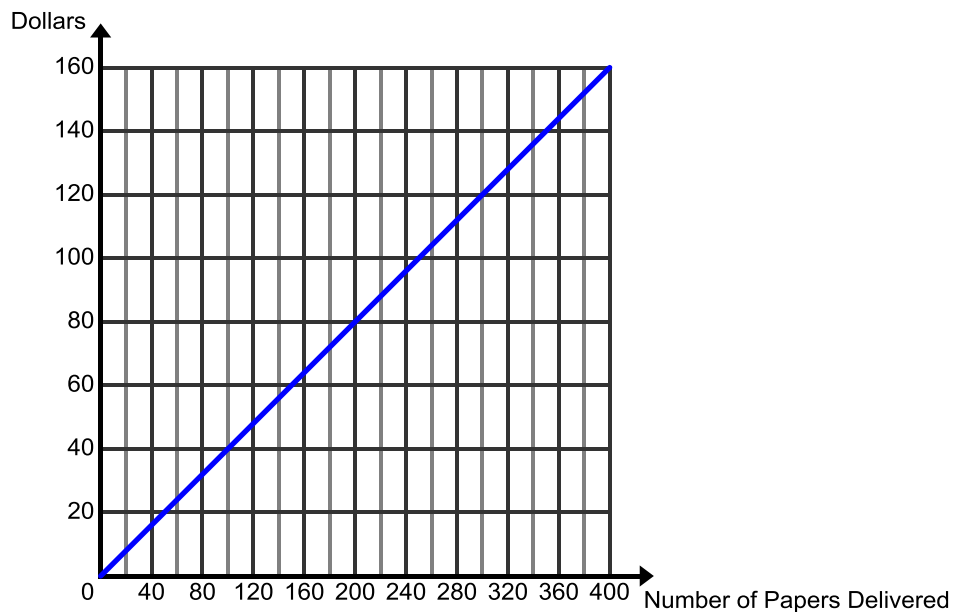
The equation below represents how much money Addy makes delivering papers. In the equation p represents the number of papers delivered and d represents the money earned.

$$d = .45p$$

- Who makes more money? How do you know?
- Create a representation for someone who delivers papers and makes more than both Addy and Rachel.

Sample Problem #3

The graph provided below shows the amount of money that Jeff earns delivering papers. Suppose that Jeff had \$75 dollars in savings before he started his job of delivering newspapers. Jeff saves all of his money earned from delivering newspapers. Graph this relationship below where x is the number of papers he delivers and y is the amount of money he has in savings. Correctly label each line has Total Savings and Money Earned.



How does the savings line compare to his money earned line? Be sure to discuss the proportionality of each graph.

Section 2.2: Linear Relations in Pattern and Context

Section Overview:

In this section, students start by writing rules for linear patterns. They use the skills and tools learned in Chapter 1 to write these equations. Students connect their rules to the geometric model and begin to surface ideas about the rate of change and initial value (starting point) in a linear relationship. Students continue to use linear patterns and identify the rate of change and initial value in the different representations of a linear pattern (table, graph, equation, and geometric model). They also begin to understand how linear functions change. Rate of change is investigated as students continue to interpret the parameters m and b in context and advance their understanding of a linear relationship. Students move fluently between the representations of a linear relationship and make connections between the representations. This conceptual foundation will set the stage for students to be able to derive the equation of a line using dilations in section 2.3.

Concepts and Skills to Master:

By the end of this section, students should be able to:

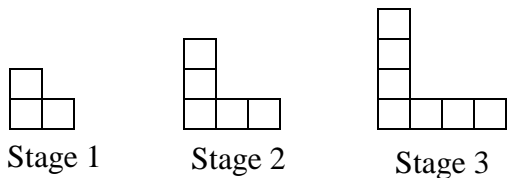
1. Write rules for linear patterns and connect the rule to the pattern (geometric model).
2. Understand how a linear relationship grows as related to rate of change and show how that growth can be seen in each of the representations.
3. Create the additional representations (table, graph, equation, context, geometric model) of a linear relationship when given one representation and make connections between them.
4. Identify the rate of change and initial value of a linear relationship in the table, graph, equation, context, and geometric model of a linear pattern.

2.2a Class Activity: Connect the Rule to the Pattern



Linear relationships can be used to illustrate many patterns. The patterns in the problems below exhibit linear relationships.

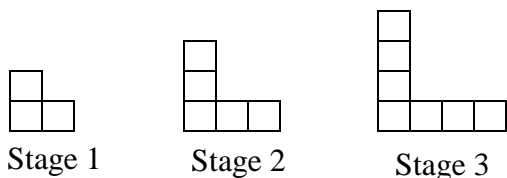
1. Use the pattern below to answer the questions that follow.



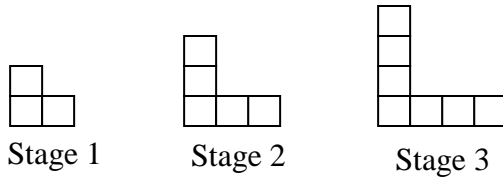
- a. Draw the figure at stage 4 in the space above. How did you draw your figure for stage 4 (explain or show on the picture how you see the pattern growing from one stage to the next)?

- b. How many blocks are in stage 4? Stage 10? Stage 100?

- c. Write a rule that gives the total number of blocks t for any stage s . Show how your rule relates to the pattern (geometric model).

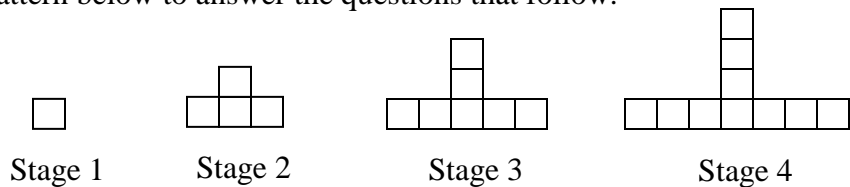


- d. Try to think of a different rule that gives the total number of blocks t for any stage, s . Show how your rule relates to the pattern (geometric model).



- e. Use your rule to determine the number of blocks in stage 100.
- f. Use your rule to determine which stage has 25 blocks.
- g. Draw or describe stage 0 of the pattern. How does the number of blocks n in stage 0 relate to the simplified form of your rule?

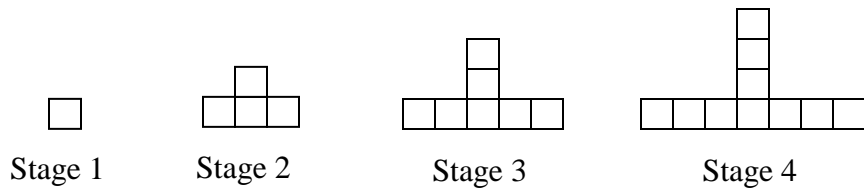
2. Use the pattern below to answer the questions that follow.



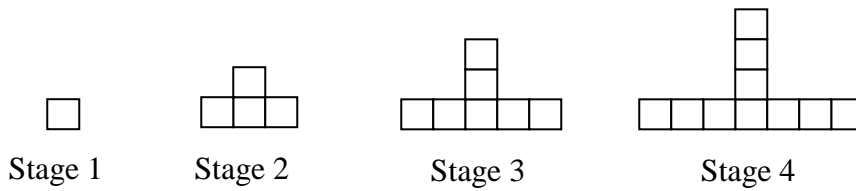
a. Draw the figure at stage 5 in the space above. How did you draw your figure in stage 5 (explain or show on the picture how you see the pattern growing from one stage to the next)?

b. How many blocks are in stage 5? Stage 10? Stage 100?

c. Write a rule that gives the total number of blocks t for any stage, s . Show how your rule relates to the pattern (geometric model).



- d. Try to think of a different rule that gives the total number of blocks t for any stage, s . Show how your rule relates to the pattern (geometric model).



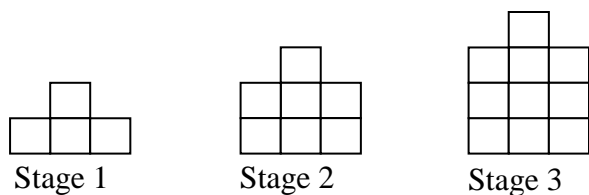
- e. Use your rule to determine the number of blocks in Stage 100.

- f. Use your rule to determine which stage has 58 blocks.

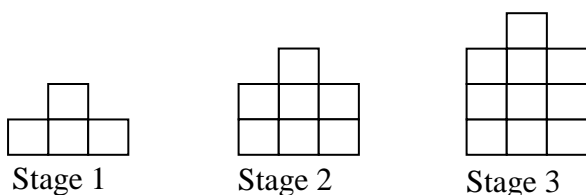
- g. Draw or describe stage 0 of the pattern. How does the number of blocks in stage 0 relate to the simplified form of your rule?

2.2a Homework: Connect the Rule to the Pattern

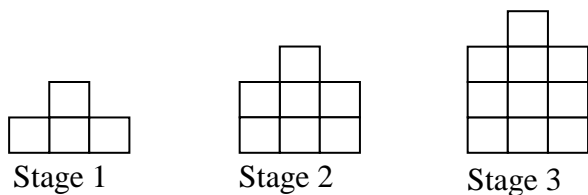
1. Use the pattern below to answer the questions that follow.



- Draw the figure at stage 4 in the space above. How did you draw your figure in stage 4 (explain or show on the picture how you see the pattern growing from one step to the next)?
- How many blocks are in stage 4? Stage 10? Stage 100?
- Write a rule that gives the total number of blocks t for any stage s . Show how your rule relates to the pattern (geometric model).

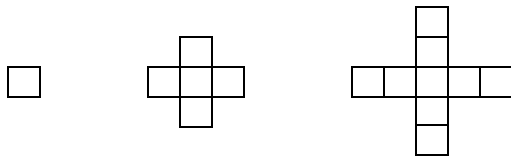


- Try to think of a different rule that gives the total number of blocks t for any stage, s . Show how your rule relates to the pattern (geometric model).



- Use your rule to determine the number of blocks in stage 100.
- Use your rule to determine which stage has 28 blocks.
- Draw or describe stage 0 of the pattern. How does the number of blocks in stage 0 relate to the simplified form of your rule?

2. Use the pattern below to answer the questions that follow.



Stage 1

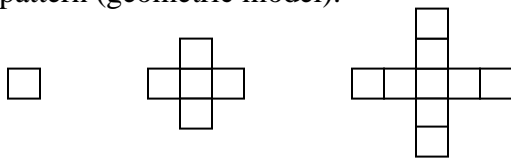
Stage 2

Stage 3

a. Draw the figure at stage 4 in the space above. How did you draw your figure in stage 4 (explain or show on the picture how you see the pattern growing from one stage to the next)?

b. How many blocks are in stage 4? Stage 10? Stage 100?

c. Write a rule that gives the total number of blocks t for any stage, s . Show how your rule relates to the pattern (geometric model).

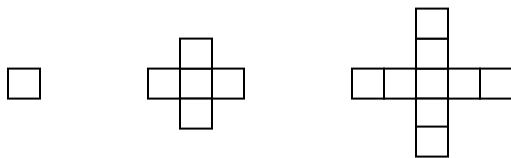


Stage 1

Stage 2

Stage 3

d. Try to think of a different rule that gives the total number of blocks t for any stage, s . Show how your rule relates to the pattern (geometric model).



Stage 1

Stage 2

Stage 3

e. Use your rule to determine the number of blocks in stage 100.

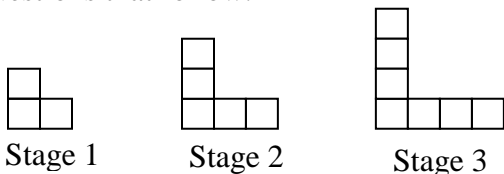
f. Use your rule to determine which stage has 37 blocks.

g. Draw or describe Stage 0 of the pattern. How does the number of blocks in stage 0 relate to the simplified form of your rule?

2.2b Class Activity: Representations of a Linear Pattern



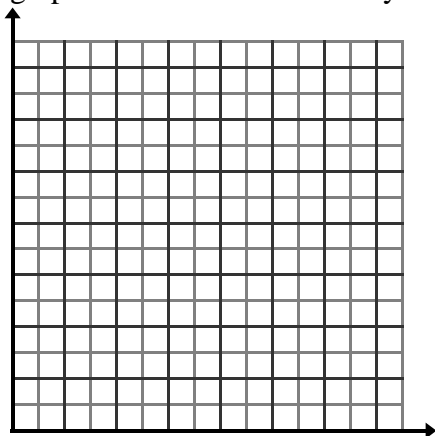
1. You studied this pattern in the previous lesson. Use your work from the previous lessons to answer the questions that follow.



- How many new blocks are added to the pattern from one stage to the next?
- Complete the table.
- Show where you see the rate of change in your table.

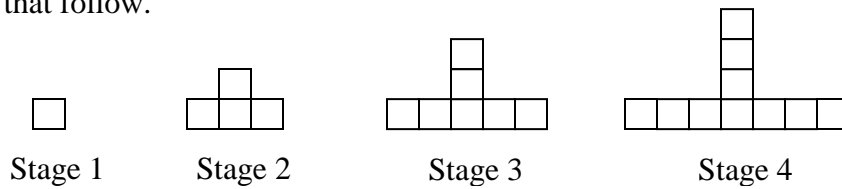
<i>Stage</i> (<i>s</i>)	<i># of</i> <i>Blocks</i> (<i>t</i>)
1	
2	
3	
4	
5	

- Create a graph of this data. Where do you see the rate of change on your graph?



- What is the simplified form of the equation that gives the number of blocks t for any stage s (see previous lesson)? Where do you see the different pieces of the equation in the geometric model, table, and graph?
- The pattern above is a **linear** pattern. Describe how a linear pattern grows. Describe what the graph of a linear pattern looks like.

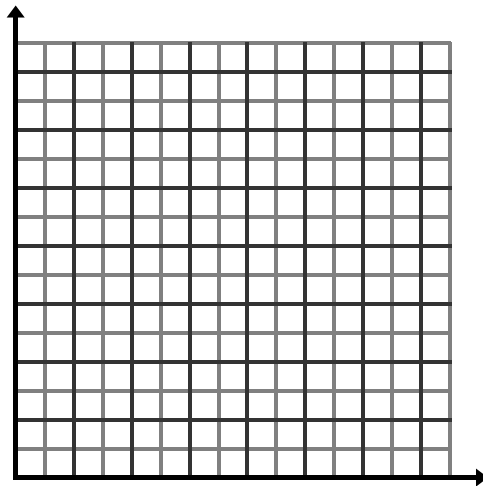
2. You studied this pattern in the previous lesson. Use your work from the previous lesson to answer the questions that follow.



- a. How many new blocks are added to the pattern from one stage to the next?
- b. Complete the table.
- c. Show where you see the rate of change in your table.

<i>Stage</i> (<i>s</i>)	<i># of</i> <i>Blocks</i> (<i>t</i>)
1	
2	
3	
4	
5	

- d. Create a graph of these data. Where do you see the rate of change on your graph?



- e. What is the equation that gives the number of blocks t for any stage s (see previous lesson)? Where do you see the different pieces of the equation in the geometric model, table, and graph?
- f. Is this pattern a linear pattern? Use supporting evidence from each of the representations to justify your answer.

3. Describe what you see in each of the representations (geometric model, table, graph, and equation) of a linear pattern. Make connections between the different representations.

Geometric Model:

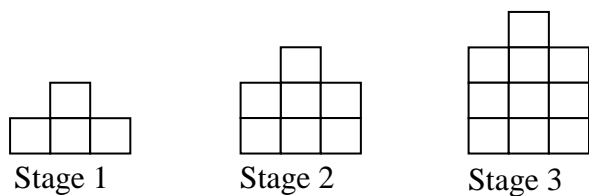
Table:

Graph:

Equation:

2.2b Homework: Representations of a Linear Pattern

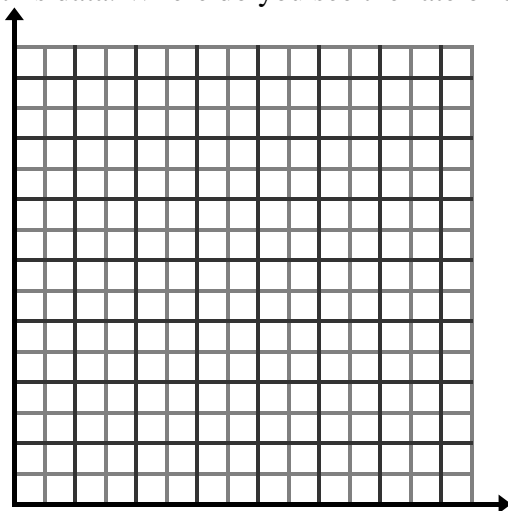
1. You studied this pattern in the previous lesson. Use your work from the previous lesson to answer the questions that follow.



- a. How many new blocks are added to the pattern from one stage to the next?
- b. Complete the table.
- c. Show where you see the rate of change in your table.

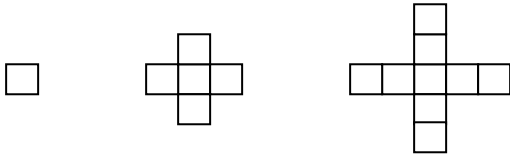
<i>Stage (s)</i>	<i># of Blocks (t)</i>
1	
2	
3	
4	
5	

- d. Create a graph of this data. Where do you see the rate of change on your graph?



- e. What is the equation that gives the total number of blocks t for any stage s (see previous lesson)? Where do you see the different pieces of the equation in the geometric model, table, and graph?
- f. Is this pattern a linear pattern? Use supporting evidence from each of the representations to justify your answer.

2. You studied this pattern in the previous lesson. Use your work from the previous lesson to answer the questions that follow.



Stage 1

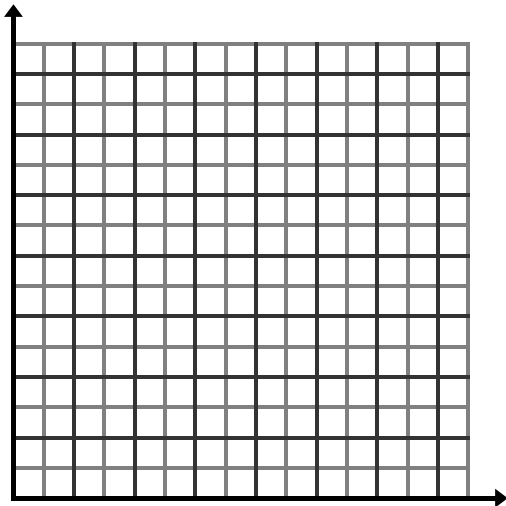
Stage 2

Stage 3

- How many new blocks are added to the pattern from one stage to the next?
- Complete the table.
- Show where you see the rate of change in your table.

<i>Stage</i> <i>(s)</i>	<i># of</i> <i>Blocks</i> <i>(t)</i>
1	
2	
3	
4	
5	

- Create a graph of this data. Where do you see the rate of change on your graph?

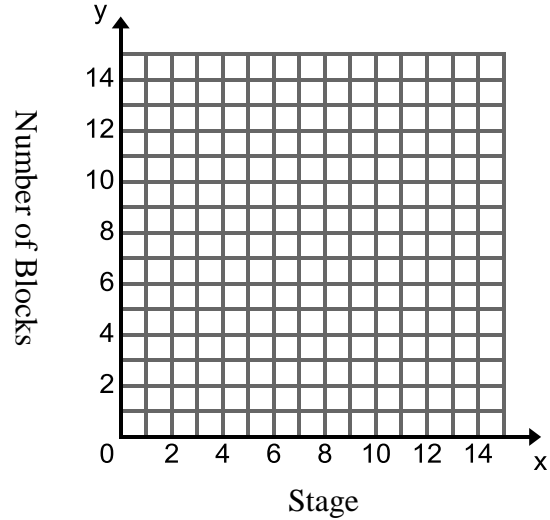


- What is the equation that gives the total number of blocks t for any stage s (see previous lesson)? Where do you see the different pieces of the equation in the geometric model, table, and graph?
- What do you notice about this pattern? Use supporting evidence from each of the representations to justify your answer.

3. Create your own geometric model of a linear pattern in the space below. Then complete the table, graph, and equation for your pattern. Use these representations to prove that your pattern is linear.



<i>Stage</i> (<i>s</i>)	<i># of</i> <i>Blocks</i> (<i>t</i>)
1	
2	
3	
4	
5	



Equation: _____

Prove that your pattern is linear using the representations (geometric model, table, graph, and equation) as evidence.

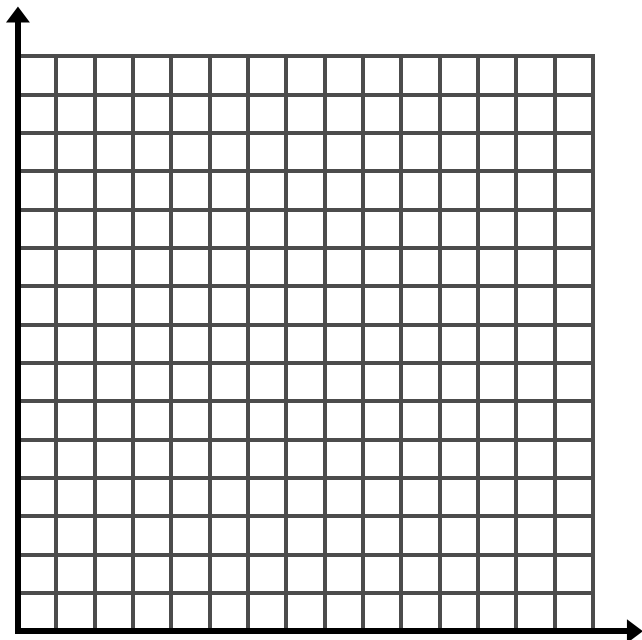
4. Draw or describe a pattern that can be represented by the equation $t = 1 + 6s$ where t is the total number of blocks and s is the stage.

2.2c Class Activity: Representations of a Linear Context



1. Courtney is collecting coins. She has 2 coins in her collection to start with and plans to add 4 coins each week.

a. Complete the table and graph to show how many coins Courtney will have after 6 weeks.



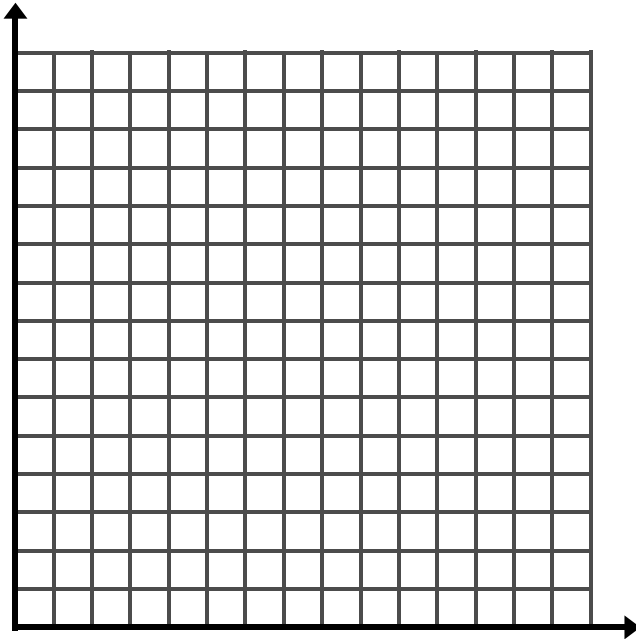
<i>Time (weeks)</i>	<i># of Coins</i>
0	
1	
2	
3	
4	
5	
6	

b. Write an equation for the number of coins c Courtney will have after w weeks.

c. Is this context linear? Use evidence from the context, graph, table, and equation to support your answer.

2. Jack is filling his empty swimming pool with water. The pool is being filled at a constant rate of four gallons per minute.

a. Complete the table and graph below to show how much water will be in the pool after 6 minutes.



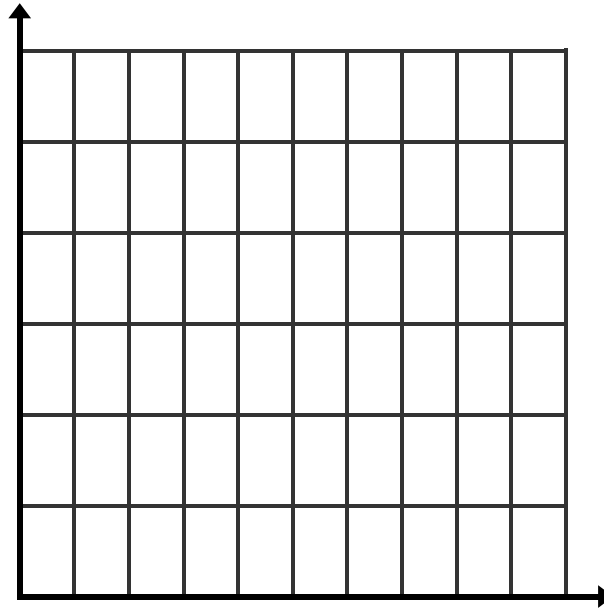
<i>Time (minutes)</i>	<i>Amount of Water (gallons)</i>
0	
1	
2	
3	
4	
5	
6	

- b. Write an equation for the number of gallons g that will be in the pool after m minutes.
- c. Is this context linear? Use evidence from the context, graph, table, and equation to support your answer.
- d. Compare this swimming pool problem to the previous problem about coins. How are the problems similar? How are they different?
- e. How would you change the coin context so that it could be modeled by the same equation as the swimming pool context?
- f. How would you change the swimming pool context so that it could be modeled by the same equation as the coin context?

3. An airplane is at an elevation of 3000 ft. The table below shows its elevation(y) for every 2 miles(x) it travels.

<i>Miles</i>	<i>Elevation</i>
0	3000
2	2000
4	1000

- a. Complete the graph to show how many miles it will take for the airplane to reach the ground.



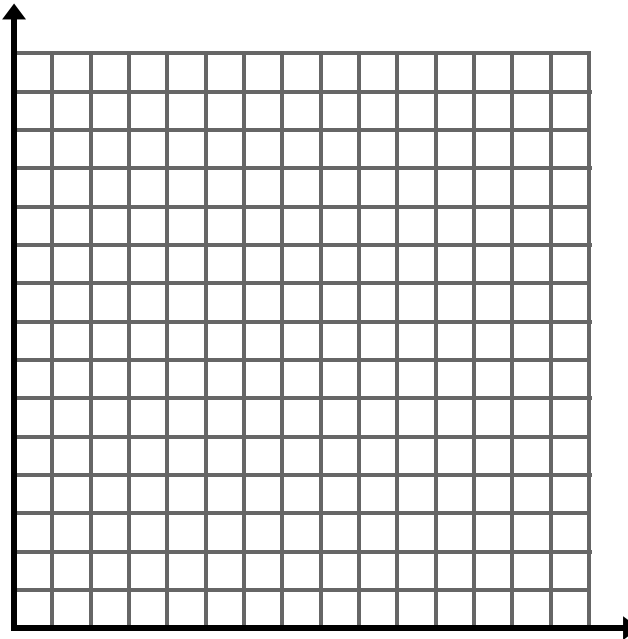
- b. Use the table and the graph to find the rate of change.
- c. Write an equation that represents this relationship
- d. Explain the how equation can be used to determine how many miles it will take for the plane to reach the ground.

2.2c Homework: Representations of a Linear Context

1. Hillary is saving money for college expenses. She is saving \$200 per week from her summer job. Currently, she does not have any money saved.

a. Complete the table and graph to show how much money Hillary will have 6 weeks from now.

<i>Time (weeks)</i>	<i>Amount Saved (dollars)</i>
0	
1	
2	
3	
4	
5	
6	

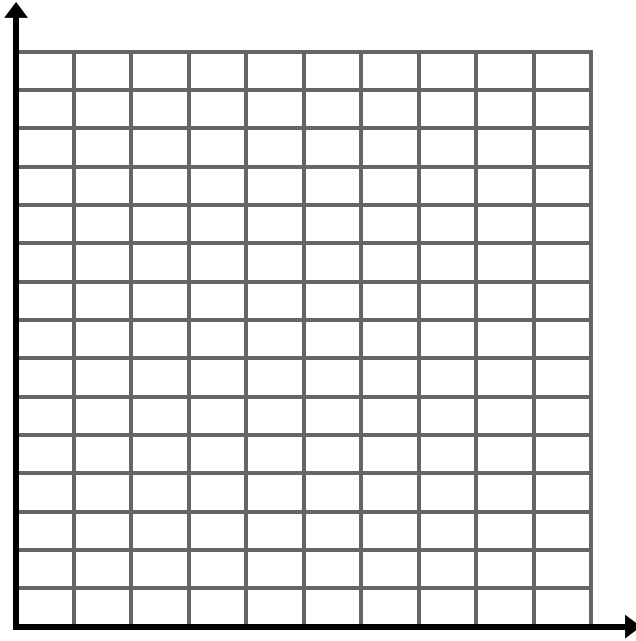


b. Write an equation for the amount of money m Hillary will have saved after w weeks if she continues saving at the same rate.

c. Is this context linear? Use evidence from the context, graph, table, and equation to support your answer.

2. The cost for a crew to come and landscape your yard is \$200 per hour. The crew charges an initial fee of \$100 for equipment.
- a. Complete the table and graph below to show how much it will cost for the crew to work on your yard for 6 hours.

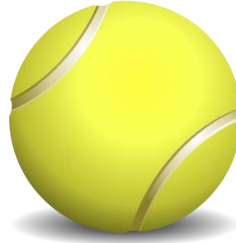
<i>Time (hours)</i>	<i>Cost (dollars)</i>
0	
2	
4	
6	



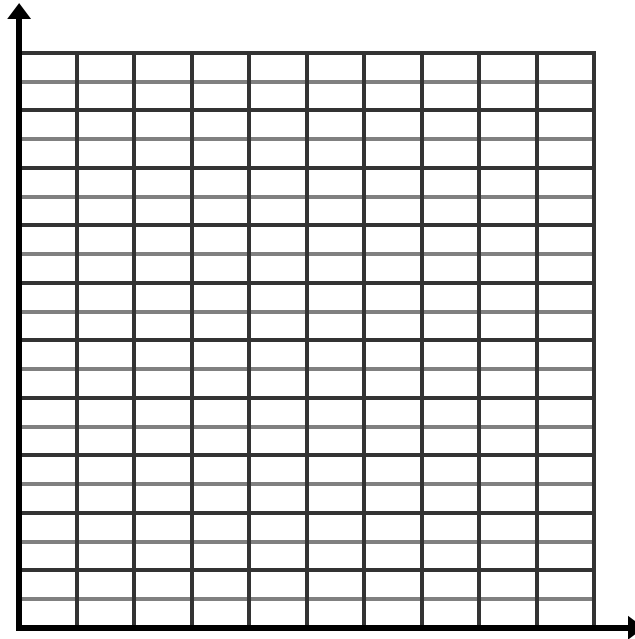
- b. Write an equation for the cost c of landscaping for h hours.
- c. Is this context linear? Use evidence from the context, graph, table, and equation to support your answer.
- d. Compare this landscaping problem to the problem with Hillary's savings. How are the problems similar? How are they different?
- e. How would you change the savings context so that it could be modeled by the same equation as the landscaping context?
- f. How would you change the landscaping context so that it could be modeled by the same equation as the savings context?

3. Linda is always losing her tennis balls. At the beginning of tennis season she has 20 tennis balls. The table below represents how many balls she has as the season progresses; where x represents the number of weeks and y represents the number of tennis balls.

<i>Weeks</i>	<i>Number of tennis balls</i>
0	20
2	16
4	12
6	8



- a. Complete the graph to show how many weeks will pass until Linda runs out of balls.



- b. Use the table and the graph to find the rate of change.
- c. Write an equation that represents this relationship.
- d. Explain how to use the equation to determine how many weeks will pass until Linda runs out of balls.

2.2d Class Activity: Rate of Change in a Linear Relationship

Explore and investigate the rate of change in linear relationships below.

1. The graph below shows the distance a cat is from his bowl of milk over time. Which sentence is a good match for the graph?

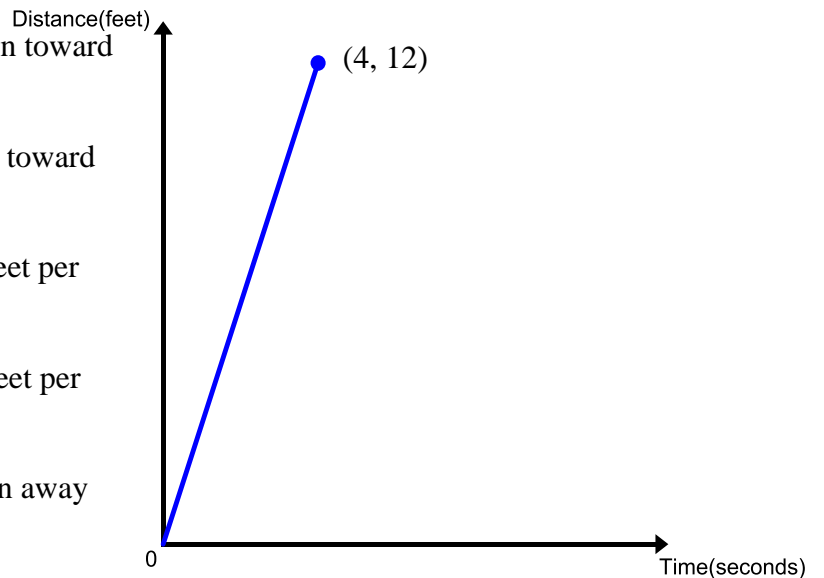
A. The cat was 12 feet away from the milk and ran toward it reaching it after 4 seconds.

B. The cat was 4 feet away from the milk and ran toward it reaching it after 12 seconds.

C. The cat ran away from the milk at a rate of 3 feet per second.

D. The cat ran away from the milk at a rate of 4 feet per second.

E. The cat was 12 feet away from the milk and ran away from it at a rate of 4 feet per second.



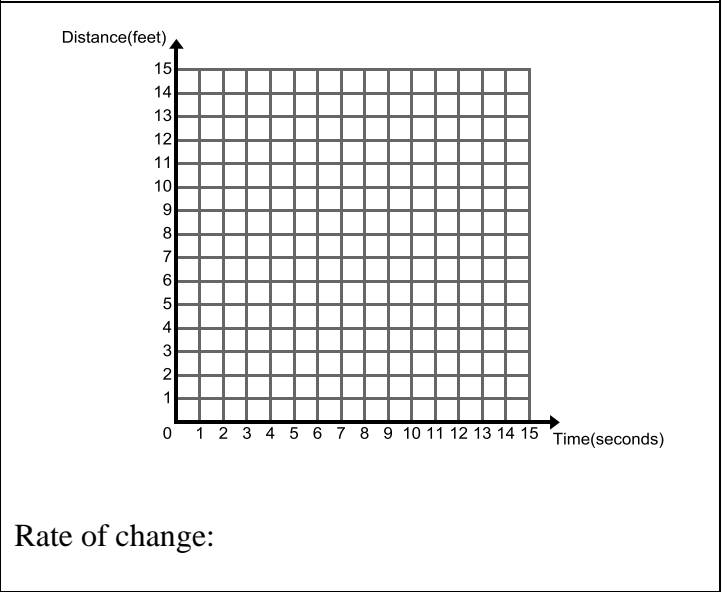
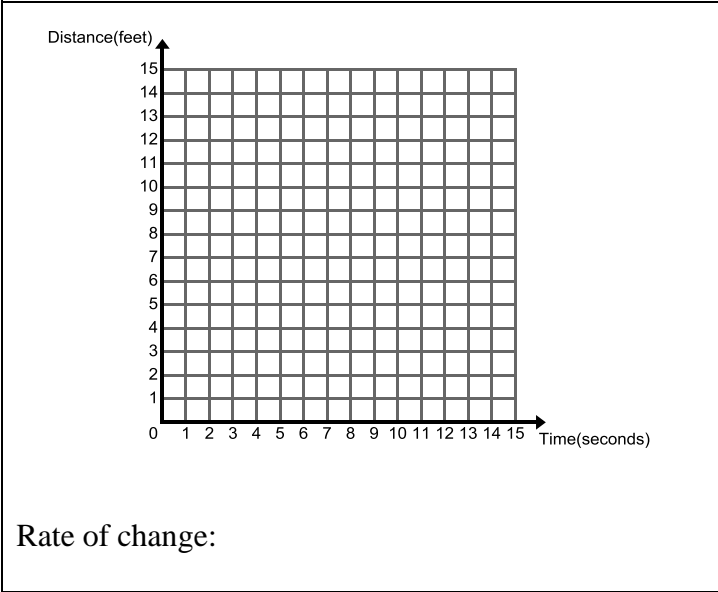
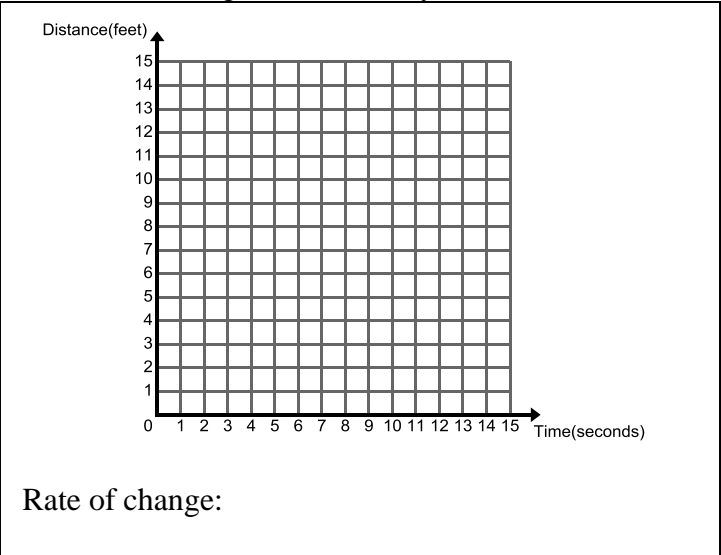
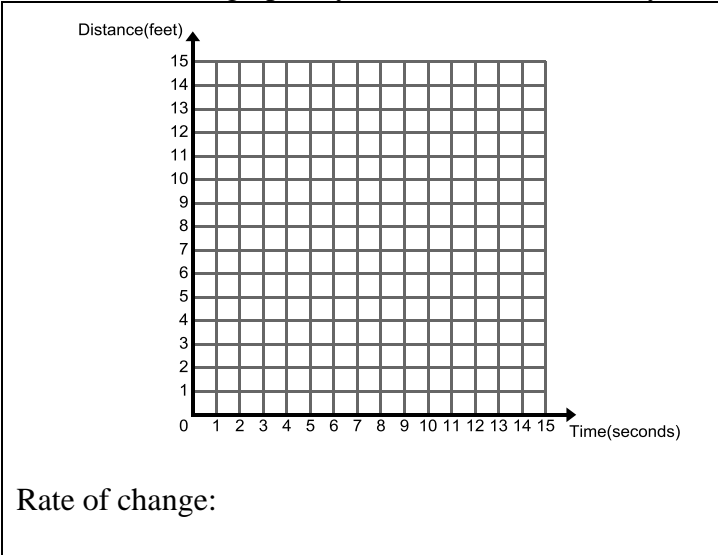
2. Write everything you can say about the cat and the distance he is from the milk during this time.

3. Create a table at the right which also tells the story of the graph.

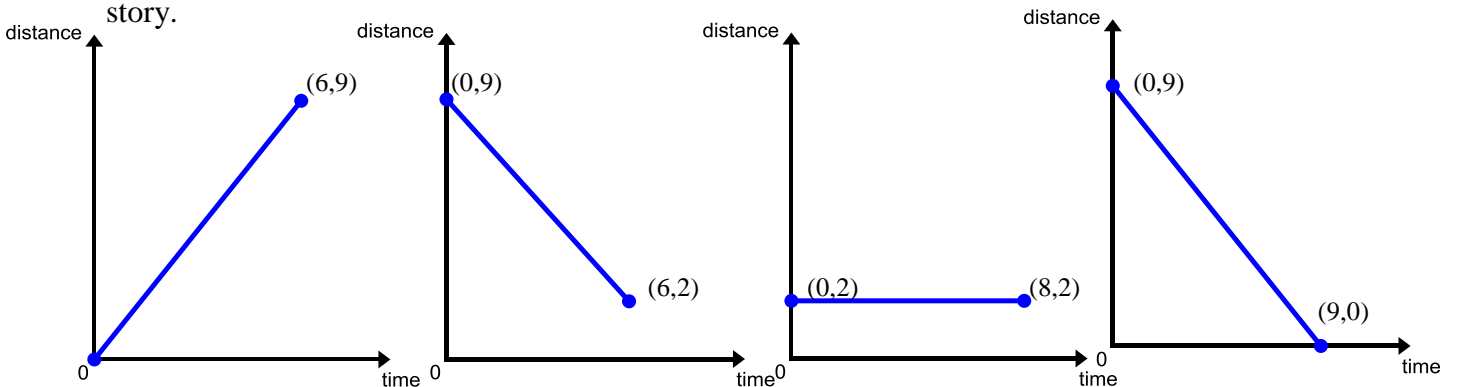
4. Is this a proportional relationship? Justify your answer.

5. Find the unit rate in this story.

6. Sketch a graph for each of the four stories from number 1 on the previous page which you didn't choose. Label the graphs by letter to match the story. Find the rate of change for each story as well.

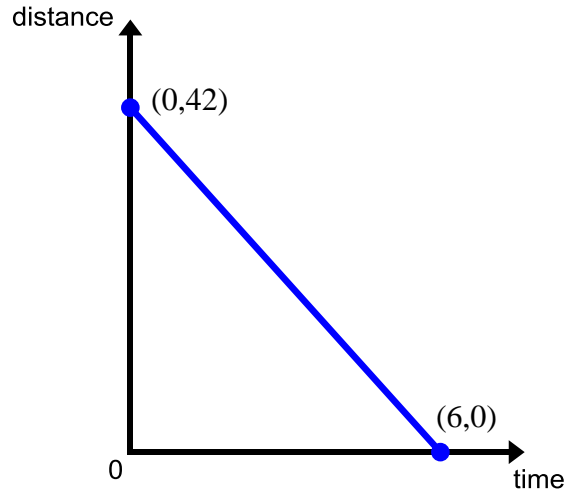


7. A baby was 9 feet from the edge of the porch. He crawled toward the edge for 6 seconds. Then his mother picked him up a few feet before he reached the edge. Circle the graph below that matches this story.



8. The graph and table below describe a runner's distance from the finish line in the last seconds of the race. Which equation tells the same story as the table and graph? Use the ordered pairs given in the table to test your chosen equation and explain your choice.

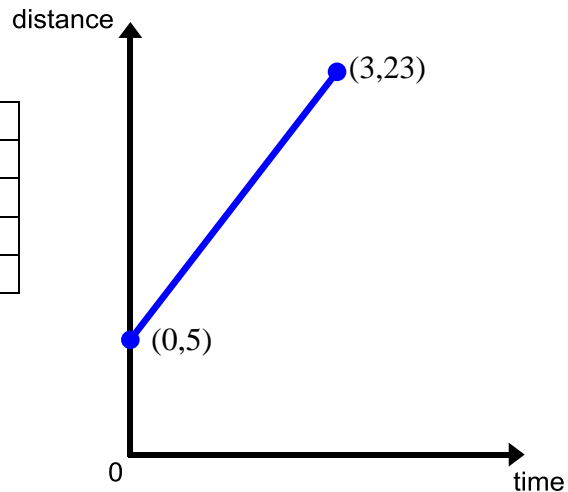
Time (seconds)	Distance (meters)
0	42
1	35
2	28
3	21
4	14
5	7
6	0



- a) $y = 7x + 42$
- b) $y = 42 - 6x$
- c) $y = 42 - 7x$
- d) $y = 42x + 6$

9. Create a table for this graph,

Time	Distance



10. Write a story for this graph.

11. Which equation matches your story, the graph and the table? Explain your choice.

- a) $y = 5 + 6x$
- b) $y = 5x + 23$
- c) $y = 3x + 5$
- d) $y = 5x + 3$

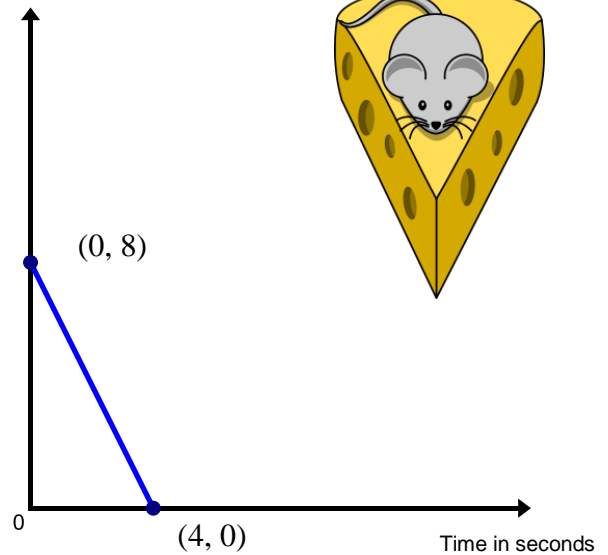
2.2d Homework: Rate of Change in a Linear Relationship

Explore and investigate the linear relationships below.

1. The graph below shows the distance a mouse is from her cheese over time. Which sentence is a good match for the graph?

- A. The mouse is 8 inches away from the cheese, she sits there and does not move.
- B. The mouse is 8 inches away from the cheese, she scurries towards it and reaches it after 4 seconds.
- C. The mouse scurries away from the piece of cheese at a rate of 2 inches per second.
- D. The mouse scurries away from the piece of cheese at a rate of 4 inches per second.
- E. The mouse is 8 inches away from the piece of cheese and scurries away from it a rate of 2 inches per second.

Distance in inches

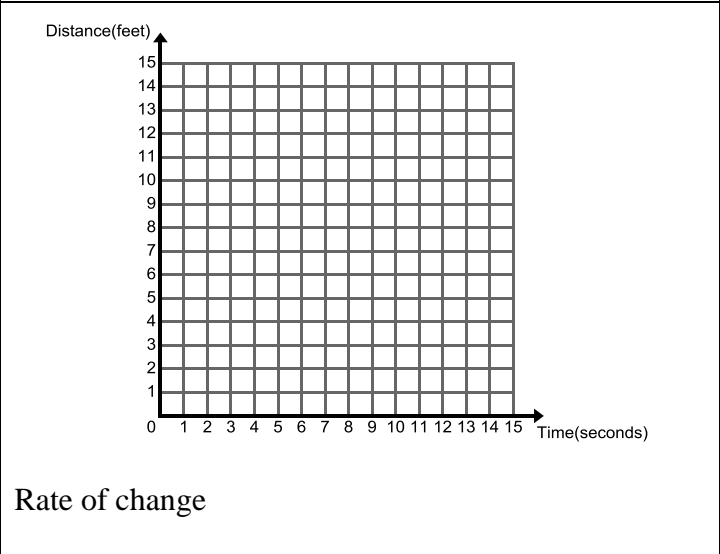
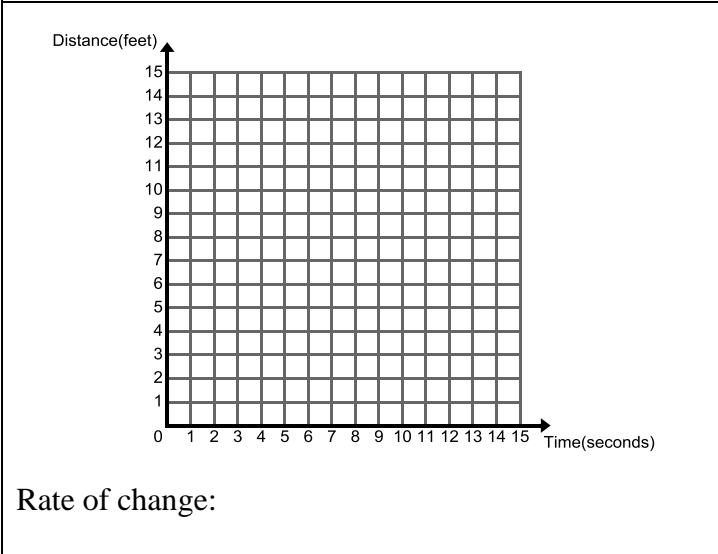
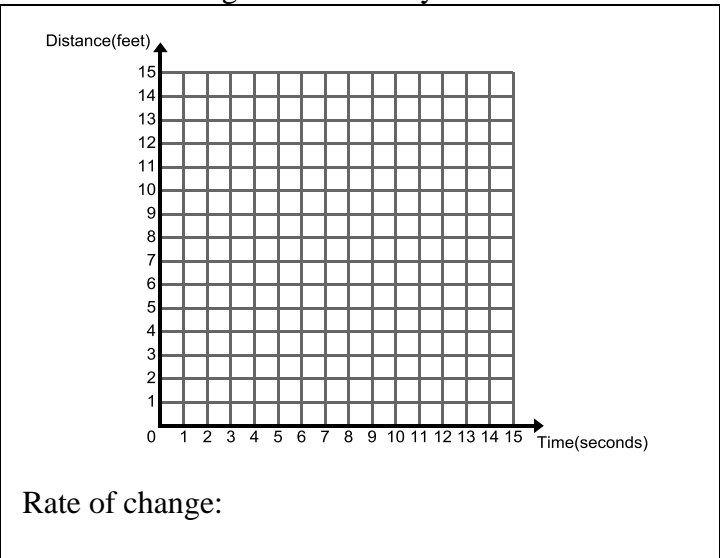
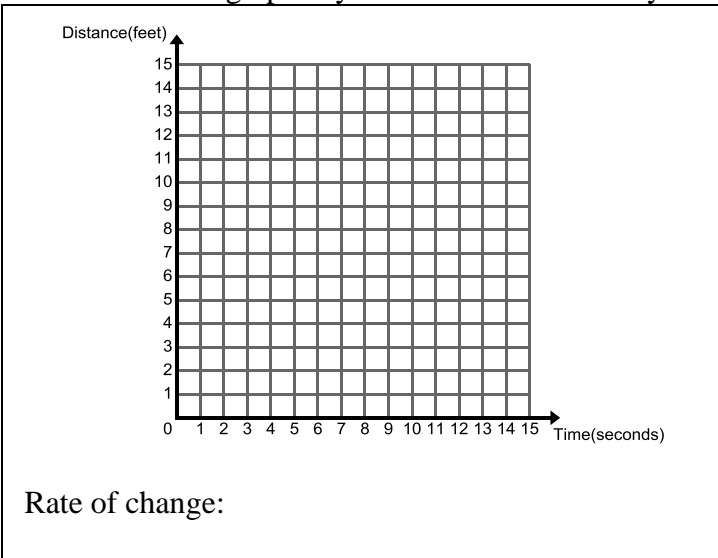


2. Write everything you can say about the mouse and the distance she is from the cheese during this time.

3. Create a table at the right which also tells the story of the graph.

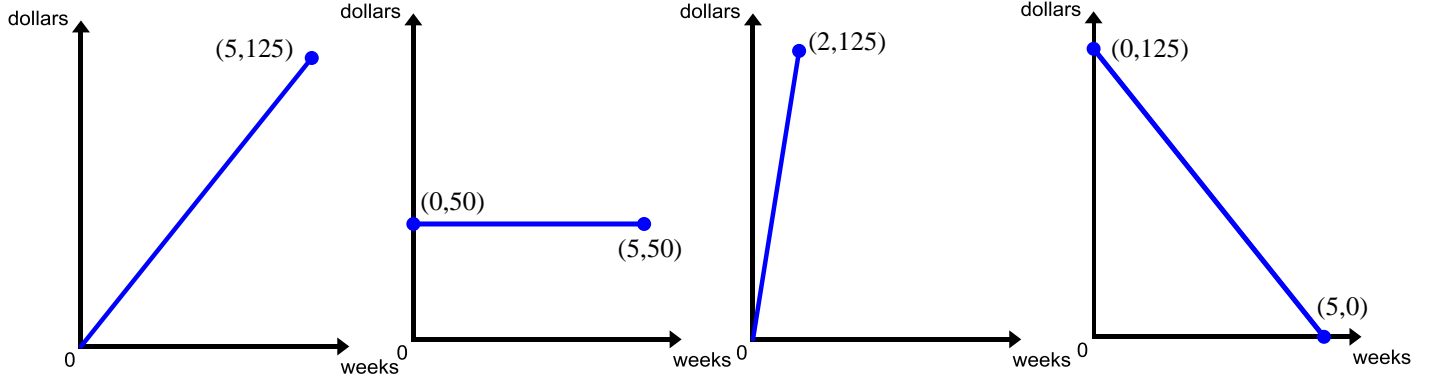
4. Is this a proportional relationship? If so what is the proportional constant?

5. Sketch a graph for the four stories from number 1 above which you didn't choose in the space provided. Label the graphs by letter to match the story. Find the rate of change for each story as well.



6. Compare the rate of change with the steepness of each line above, how does the rate of change relate to the steepness of the line?

7. Triss opens a bank account and adds \$25 to the account every week. Circle the graph below that matches this story.



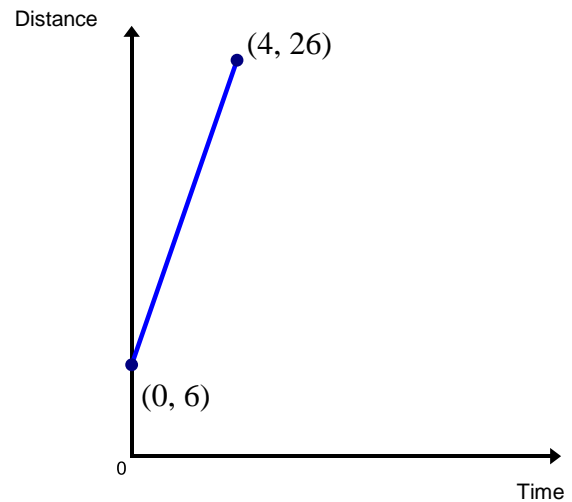
8. Write a story for each of the remaining graphs above.

Use the graph at the right to complete the following.

9. Create a table for this graph,

Time	Distance
0	
1	
2	
3	
4	

10. Write a story for this graph.



11. Which equation matches your story, the graph and the table?

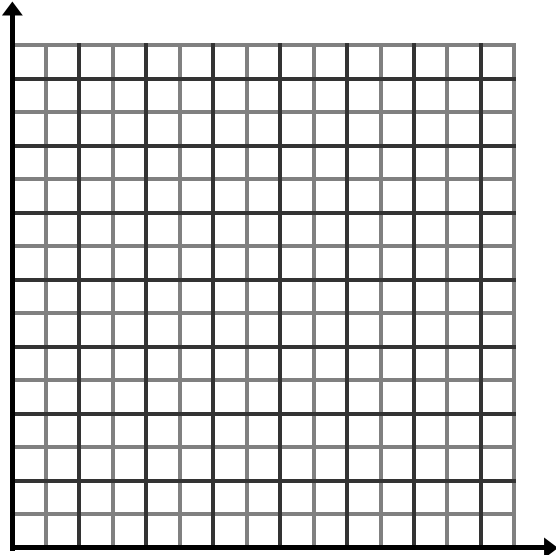
- a) $y = 5 + 6x$
- b) $y = 4x + 26$
- c) $y = 5x + 6$
- d) $y = 6x + 4$

2.2e Class Activity: More Representations of a Linear Context



Directions: In each of the following problems, you are given one of the representations of a linear relationship. Complete the remaining 3 representations. Be sure to label the columns in your table and the axes on your graph.

1. The State Fair

<p><u>Context</u></p> <p>You and your friends go to the state fair. It costs \$6 to get into the fair and \$2 each time you go on a ride. Consider the relationship between number of rides and total cost.</p>	<p><u>Table</u></p> <table border="1" style="width: 100%; border-collapse: collapse; margin-top: 10px;"> <tr><td style="width: 50%; height: 20px;"></td><td style="width: 50%; height: 20px;"></td></tr> <tr><td style="height: 20px;"></td><td style="height: 20px;"></td></tr> <tr><td style="height: 20px;"></td><td style="height: 20px;"></td></tr> <tr><td style="height: 20px;"></td><td style="height: 20px;"></td></tr> <tr><td style="height: 20px;"></td><td style="height: 20px;"></td></tr> <tr><td style="height: 20px;"></td><td style="height: 20px;"></td></tr> <tr><td style="height: 20px;"></td><td style="height: 20px;"></td></tr> <tr><td style="height: 20px;"></td><td style="height: 20px;"></td></tr> </table>																
<p><u>Graph</u></p> 	<p><u>Equation</u></p>																

- a. What is the rate of change in this problem? What does the rate of change represent in the context?
- b. What is the y-intercept of your graph? Where do you see the y-intercept in the table and in the equation? What does the y-intercept represent in the context?
- c. How would you change the context so that the relationship between total cost and number of rides can be modeled by the equation $y = 2x$?
- d. How would you change the context so that the relationship between total cost and number of rides can be modeled by the equation $y = 6$?

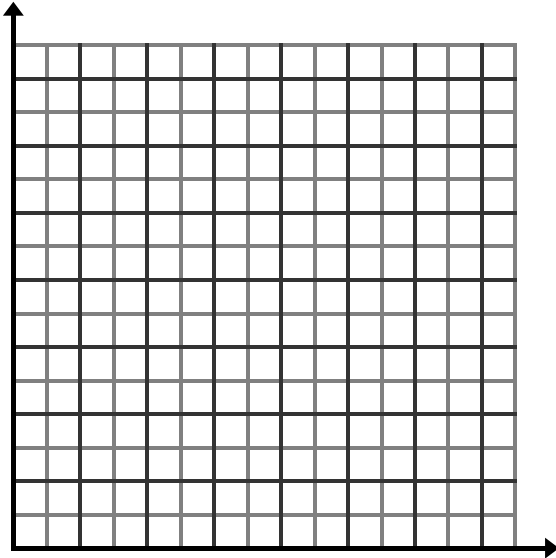
2. Road Trip

Context

You are taking a road trip. You start the day with a full tank of gas. Your tank holds 16 gallons of gas. On your trip, you use 2 gallons per hour. Consider the relationship between time in hours and amount of gas remaining in the tank.

Table

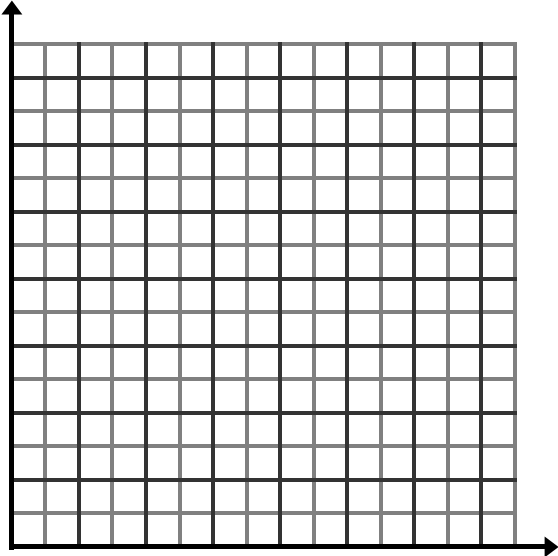
Graph



Equation

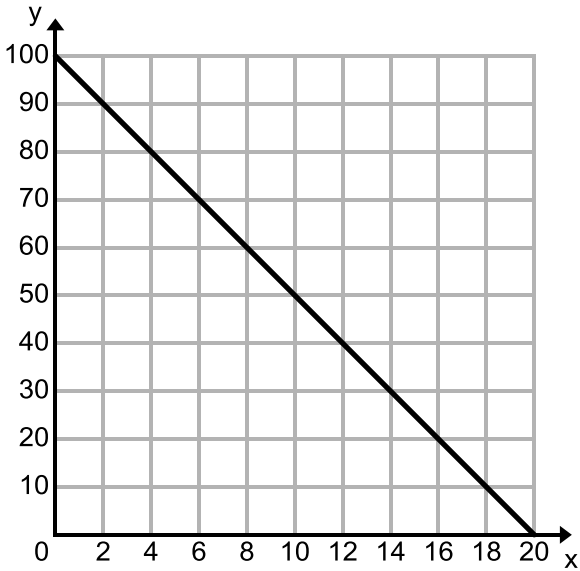
- What is the rate of change in this problem? What does the rate of change represent in the context?
- What is the y-intercept of your graph? Where do you see the y-intercept in the table and in the equation? What does the y-intercept represent in the context?
- How would your equation change if your gas tank held 18 gallons of gas and used 2.5 gallons per hour of driving? What would these changes do to your graph?

3.

<p><u>Context</u></p>	<p><u>Table</u></p> <table border="1" data-bbox="943 205 1326 579"><thead><tr><th>Time (hours)</th><th>Cost (dollars)</th></tr></thead><tbody><tr><td>0</td><td>7</td></tr><tr><td>1</td><td>9</td></tr><tr><td>2</td><td>11</td></tr><tr><td>3</td><td>13</td></tr><tr><td>4</td><td>15</td></tr><tr><td>5</td><td>17</td></tr></tbody></table>	Time (hours)	Cost (dollars)	0	7	1	9	2	11	3	13	4	15	5	17
Time (hours)	Cost (dollars)														
0	7														
1	9														
2	11														
3	13														
4	15														
5	17														
<p><u>Graph</u></p> 	<p><u>Equation</u></p>														

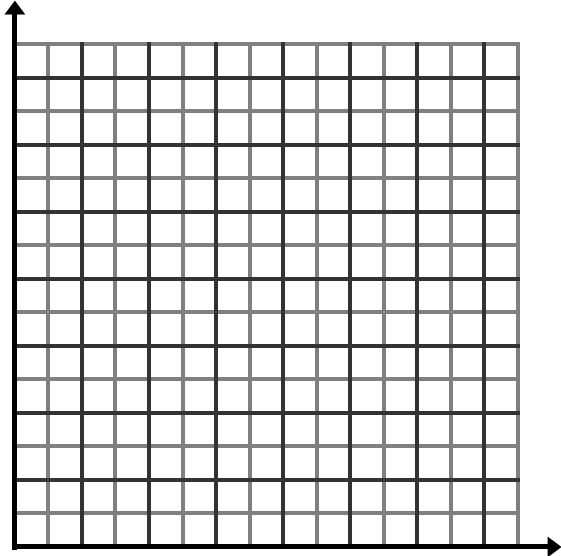
- What is the rate of change in this problem? What does the rate of change represent in your context?
- What is the y-intercept of your graph? Where do you see the y-intercept in the table and in the equation? What does the y-intercept represent in your context?
- How would your context change if the rate of change was 3?

4.

<p><u>Context</u></p>	<p><u>Table</u></p> <table border="1" data-bbox="954 205 1338 541"><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table>																
<p><u>Graph</u></p> 	<p><u>Equation</u></p>																

- What is the rate of change in this problem? What does the rate of change represent in your context?
- What is the y -intercept of the graph? Where do you see the y -intercept in the table and in the equation? What does the y -intercept represent in your context?
- How would your context and equation change if the y -intercept of the graph was changed to 75? How would this change affect the graph?
- How would your context and equation change if the rate of change in this problem was changed to -2 ? Would the graph of the new line be steeper or less steep than the original?

5.

<p><u>Context</u></p>	<p><u>Table</u></p> <table border="1" data-bbox="943 205 1326 541"><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table>																
<p><u>Graph</u></p> 	<p><u>Equation</u></p> $y = 4x + 10$																

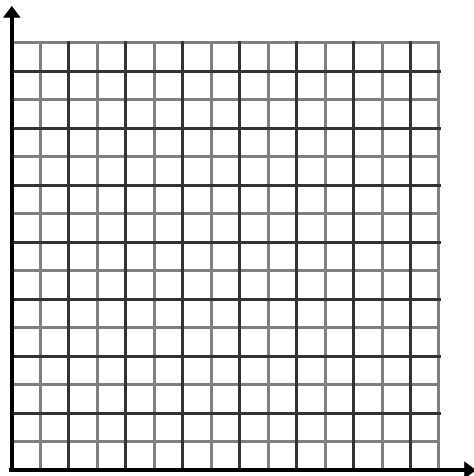
- a. How would your context change if the equation above was changed to $y = 2x + 10$? How would this change affect the graph?
- b. How would your context change if the equation above was changed to $y = 4x + 8$? How would this change affect the graph?

6. Describe in your own words what a linear relationship is composed of. Think about all of the equations that you have written to represent a linear relationship, what do they have in common, what do the different parts of the equations represent?

2.2e Homework: More Representations of a Linear Context

Directions: In each of the following problems, you are given one of the representations of a linear relationship. Complete the remaining 3 representations. Be sure to label the columns in your table and the axes on your graph.

1. A Community Garden

<p><u>Context</u></p> <p>Gavin is buying tomato plants to plant in his local community garden. Tomato plants are \$9 per flat (a flat contains 36 plants). Consider the relationship between total cost and number of flats purchased.</p>	<p><u>Table</u></p> <table border="1"><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table>																
<p><u>Graph</u></p> 	<p><u>Equation</u></p>																

- What is the rate of change in this problem? What does the rate of change represent in the context?
- What is the y-intercept of your graph? What does the y-intercept represent in the context?
- How would you change the context so that the relationship between total cost c and number of flats f purchased was $c = 12f$?

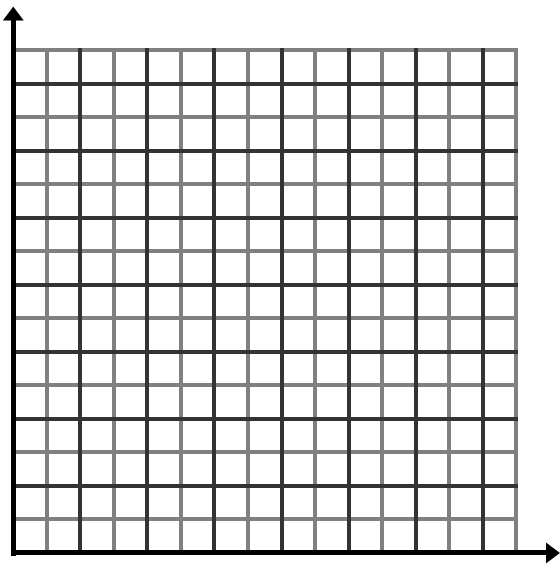
2. Enrollment

Context

The number of students currently enrolled at Discovery Place Preschool is 24. Enrollment is going up by 6 students each year. Consider the relationship between the number of years from now and the number of students enrolled.

Table

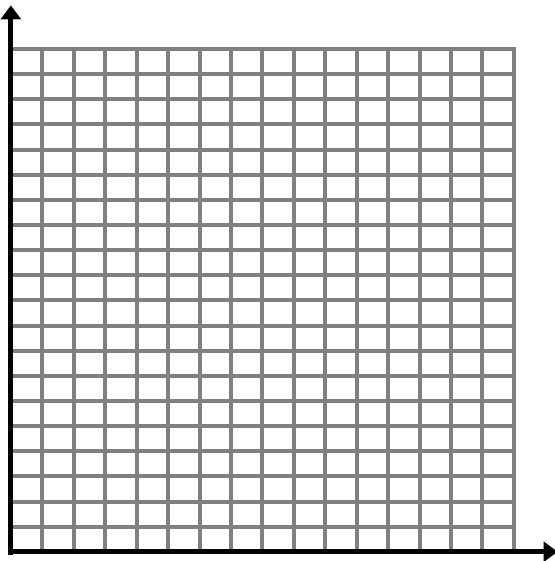
Graph



Equation

- What is the rate of change in this problem? What does the rate of change represent in the context?
- What is the y-intercept of your graph? What does the y-intercept represent in the context?
- How would you change the context so that the relationship between number of years and number of students enrolled was $y = 40 + 6x$?

3. Write your own context

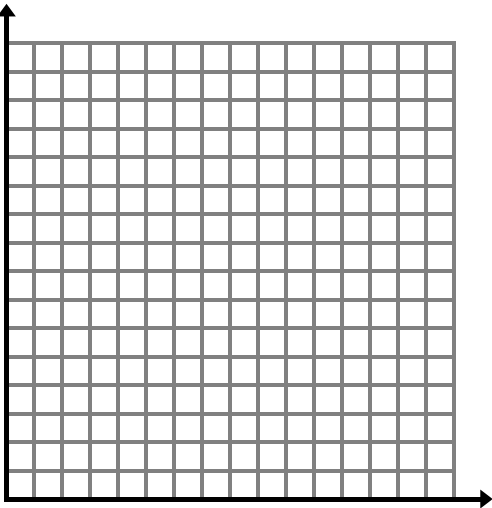
<p><u>Context</u></p>	<p><u>Table</u></p> <table border="1"><thead><tr><th>Time (hours)</th><th># of Fish</th></tr></thead><tbody><tr><td>0</td><td>300</td></tr><tr><td>2</td><td>270</td></tr><tr><td>4</td><td>240</td></tr><tr><td>6</td><td>210</td></tr><tr><td>8</td><td>180</td></tr><tr><td>10</td><td>150</td></tr></tbody></table>	Time (hours)	# of Fish	0	300	2	270	4	240	6	210	8	180	10	150
Time (hours)	# of Fish														
0	300														
2	270														
4	240														
6	210														
8	180														
10	150														
<p><u>Graph</u></p> 	<p><u>Equation</u></p>														

- a. What is the rate of change in this problem? What does the rate of change represent in the context?

- b. What is the y-intercept of your graph? What does the y-intercept represent in the context?

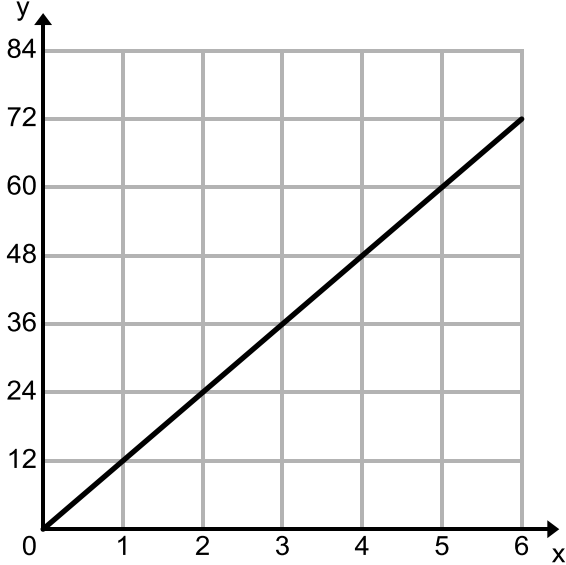
- c. How would you change your context so that the amount of fish remaining y after x hours could be represented by the equation $y = 300 - 30x$?

4. Write your own context

<p><u>Context</u></p>	<p><u>Table</u></p> <table border="1" data-bbox="1117 205 1497 541"><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table>																
<p><u>Graph</u></p> 	<p><u>Equation</u></p> $y = 30 + 5x$																

- a. What is the rate of change in this problem? What does the rate of change represent in the context?
- b. What is the y-intercept of your graph? What does the y-intercept represent in the context?

5. Write your own context

<p><u>Context</u></p>	<p><u>Table</u></p> <table border="1" data-bbox="954 205 1338 541"><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table>																
<p><u>Graph</u></p> 	<p><u>Equation</u></p>																

- a. What is the rate of change in this problem? What does the rate of change represent in the context?

- b. What is the y-intercept of your graph? What does the y-intercept represent in your context?

- c. How would your context and equation change if the rate of change in this problem was changed to 10? Would the graph of the new line be steeper or less steep?

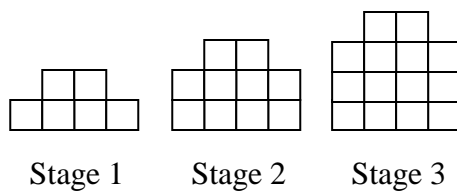
2.2f Self-Assessment: Section 2.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Some sample criteria are provided as well as sample problems on the following page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Write rules for linear patterns and connect the rule to the pattern (geometric model). <i>See sample problem #1</i>	I don't know how to write a rule for the pattern and I don't know how to find the number of blocks in the 50 th stage.	I don't know how to write a rule for the pattern. The only way I know how to find the number of blocks in the 50 th stage is by adding the rate of change 50 times.	I can write a rule that describes the pattern but don't know how my rule connects to the pattern.	I can write a rule that describes the pattern and explain how my rule connects to the pattern. I can also use the rule to predict the number of blocks in stage 50.
2. Understand how a linear relationship grows as related to rate of change and show how that growth can be seen in each of the representations. <i>See sample problem #2</i>	I can find the rate of change in only one of the representations.	I can find the rate of change in some of the representations.	I know how to find the rate of change in all the representations but have a hard time explaining how you can see the growth.	I know how to find the rate of change in all the representations. I can also show how the rate of change can be seen.
3. Create the additional representations (table, graph, equation, context, geometric model) of a linear relationship when given one representation and make connections between them. <i>See sample problem #3</i>	I can only make one linear representation.	I can make some of the linear representations.	I can make all the different representations of a linear pattern but I don't know how they are connected.	I can fluently move between the different representations of a linear relationship and make connections between them.
4. Identify the rate of change and initial value of a linear relationship in the table, graph, equation, context, and geometric model of a linear pattern. <i>See sample problem #4</i>	I can identify the rate of change and initial value in 2 or less of the representations of a linear model.	I can identify the rate of change and initial value in 3 of the representations of a linear model.	I can identify the rate of change and initial value in 4 of the representations of a linear model.	I can identify the rate of change and initial value in all of the representations of a linear model.

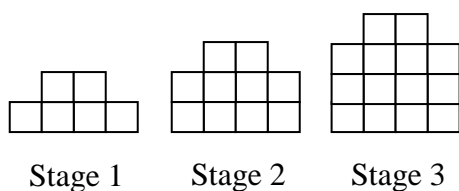
Sample Problem #1

Write a rule that describes the pattern given below where s is the stage number and t is the total number of blocks. Be sure to explain how your rule connects to the pattern. Then use the rule to predict the number of blocks in the 50th stage.

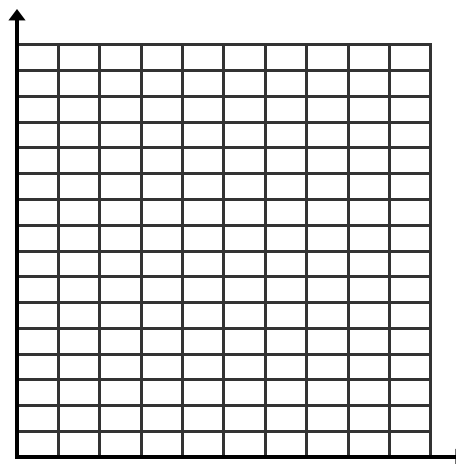


Sample Problem #2

For the pattern given below find the number of blocks in the next stage by determining the rate of change. Fill in the table and graph and explain how you can see the pattern grow in each of these representations.



<i>Stage</i> (s)	<i># of</i> <i>Blocks</i> (b)
1	
2	
3	
4	
5	



Sample Problem #3

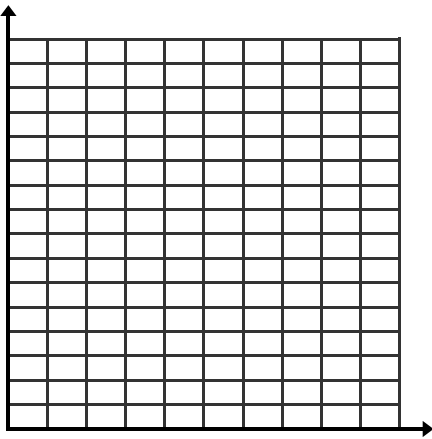
Given the following equation create a table, graph, and a context or geometrical model that the equation could possibly describe. Write a sentence that describes how the different representations are related.

$$y = -3x + 12$$

Table:

x	y

Graph:



Context/Model:

Sample Problem #4

For each of the representations given below identify the rate of change and initial value.

1. $y = \frac{1}{2}x - 3$

Rate of change:

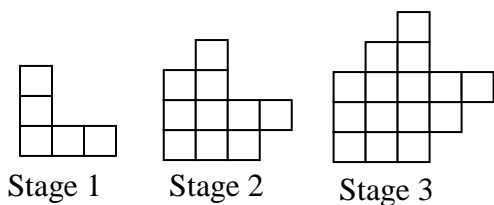
Initial value:

2. The local community center charges a monthly fee of \$15 to use their facilities plus \$2 per visit.

Rate of change:

Initial Value:

3.



Rate of change:

Initial Value:

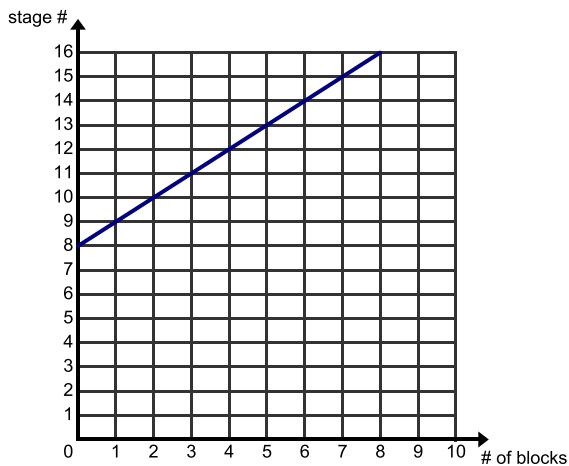
4.

x	y

Rate of Change:

Initial Value:

5.



Rate of Change:

Initial Value:

Section 2.3: Investigate The Slope of a Line

Section Overview:

This section uses proportionality to launch an investigation of slope. Transformations are integrated into the study of slope by looking at the proportionality exhibited by dilations. Students dilate a triangle on lines to show that a dilation produces triangles that have proportional parts and thus the slope is the same between any two distinct points on a non-vertical line. They further their investigation of slope and proportional relationships and derive the slope formula. Adequate time and practice is given for students to solidify their understanding of this crucial aspect of linear relationships. The last few lessons provide an opportunity for students to use all of the knowledge and tools acquired throughout the chapter to formally derive the equations $y=mx$ and $y=mx+b$. They will use proportionality produced by a dilation to do this derivation.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Show that the slope of a line can be calculated as rise/run. Also explain why the slope is the same between any two distinct points on the line.
2. Find the slope of a line from a graph, set of points, and table. Recognize when there is a slope of zero or when the line is undefined.
3. Given a context, find slope from various starting points (2 points, table, line, equation).
4. Recognize that m in $y=mx$ and $y=mx+b$ represents the rate of change or slope of a line. Understand that b is where the line crosses the y -axis or is the y -intercept.
5. Derive the equation $y=mx$ and $y=mx+b$ using dilations and proportionality.

2.3a Class Activity: Building Stairs and Ramps.

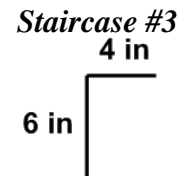
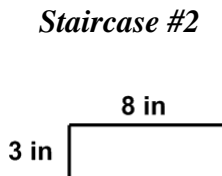
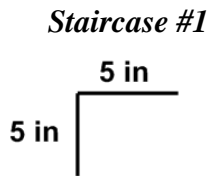
In the previous section you saw that a constant rate of change is an attribute of a linear relationship. When a linear relationship is graphed on a line you call the constant rate of change of the line the **slope** of the line.

The **slope** of a line describes how steep it is. It describes the change in y values compared to the change in the x values.

The following investigation will examine how slope is measured.

On properly built staircases all of the stairs have the same measurements. The important measurements on a stair are what we call the *rise* and the *run*. When building a staircase these measurements are chosen carefully to prevent the stairs from being too steep, and to get you to where you need to go.

One step from three different staircases has been given below.



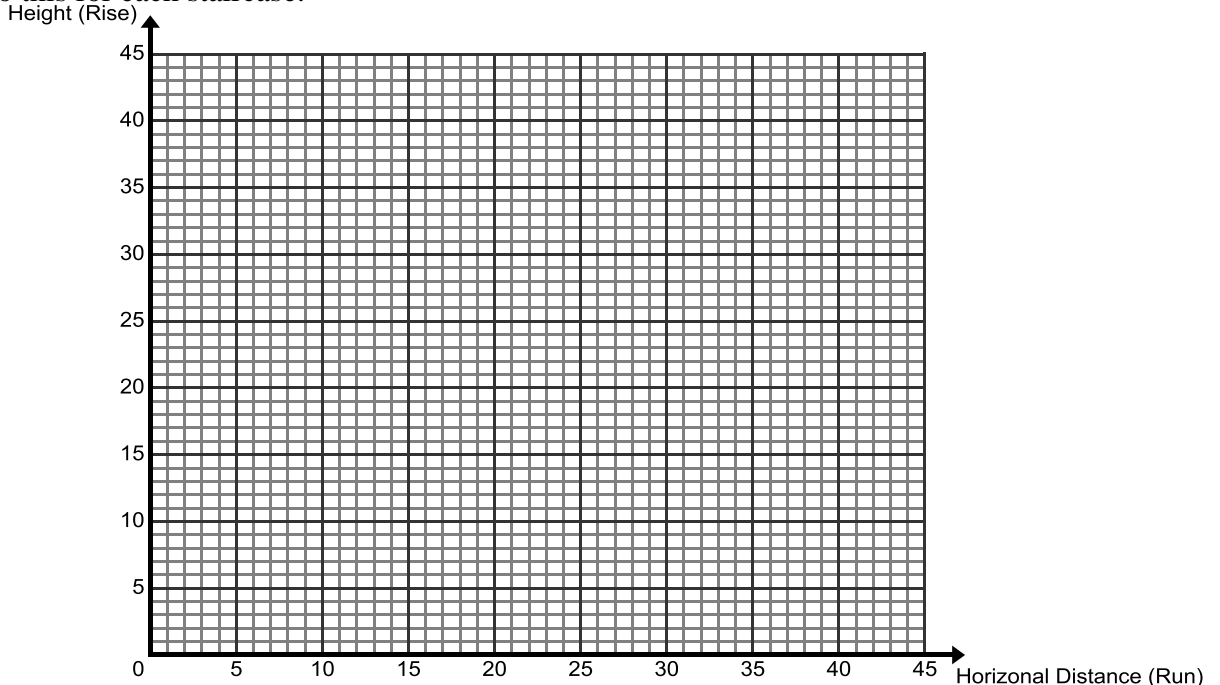
The **vertical** measurement is the “rise”.

The **horizontal** measurement is the “run”.

- State the rise and run for each staircase.

a. <i>Staircase #1</i>	b. <i>Staircase #2</i>	c. <i>Staircase #3</i>
rise = ____ run = ____	rise = ____ run = ____	rise = ____ run = ____

- Using the run and rise for each step, graph the height a person will be at after each step for the first 5 steps. Do this for each staircase.



3. Which staircase is the steepest? _____

Just like staircases, the measurement of the steepness of a line is also very important information. On the graph on the previous page draw a connecting line from the origin(0,0) through the tip of each stair step.

Find the slope of each line representing a staircase using the ratio: $\frac{\text{rise}}{\text{run}}$ or $\frac{y}{x}$, and by simplifying this fraction.

4. Calculate the slope ratio for each staircase.

a. Staircase #1: _____

b. Staircase #2: _____

c. Staircase #3: _____

5. If you didn't have the graph to look at, only the ratios you just calculated, how would you know which staircase would be the steepest?

6. Calculate the slope for climbing 1, 2, & 3 steps on each of the staircases.

	<u>Staircase #1</u>			<u>Staircase #2</u>			<u>Staircase #3</u>		
	Total Rise	Total Run	Slope (rise/run)	Total Rise	Total Run	Slope (rise/run)	Total Rise	Total Run	Slope (rise/run)
1 step									
2 steps									
3 steps									

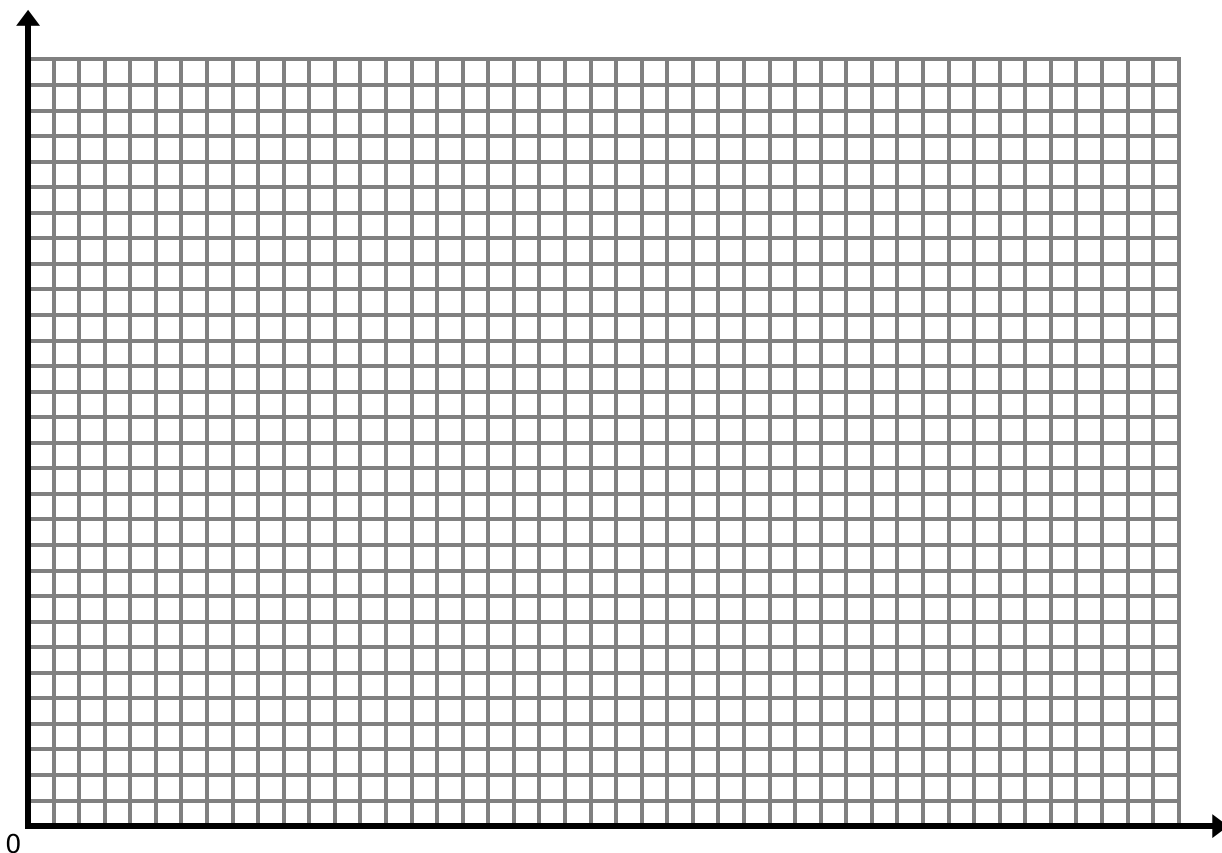
7. Does the slope of the staircase change as you climb each step?

8. Using your knowledge of how slope is calculated, see if you can figure out the slope of the ramp found at your school. Take measurements at two locations on the ramp. Use the table below to help you.

	Rise	Run	Slope
1 st measurement			
2 nd measurement			

2.3a Homework: Measuring the Slope of Stairs and Ramps

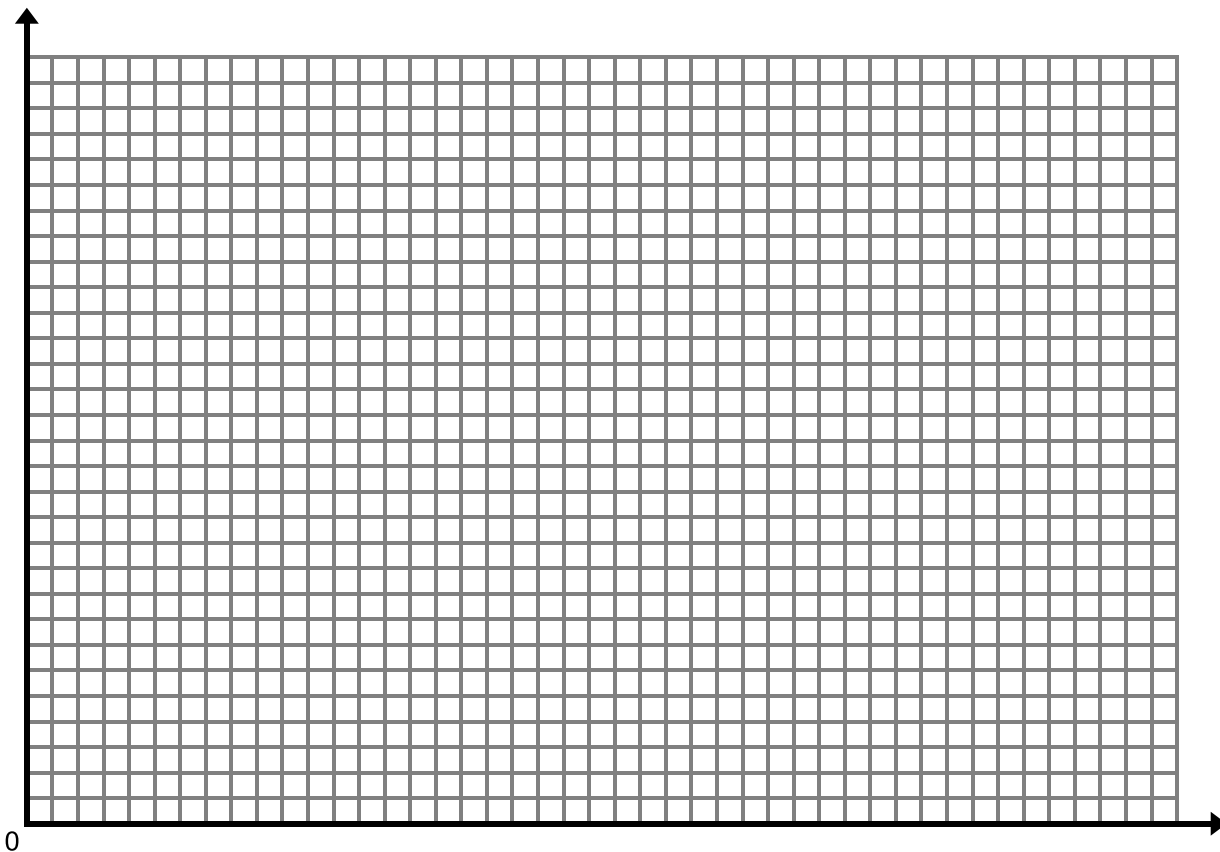
1. Your task is to design a set of stairs and a wheelchair ramp at the side. Both the stairs and the ramp will begin at the same place (at ground level) and end at the height of 3 feet. Answer the following questions as you develop your design.
 - How many steps do you want or need?
 - How deep should each step be? Why do you want this run depth?
 - How tall will each step be? Why do you want this rise height?
 - What is the total distance (total depth for all steps) you will need (at the base) for all of the stairs—this would be a measurement at ground level from stair/ramp start point to stair/ramp end point?
2. Sketch the ramp (as viewed from the side) on graph paper below. Label and sketch the base and height, for example: Ramp-base (in inches or feet) and Height (in inches or feet).



3. From the sketch of the ramp, find and record the following measurements.

	Rise height (total height you've climbed on the ramp at this point)	Run depth (total distance – at ground level – covered from stair-base beginning)	Ratio $\frac{rise}{run}$
12 inches in from the start of the ramp			
24 inches in from the start of the ramp			
Where the ramp meets the top			

4. Sketch the stairs (as viewed from the side) on graph paper below. Label the sketch base and height, for example: Stair-base (in inches) and Height (in inches).



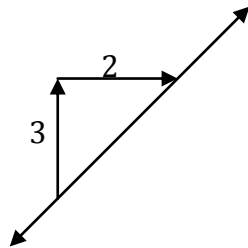
From the sketch above, find and record the following measurements.

	Rise height (total height you've climbed at this point)	Run depth (total distance covered from stair-base beginning)	Ratio $\frac{rise}{run}$	Reduced ratio
At the first step				
At the third step				
At the last step				

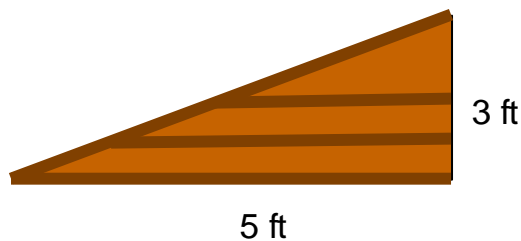
5. What do you notice about the Ratio column for both the ramp and the stairs?
6. On your stair drawing, draw a line from the origin (0,0) through the very tip of each stair. Now look at your ramp drawing. What do you observe?
7. Explain what would happen to the slope of the line for your stairs if the rise of your stairs was higher or lower?
8. Fill in the blanks

a. Slope = _____

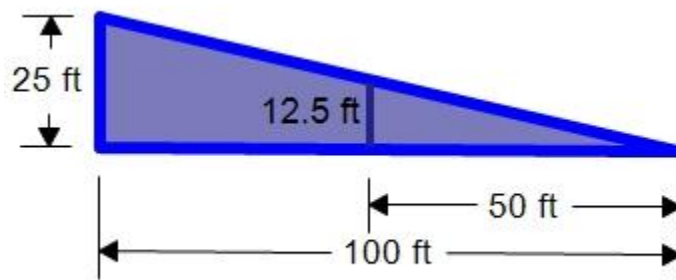
b. In the picture below the rise is _____ units, and the run is _____ units. Thus, the slope of this line is _____.



9. Suppose you want to make a skateboard ramp that is **not** as steep as the one show below. Write down two different slopes that you could use.



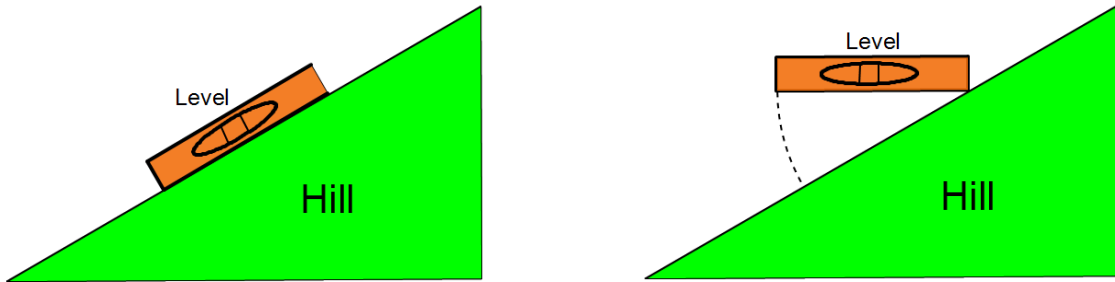
10. Find the slope of the waterslide. Calculate it in two different ways.



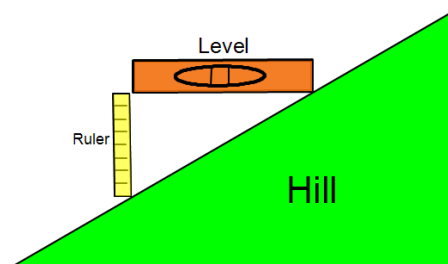
Extra Practice: Measure the Slope of a Hill

Instructions for measuring the grade of a hill or a road:

Rest the level (or clear bottle filled with water) on the ground. Lift up the lower end of the level/bottle (i.e, the end nearest the bottom of the hill) until the level measures level. (If using a bottle, the water level in the bottle should be parallel to the side of the bottle.)



While holding the level/bottle still in this position, measure the distance between the end of the level and the ground using the ruler, as shown in the image below.



1. Calculate the grade by dividing the distance measured with the ruler (the "rise") by the length of the level or straight edge (the "run") and multiplying by 100:
2. Roadway signs such as the one to the right are used to warn drivers of an upcoming steep down grade that could lead to a dangerous situation. What is the grade, or slope, of the hill described on the sign? (Hint: Change the percent to a decimal and then change the decimal to a fraction)



2.3b Class work: Dilations and Proportionality

When an object, such as a line, is moved in space it is called a transformation. A special type of transformation is called a dilation. A dilation transforms an object in space from the center of dilation, usually the origin, by a scale factor called r . The dilation moves every point on the object so that the point is r times away from the center of dilation as it was originally. This means that the object is enlarged or reduced in size.

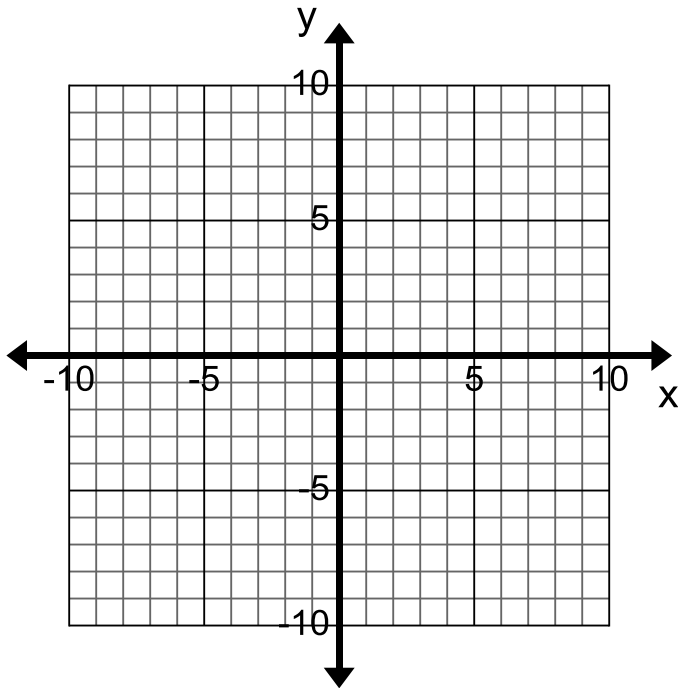
For example, if you dilate the set of points $(0,0)$, $(0,4)$, $(3, 0)$, and $(3,4)$ with a scale factor of 2 and the center of dilation is at the origin $(0,0)$ the distance of each point from the center will be 2 times as long as it was originally. Algebraically this means that you multiply each point by 2.

$$(x, y) \rightarrow (2x, 2y)$$

To confirm this, investigate this transformation below.

1.

- a. Graph and connect the ordered pairs $(0,0)$, $(0,4)$, $(3, 0)$, and $(3,4)$.



- b. Find the length of each segment and dilate it by a scale factor of two. Draw and label the new lengths from the center of dilation (the origin) in a different color on the coordinate plane above.

- c. Compare the size of the pre-image with the image.

- d. Dilate by a scale factor of 2 algebraically using the ordered pairs. Algebraically this is written as $(x, y) \rightarrow (2x, 2y)$. Write the ordered pairs below.

- e. Graph and connect your new ordered pairs for the image on the coordinate plane in part a.

- f. What do you observe about the transformation when you do it graphically and algebraically?

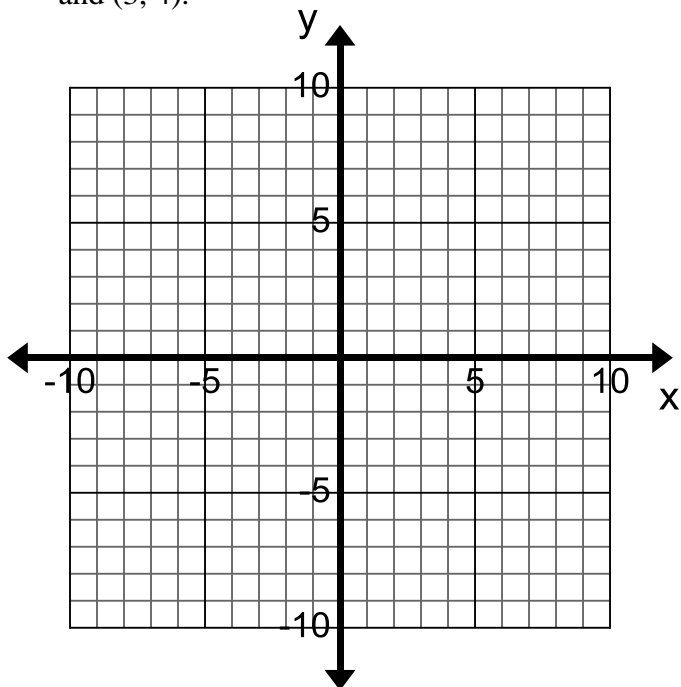
- g. How do the lines that correspond to one another in the image and pre-image compare?

The original object is called the *pre-image* and the transformed object is called the *image*.

A special notation is used to differentiate between the pre-image and the image. If the pre-image is called A then the image is called A' , pronounced “A prime”.

2. Try another shape to see what kind of relationship exists between the pre-image and the image.

- a. Graph and connect the ordered pairs $(0,0)$, $(3,0)$, and $(3, 4)$.



- b. Find the length of each segment and dilate it by a scale factor of $\frac{1}{2}$. (The length of the hypotenuse is 5) Draw the new lengths from the center of dilation (the origin) in a different color.

- c. Label the pre-image A and the image A' . Compare the size of the pre-image with the image.

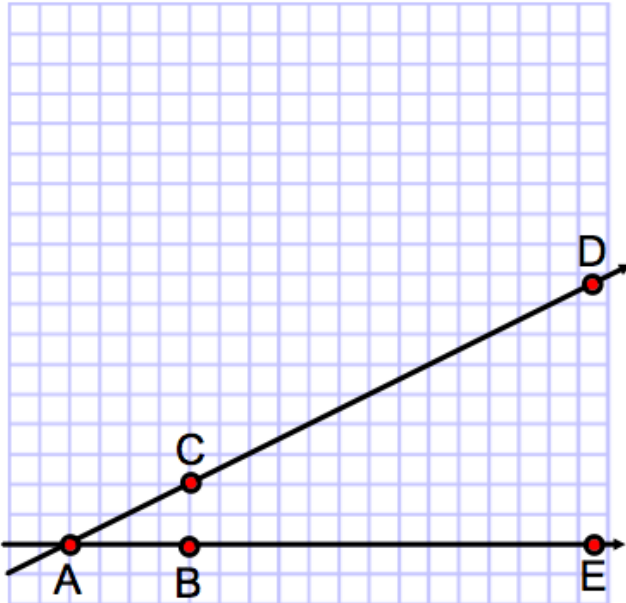
- d. Dilate by a scale factor of $\frac{1}{2}$ algebraically using the ordered pairs. That it is $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$. Write the ordered pairs below.

- e. Graph your new ordered pairs for the image.

- f. What do you observe about the transformation when you do it graphically and algebraically?

- g. How do the lines that correspond to one another in the image and pre-image compare?

Now that you know how a dilation works you will investigate what kind of relationship is formed between the pre-image and the image after the dilation.

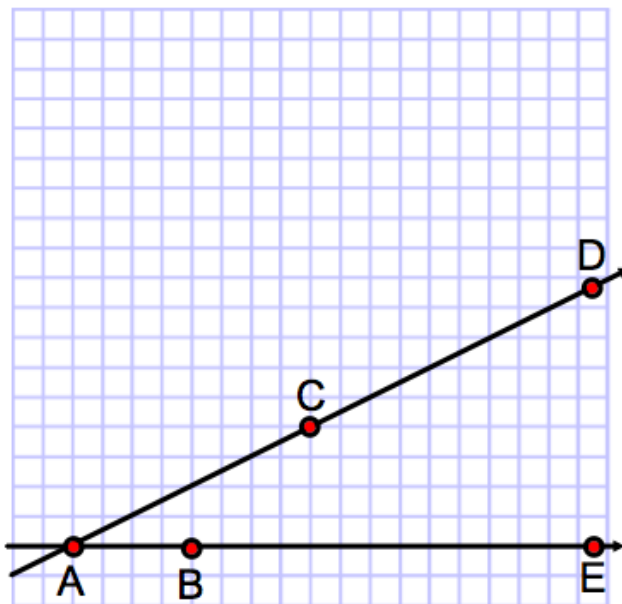
<p>3. Using the figure below.</p>	
<p>a. Connect point B to C</p> <p>b. Double the length of \overline{AB} on the line \overline{AE}. Label the new segment $\overline{AB'}$</p> <p>c. Double the length of \overline{AC} on the line \overline{AD}. Label the new segment $\overline{AC'}$.</p> <p>d. Connect B' to C'.</p>	

4. What do you notice about $\overline{B'C'}$ in relationship to \overline{BC} ?
5. What do you notice about the size of the two triangles?
6. Write a ratio that compares the corresponding parts of the pre-image with the image.
7. What kind of relationship exists between the pre-image and image?

2.3b Homework: Dilations and Proportionality

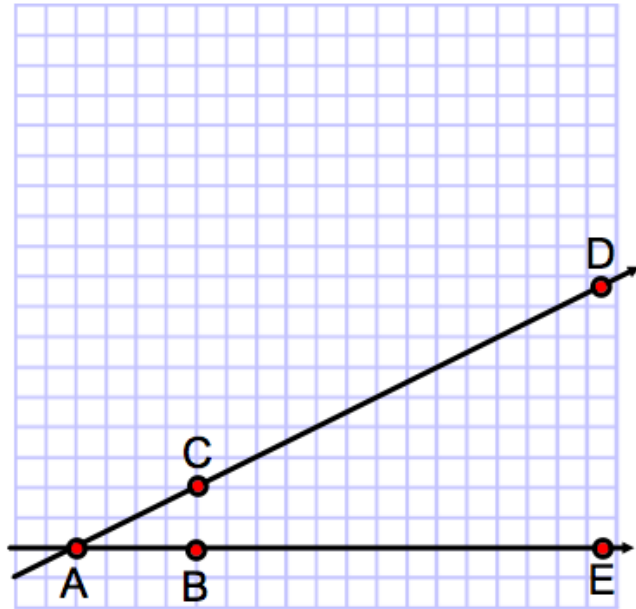
Continue your investigation of dilations and the relationships between the pre-image and image below.

1. Using the figure below:
 - a. Connect point B to C
 - b. Double the length of \overline{AB} on the line \overline{AE} . Label the new segment $\overline{AB'}$
 - c. Double the length of \overline{AC} on the line \overline{AD} . Label the new segment $\overline{AC'}$.
 - d. Connect B' to C'.
2. What do you notice about $\overline{B'C'}$ in relationship to \overline{BC} ?



3. What kind of relationship exists between the pre-image and image?

4. Using the figure below:
 - a. Connect point B to C.
 - b. HALF the length of \overline{AB} on the line \overline{AE} . Label the new segment $\overline{AB'}$.
 - c. HALF the length of \overline{AC} on the line \overline{AD} . Label the new segment $\overline{AC'}$.
 - d. Connect B' to C'.



5. What do you notice about $\overline{B'C'}$ in relationship to \overline{BC} ?

6. What kind of relationship exists between the pre-image and image?

7. Using the figure below:

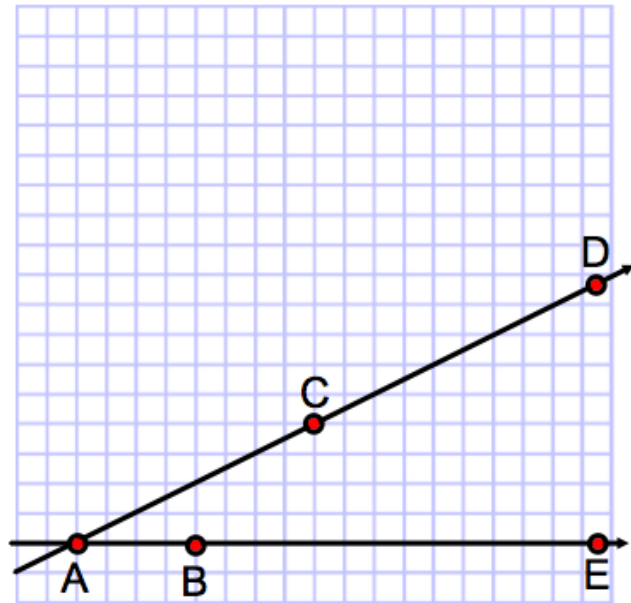
a. Connect point B to C.

b. HALF the length of \overline{AB} on the line \overline{AE} . Label the new segment $\overline{AB'}$.

c. HALF the length of \overline{AC} on the line \overline{AD} . Label the new segment $\overline{AC'}$.

d. Connect B' to C'.

8. What do you notice about $\overline{B'C'}$ in relationship to \overline{BC} ?



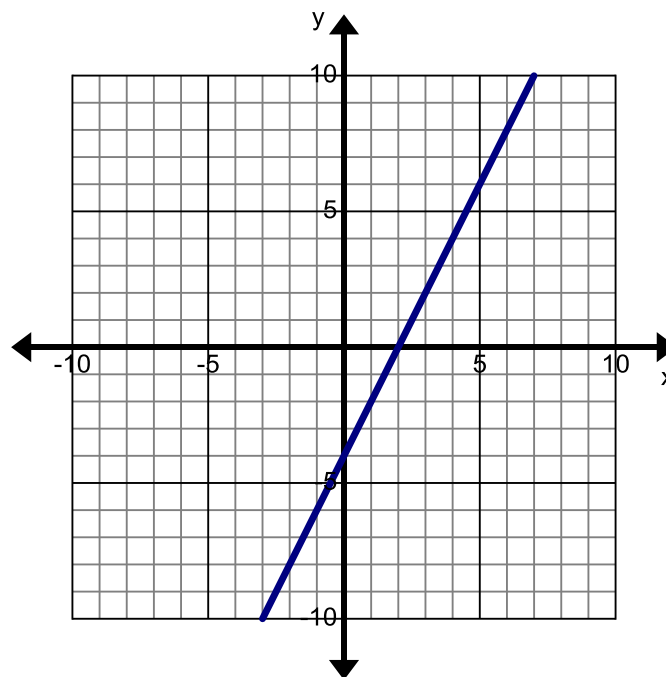
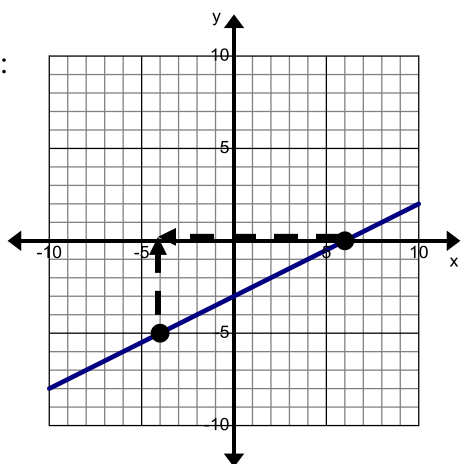
9. What kind of relationship exists between the pre-image and image?

2.3c Class Activity: Proportional Triangles and Slope

1. On the line to the right choose any two points that lie on the line. (To make your examination easier choose two points that lie on an intersection of the gridlines).

From the two points create a right triangle, the line itself will be the hypotenuse and the legs will extend from the two points and meet at a right angle. An example is shown below.

Example:



2. Compare the points that you choose and your triangle with someone in your class. Discuss the following;
 - Did you both chose the same points?
 - How are your triangles the same?
 - How are your triangles different?
 - What relationship exists between your triangles?

Given any two triangles with hypotenuse on the given line and legs horizontal and vertical, then there is a dilation that takes one on the other. In particular, the lengths of corresponding sides are all multiplied by the factor of the dilation, and so the ratio of the length of the vertical leg to the horizontal leg is the same for both triangles.

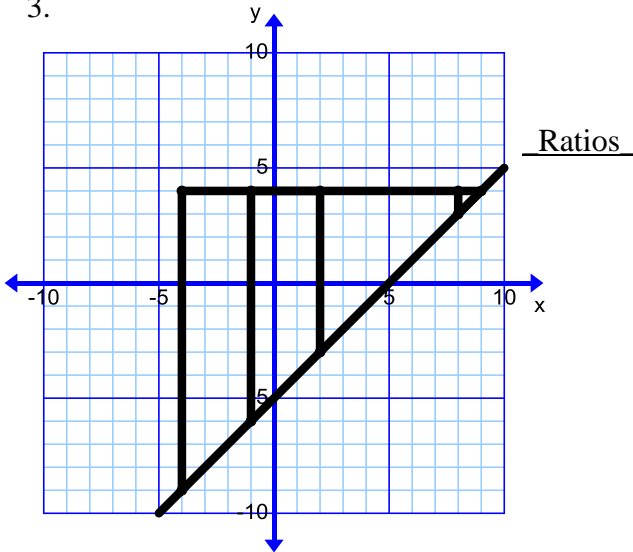
The graphs given below have several different triangles formed from two points that lie on the line. These triangles are dilations of one another and their corresponding parts are proportional. Answer the questions below about these lines to observe what this tells us about slope.

- In each graph below, how many right triangles do you see?
- Trace each triangle you see with a different color.
- For each triangle write a ratio comparing the lengths of its legs or $\frac{\text{height}}{\text{base}}$. Then simplify the ratio

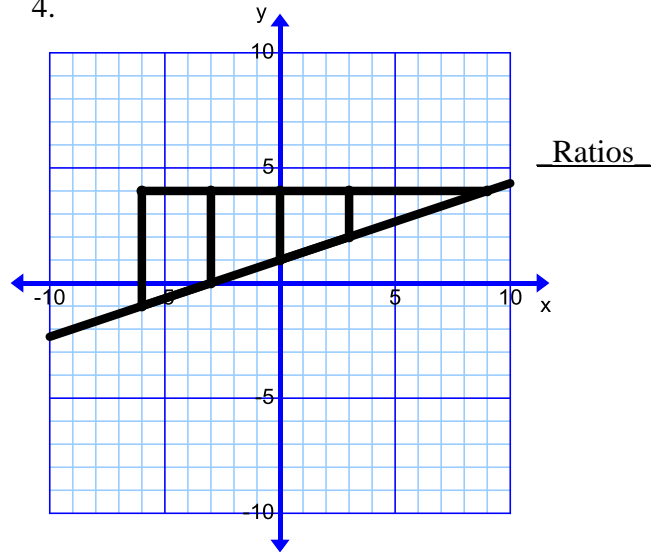
$$\frac{\text{height}}{\text{base}} = \underline{\hspace{2cm}}.$$

In the future we will refer to the ratio as $\frac{\text{rise}}{\text{run}}$, instead of $\frac{\text{height}}{\text{base}}$.

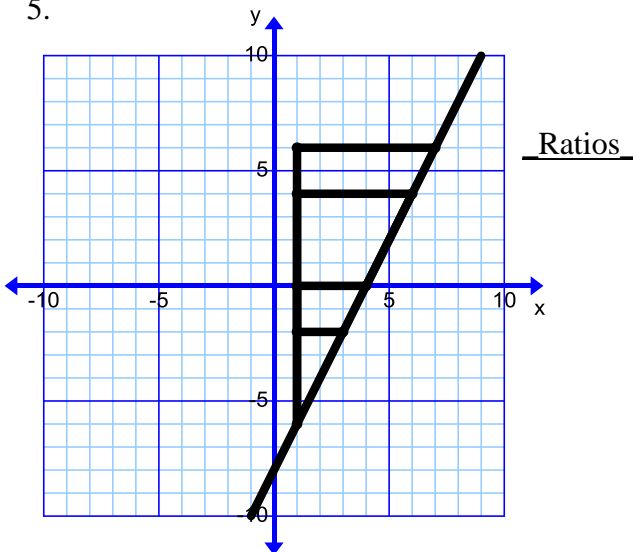
3.



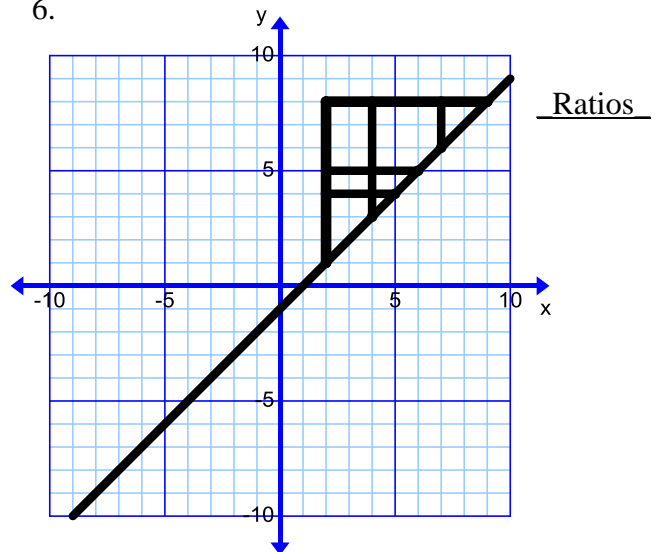
4.



5.

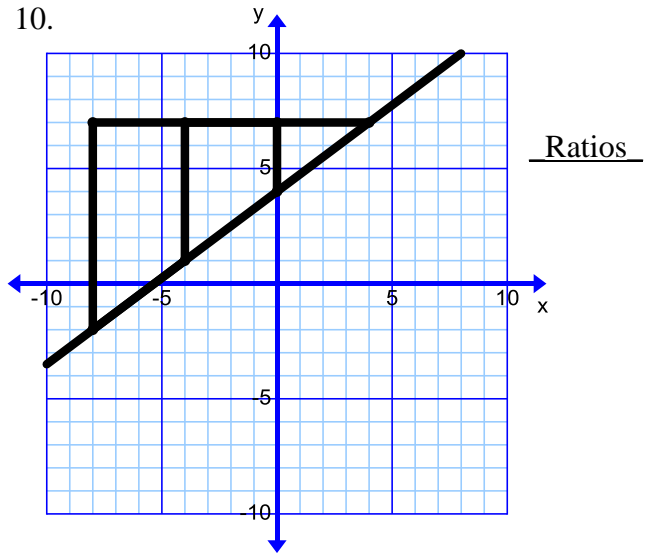
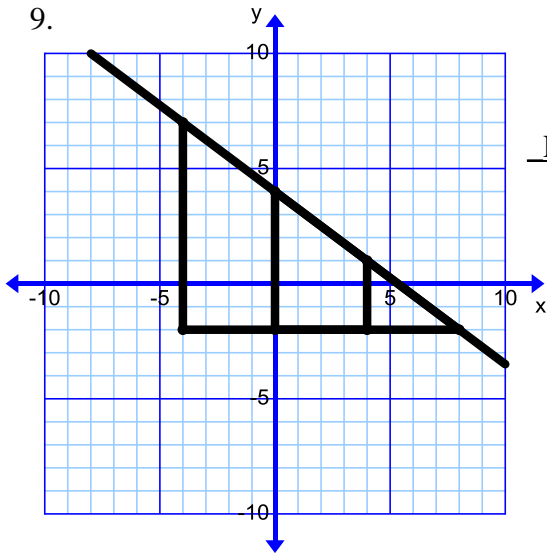


6.



7. Do the ratios (rise to run) always simplify to the same fraction (even quadrants with negative ordered pairs)? Why or why not?

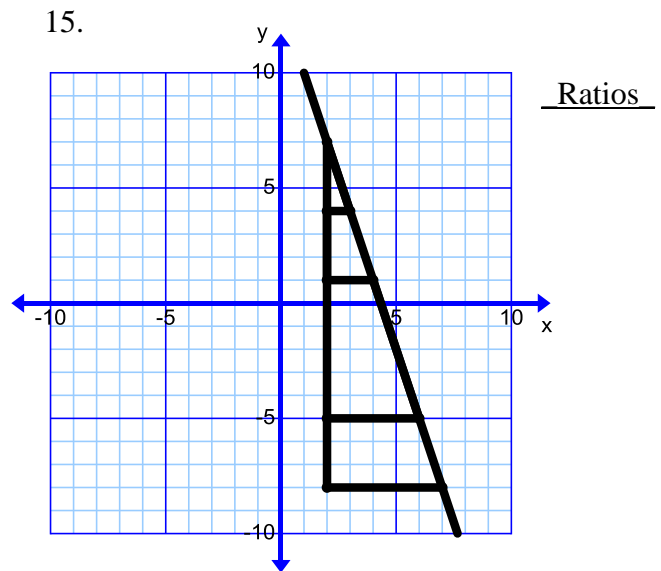
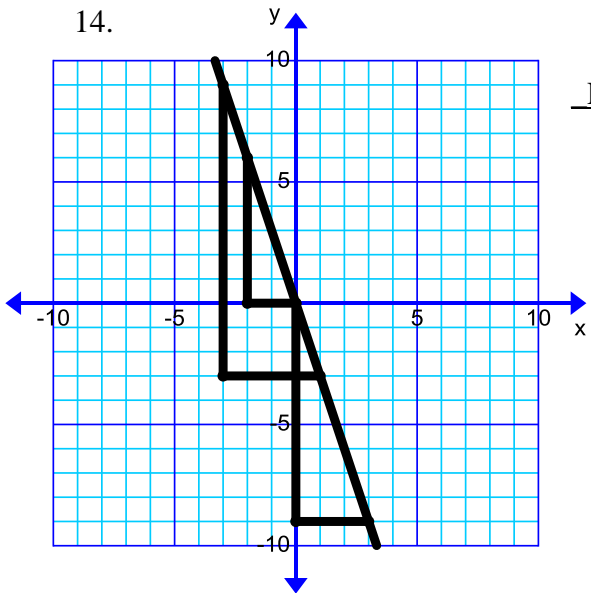
8. How does the “rise over run ratio” describe the steepness of the line?



11. Why do graphs 9 and 10 have the same ratio but they are different lines?

12. How could you differentiate between the slopes of these lines?

13. How does the rise related to the run of a negative slope affect the steepness of the line?



16. Are the ratios for graphs 14 and 15 positive or negative? How do you know?

17. Why are the slopes for graphs 14 and 15 the same if they are different lines?

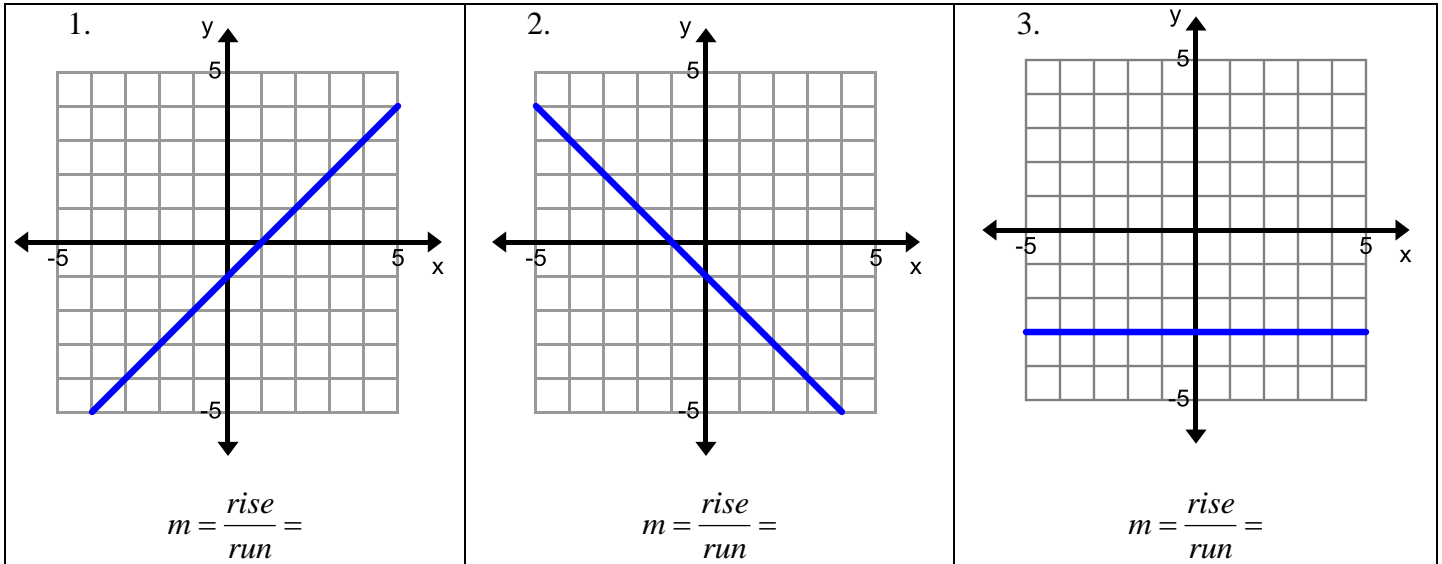
2.3c Homework: Similar Triangles and Slope

For each line graphed below,

- Draw a Right Triangle to calculate the slope of the line. The slope of a line is denoted by the letter m .

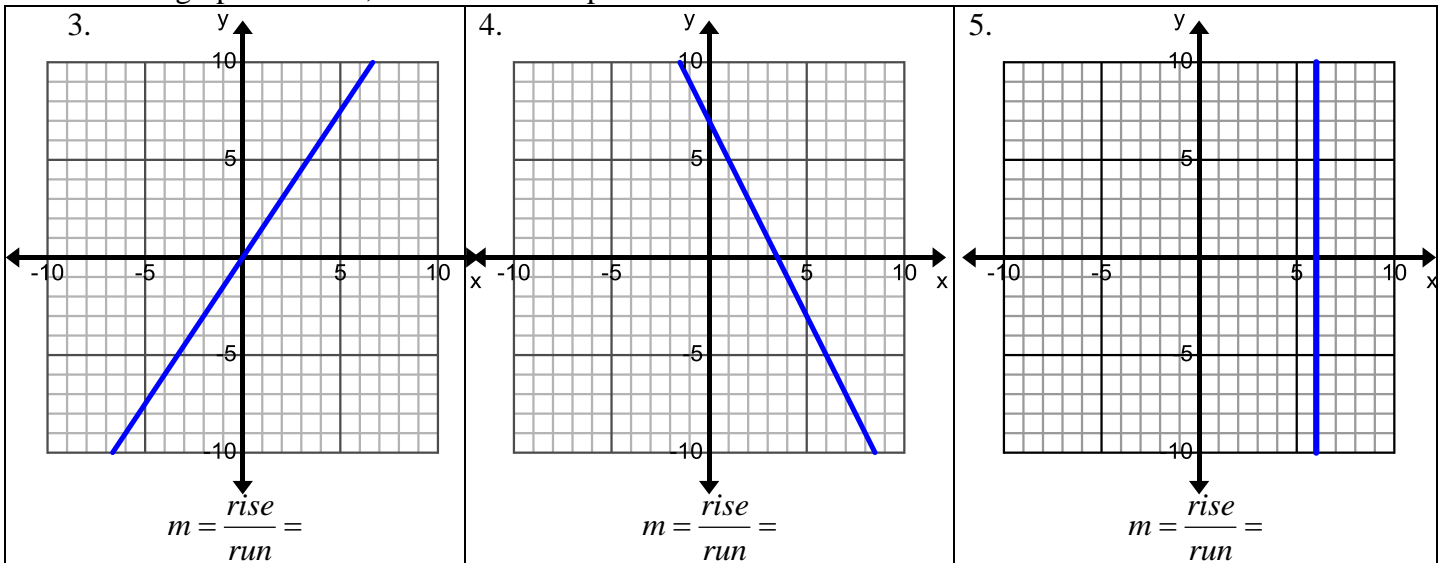
Thus $slope = m = \frac{rise}{run}$

- Label each triangle with a ratio and simplify the ratio $m = \frac{rise}{run} = \underline{\hspace{2cm}}$



2. What does the sign of the slope tell us about the line?

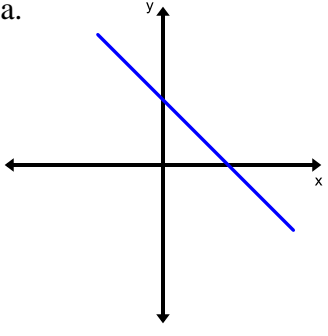
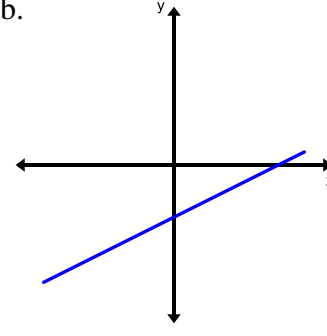
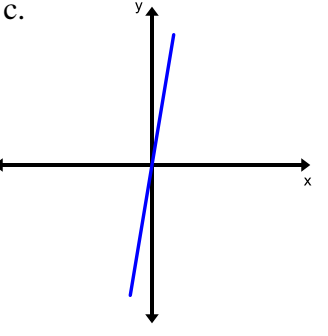
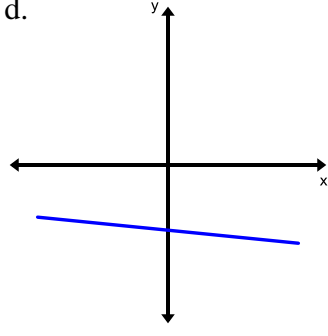
For each line graphed below, calculate the slope of the line.



6. Briefly, explain how to calculate slope when looking at a graph.

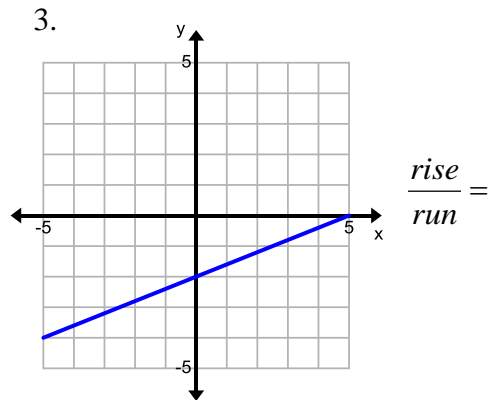
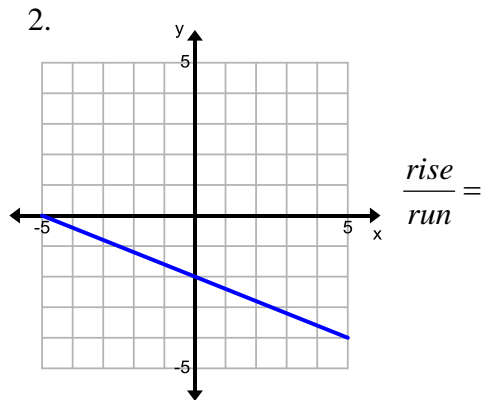
2.3d Class Activity: Finding Slope from Graphs

1. Do the graphs below have positive or negative slopes? How do you know?

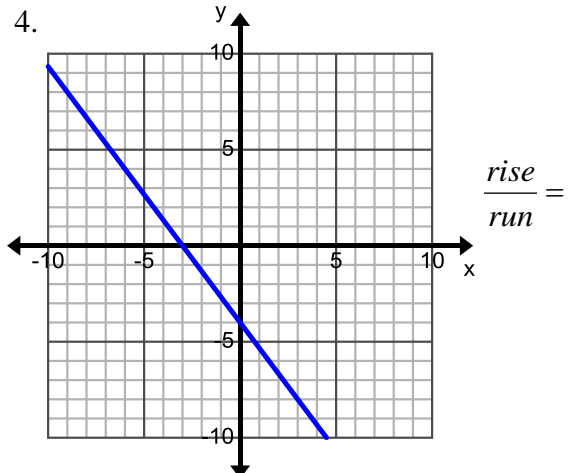
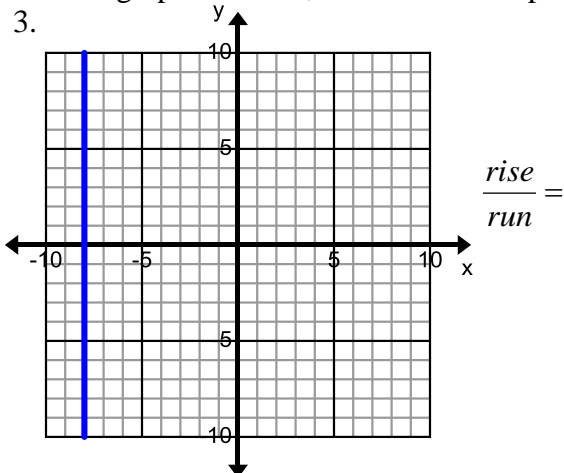
<p>a.</p> 	<p>b.</p> 	<p>c.</p> 	<p>d.</p> 
<p>Positive or negative?</p>			
<p>e. Explain how you know whether a line of a graph has a positive or negative slope.</p>			

For each line graphed below,

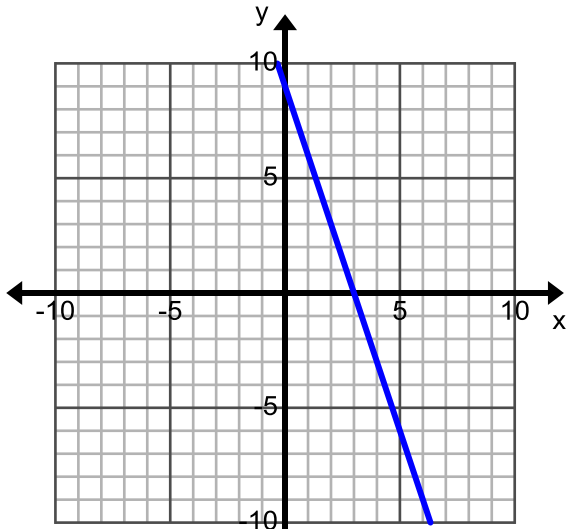
- Draw a right triangle to calculate the slope of the line.
- Label each triangle with a ratio and simplify the ratio $m = \frac{\text{rise}}{\text{run}} = \text{---}$



For each line graphed below, calculate the slope of the line.

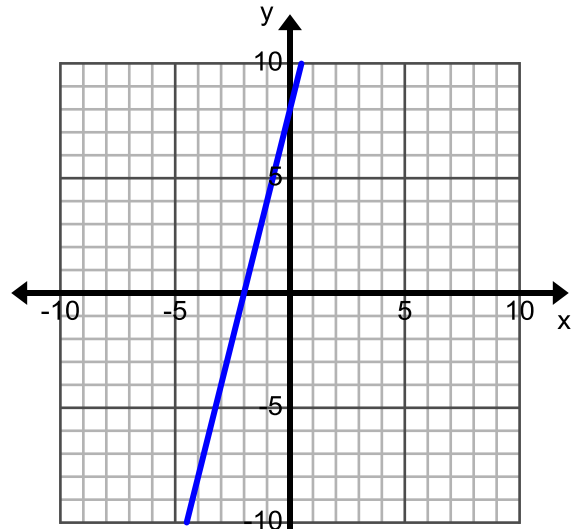


5.



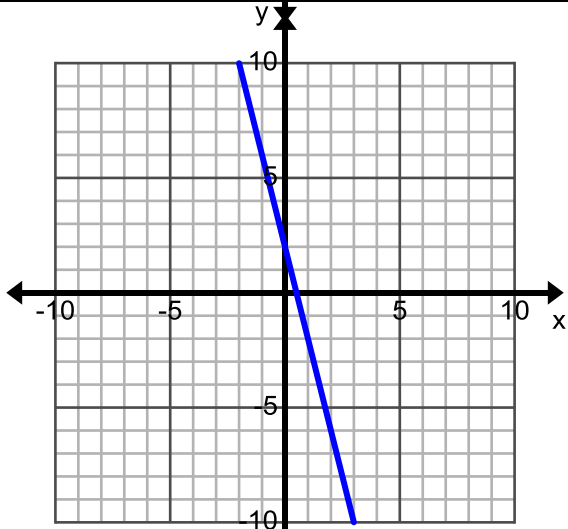
$m =$

6.



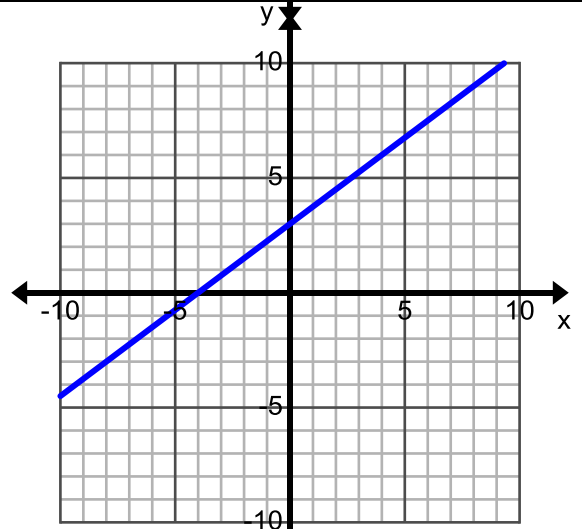
$m =$

7.



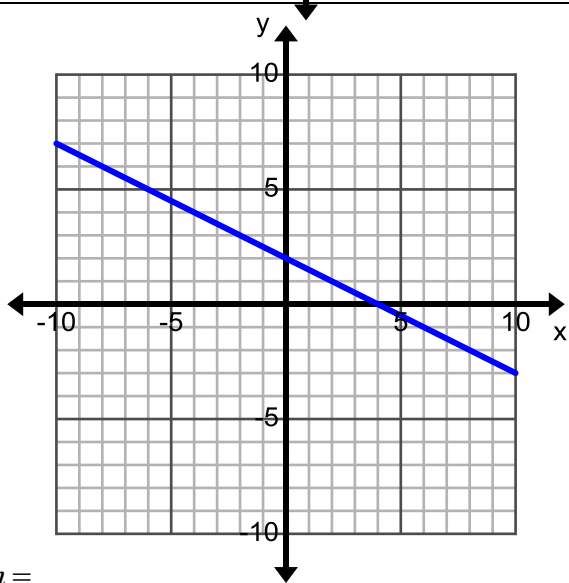
$m =$

8.



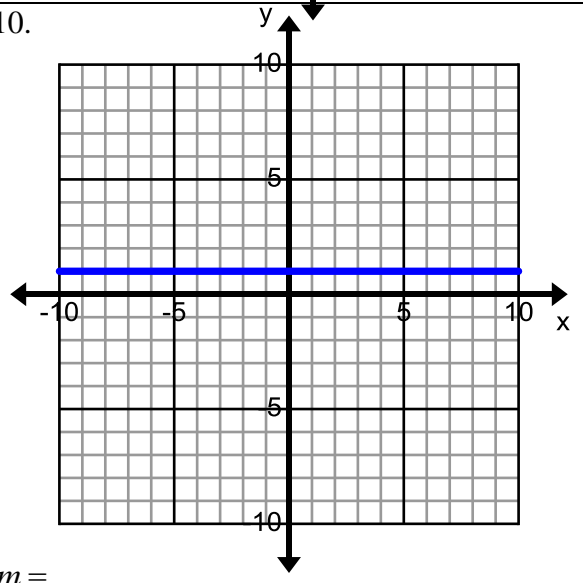
$m =$

9.



$m =$

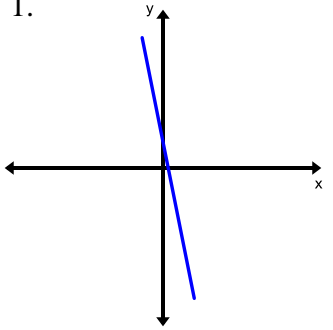
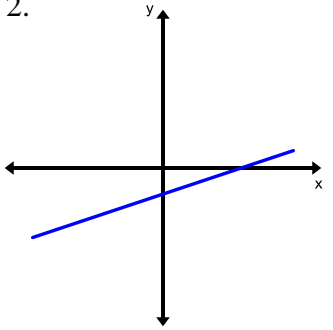
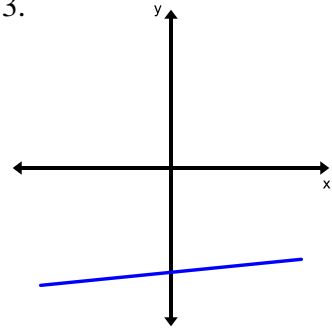
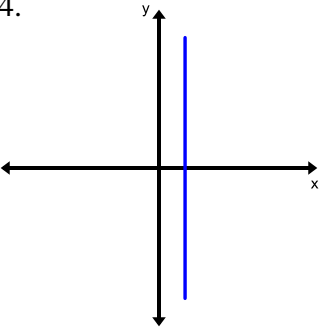
10.



$m =$

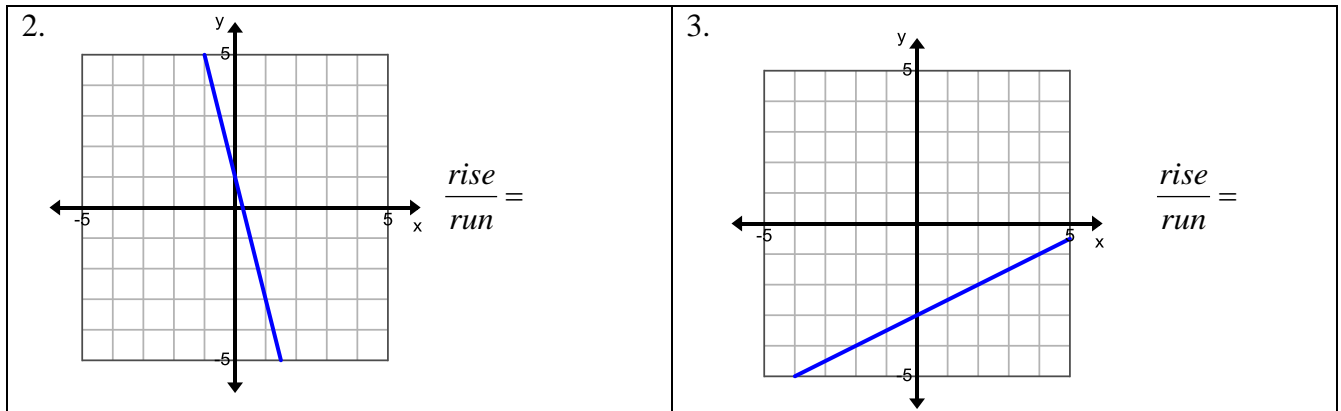
2.3d Homework: Finding Slope from Graphs

1. Do the graphs below have positive or negative slopes? How do you know?

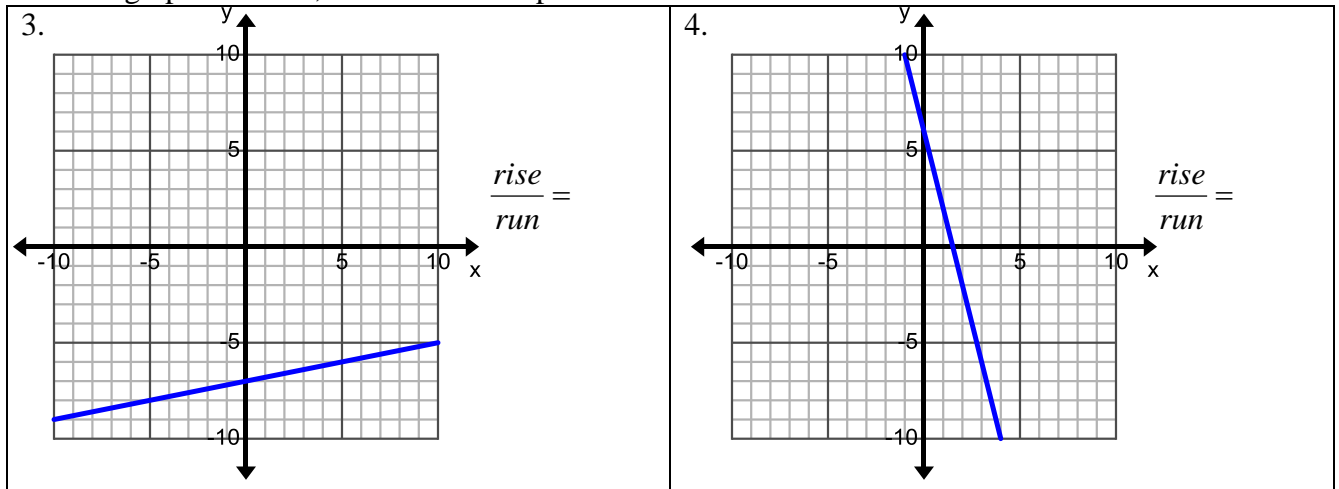
1. 	2. 	3. 	4. 
Positive or negative?			
5. Explain how you know whether a line of a graph has a positive or negative slope.			

For each line graphed below,

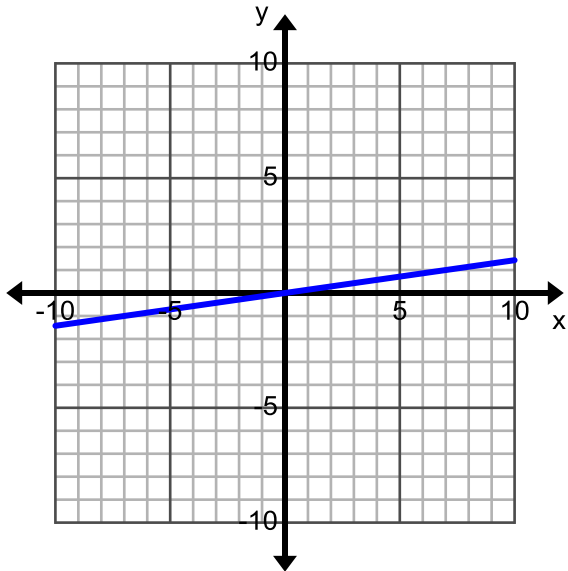
- Draw a right triangle to calculate the slope of the line.
- Label each triangle with a ratio and simplify the ratio $m = \frac{\text{rise}}{\text{run}} = \text{_____}$



For each line graphed below, calculate the slope of the line.

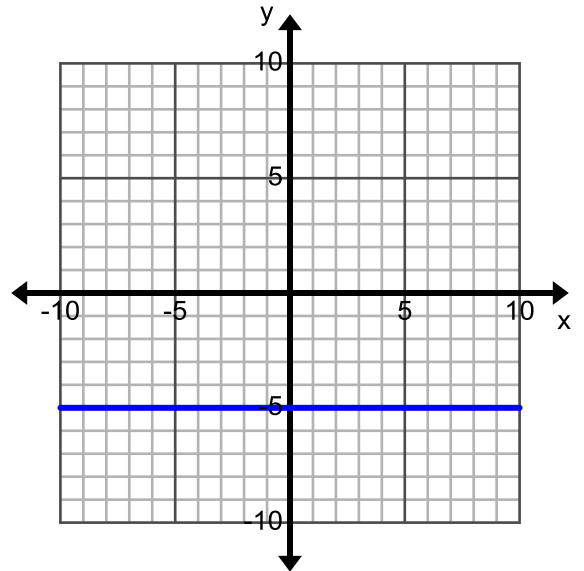


5.



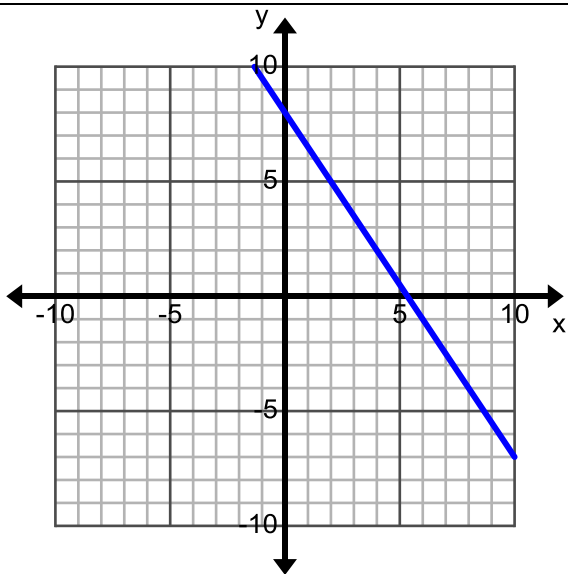
$m =$

6.



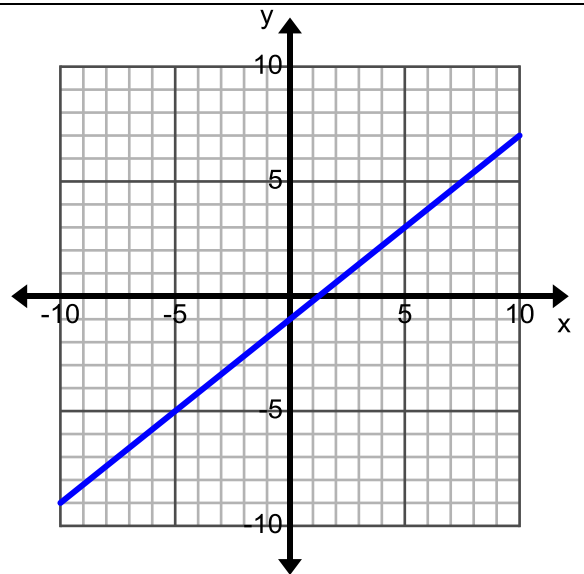
$m =$

7.



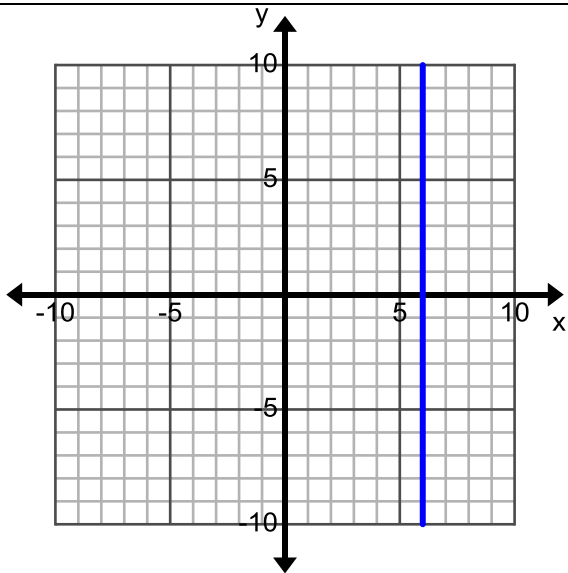
$m =$

8.



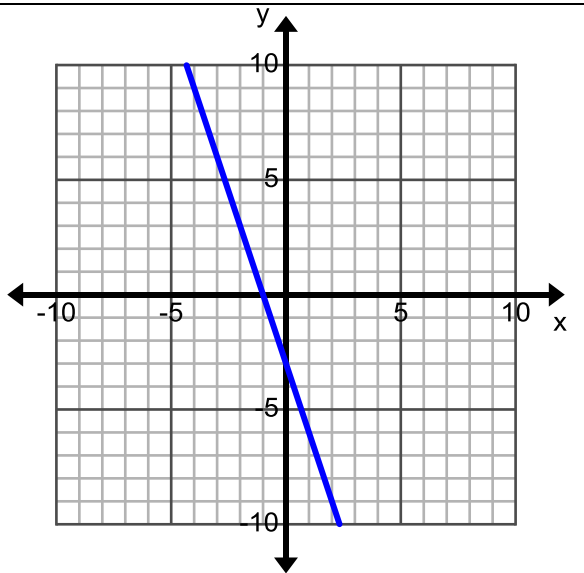
$m =$

9.



$m =$

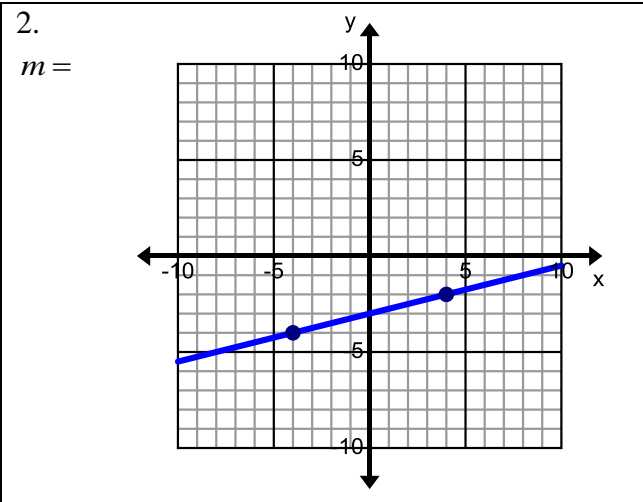
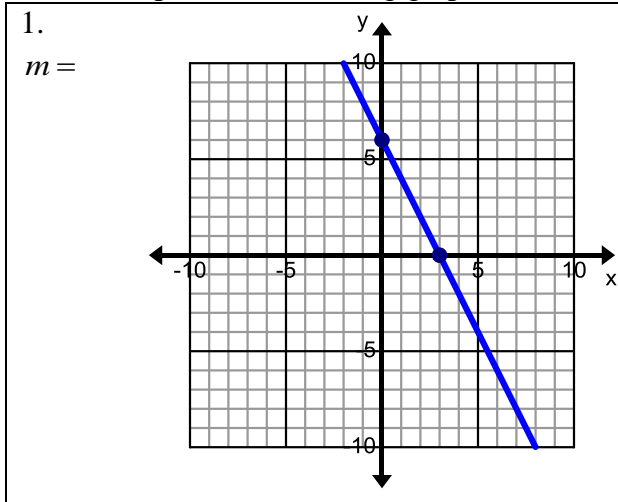
10.



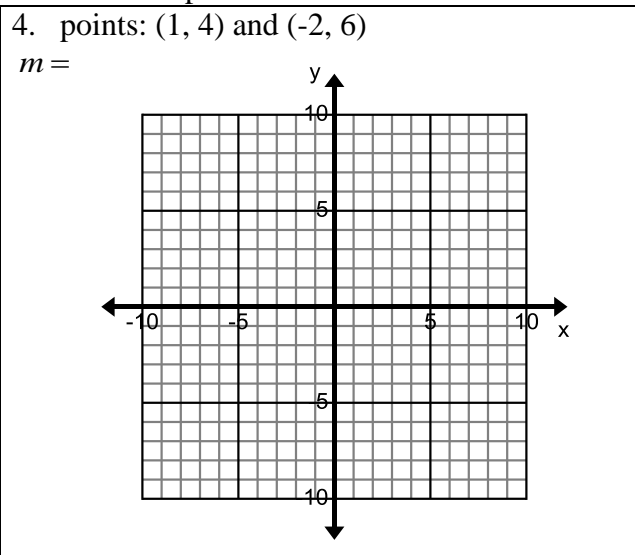
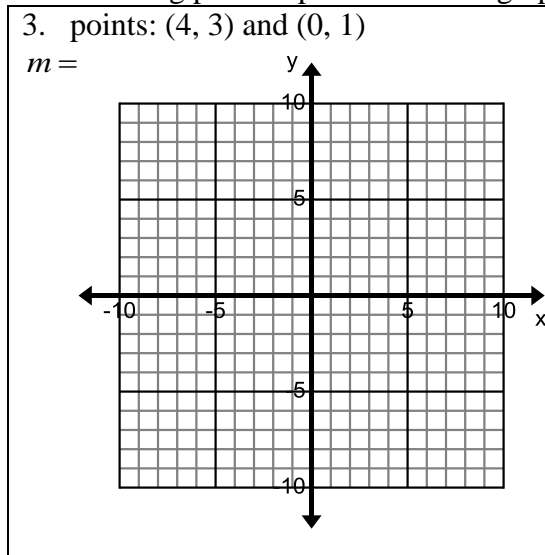
$m =$

2.3e Class Activity: Finding Slope from Two Points

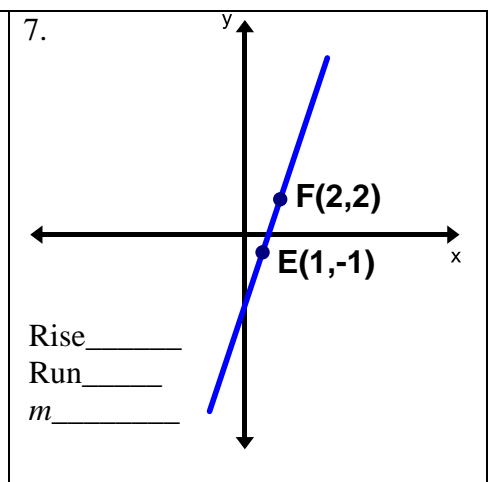
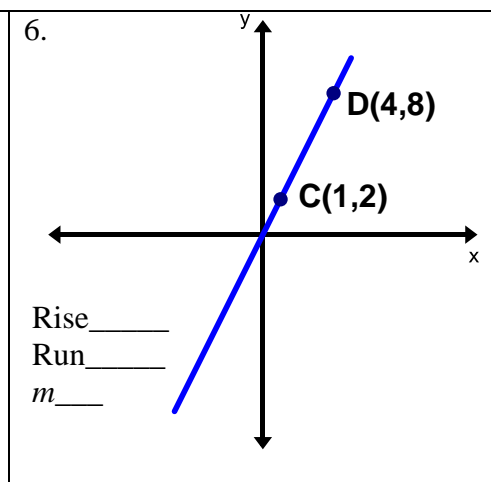
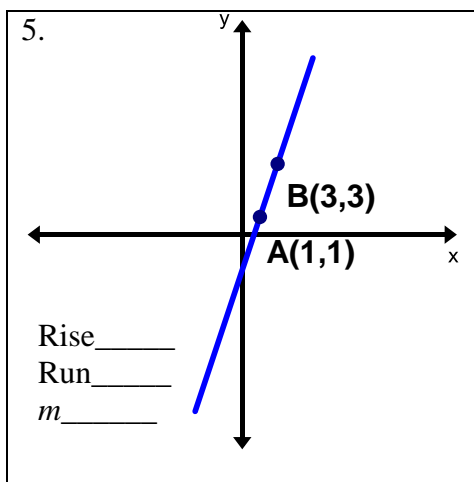
Calculate the slope of the following graphs:

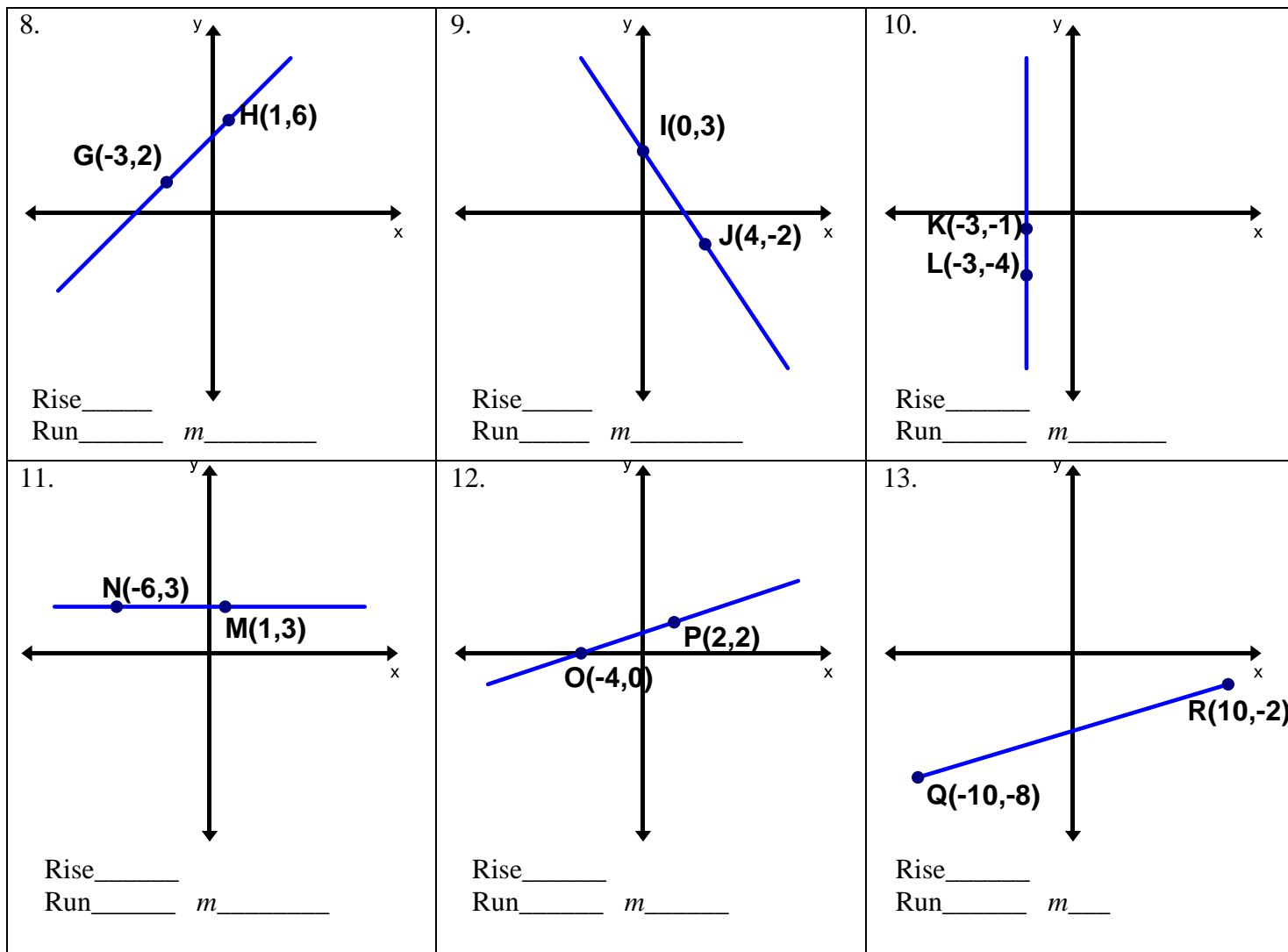


Graph the following pairs of points. Use the graph to determine the slope.



Find the rise and run and slope of each line shown below. You will have to think of a way to use the coordinate points to find the rise and run.





14. Graphing points can be time-consuming. Develop a procedure for calculating the slope without graphing each point. Explain your procedure below. Show that it works for problems 1-4 above.

Discuss and compare your method for calculating slope without using right triangles on a graph with someone else.

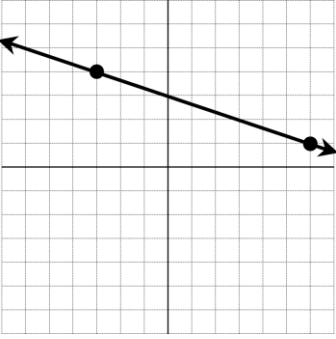
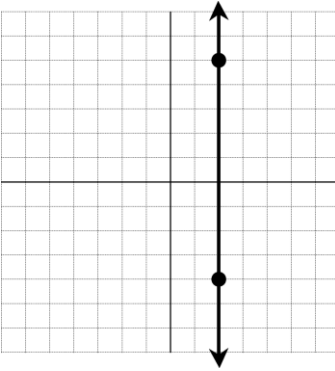
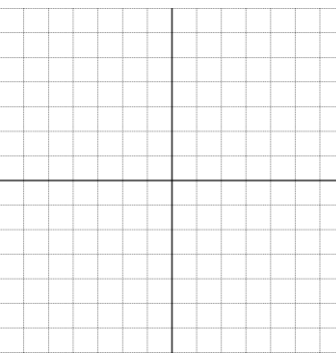
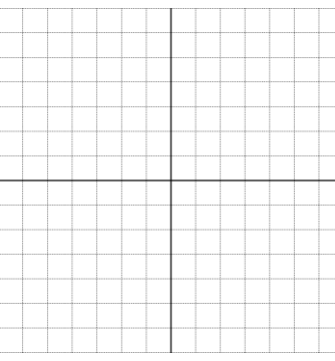
15. Now discuss this formula: $slope = m = \frac{y_2 - y_1}{x_2 - x_1}$ What does it mean? How does it work?

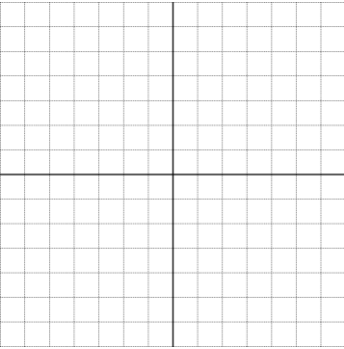
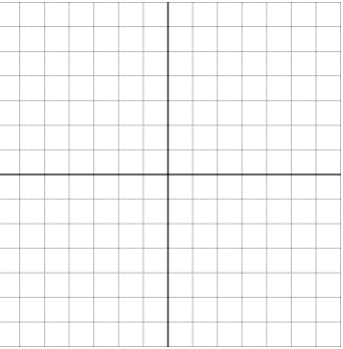
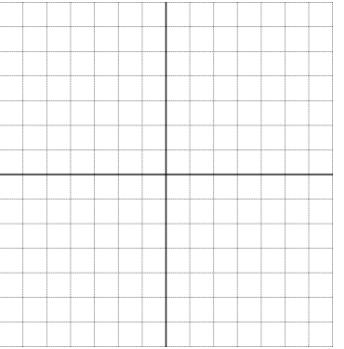
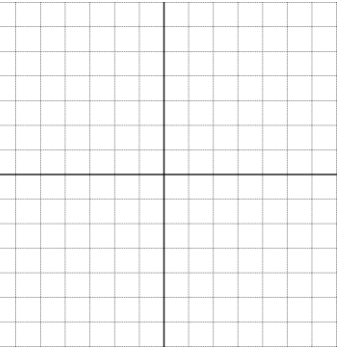
Fill in the missing information in the problems below. Use the empty box to calculate slope using the formula,

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}. \text{ The first one has been done for you.}$$

17.	18.	19.	20.
$(-4, -4) (1, 6)$	$(-3, -6) (6, 0)$	$(4, -3) (-5, 9)$	
$m = \frac{6 - -4}{1 - -4}$			
$\frac{\Delta y}{\Delta x} = \frac{10}{5} = 2$	$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} =$

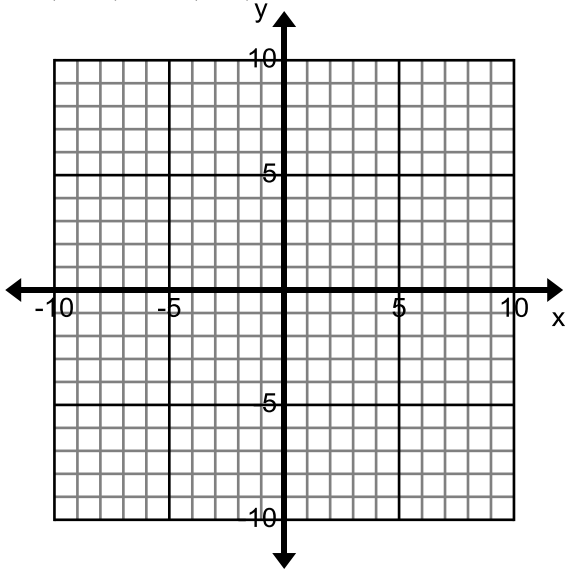
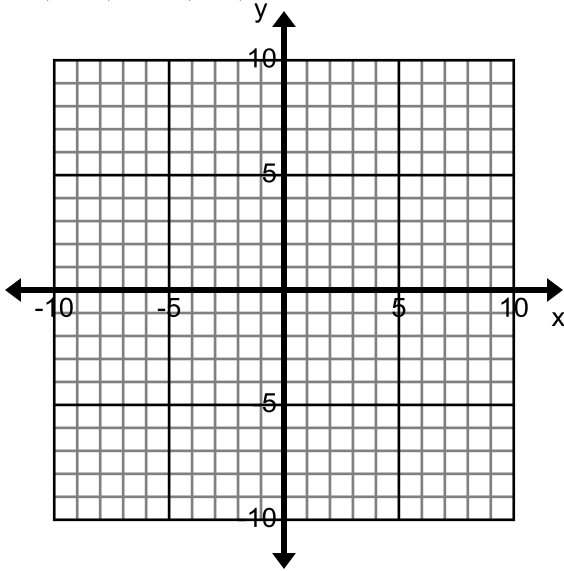
21.	22.	23.	24.
	$(-5, -7) (-5, -3)$		$(-1, -7)$
$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} = 5$

25.	26.	27.	28.
			
		$(-2, -7) (-1, -3)$	$(-7, 6) (6, -7)$
$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} =$

29.	30.	31.	32.
			
$(-6, 1)$	$(-1, 6) (-4, 6)$		$(0, -2)$
$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} = \frac{3}{2}$	$\frac{\Delta y}{\Delta x} = \frac{-3}{4}$

2.3e Homework: Finding Slope from Two Points

Graph the following pairs of points. Use the graph to determine the slope.

<p>1. $(-4, 3)$ and $(2, 6)$</p>  <p>$m =$</p>	<p>2. $(-1, 4)$ and $(0, 1)$</p>  <p>$m =$</p>
--	---

Calculate the slope of the line connecting each pair of points.

3. $(1, 42)$ and $(4, 40)$

4. $(-21, -2)$ and $(-20, -5)$

5. $(3, -10)$ and $(-6, -10)$

6. $(10, -11)$ and $(11, -12)$

7. $(5, 1)$ and $(-7, 13)$

8. $(14, -3)$ and $(14, -7)$

9. $(8, 41)$ and $(15, 27)$

10. $(17, 31)$ and $(-1, -5)$

11. $(-5, 36)$ and $(-4, 3)$

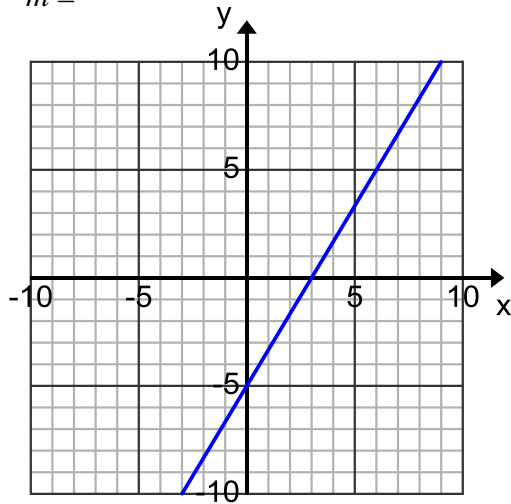
12. $(32, -23)$ and $(-6, -2)$

2.3f Class Activity: Practice Finding the Slope of a Line

Calculate the slope of the line on each graph.

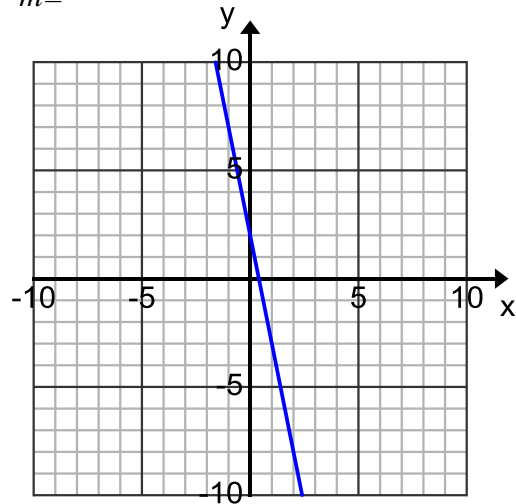
a.

$m =$



b.

$m =$



For each pair of points,

- Calculate the slope of the line passing through each pair.
- Find one other point that lies on the line containing the given points.

3. $(10, -6)$ and $(-5, 4)$

4. $(7, 3)$ and $(-3, 0)$

5. $(0, 4)$ and $(1, 0)$

6. $(-5, 1)$ and $(-5, -2)$

Calculate the slope of the line that contains the points given in each table. Calculate the slope twice, one time by using the Slope Formula with two points and the other time by finding the rate of change or unit rate in the table.

7.

x	y
3	4
4	5
5	6
6	7

$m =$ _____

$m =$ _____

8.

x	y
0	4
1	9
2	14
3	19

$m =$ _____

$m =$ _____

9.

x	y
0	9
3	12
6	15
9	18

$m =$ _____

$m =$ _____

10.

x	y
2	4
4	12
6	20
8	28

$m =$ _____

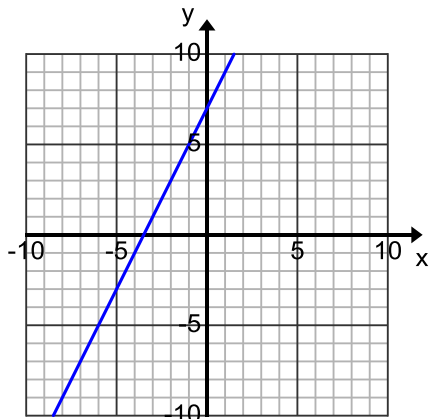
$m =$ _____

11. Why are the slopes the same no matter what two points you use to find the slope?

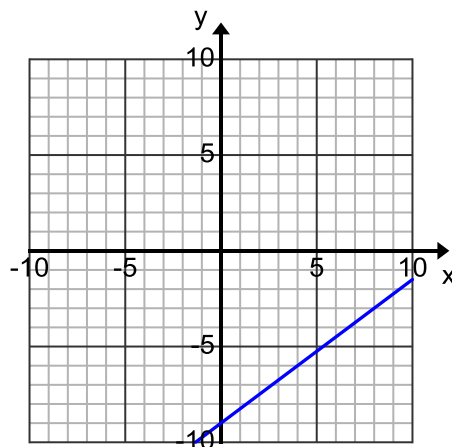
2.3f Homework: Practice Finding the Slope of a Line

Calculate the slope of the line on each graph.

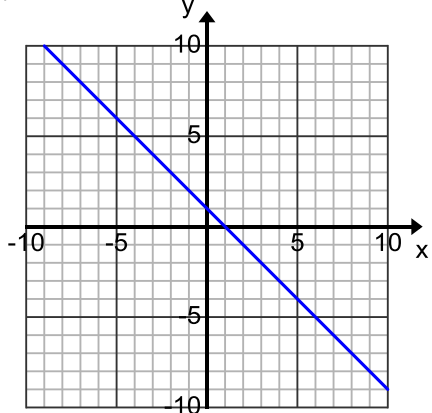
1. $m =$



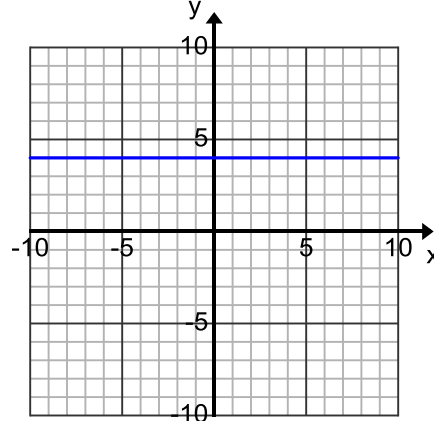
2. $m =$



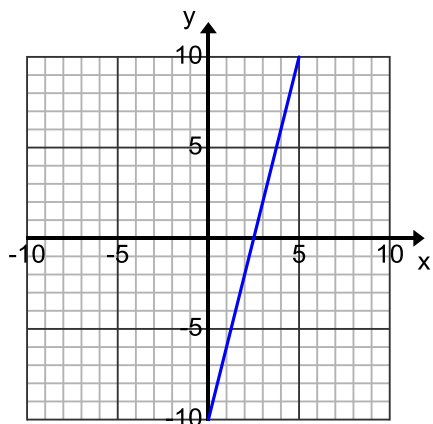
3. $m =$



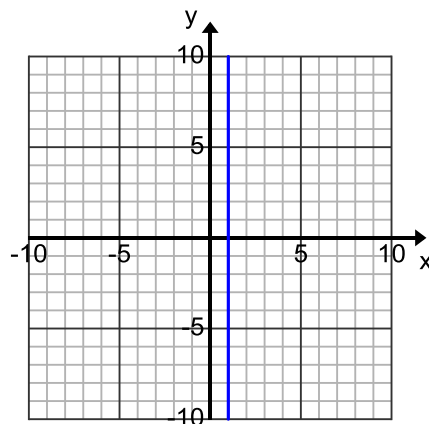
4. $m =$



5. $m =$



6. $m =$



Calculate the slope of the line passing through each pair of points.

7. (3, 9) and (4, 12)

8. (5, 15) and (6, 5)

9. (6, 9) and (18, 7)

10. (-8, -8) and (-1, -3)

For numbers 11 and 12;

- Calculate the slope of the line passing through each pair
- Find one other point that lies on the line containing the given points

11. (-6,-5) and (4,0)

12. (4,1) and (0,7)

Calculate the slope of the line that contains the points given in each table.

13.

x	y
3	-9
5	-15
9	-27

14.

x	y
8	1
6	3
2	7
-4	13

15.

x	y
0	4
1	-5
2	-14
3	-23

16.

x	y
10	1
8	1
-12	1
-14	1

17.

x	y
-5	2
5	6
10	8

18.

x	y
-3	5
-3	10
-3	15
-3	20

19. Why doesn't it matter which two points you use to find the slope?

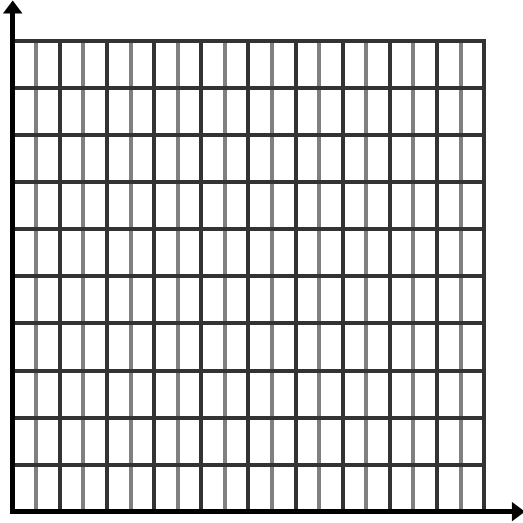
2.3g Class Activity: Finding Slope from a Context

1. Gourmet jellybeans cost \$9 for 2 pounds.

a. Complete the table.

Pounds	.5		3	4		8	10		20
Total Cost		\$9			\$27				

b. Graph and label the relationship.



c. What is the slope of the line?

d. Write the slope of the line as a rate of change that describes what the slope means in this context.

e. Write an equation to find the cost for any amount of jellybeans.

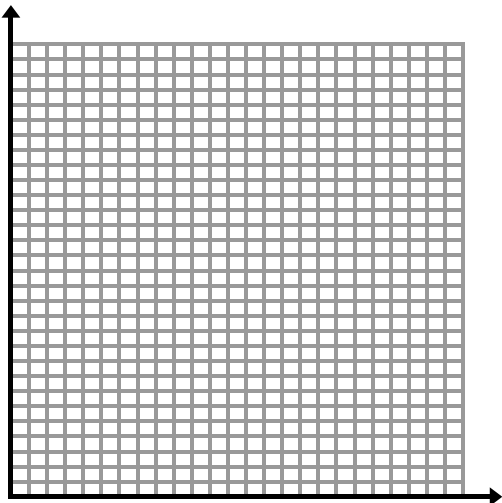
f. Why is the data graphed only in the first quadrant?

2. Kaelynn takes the same amount of time to solve each of the equations on her math homework. She can solve 10 equations in 8 minutes.

a. Complete the table.

Minutes	2		8			20	
Equations Solved		5		15			30

b. Graph and label the relationship.



c. What is the slope of the line?

d. Write the slope of the line as a rate of change that describes what the slope means in this context.

e. Write an equation to find the number of equations solved for any number of minutes.

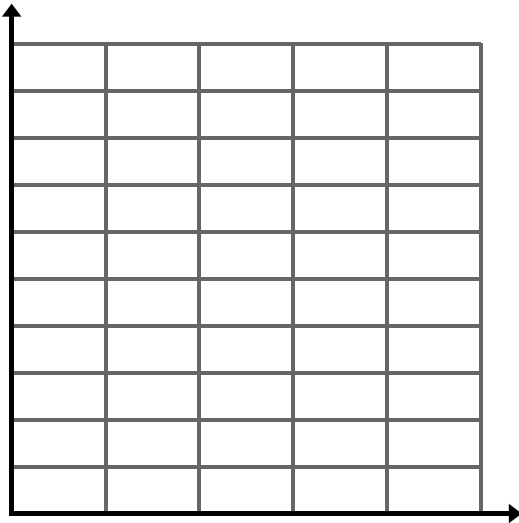
3. Mr. Irving and Mrs. Hendrickson pay babysitters differently.

a. Examine the table. Describe the difference.

Hours	Irving	Hendrickson
0	\$0	\$4
1	\$9	\$12
3	\$27	\$28
5	\$45	\$44

b. Are both relationships proportional?

c. Graph and label the two pay rates (two different lines).



d. What is the slope of each line?

Irving: Hendrickson:

e. Write each slope as a rate of change to interpret its meaning?

Irving:

Hendrickson:

f. Explain the different y -intercepts.

g. Is one babysitting job better than the other? Why or why not?

h. Write an equation for each situation.

- Irving:

- Hendrickson:

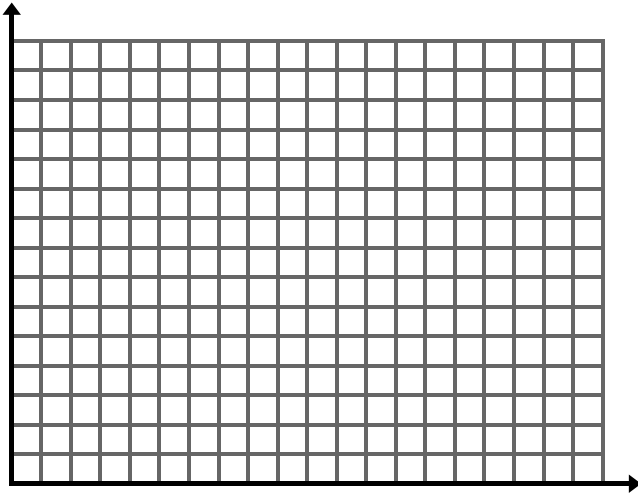
2.3g Homework: Finding Slope from a Context

1. The soccer team is going out for hot dogs. Greg's Grill is having a special on hot dogs: four hot dogs for three dollars. Each hot dot costs the same amount of money.

a. Complete the table.

Hot Dogs	1			16		28	40
Total Cost		\$3	\$9		\$18		

b. Label the graph axes and then graph the relationship.



c. What is the slope of the line?

d. Write a sentence with correct units that describes what the slope means.

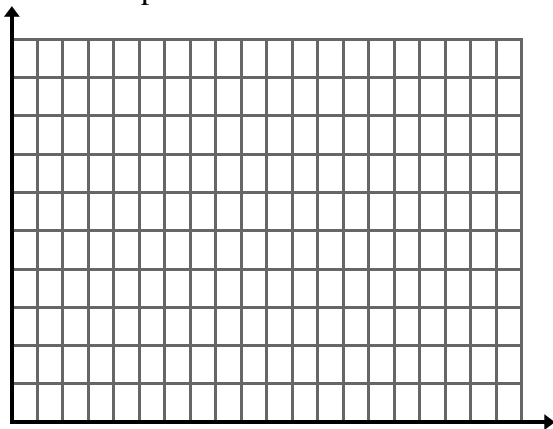
e. Write an equation to find the cost for any amount of hot dogs.

2. The state fair costs \$2 to get in plus \$.50 per ticket to go on rides. Complete the following table, showing the cost for getting into the fair with additional tickets for rides.

a. Complete the table.

Tickets	0	1		10		30	
Total Expense	2		\$4.50		\$12		\$20

b. Label the graph axes and then graph the relationship.



c. What is the slope of the line?

d. Write a sentence that describes what the slope of the line means.

e. Why does the line not pass through 0?

f. Write an equation to find the total expense at the fair with any amount of tickets purchased.

3. Excellent Bakers and Delicious Delights bakeries charge differently for sandwiches for business lunches.

a. Examine the table. Describe the difference.

Sandwich Meals	Excellent Bakers	Delicious Delights
0	\$0	\$15
1	\$20	\$30
3	\$60	\$60
5	\$100	\$90

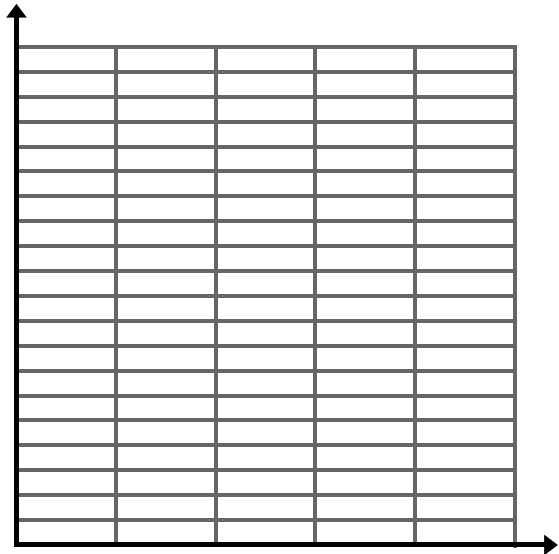
b. Are both relationships proportional?

c. Find the rate of change for each relationship.

Excellent:

Delicious :

d. Graph the two pay rates (two different lines). Label.



e. What is the slope of each line?

Excellent: Delicious:

f. What does the slope tell you about these two situations?

g. Explain the different y intercepts.

h. Is one situation better than the other? Why or why not?

i. Write an equation for each situation.

• Excellent Bakery _____

• Delicious Delights _____

2.3h Class Activity: The Equation of a Linear Relationship

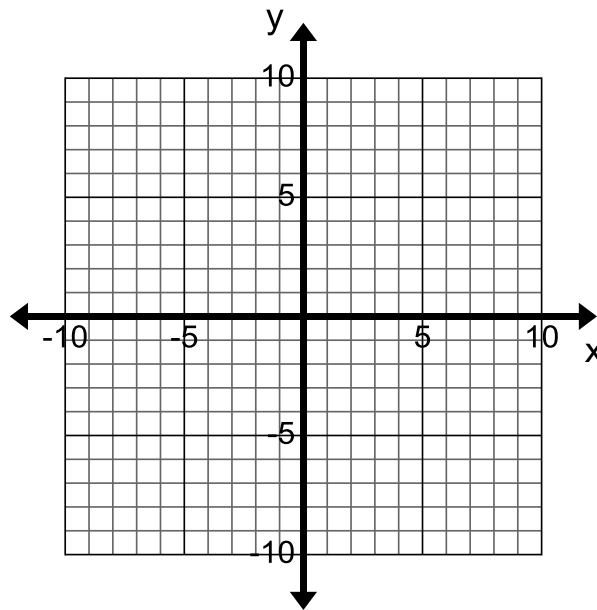
Throughout this chapter the components of a linear relationship have been investigated. A linear relationship is defined by the constant rate of change that it possesses and it can be represented in many ways. In this lesson the focus will be more on the equation that represents a linear equation.

The equation given below represents a linear relationship.

$$y = 2x + 5$$

1. Graph this equation of this line by making a table of values that represent solutions to this equation. (This is often called a T-chart).

x	y
-1	
0	
1	
2	



2. Find the slope of this line?
3. Where do you see the slope in the equation?

Often the letter b is used to denote the y-intercept.

4. What is the y-intercept of this line?

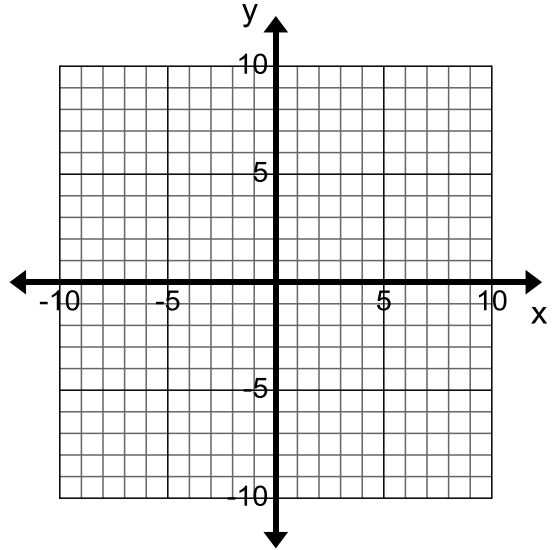
$$b =$$

5. Where do you see the y-intercept in the equation?

For each of the equations given below, make a table of values to help you graph the line. Then identify the slope and y-intercept. Circle the slope in your equation and put a star next to the y-intercept.

6. $y = 5x - 2$

x	y

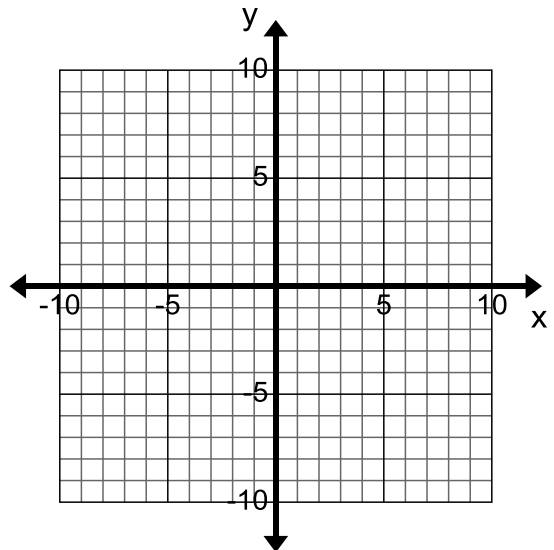


Slope(m):

y-intercept(b):

7. $y = 3x + 1$

x	y

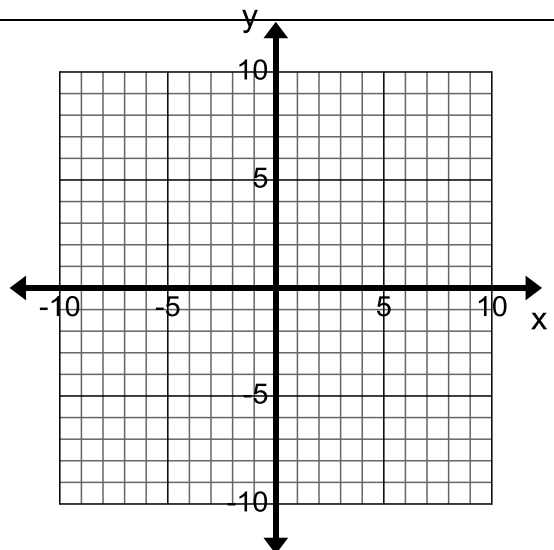


Slope(m):

y-intercept(b):

8. $y = 4x$

x	y

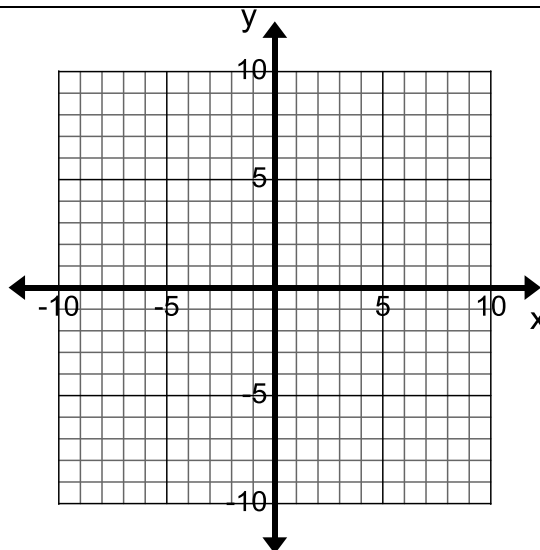


Slope(m):

y-intercept(b):

9. $x + y = 3$

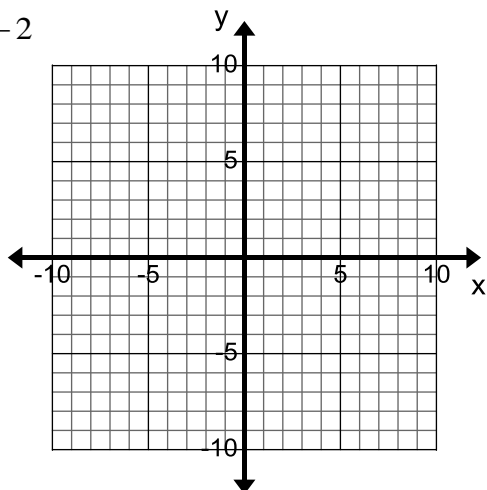
x	y



Slope(m):

y-intercept(b):

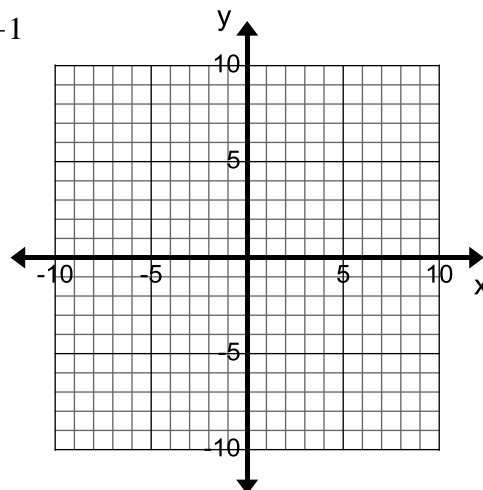
10. $y = \frac{1}{2}x - 2$



Slope(m):

y-intercept(b):

11. $y = -\frac{4}{3}x + 1$



Slope(m):

y-intercept(b):

12. Examine the graphs and equations given above. Describe the general form of a linear equation. In other words, in general, how is a linear equation written? What are its different parts?

13. Write down the general form of a linear equation in the box below based off of your class discussion.

Slope-intercept form of a linear equation is

where m represents the slope (rate of change)
and b represents the y-intercept (initial value or starting point)

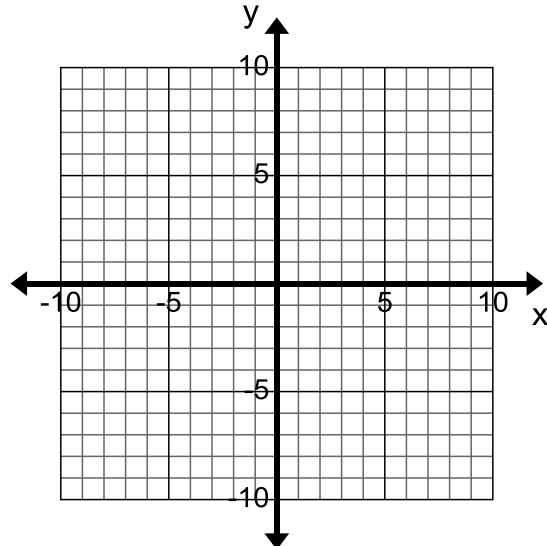
14. What if the y-intercept is zero, how do you write the general form of the equation?

2.3h Homework: The Equation of a Linear Relationship

For each of the equations given below, make a table of values to help you graph the line. Then identify the slope and y-intercept. Circle the slope in your equation and put a star next to the y-intercept.

1. $y = 3x - 3$

x	y

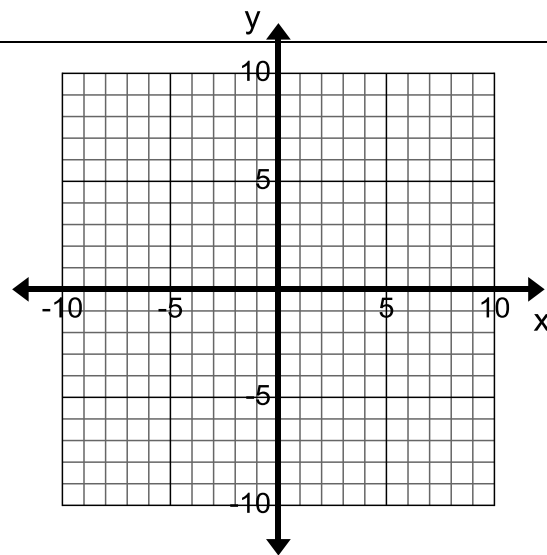


Slope(m):

y-intercept(b):

2. $y = x + 4$

x	y

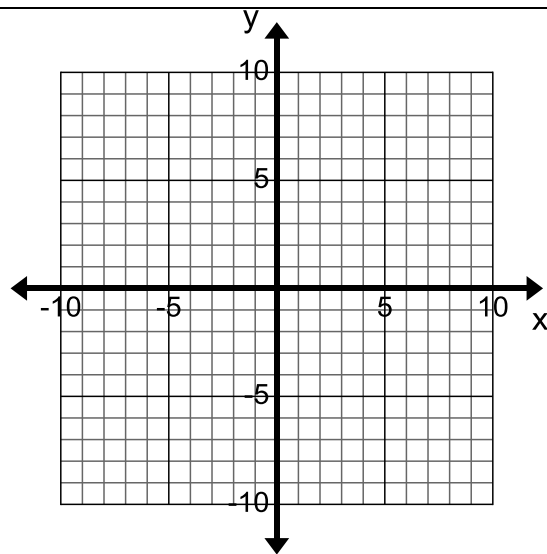


Slope(m):

y-intercept(b):

3. $y = -5x$

x	y

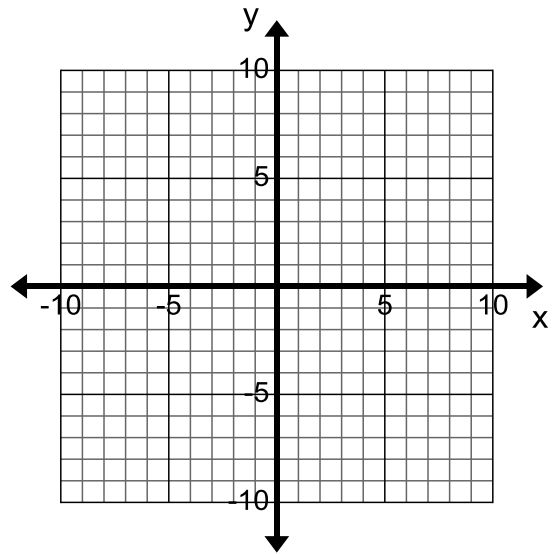


Slope(m):

y-intercept(b):

4. $y = -\frac{1}{4}x$

x	y

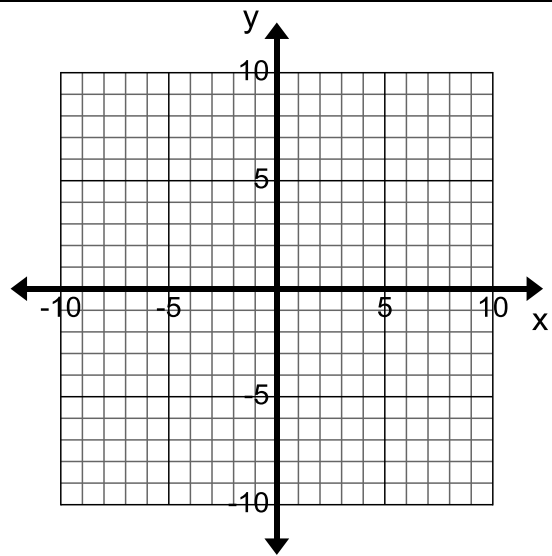


Slope(m):

y-intercept(b):

5. $x + y = 2$

x	y

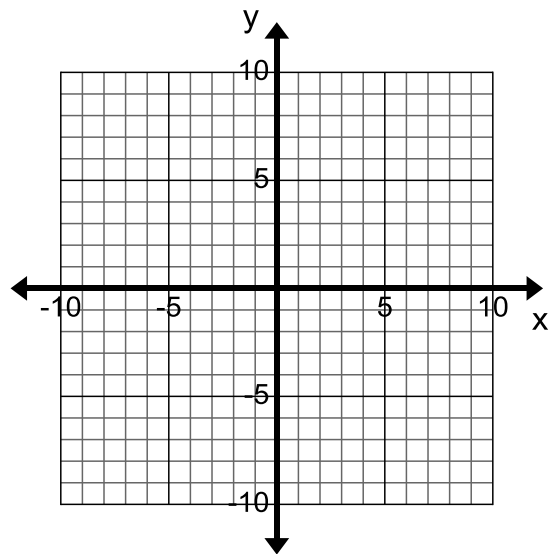


Slope(m):

y-intercept(b):

6. $-x + y = -1$

x	y

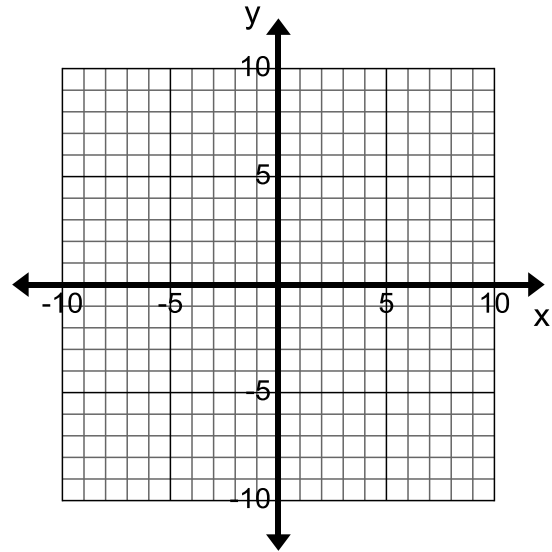


Slope(m):

y-intercept(b):

7. $y = -2x + 6$

x	y

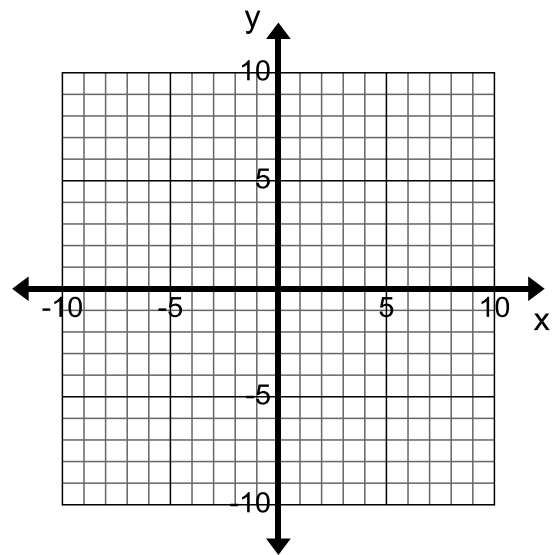


Slope(m):

y-intercept(b):

8. $y = -\frac{4}{3}x - 2$

x	y

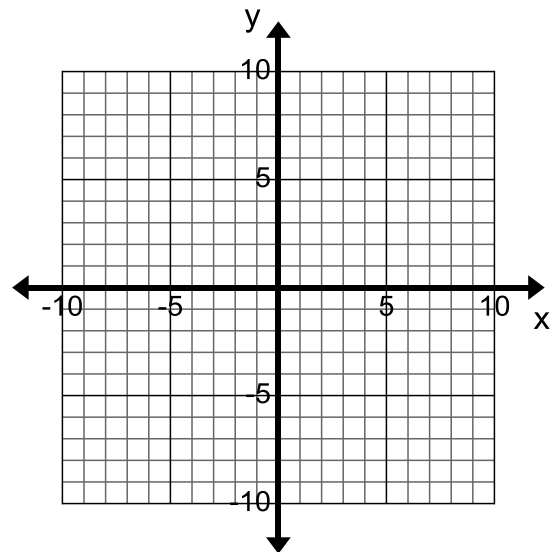


Slope(m):

y-intercept(b):

9. $y = -x$

x	y



Slope(m):

y-intercept(b):

2.3i Class Activity: Use Dilations and Proportionality to Derive the Equation $y = mx$.



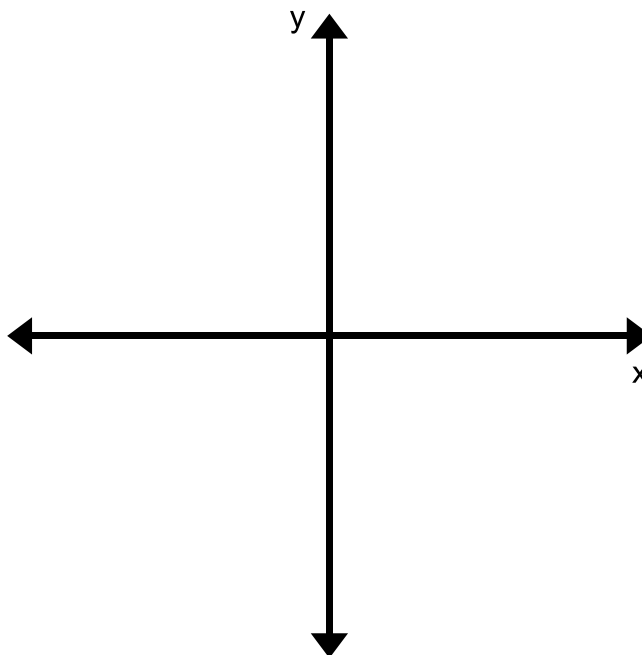
Up to this point you have been investigating how to describe many patterns and stories with a linear relationship. You have begun to get a sense of how linear relationships are formed and described.

Write down what you know about linear relationships below.

Now write down what you know about linear proportional relationships below.

You are going to use the facts listed above to derive the equation $y=mx$ using dilations. Begin by looking at the example below.

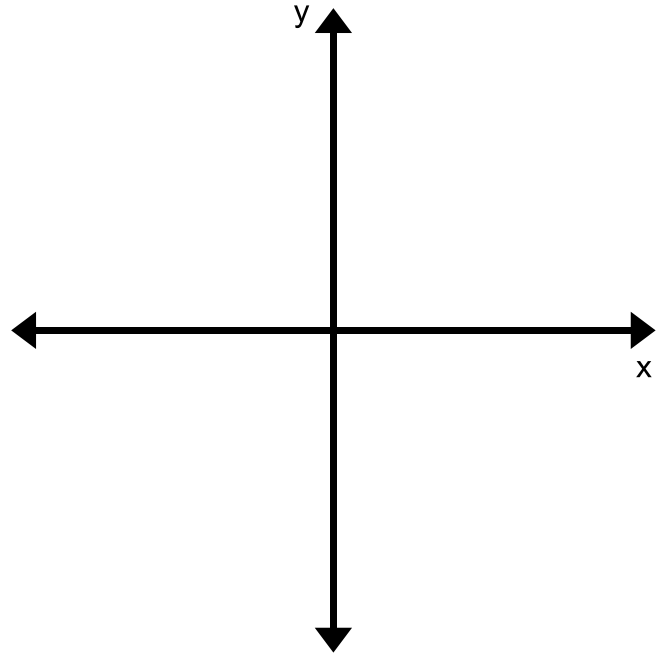
1. Graph a line on the coordinate plane to the right that goes through the origin and has a slope of $\frac{2}{3}$. Label the rise and run on your graph with a right triangle. Redraw and label this triangle in the space provided below the graph.
2. Does this line describe a proportional relationship?
3. Choose any point (x,y) on your line and draw a slope triangle that describes the rise and run. Redraw and label this triangle in the space provided below the graph.



- Write a proportional statement with your ratios.
- Solve the equation that you wrote above for y .

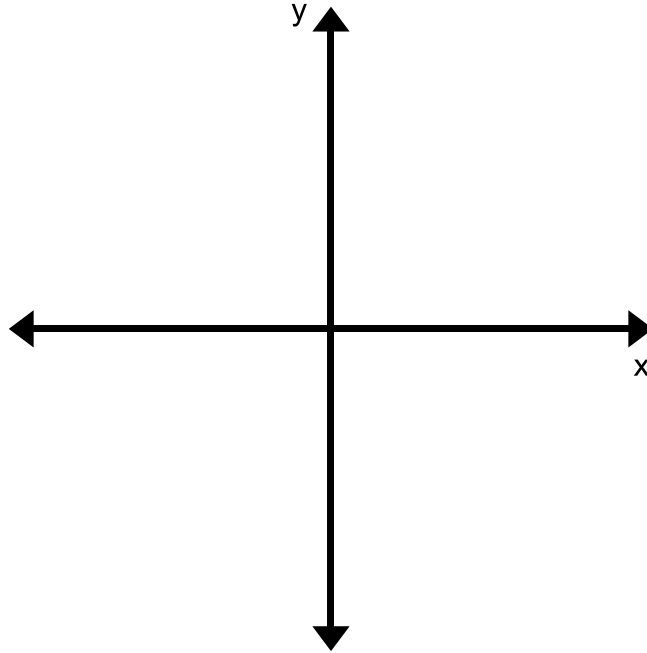
Now look at the general case for the form $y=mx$.

- Graph a line on the coordinate plane to the right that goes through the origin and has a slope of m ; **remember that slope is the same as a unit rate which compares your y -value to an x value of 1**. Label the rise and run on your graph with a right triangle. Redraw and label this triangle in the space provided below the graph.
- Does this line describe a proportional relationship?
- Choose any point (x,y) on your graph and draw a slope triangle that describes that rise and run. Redraw and label this triangle in the space provided below the graph.
- Write a proportional statement with your ratios.
- Solve the equation that you wrote above for y .



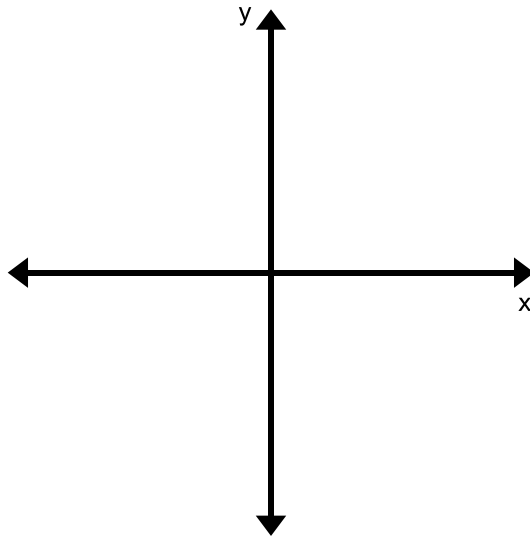
Notice that this is the equation $y=mx$ that describes a proportional relationship.

11. Show that the equation for a line that goes through the origin and has a slope of $\frac{2}{5}$ is $y = \frac{2}{5}x$ using dilations and proportionality.

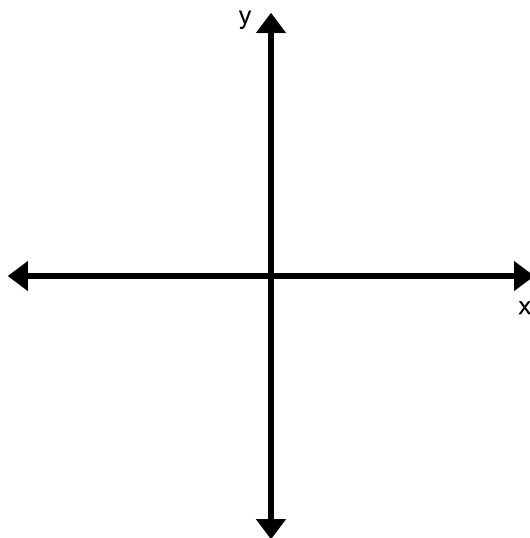


2.3i Homework: Use Dilations and Proportionality to Derive the Equation $y = mx$.

1. Show that the equation for a line that goes through the origin and has a slope of $\frac{1}{3}$ is $y = \frac{1}{3}x$ using dilations and proportionality.



2. Show that the equation for a line that goes through the origin and has a slope of $-\frac{3}{5}$ is $y = -\frac{3}{5}x$ using dilations and proportionality.



2.3j Class Activity: Use Dilations and Proportionality to Derive the Equation $y = mx + b$ n#

What about linear relationships that are not proportional? You are going to further investigate the general form of a linear equation with a transformation. A geometric transformation can relate a linear proportional relationship to a linear non-proportional relationship.

In previous sections we used the geometric transformation called a dilation. Another type of transformation is called a translation. Shifting a line or moving all the points on the line the same distance and direction is a transformation is a **translation**.

For example, if you transform the line $y = \frac{1}{2}x$ upwards by 3 units, every ordered pair that lies on that line gets moved up 3 units. Algebraically that means that you add 3 to every y value since this is a vertical shift.

$$(x, y) \rightarrow (x, y + 3)$$

To confirm this, investigate this transformation below.

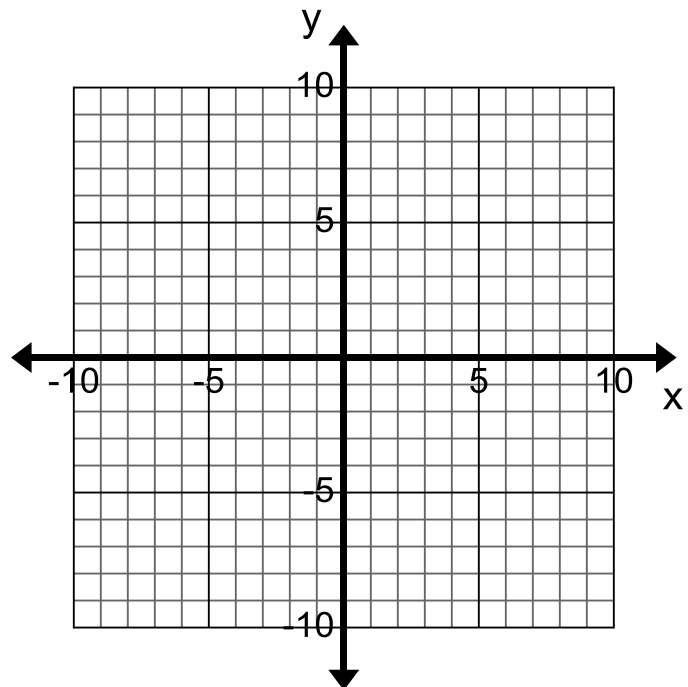
Consider the relation $y = \frac{1}{2}x$

1. Make a table of values for this relation.

x	y

2. Graph the relation on the coordinate plane to the right.
3. On the same coordinate plane translate every point 3 units up and draw the new graph or the image in a different color.
4. Using your table of values add 3 to every y value. $(x, y) \rightarrow (x, y + 3)$

x	y	$y+3$



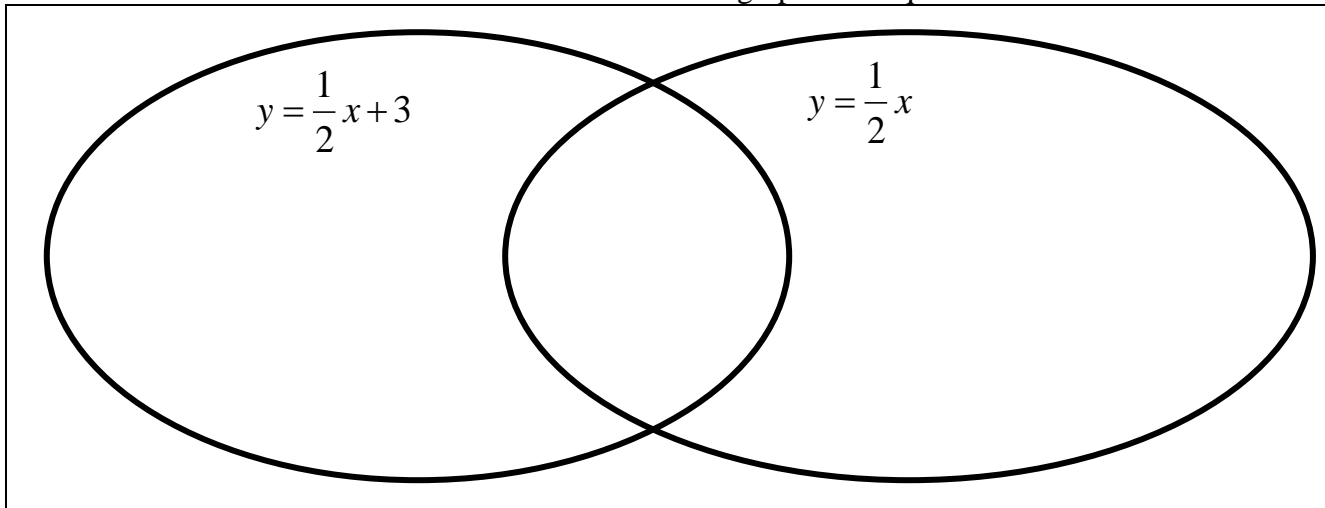
5. Graph your new ordered pairs.
6. Write an equation for your image and compare it with your equation for the pre-image.

In section 2.3h you investigated the general form for a linear equation given below.

The general form of a linear equation is $y=mx+b$ where m is the slope or rate of change and b is the initial value or y-intercept.

It is also seen in the example above. The equation $y = \frac{1}{2}x + 3$ represents the linear relationship where the slope is $\frac{1}{2}$ and 3 is the y-intercept or initial value.

Use the information on the previous page to compare and contrast the two relations using the Venn Diagram below. Be sure to list similarities and differences about their graphs and equations.

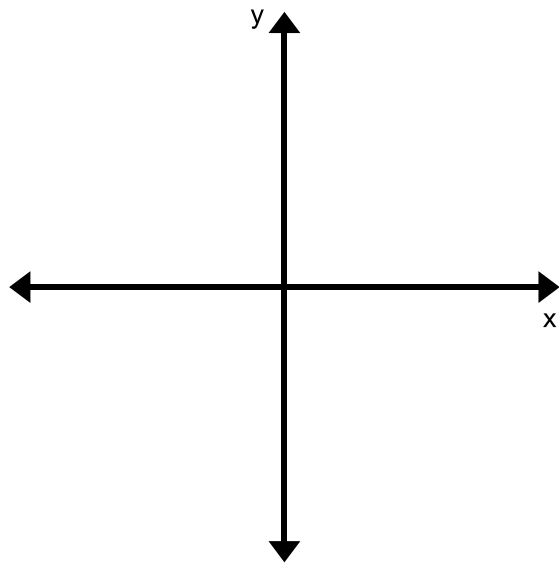


So how is this general form, $y=mx+b$, for a linear equation derived? Start with an example.

17. Graph a line on the coordinate plane to the right that goes through the point (0,4) and has a slope of $\frac{2}{3}$. Label the rise and run on your graph with a right triangle. Redraw and label this triangle in the space provided below the graph.

18. Does this line describe a proportional relationship? Explain.

19. Choose any point (x,y) on your graph and draw a slope triangle that describes the rise and run. Redraw and label this triangle in the space provided below the graph.

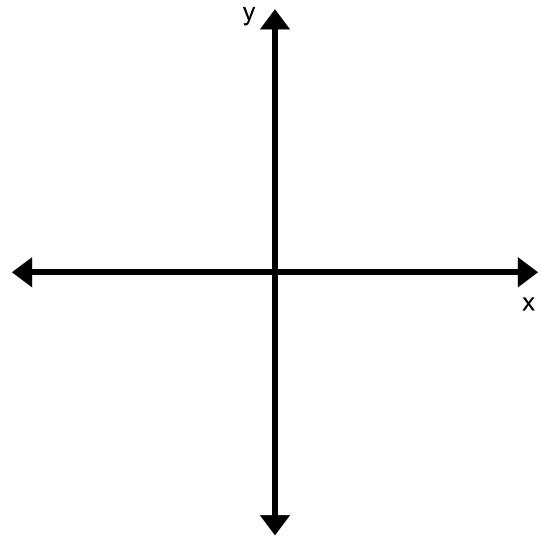


20. Write a proportional statement with your ratios.

21. Solve the equation that you wrote above for y .

Now look at the general case for the form $y = mx + b$.

22. Graph a line on the coordinate plane to the right that goes through any point on the y -axis (call the y -intercept b) and has a slope of m . Label the rise and run on your graph with a right triangle. Redraw and label this triangle in the space provided below the graph.



23. Does this line describe a proportional relationship? Explain.

24. Choose any point (x, y) on your graph and draw a slope triangle that describes the rise and run. Redraw and label this triangle in the space provided below the graph.

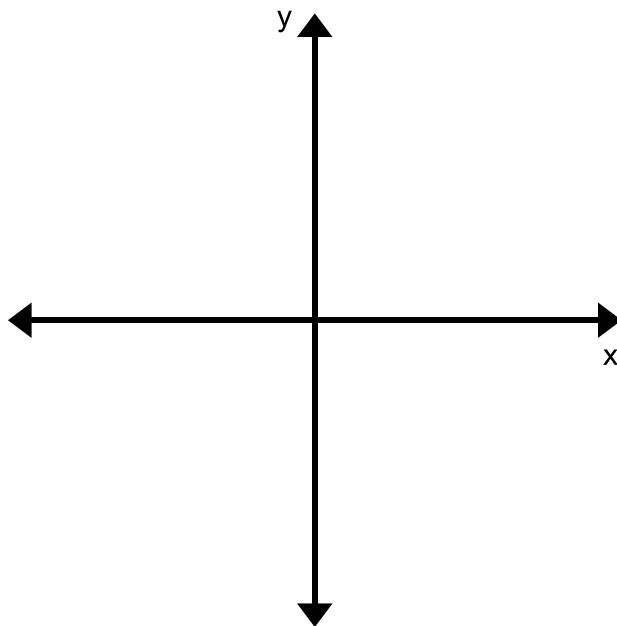
25. Write a proportional statement with your slope ratios.

26. Solve the equation that you wrote above for y .

Notice that this is the general form of a linear equation. This form is called Slope-Intercept form.

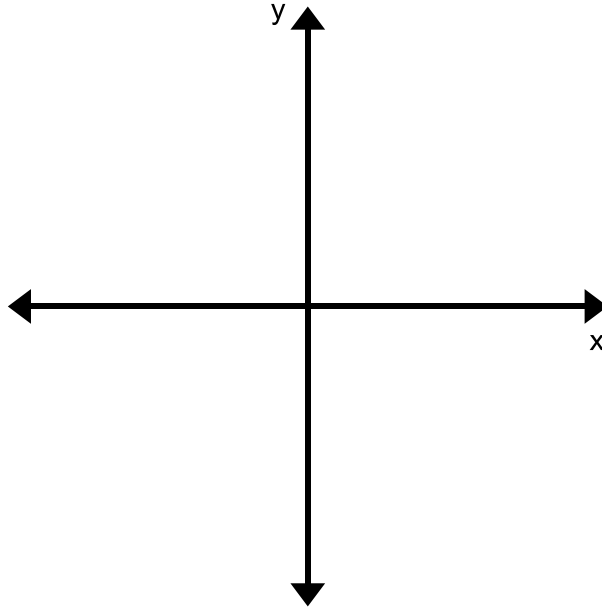
Slope-Intercept form of a linear equation is
$$y = mx + b$$
where m represents the slope (rate of change)
and b represents the y -intercept (initial value or starting point)

27. Show that the equation of a line that goes through the point $(0,3)$ and has a slope of $\frac{1}{6}$ is $y = \frac{1}{6}x + 3$.

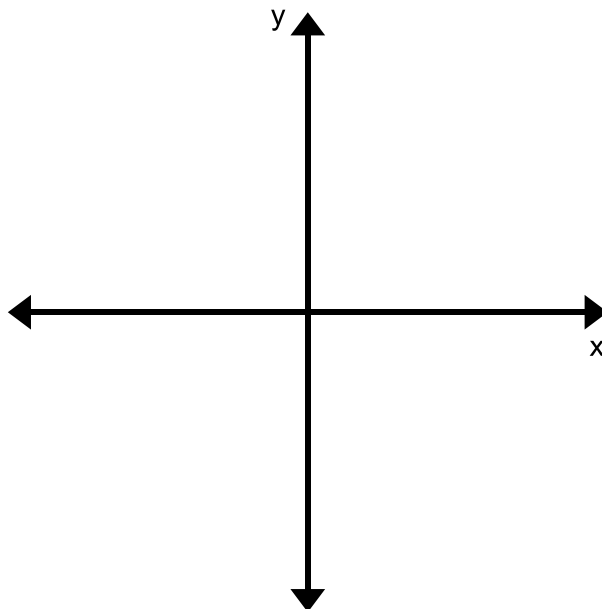


2.3j Homework: Use Similar Triangles to Derive the Equation $y = mx + b$

1. Show that the equation of a line that goes through the point $(0, -2)$ and has a slope of $\frac{3}{2}$ is $y = \frac{3}{2}x - 2$.



2. Show that the equation of a line that goes through the point $(0,4)$ and has a slope of -2 is $y = -2x + 4$.



2.3k Self-Assessment: Section 2.3

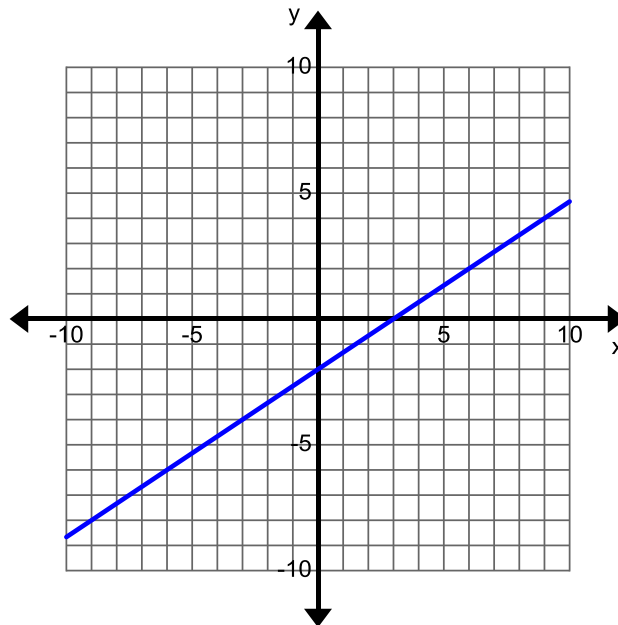
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provided on the next page that match each skill/concept

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
<p>1. Show that the slope of a line can be calculated as rise/run. Also explain why the slope is the same between any two distinct points on the line.</p> <p><i>See sample problem #1</i></p>	<p>I can find the slope of the line but do not know how I did it.</p>	<p>I can find the slope of the line by finding the rise and run but do not know how to show how you can find the slope between any two points on the line.</p>	<p>I can find the slope of the line by finding the rise and run. I can explain how to find the slope using any two points that lie on gridlines.</p>	<p>I can find the slope of the line by finding the rise and run and show or explain why the slope can be found from any two points that fall on the line.</p>
<p>2. Find the slope of a line from a graph, set of points, and table. Recognize when there is a slope of zero or when the slope of the line is undefined.</p> <p><i>See sample problem #2</i></p>	<p>I can find the slope of the line from one of the three representations.</p>	<p>I can find slope of the line from two of the three representations.</p>	<p>I can find the slope of the line from the graph, points, and table but I did not simplify all of my answers.</p>	<p>I can accurately find the slope of the line from the graph, points, and table. I can also express my answers in simplest form.</p>
<p>3. Given a context, find slope from various starting points (2 points, table, line, equation).</p> <p><i>See sample problem #3</i></p>	<p>I can only find the slope of the line from one of the starting points. I don't know what the slope means given then context.</p>	<p>I can find the slope of the line from some of the starting points but I don't know what the slope means given the context.</p>	<p>I can find the slope of a line from any starting point but I don't know what the slope means in the given context.</p>	<p>I can find the slope of the linear relationship from any starting point and interpret what it means given the context.</p>
<p>4. Recognize that m in $y = mx$ and $y = mx + b$ represents the rate of change or slope of a line. Understand that b is where the line crosses the y-axis or is the y-intercept.</p> <p><i>See sample problem #4</i></p>	<p>I do not know how to find the slope and y-intercept given in the equation.</p>	<p>I can identify the y-intercept but not the slope in this equation.</p>	<p>I can identify the slope but not the y-intercept in the equation.</p>	<p>I can identify the slope and y-intercept in the equation.</p>

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
5. Derive the equation $y = mx$ and $y = mx + b$ using dilations and proportionality. <i>See sample problem #5</i>	I can show the y-intercept and slope for both equations but I don't know how they relate to dilations of triangles or proportions.	I can derive only one of the equations using similar triangles.	I can accurately go through the steps of deriving both of the equations of the line using similar triangles and proportionality.	I can accurately go through the steps of deriving both of the equations of the line using dilations of triangles and proportionality. I can explain the reasoning at each step in my own words.

Sample Problem #1

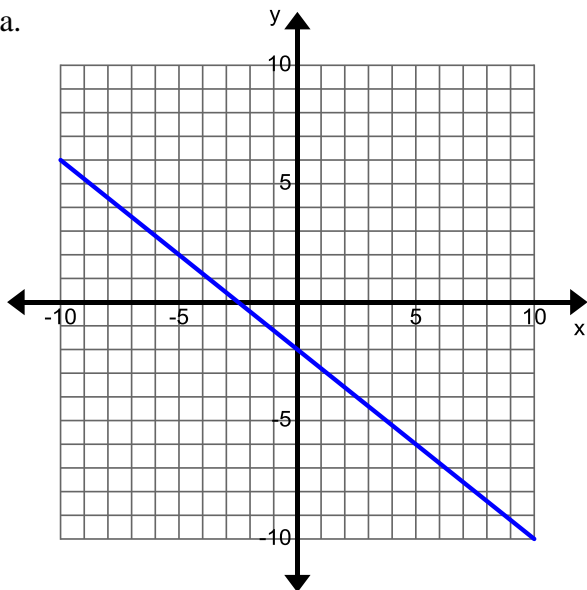
Find the slope of the line given below by finding the rise and run. Also show or explain why the slope can be found from any two points that fall on the line.



Sample Problem #2

Find the slope of each line given in the graph, set of points and table below.

a.



b. Find the slope of the line that goes through the points $(-2, 2)$, $(6, -10)$.

c. Find the slope of the line that goes through the points in the table given below.

x	y
4	-8
6	-8
8	-8

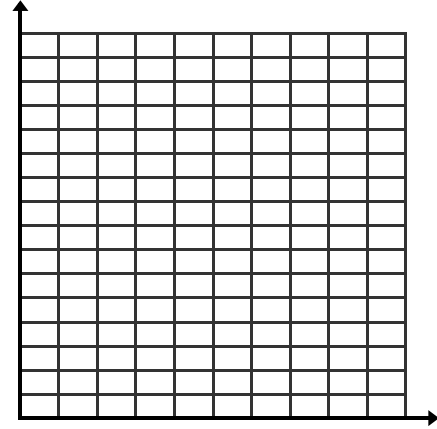
Sample Problem #3

Find the slope of the line from each context given below.

- a. The cost to fix a car at Bubba's Body Shop is shown in the table below. What is the slope of this linear relationship, be sure to explain what it means given the context.

Total Cost	
<i>t</i> (time in hours)	<i>C</i> (cost in dollars)
0	0
1	25
2	50
3	75

- b. The graph given below describes the height over time of a Norfolk Pine that is planted in Rudy's backyard. Find and describe the slope of this relationship as related to the context.



- c. The equation $y=2x+10$ describes the monthly cost to rent a movie at the local video kiosk, where x represents the number of movies rented and y represents the total cost. Find and describe the slope of this relationship.

- d. The points $(1, 16)$ $(4, 15)$ describe the height of a lit candle that is burning at two different times of day. Here x represents the number of hours it has been burning and y -represents the candle's height in inches. Find the slope of this relationship and describe what it means given the context.

Sample Problem #4

For the equation given below identify and the rate of change or slope and the y -intercept.

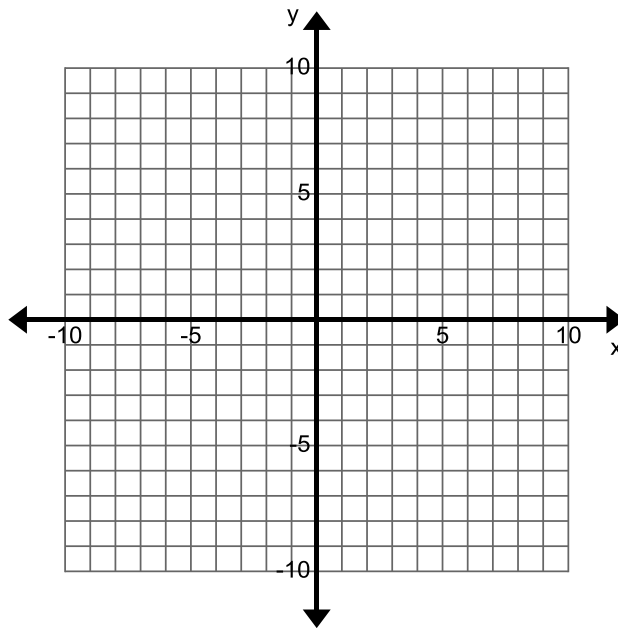
$$y = -\frac{1}{3}x + 5$$

Slope(m):

y -intercept(b):

Sample Problem #5

Show that the equation for a line that goes through the origin and has a slope of $\frac{5}{4}$ is $y = \frac{5}{4}x$ using dilations and proportionality.



Show that the equation for a line that goes through (0,3) and has a slope of 2 is $y = 2x + 3$.

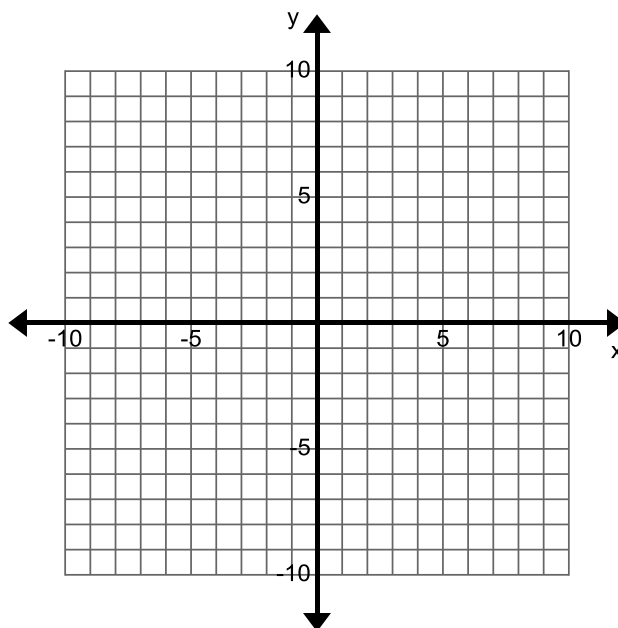


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Chapter 3: Representations of a Line (4 weeks)

Utah Core Standard(s):

- Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line. (8.F.3)
- Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (8.F.4)
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. (8.F.2)

Vocabulary: constant difference, context, difference table, equation, geometric model, graph, horizontal, initial value, linear, parallel, perpendicular, rate of change, reflection, rotation, slope, slope-intercept form, table, transformation, translation, unit rate, vertical, y-intercept.

Chapter Overview:


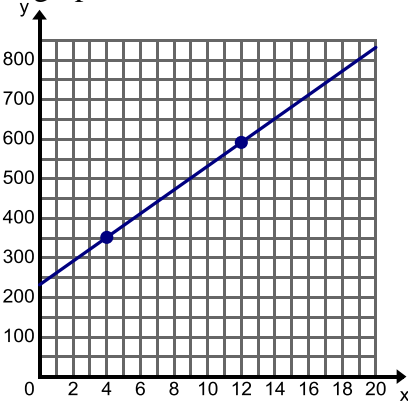


In this chapter, students solidify their understanding of the slope-intercept form of a linear equation. They write the equation for a linear relationship in slope-intercept form given a slope and y-intercept, two points, or a graph. They also write equations in slope-intercept form from a given context. In conjunction with writing equations students will graph equations given a variety of conditions. They may be given an equation in a variety of forms to graph, a slope and point, or a context. This chapter mainly focuses on the procedural process of graphing and writing equations for linear relationships. The transition from equation to relation to function is an important and difficult one. Chapter 5 will specifically address this transition and help students make the change in thinking.


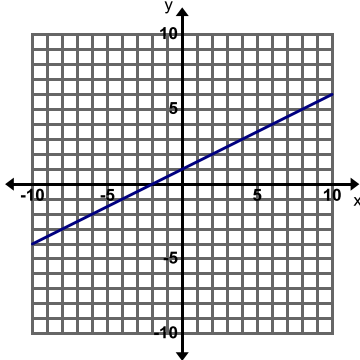


Connections to Content:

Prior Knowledge: Up to this point, students have been studying what makes a linear relationship and how it is composed. They have graphed linear equations by plotting ordered pairs generated in a table as solutions to an equation. They have written linear relationships by focusing on how the relationship grows by a constant rate of change and looking at an initial value. For the most part the slope-intercept form of a linear equation has been addressed mostly on a conceptual level. This chapter allows students to further their study of linear relationships by focusing on the procedural methods for graphing and writing linear equations.

Future Knowledge: In this chapter students will gain the skills and knowledge to work with simultaneous linear relationships in Chapter 4. This chapter sets them up to be able to write and graph a system of linear equations in order to examine its solution. In addition this chapter is the building block for a student's understanding of the idea of function. In Chapter 5, students will solidify the concept of function, construct functions to model linear relationships between two quantities, and interpret key features of a linear function. This work will provide students with the foundational understanding and skills needed to work with other types of functions in future courses.

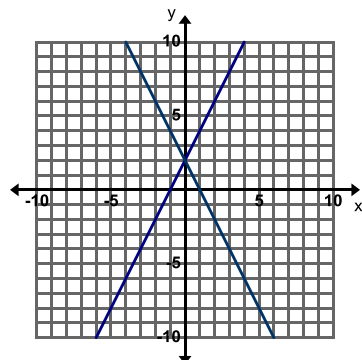
MATHEMATICAL PRACTICE STANDARDS (emphasized):

	<p>Make sense of problems and persevere in solving them.</p>	<p>The graph below shows the weight of a baby elephant where x is the time (in weeks) since the elephant's birth and y is the weight (in pounds). At 4 weeks, the elephant weighed 352 lbs. and at 12 weeks, the elephant weighed 592 lbs. Label the axes of the graph.</p>  <p>Use your graph and equation to tell the story of this elephant. <i>Students are using the skills they learned for writing equations of lines to solve a real world problem. They are translating between the different representations of a line and recognizing important features of the representations. Following this work, the teacher is prompted to ask the students if it really makes sense that this elephant gains exactly 30 pounds each week, leading to a conversation about real world data and statistics.</i></p>
	<p>Reason abstractly and quantitatively.</p>	<p>The cost to rent a jet ski is \$80 per hour. The cost also includes a flat fee for a lesson on how to use the jet ski. Steve rented a jet ski for 3 hours and the total bill was \$265. Write the equation for the total cost of renting a jet ski y for x hours. Use your equation to add more details to the story about renting a jet ski.</p> <p><i>Throughout this chapter students must write equations for many linear relationship contexts. In some cases it might be abstracting an initial value as the y-intercept or by looking at the constant rate of change as the slope. In other cases students must comprehend the intended meaning of given quantities not just how to compute them. For example, in the problem above a student must discern that they can represent renting a jet ski for 3 hours at a costs of \$265 as the point (3, 265). They must then recognize how to use the point to write an equation.</i></p>
	<p>Look for and make use of structure</p>	<p>Brainstorm ideas on how you could write the equation of the line without graphing when you are given a point and the slope. Consider how you could use the equation $y = mx + b$ to find the y-intercept if you know the slope and a point on the line. Using an example from the previous page may help you work through the process.</p> <p><i>In the problem above, students make use of the structure of the slope-intercept form of an equation. They recognize that by substituting the ordered pair values of x and y and the slope into the equation they can obtain the value for b, the y-intercept. To do so they must recognize that they can change the structure of the equation (solve for b) and obtain their intended value. This process allows them to then write an equation in slope-intercept form.</i></p>

	<p>Construct viable arguments and critique the reasoning of others.</p>	<p>Find, Fix, and Justify: Kevin was asked to graph the line $y = -\frac{1}{2}x + 1$. Kevin graphed the line below and made a common error. Describe Kevin's error and then graph the line correctly on the grid.</p>  <p><i>There are many Find, Fix, and Justify problems in this chapter where a student must find a common mistake and fix it. As student's critique another student's work they must analyze the key components of a linear relationship. This reinforces a student's understanding and often clears up common misconceptions.</i></p>
	<p>Model with Mathematics.</p>	<p>A Health Teacher is writing a test with two sections. The entire test is worth 40 points. He wants the questions in Section A to be worth 2 points each and the questions in Section B to be worth 4 points each. Let x represent the number of questions in Section A and y represent the number of questions in Section B.</p> <ol style="list-style-type: none"> Write an equation that describes all the different combinations of number of questions in Section A and B. Graph this equation to show all possible numbering outcomes for this test. <p><i>This question is asking students to model a situation with an equation and graph. The equation allows the student to see how the unknown values are related to each other. The graph provides a pictorial representation of all of the possible solutions. Through modeling this situation with mathematics, students better understand how to represent more than one solution algebraically and graphically.</i></p>
	<p>Use appropriate tools strategically</p>	<p>Use a graphing calculator to graph the following equation. $x - 3y = -9$</p> <p><i>A graphing calculator is not only a useful tool in graphing an equation but also helpful when used to check your work. A teacher may allow students to only use the graphing calculator when checking their work. Another strategic way to use the graphing calculator is to examine how a line can change if the quantities in the equation are changed. In the example above, a student must employ the strategy of changing the equation into slope-intercept form before entering it into the graphing calculator.</i></p>



Attend to Precision



Upon first glance these lines appear to be perpendicular. But by calculating the slopes of the lines it is determined that they are not. This is a good example of attending to precision. Students must rely upon accurately and efficiently calculating the slope of each line to see that the slopes are not opposite reciprocals of each other.



Look for and express regularity in repeated reasoning

Graph the equation $y = x + 3$ and label the line with the equation.

- Predict how the graph of $y = x + 1$ will compare to the graph of $y = x + 3$.
- Predict how the graph of $y = x - 3$ will compare to the graph of $y = x + 3$.
- Graph the following equations on the same grid and label each line with its equation.
 $y = x + 1$
 $y = x - 3$
- Were your predictions correct? Why or why not?
- What is the relationship between the lines $y = x + 3$, $y = x + 1$, and $y = x - 3$?
- Write a different equation that would be parallel to the equations in this problem.
- Describe the movement of a line when b is increased or decreased while m is held constant.

In section 3.2 students investigate how changing different parts of the equation result in different transformations of a line. They do this through repeatedly examining how these changes affect the line. For example a student might state, "Every time that you change the y-intercept in the equation the line keeps the same slope but moves up or down the y-axis."

Section 3.0 Anchor Problem: Solutions to a Linear Equation

Recall from Chapter 1 that you wrote and solved equations with one variable. Find the solution to each equation below.

1. $x + 7 = 10$

2. $5y = 15$

3. $4x - 6 = 10$

4. $3x - 11 = 2x + 9$

5. In your own words describe what a solution is.

Talk with your neighbor about what they think a solution is.

6. Refine your definition of a solution now that we have discussed it as a class.

A solution is:

7. Can there be more than one solution to an equation.

Now find the solutions to each equation below (it is okay to guess).

8. $x + y = 12$

9. $m - n = 12$

10. $xy = 24$

11. $y = 5x$

12. $y = x^2$

Compare your solutions with your neighbor.

13. Is the definition for a solution the same if you have two different variables in your equation as opposed to above where we have only one variable?

14. How many total solutions are there for an equation with more than one variable?

Find at least four solutions to each equation. Write the solutions as ordered pairs.

15. $y = 2x$

x	y	(x,y)

16. $x + y = 5$

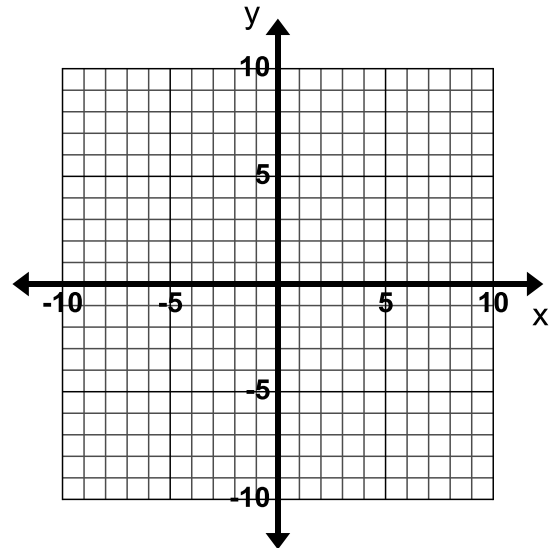
x	y	(x,y)

17. Is it possible to list every possible solution to these equations?

18. Why do you think the instructions prompted you to write your answers as ordered pairs?

19. To show every solution to an equation with two different variables you _____.

20. Show the solutions to the equation $y = 3x$ on the graph below and describe in detail how the graph shows all the solutions to this equation.



21. Why do you draw arrows on your graph?

22. Put a star on the graph where a solution is a fraction.

23. Put a smiley on the graph where there is a negative solution.

Section 3.1: Graph and Write Equations of Lines

Section Overview:

Now that students have an understanding of the parameters m and b in the slope-intercept form of a linear equation, this section will transition students into the procedural work of being able to write and graph the equation of a line from any set of givens. Students apply the skills they have learned to write linear equations that model real world situations.

Concepts and Skills to Master:

By the end of this section, students should be able to:

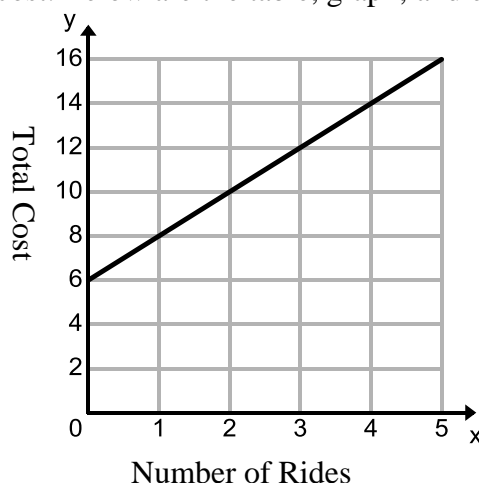
1. Write a linear equation in the form $y = mx + b$ given any of the following:
 - slope and y-intercept
 - slope and a point
 - two points
 - a table
 - a graph of a linear relationship
 - a context of a real world situation
2. Graph linear relationships given any of the following:
 - an equation
 - slope and a point

3.1a Class Activity: Write Equations in Slope-Intercept Form **n#**

Revisit a situation from the previous chapter:

You and your friends go to the state fair. It costs \$6 to get into the fair and \$2 each time you go on a ride. Consider the relationship between number of rides and total cost. Below are the table, graph, and equation that model this linear relationship.

Number of Rides (x)	Total Cost (y)
0	6
1	8
2	10
3	12
4	14
5	16



You modeled this situation with the equation $y = 2x + 6$

Discuss the following questions with a partner. Highlight your answers on the table, graph and equation above.

What is the slope of the graph? Where do you see the slope in the equation? What does the slope represent in the context?

What is the y-intercept of the graph? Where do you see the y-intercept in the equation? What does the y-intercept represent in the context?

By looking at the problems done in the previous chapter, you can see that one way to represent a linear equation is in slope-intercept form. In the previous chapter you also derived the equation $y=mx+b$.

Slope-intercept form of a linear equation is
$$y = mx + b$$
where m represents the slope (rate of change)
and b represents the y-intercept (initial value or starting point)

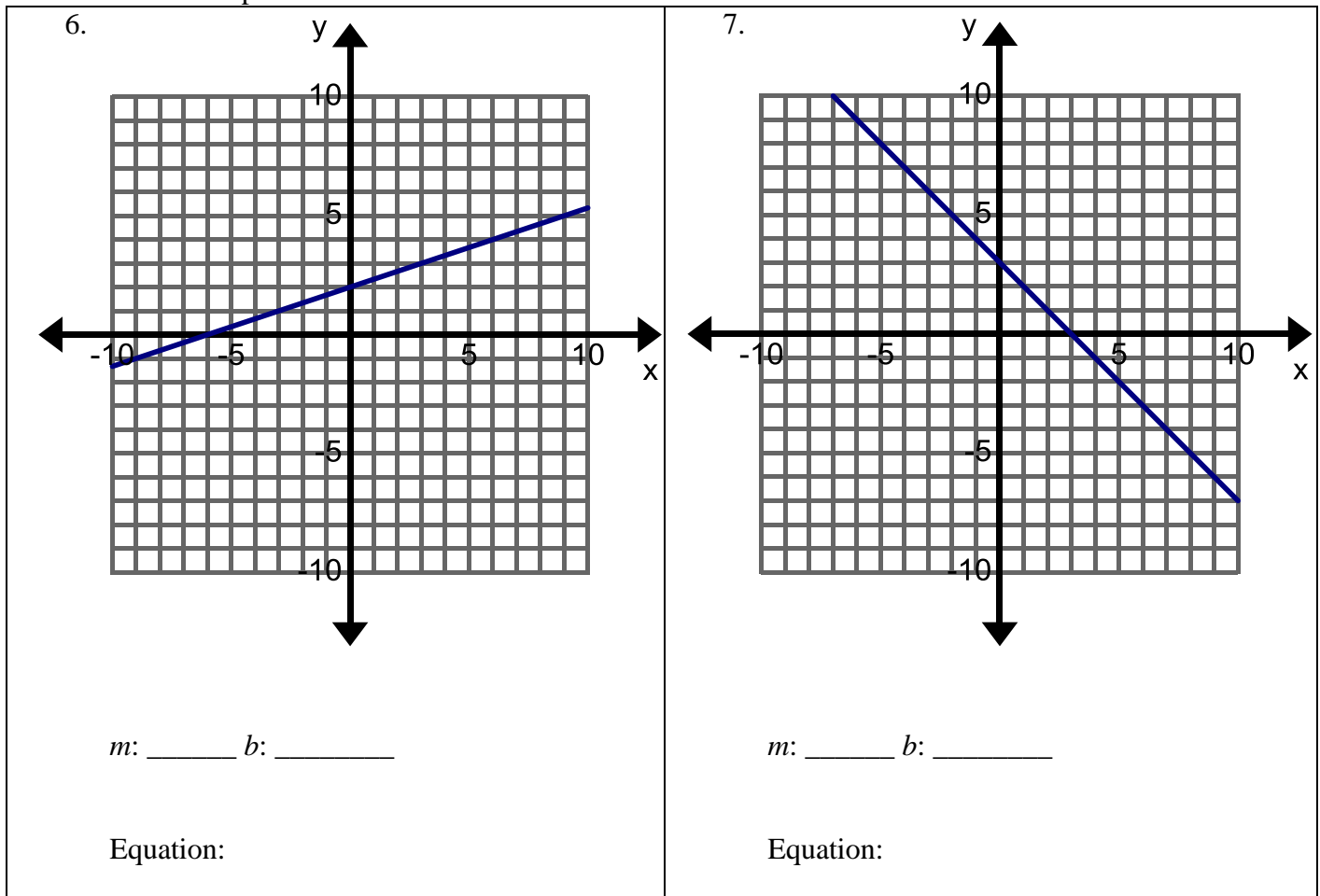
If you are given a representation of a linear relationship, you can write the equation for the relationship in slope-intercept form by finding the slope (m) and y-intercept (b) and substituting them into the slope-intercept form of a linear equation shown above.

Directions: Write the equation of each line in slope-intercept form.

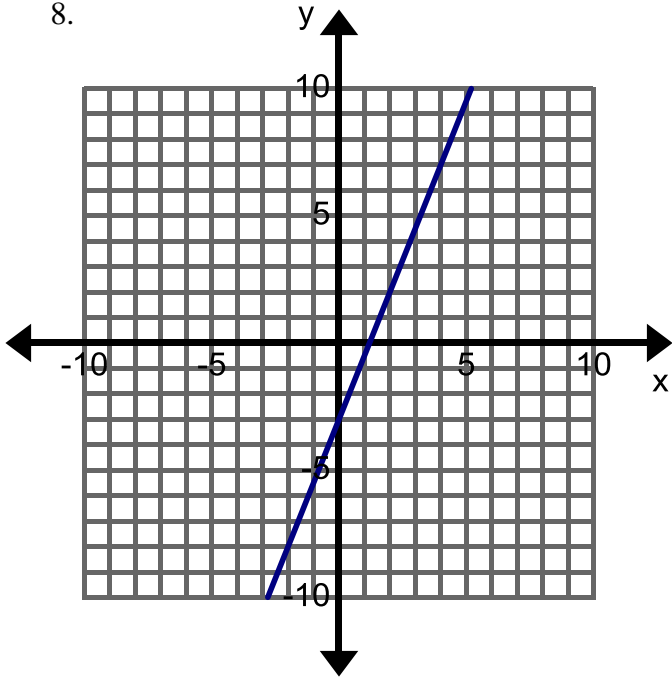


1. The slope of the line is 3. The y-intercept is (0, 4).
2. The slope of the line is -2 . The y-intercept is (0, 0).
3. The slope of the line is $\frac{1}{2}$. The y-intercept is (0, -2).
4. The slope of the line is $-\frac{4}{3}$. The y-intercept is (0, -1).
5. The slope of the line is 0. The y-intercept is (0, 2).

Directions: Find the slope and y-intercept from the graph, table, or story below. Then write the equation of each line in slope-intercept form. If you have a hard time determining where the line intersects a point be sure to check at least three points.



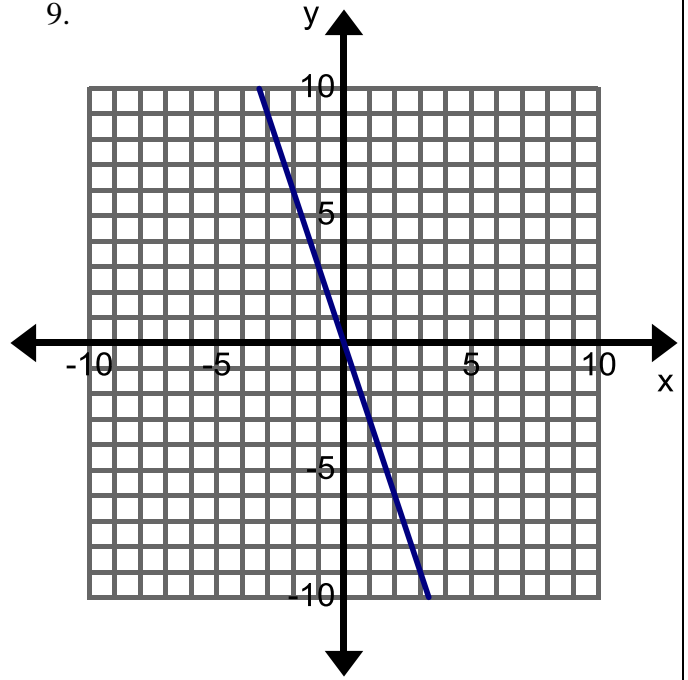
8.



m : _____ b : _____

Equation:

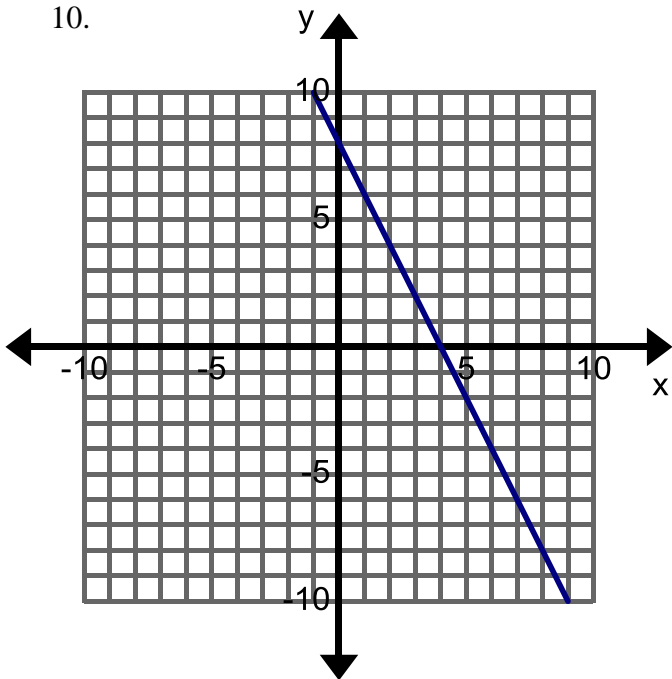
9.



m : _____ b : _____

Equation:

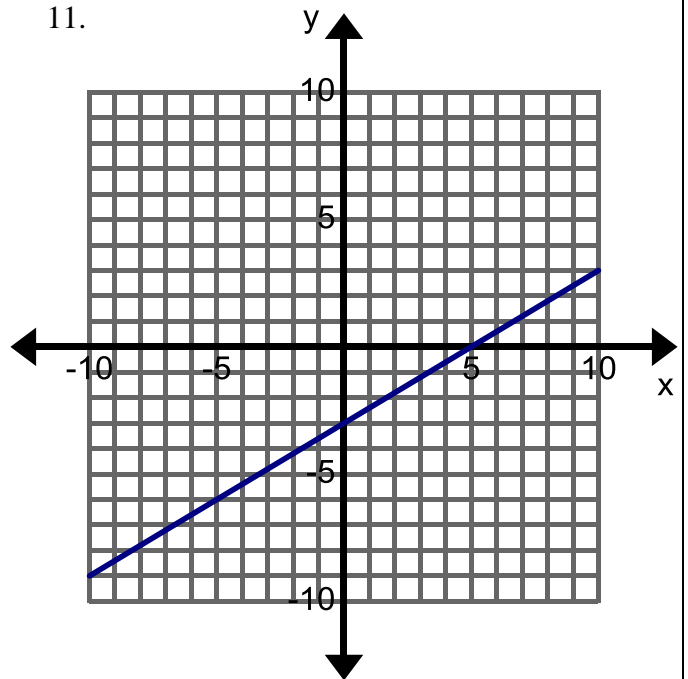
10.



m : _____ b : _____

Equation:

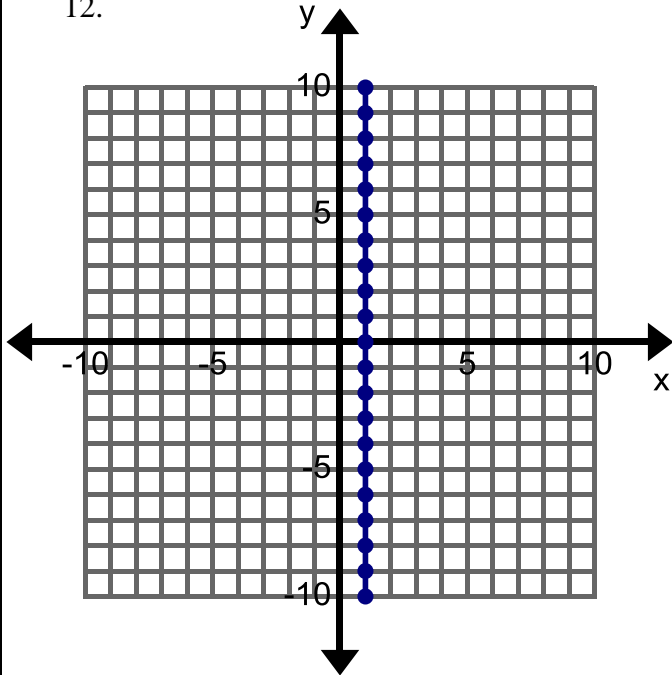
11.



m : _____ b : _____

Equation:

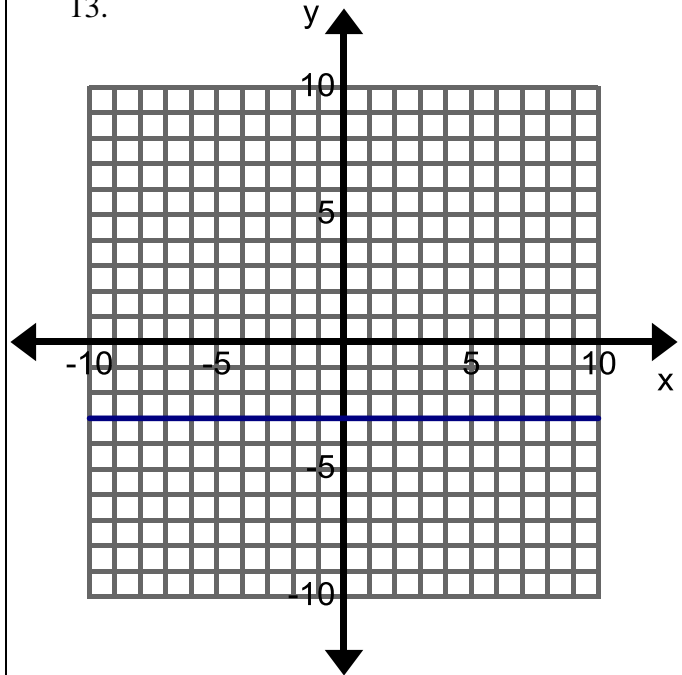
12.



m : _____ b : _____

Equation:

13.



m : _____ b : _____

Equation:

14.

x	y
0	4
1	6
2	8
3	10

m : _____ b : _____

Equation:

15.

x	y
-1	-1
0	-2
1	-3
2	-4

m : _____ b : _____

Equation:

16.

x	y
-2	-1
0	0
2	1
4	2

m : _____ b : _____

Equation:

17. You are going on a road trip with your family. You are already 30 miles into your trip and the speed limit is 75 miles per hour on the freeway. Let x be the number of hours from now and y be the total distance traveled.

m : _____ b : _____

Equation:

18. A Basset Hound weighs 100 pounds and is on a special diet to lose 4 pounds per month. Let x represent the number of months passed and y the weight of the dog.

m : _____ b : _____

Equation:

19. Your cell phone plan has a flat rate of \$30 each month. For each text you send it costs \$0.20. Let x represent the number of texts that you send and y your total monthly bill.

m : _____ b : _____

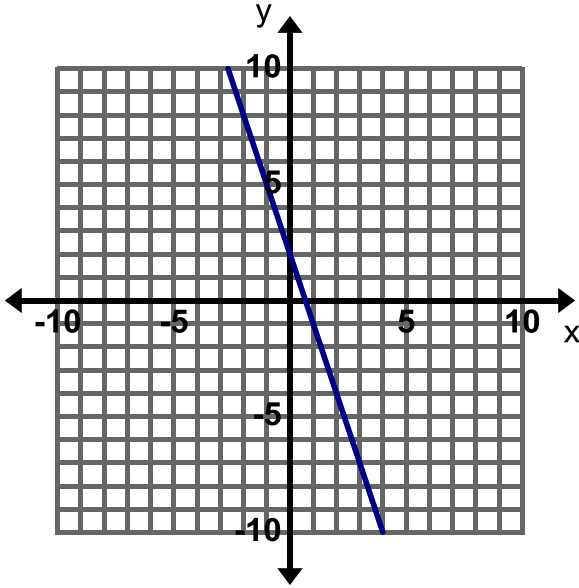
Equation:

Find, Fix, and Justify: In each of the following problems, a common mistake of writing the equation of a line has been made. Describe the error and write the correct equation.



has been made. Describe the error and write the correct equation.

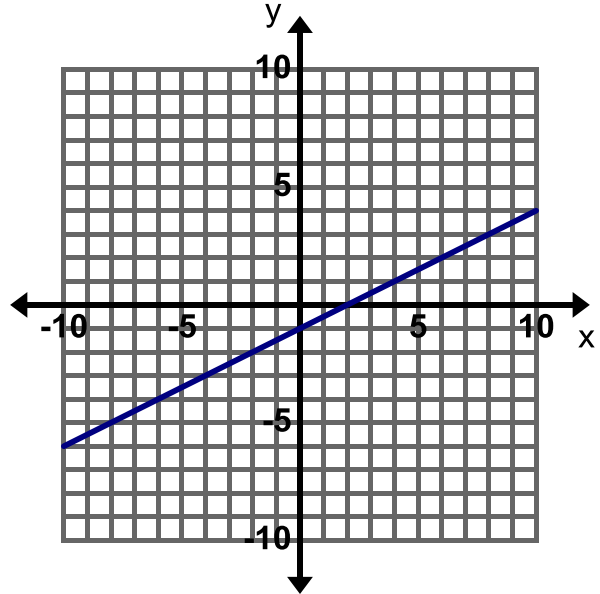
20. Incorrect Equation: $y = 3x + 2$



Mistake:

Correct Equation:

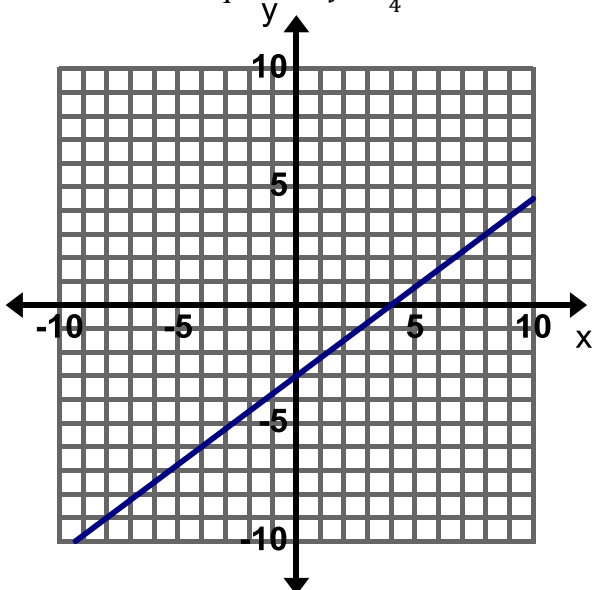
21. Incorrect Equation: $y = 2x - 1$



Mistake:

Correct Equation:

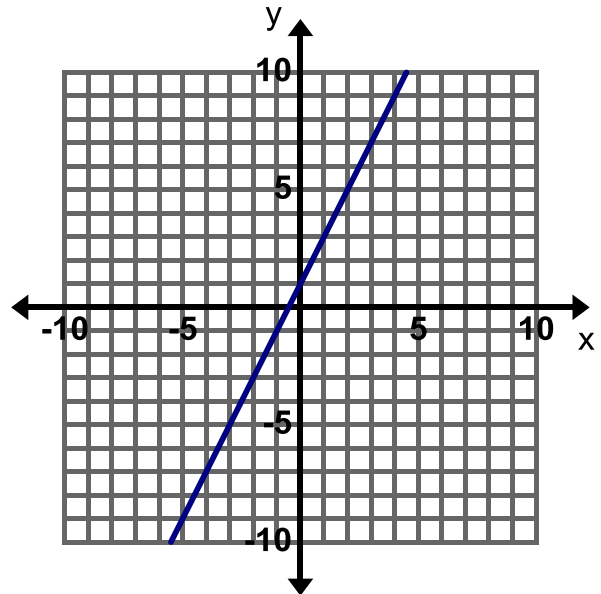
22. Incorrect Equation: $y = \frac{3}{4}x + 4$



Mistake:

Correct Equation:

23. Incorrect Equation: $y = x + 2$



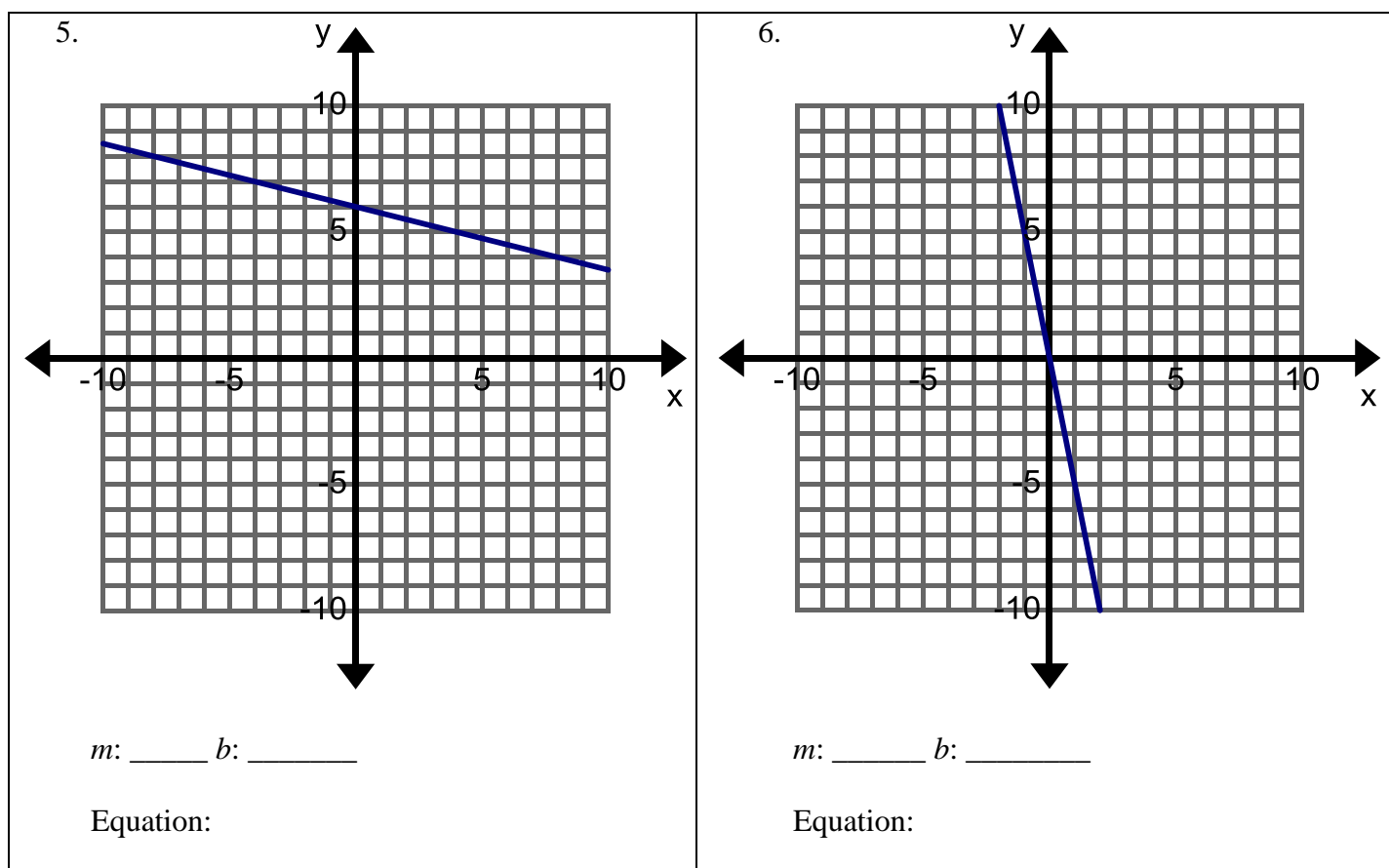
Mistake:

Correct Equation:

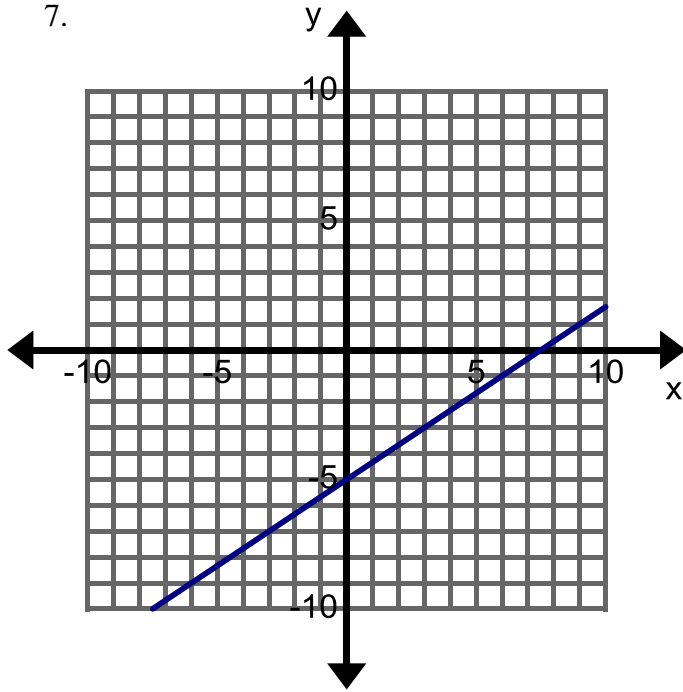
3.1a Homework: Write Equations in Slope-Intercept Form

Directions: Write the equation of each line in slope-intercept form.

1. The slope of the line is 5. The y-intercept is $(0, -1)$.
2. The slope of the line is -1 . The y-intercept is $(0, -6)$.
3. The slope of the line is $\frac{1}{4}$. The y-intercept is $(0, 0)$.
4. The slope of the line is $-\frac{3}{5}$. The y-intercept is $(0, 10)$.



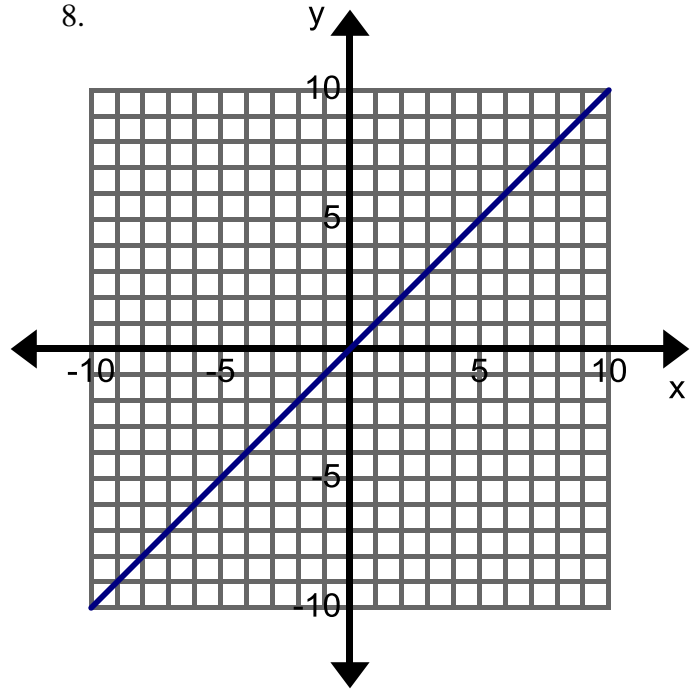
7.



m : _____ b : _____

Equation:

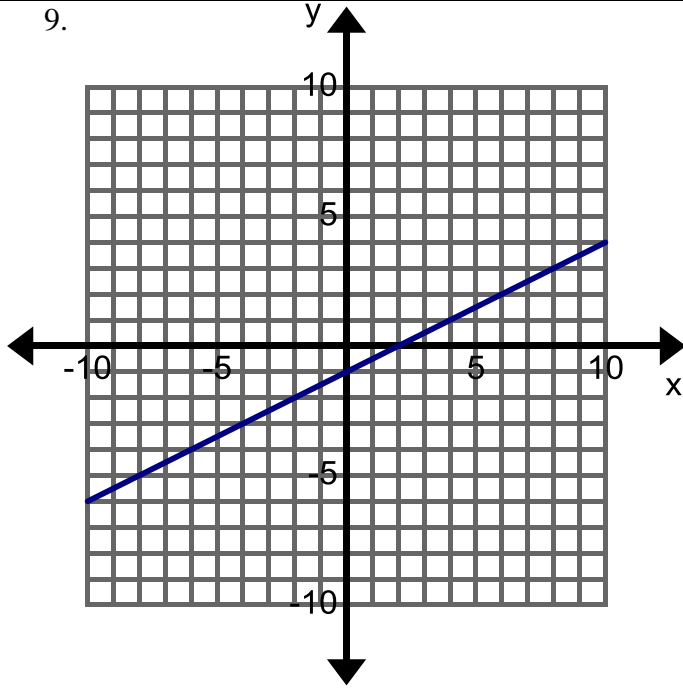
8.



m : _____ b : _____

Equation:

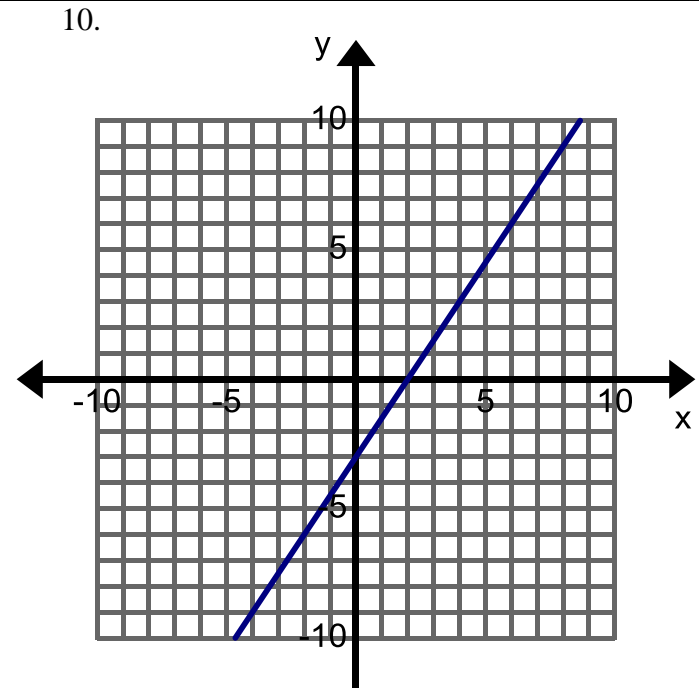
9.



m : _____ b : _____

Equation:

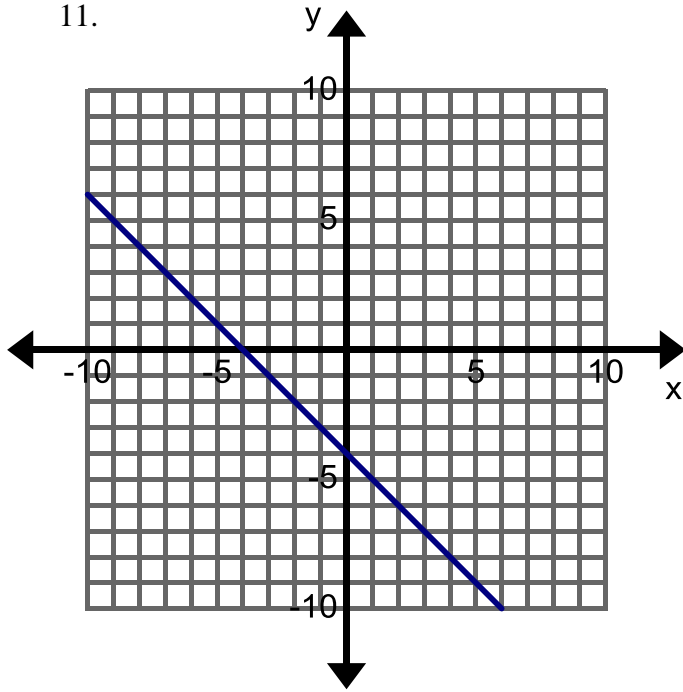
10.



m : _____ b : _____

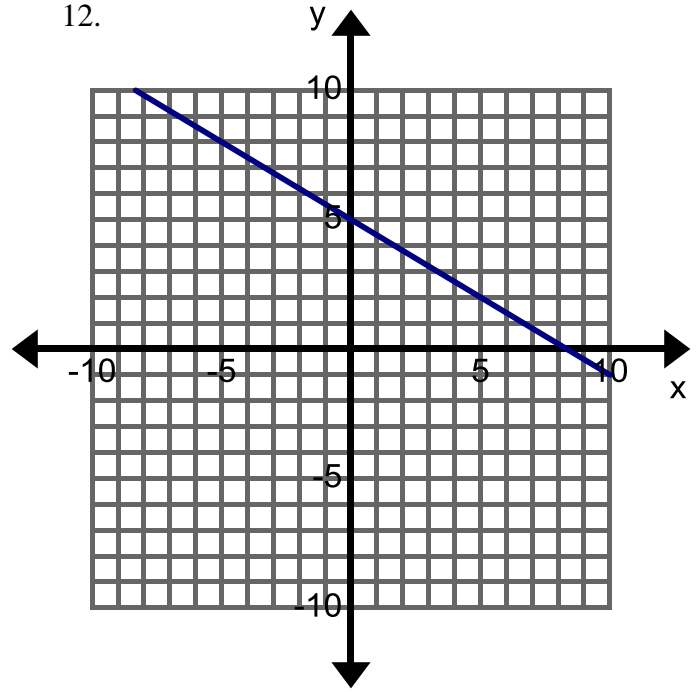
Equation:

11.

 m : _____ b : _____

Equation:

12.

 m : _____ b : _____

Equation:

13.

x	y
0	2
1	0
2	-2
3	-4

 m : _____ b : _____

Equation:

14.

x	y
-3	3
0	4
3	5
6	6

 m : _____ b : _____

Equation:

15.

x	y
-1	5
0	5
1	5
2	5

 m : _____ b : _____

Equation:

16. You want to ship Science Textbooks from Florida. The textbooks cost \$60 each plus \$150 for shipping costs. Let x represent the number of textbooks shipped and y the total cost.

 m : _____ b : _____

Equation:

17. Velma has \$450 in her checking account and withdraws \$25 each week. Let x represent the number of weeks that have past and y the total amount of money in her account.

 m : _____ b : _____

Equation:

16. Jarius is 15 feet away from his car and walks toward at a rate of 2 feet per second. Let x represent the number of seconds that have passed and y the distance away from the car.

 m : _____ b : _____

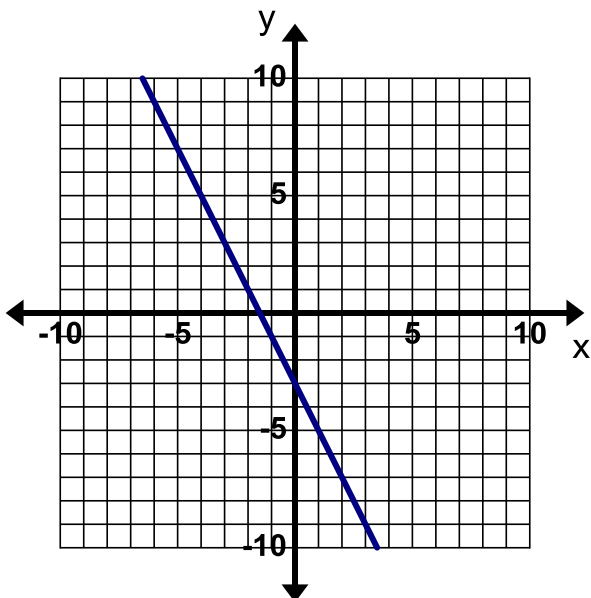
Equation:

Find, Fix, and Justify: In each of the following problems, a common mistake of writing the equation of a line has been made. Describe the error and write the correct equation.



has been made. Describe the error and write the correct equation.

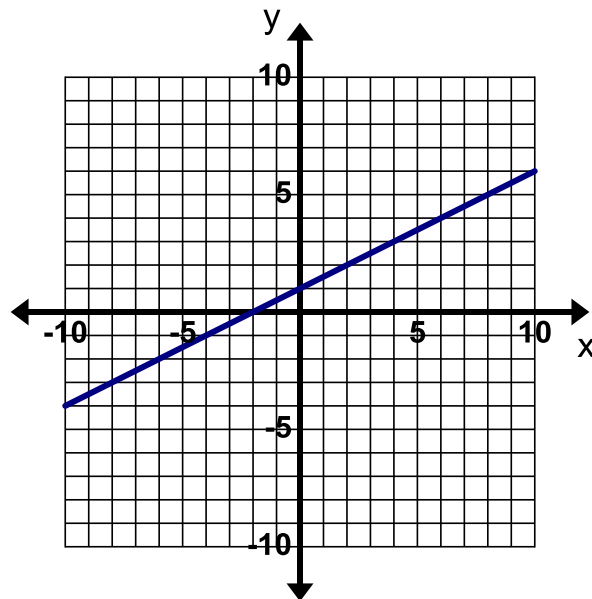
20. Incorrect Equation: $y = -2x + 3$



Mistake:

Correct Equation:

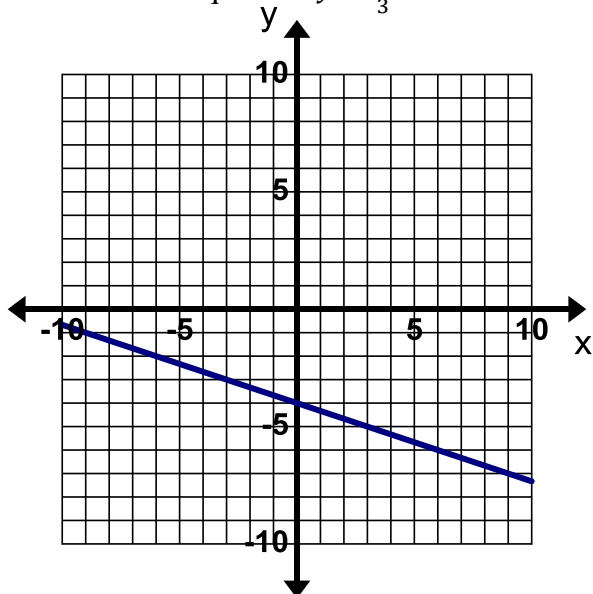
21. Incorrect Equation: $y = x + \frac{1}{2}$



Mistake:

Correct Equation:

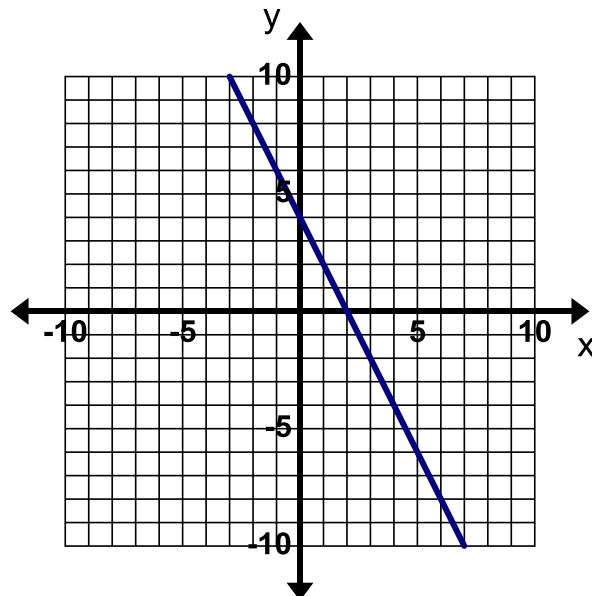
22. Incorrect Equation: $y = \frac{1}{3}x - 4$



Mistake:

Correct Equation:

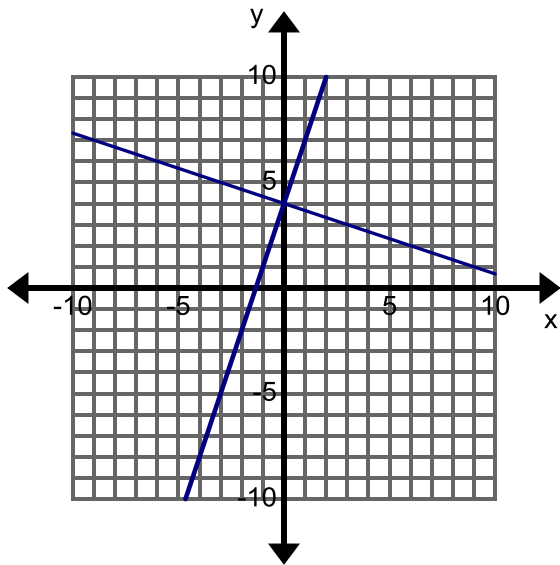
23. Incorrect Equation: $y = -2x + 2$



Mistake:

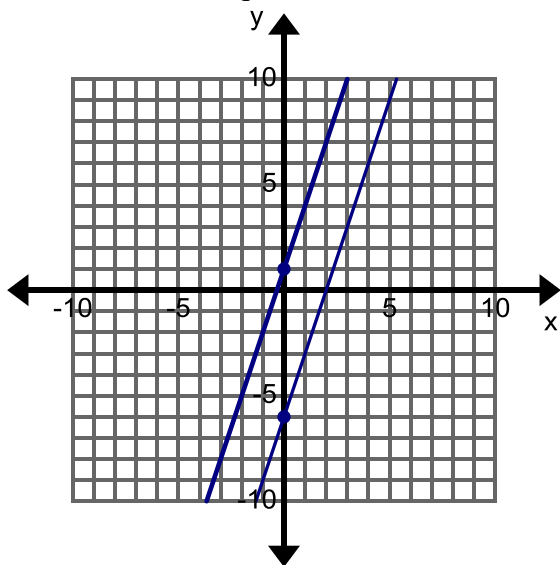
Correct Equation:

24. Write the equation of each line in the graph below. Label each line with its equation.



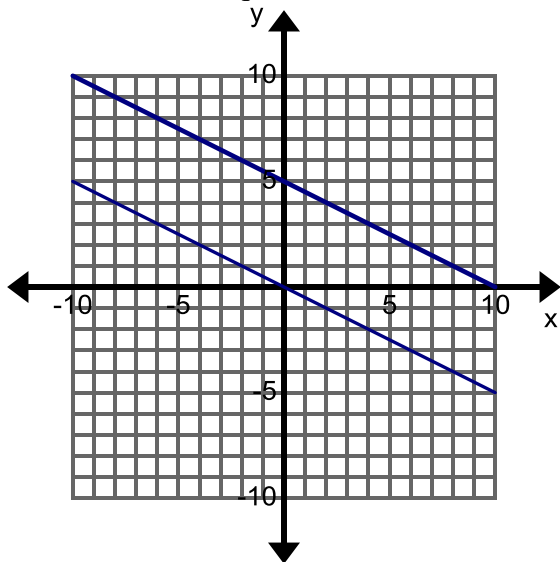
Compare the two lines. What is the same?
What is different?

25. Write the equation of each line in the graph below. Label each line with its equation.



Compare the two lines. What is the same?
What is different?

26. Write the equation of each line in the graph below. Label each line with its equation.



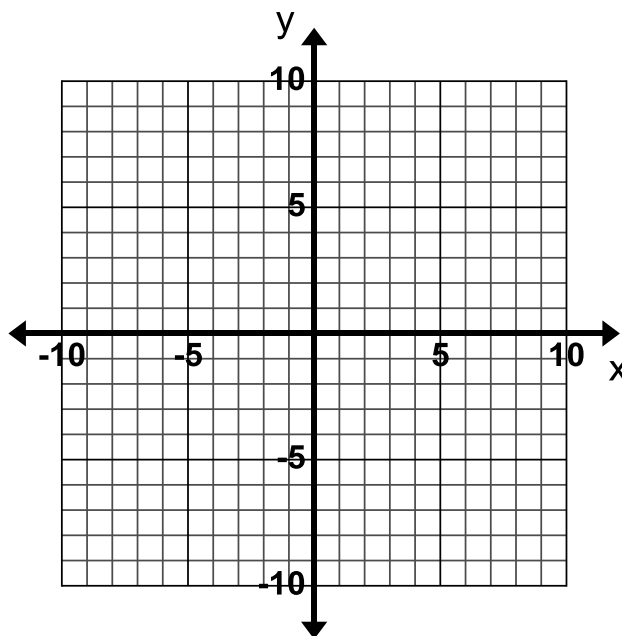
Compare the two lines. What is the same?
What is different?

3.1b Classwork: Graph from Slope-Intercept Form

1. On the following coordinate plane, draw a line with a slope of $\frac{1}{3}$.

a. How do you know that your line has a slope of $\frac{1}{3}$?

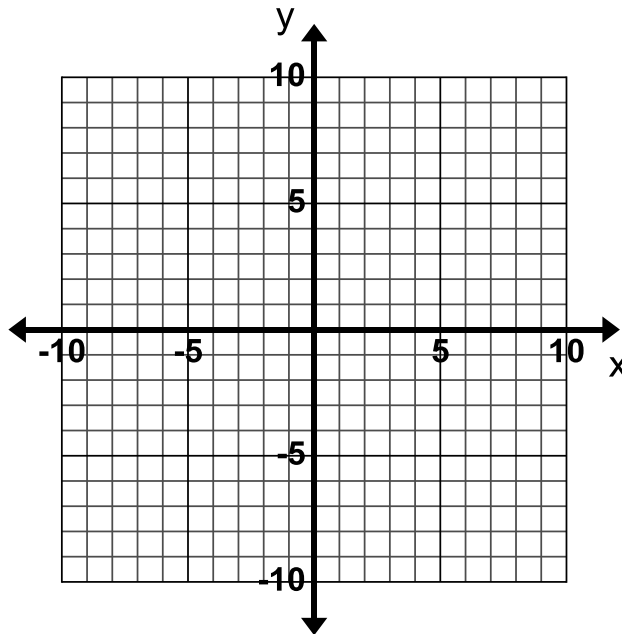
b. What did you do to draw your line to ensure that you ended with a slope of $\frac{1}{3}$?



2. Draw a line with a slope of -2.

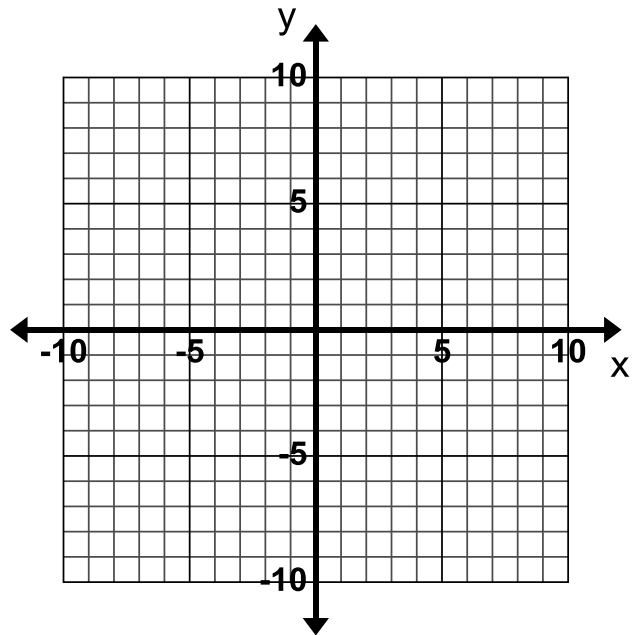
a. How do you know that your line has a slope of -2?

b. What did you do to draw your line to ensure that you ended with a slope of -2?



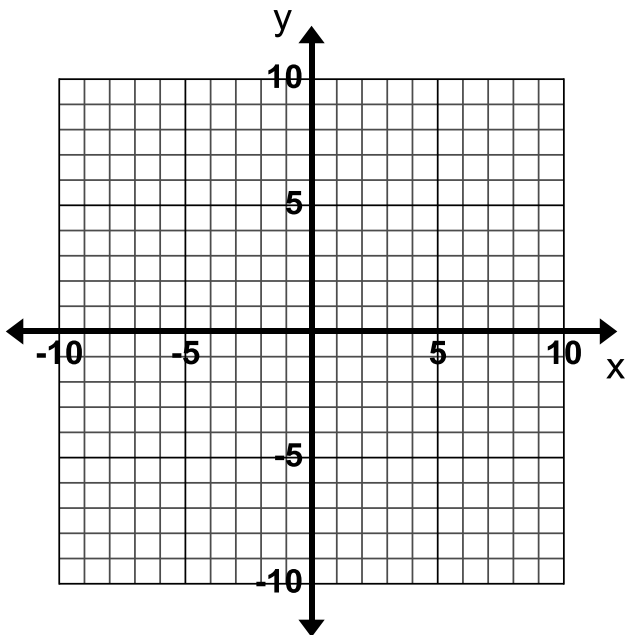
3. Consider the following equation $y = \frac{2}{3}x - 1$.

- What is the y-intercept?
- What is the slope?
- Graph the line on the grid to the right by first plotting the y-intercept and then drawing a line with the slope that goes through the y-intercept. Use what you wrote in the first two questions to help you.



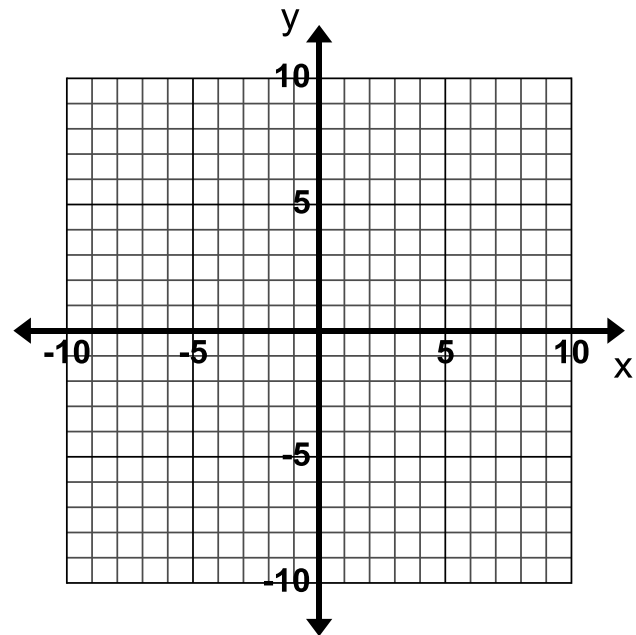
4. Graph $y = 3x + 1$

- What is the y-intercept?
- What is the slope?

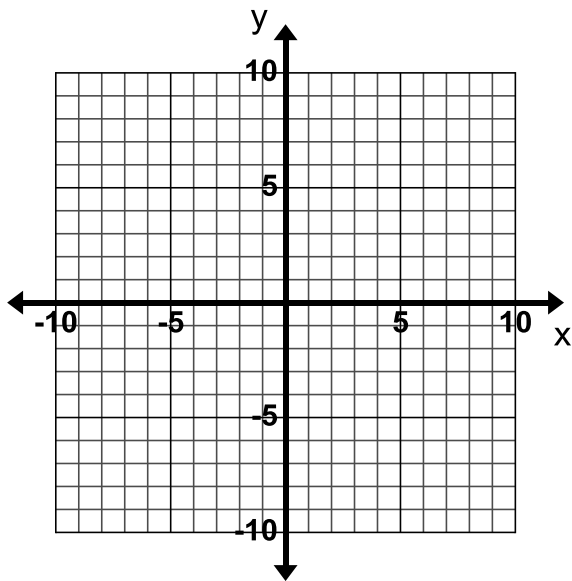


5. Graph $y = -\frac{3}{4}x$

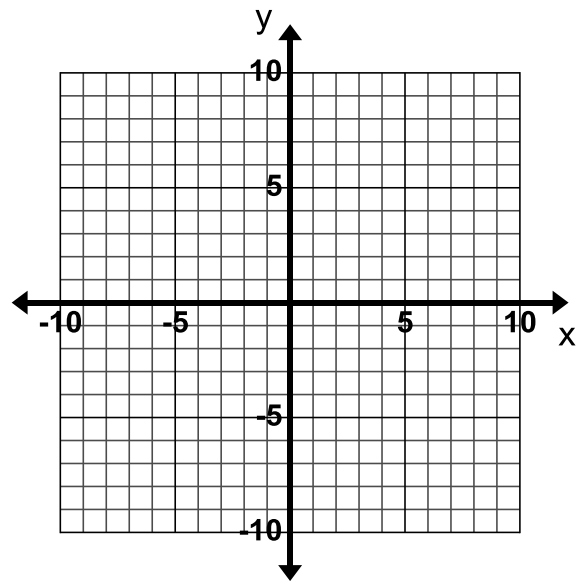
- What is the y-intercept?
- What is the slope?



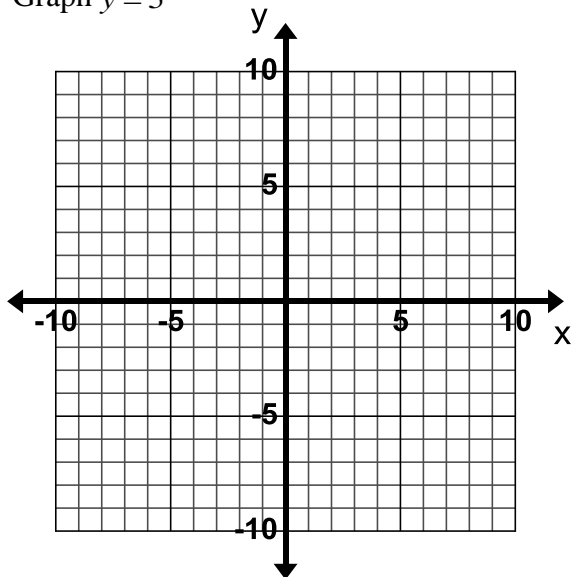
6. Graph $y = -2 + x$



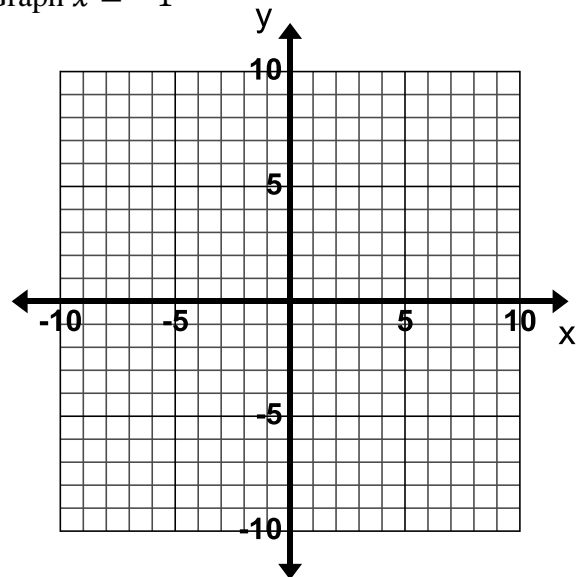
7. Graph $y = -2x + 3$



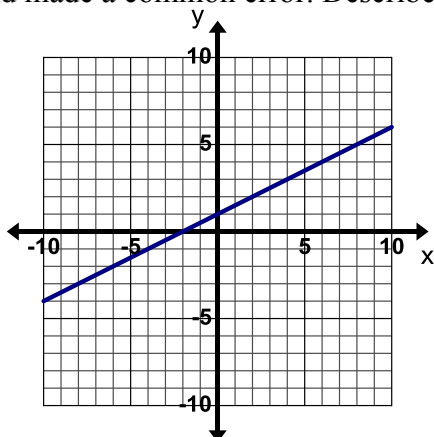
8. Graph $y = 5$



9. Graph $x = -1$

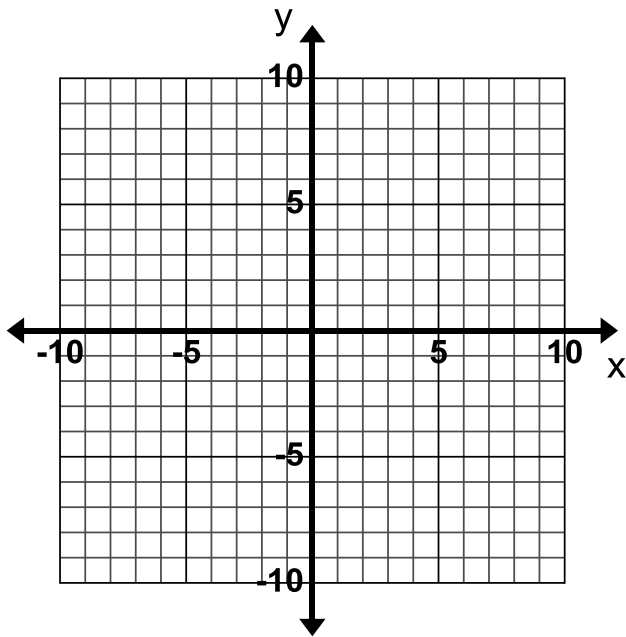


10. **Find, Fix, and Justify:** Kevin was asked to graph the line $y = -\frac{1}{2}x + 1$. Kevin graphed the line below and made a common error. Describe Kevin's error and then graph the line correctly on the grid.

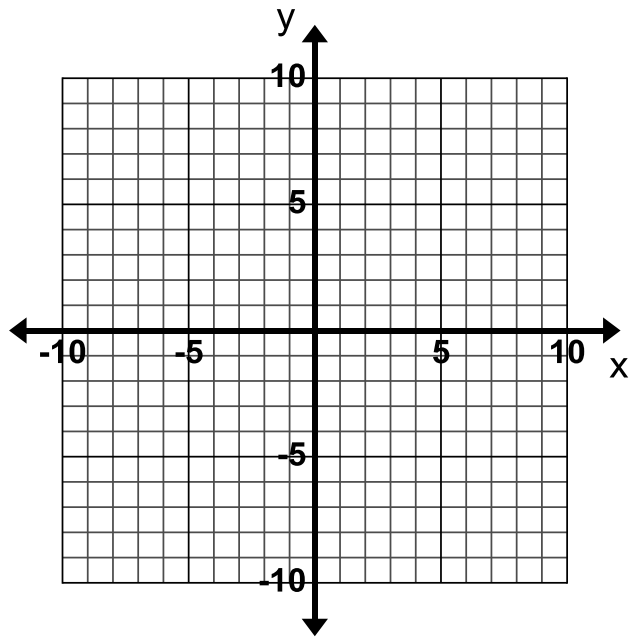


3.1b Homework: Graph from Slope-Intercept Form

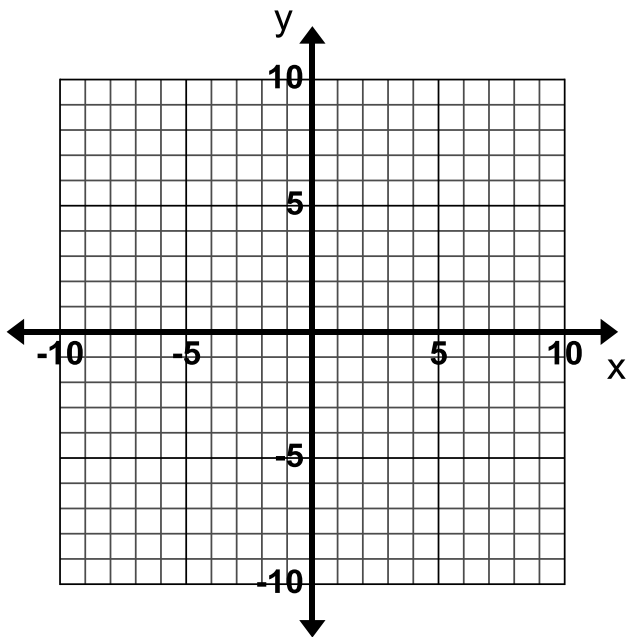
1. Graph $y = -2x + 3$



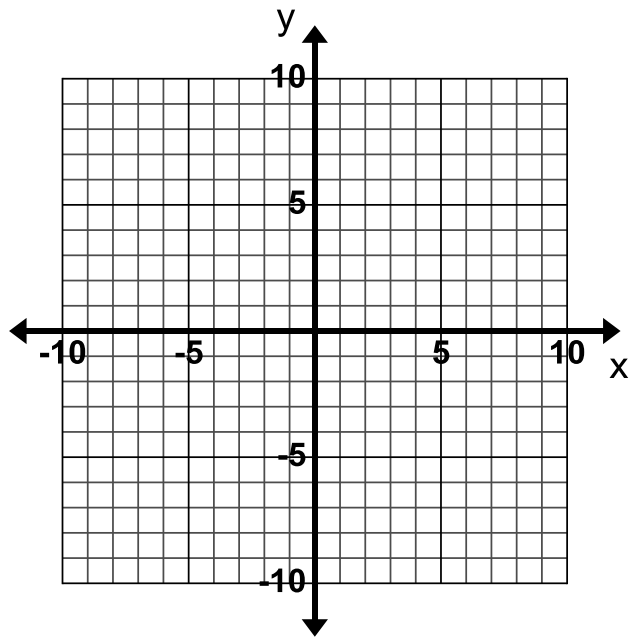
2. Graph $y = 4x - 3$



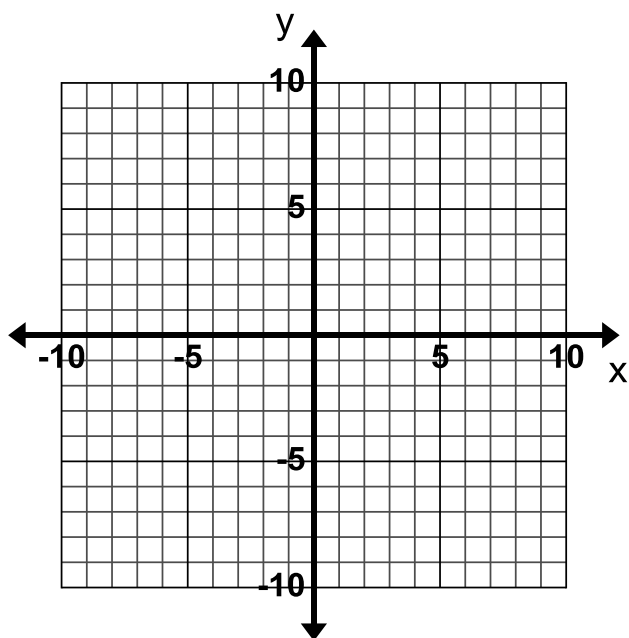
3. Graph $y = -4x$



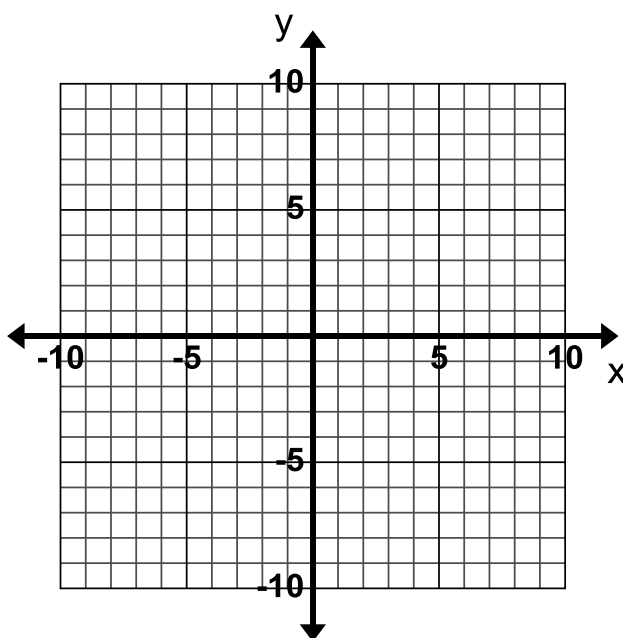
4. Graph $y = 5 + \frac{2}{3}x$



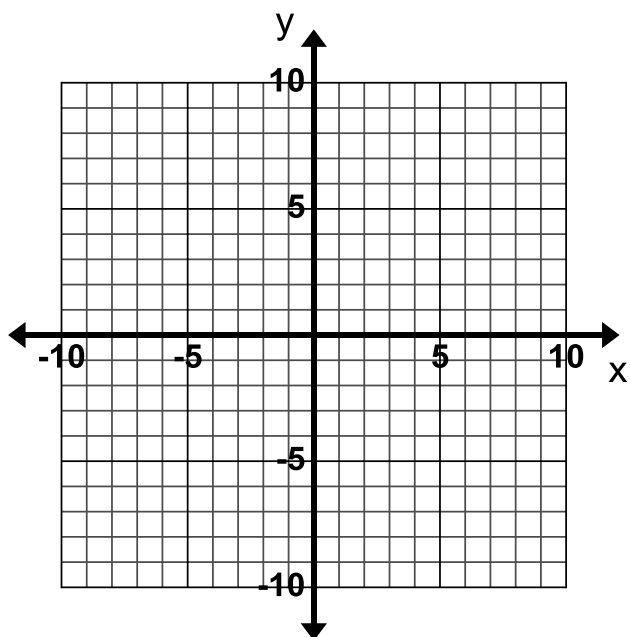
5. Graph $y = -\frac{2}{3}x - 1$



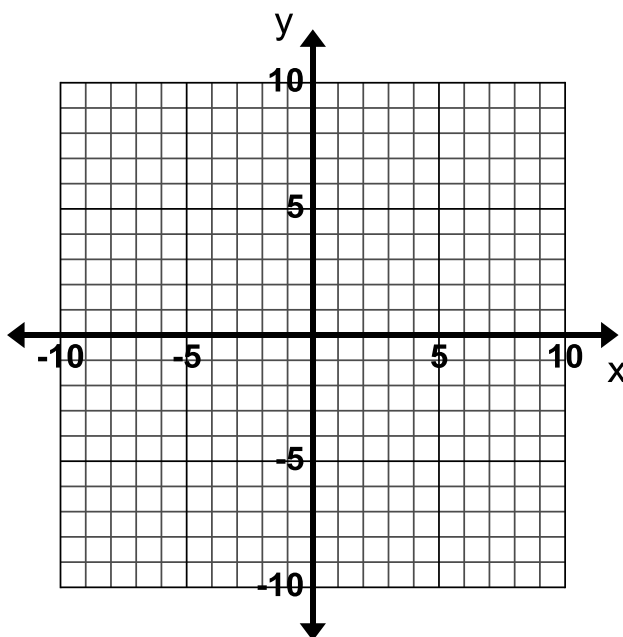
6. Graph $y = 3 + x$



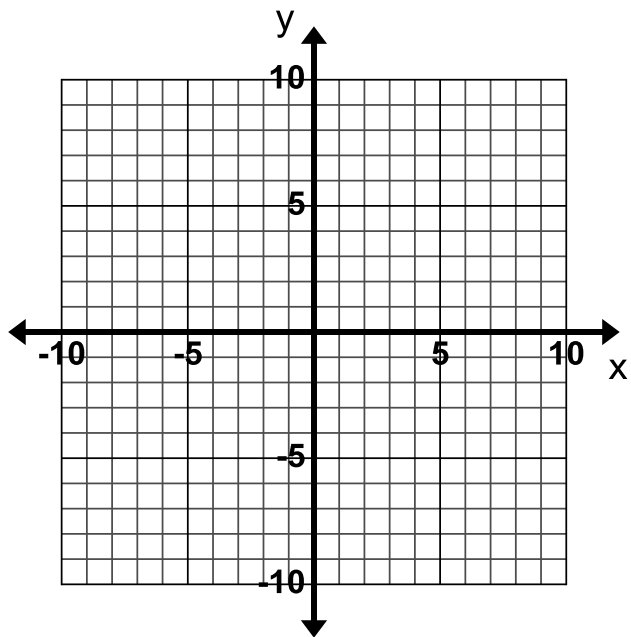
7. Graph $y = 2x - 9$



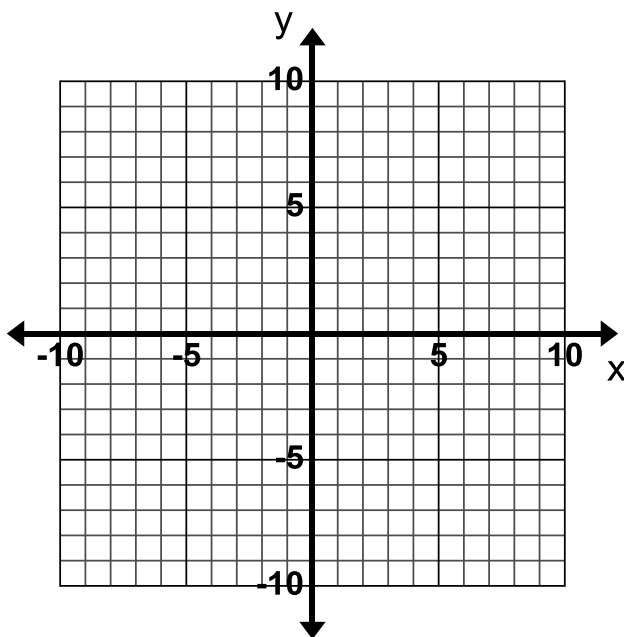
8. Graph $y = 9 - x$



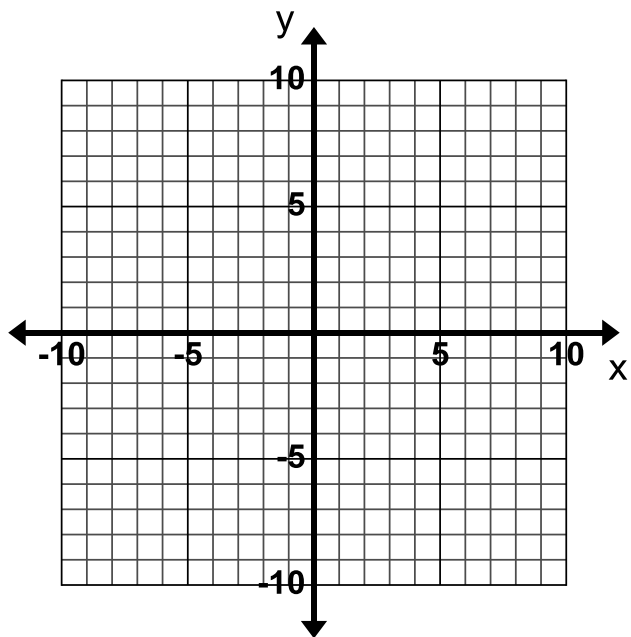
9. Graph $y = 3x - 2$



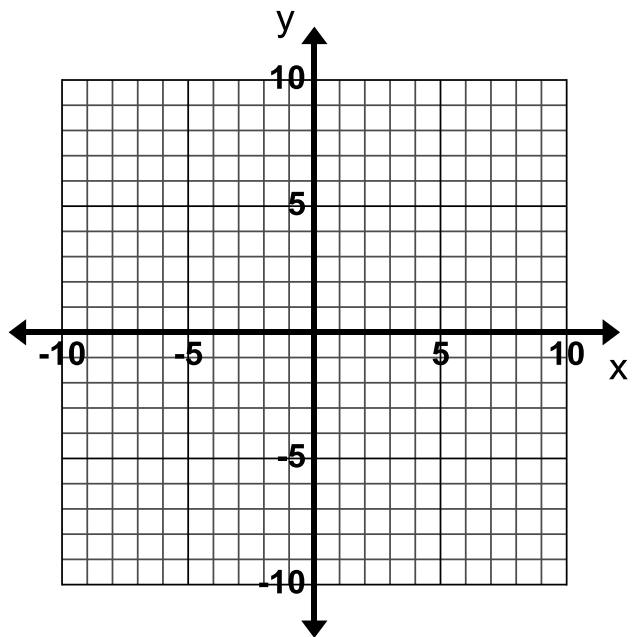
10. Graph $y = 4 - \frac{1}{3}x$



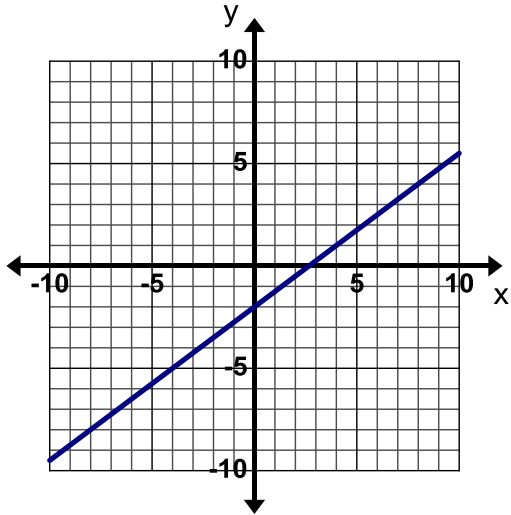
11. Graph $y = 0$



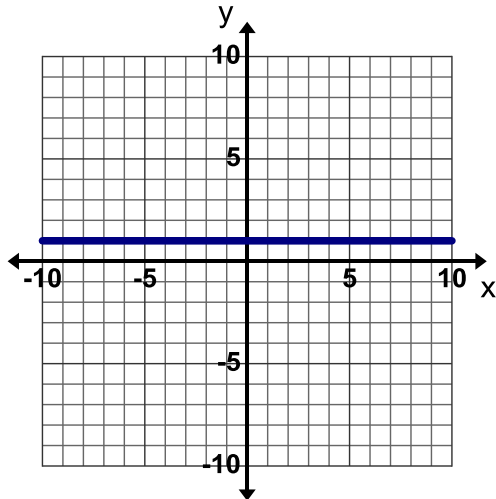
12. Graph $x = 1$



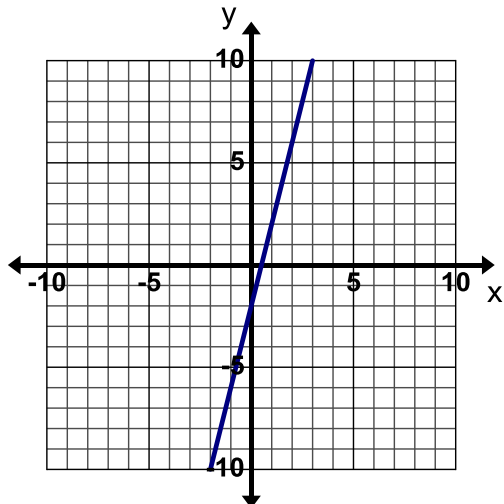
13. **Find, Fix, and Justify:** Lani was asked to graph the line $y = \frac{4}{3}x - 2$. Lani graphed the line below and made a common error. Describe Lani's error and then graph the line correctly on the grid.



14. **Find, Fix, and Justify:** Janeen was asked to graph the line $x = 1$. Janeen graphed the line below and made a common error. Describe Janeen's error and then graph the line correctly on the grid.



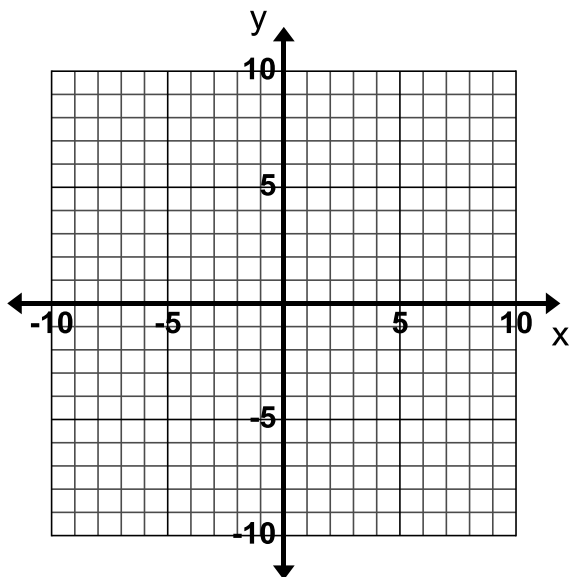
15. **Find, Fix, and Justify:** Zach was asked to graph the line $y = 4 - 2x$. Zach graphed the line below and made a common error. Describe Zach's error and then graph the line correctly on the grid.



3.1c Class Activity: Write and Graph in Slope-Intercept Form

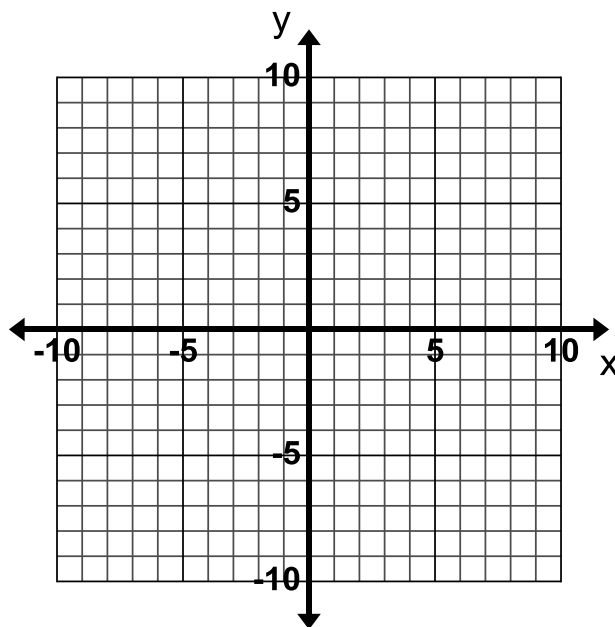
Graph the equation given below. Be ready to discuss your ideas with the class.

$$4x+2y=8$$

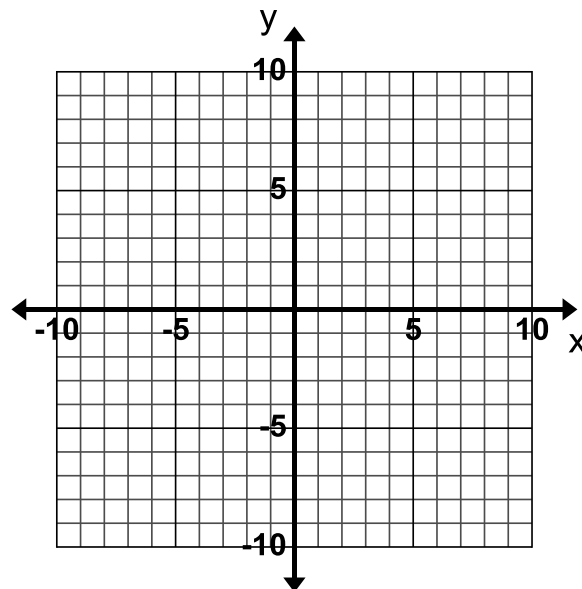


Write down how to graph an equation that is not written in Slope-Intercept Form in the space below.

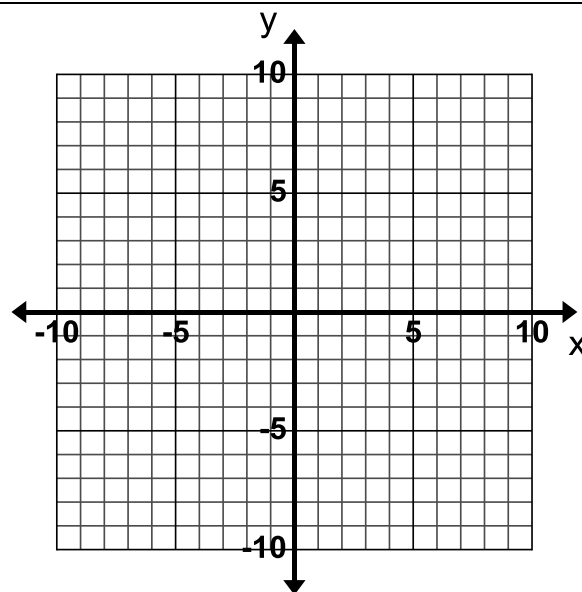
1. Graph $x-5y=10$



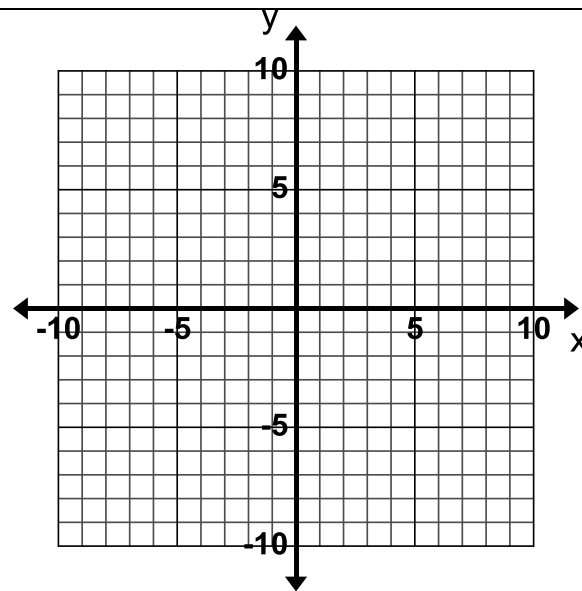
2. Graph $3x + 2y = -6$



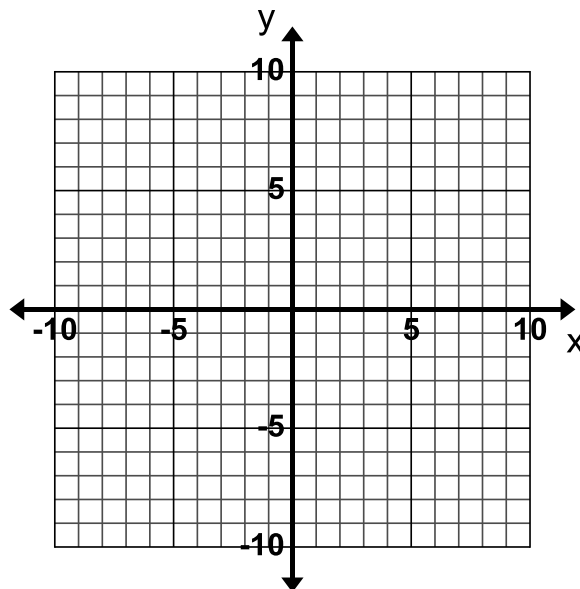
3. Graph $4x + 8y = -24$



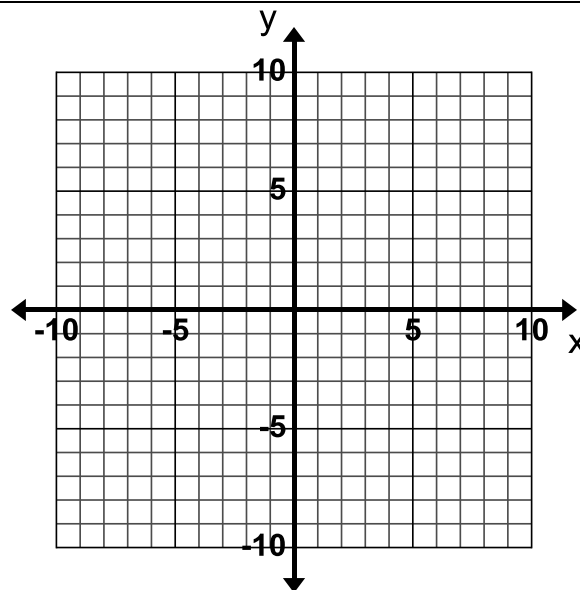
4. Graph $3x + y = 9$



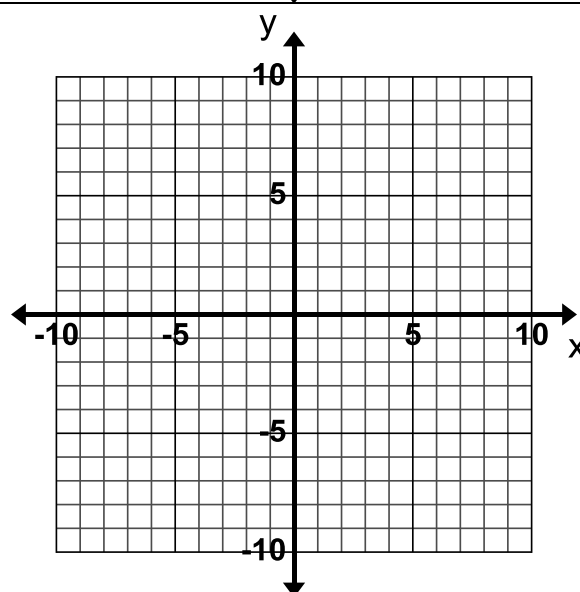
5. Graph $5x - y = -5$



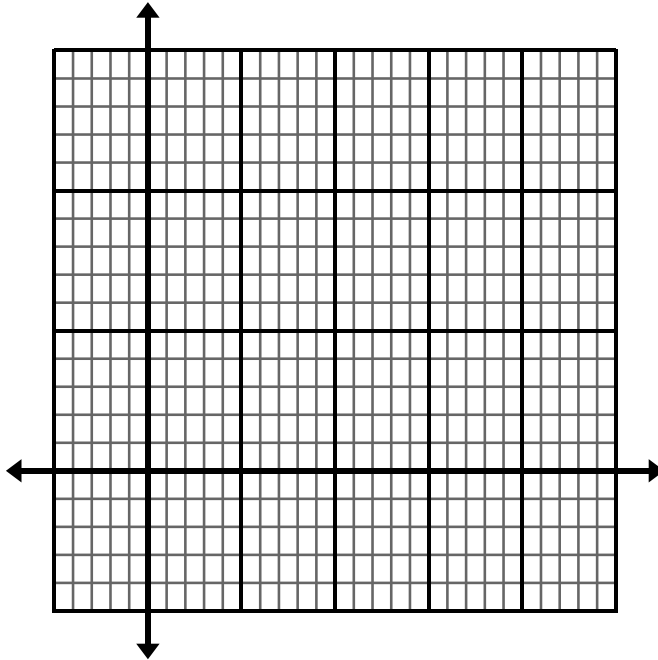
6. Graph $x + 2y = -8 + x$



7. Graph $2(y - x) = 2y - 14$



8. A Health Teacher is writing a test with two sections. The entire test is worth 40 points. He wants the questions in Section A to be worth 2 points each and the questions in Section B to be worth 4 points each. Let x represent the number of questions in Section A and y represent the number of questions in Section B.
- Write an equation that describes all the different combinations of number of questions in Section A and B.
 - Graph this equation to show all possible numbering outcomes for this test.



- Highlight an ordered pair that falls on the line and explain what it represents.

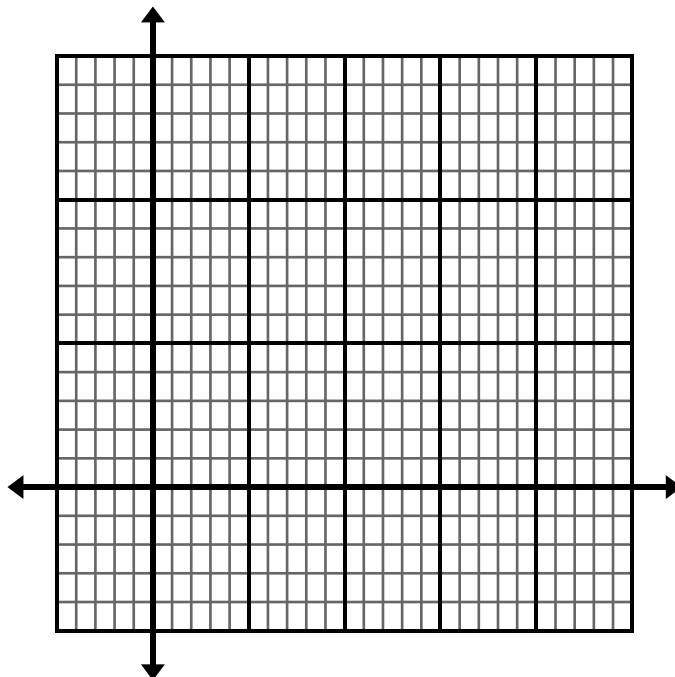
Use the graph to answer the following questions.

- If you have 16 questions in Section A of the test how many questions will be in Section B of the test?
- If you have 8 questions worth 4 points each, how many questions will be worth 2 points each?
- Is it realistic for there to be 9 questions in Section A on the test? Explain your answer.

9. The Hernandez family wants to eat out on Monday night. Salads cost \$8.00 each and sandwiches cost \$6.00 each. They have a gift card for \$42 and want to spend all of it. Let x represent the number of salads that the family can buy and y represent the number of sandwiches that they can buy.

a. Write an equation that represents all the possible combinations of salads and sandwiches that they can buy with \$42.

b. Graph this equation to show all the different salad and sandwich combinations.

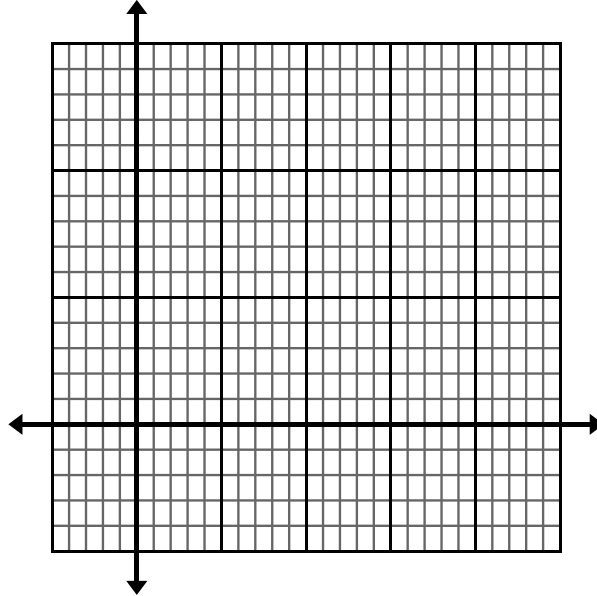


c. What do the ordered pairs on the graph represent?

d. List the realistic combinations for the order. Mark the ordered pairs on the graph that represent these combinations. Explain why these are the only solutions that would work in the real world.



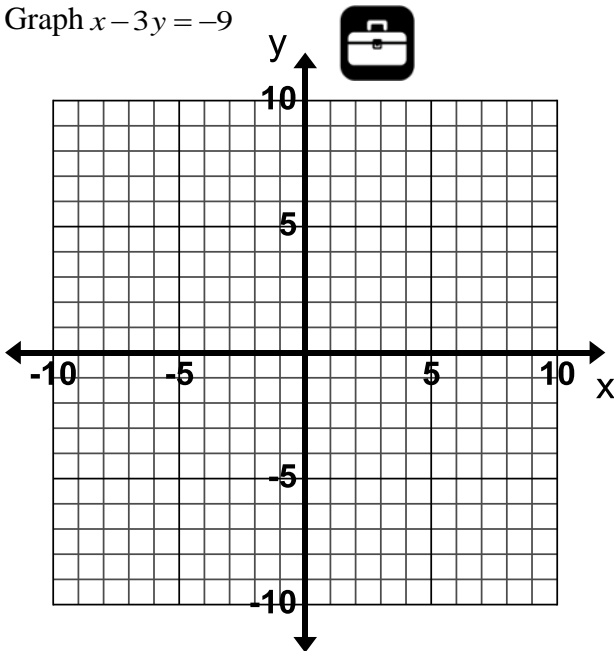
10. The difference between Eugene's age and Wyatt's age's is 5 years. Eugene is older than Wyatt. Let x represent Eugene's age and y represent Wyatt's age.
- Write an equation that represents all the possible different ages that Eugene and Wyatt can be.
 - Graph this equation to show all the age combinations.



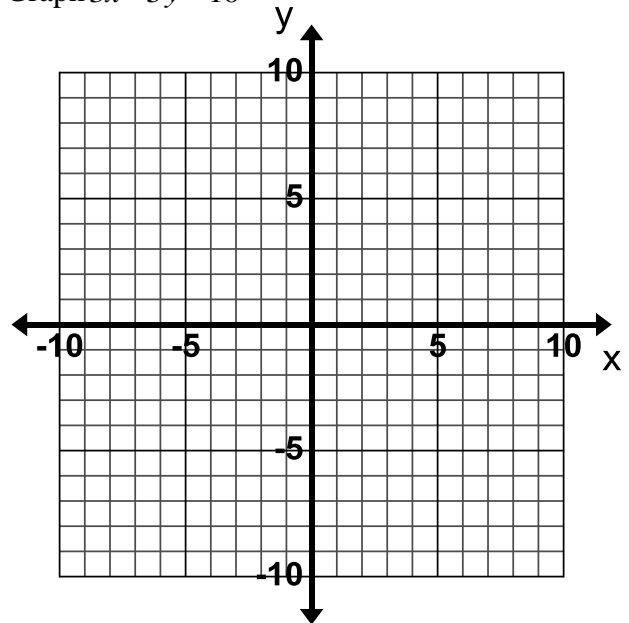
- Mark an ordered pair on the graph that represents the ages of Eugene and Wyatt if Eugene is the same age as you.
- Why does the graph only include mainly the first quadrant?

Graph each equation first by hand and then use a graphing calculator to check your line.

11. Graph $x - 3y = -9$

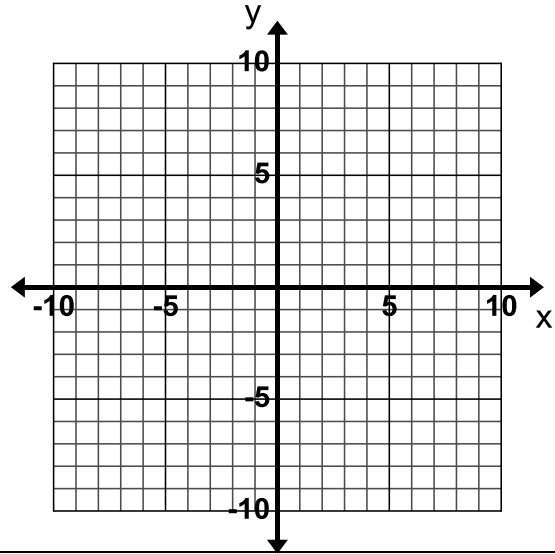


12. Graph $3x - 5y = 10$

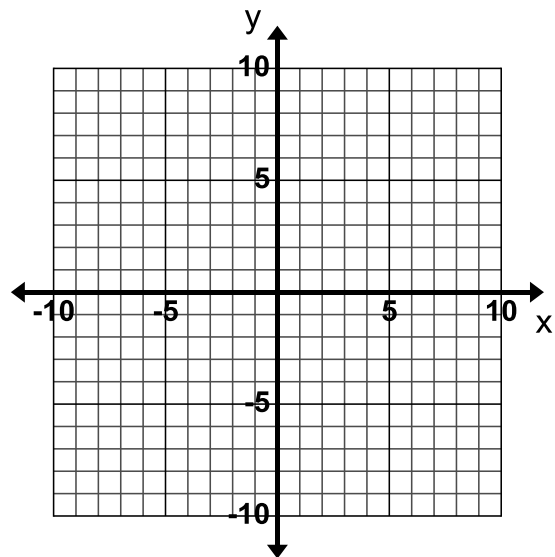


3.1c Homework: Write and Graph in Slope-Intercept Form

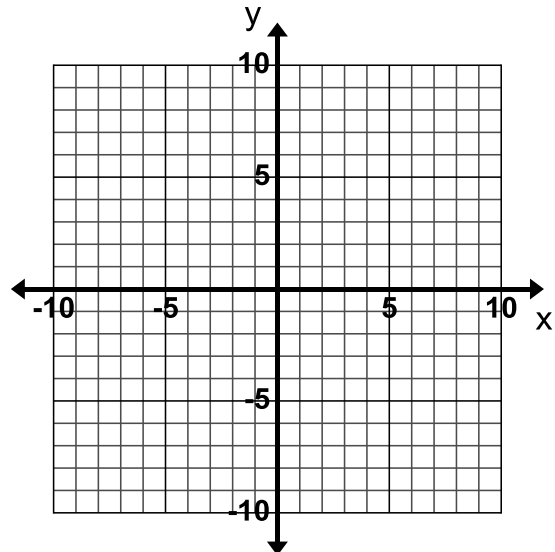
1. Graph $x + y = 4$



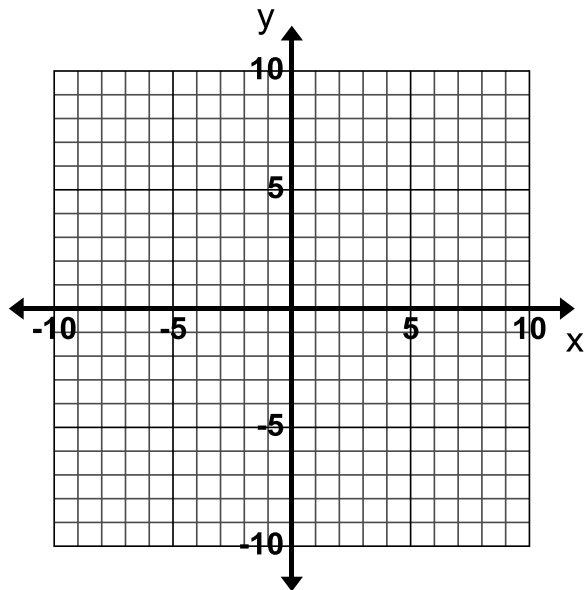
2. Graph $2x + 3y = -12$



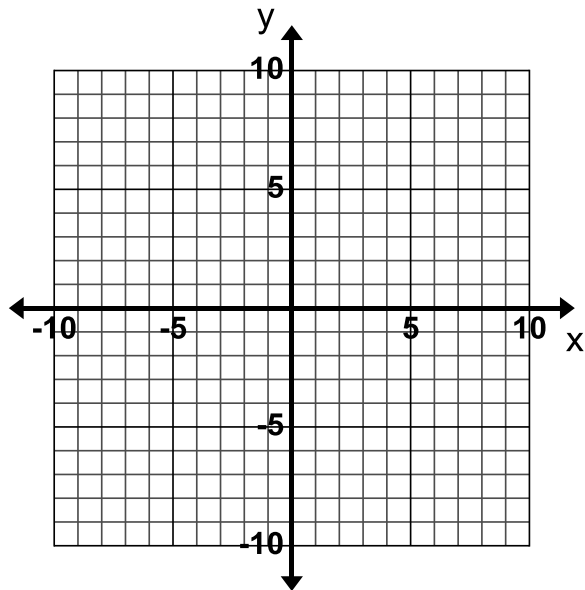
3. Graph $-2x + y = 3$



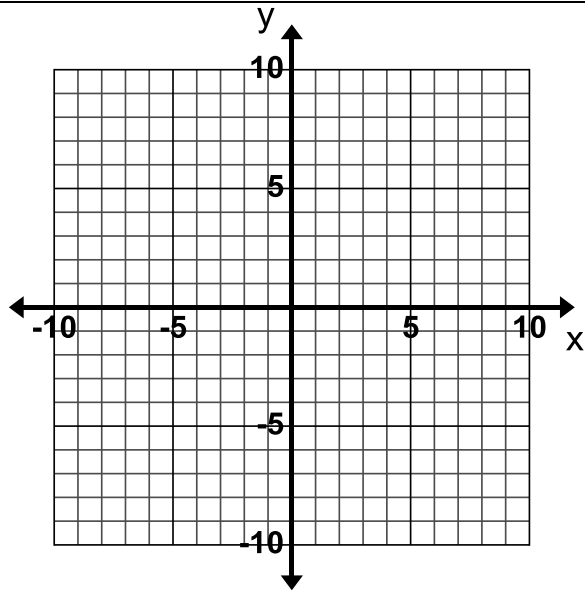
4. Graph $x - y = 10$



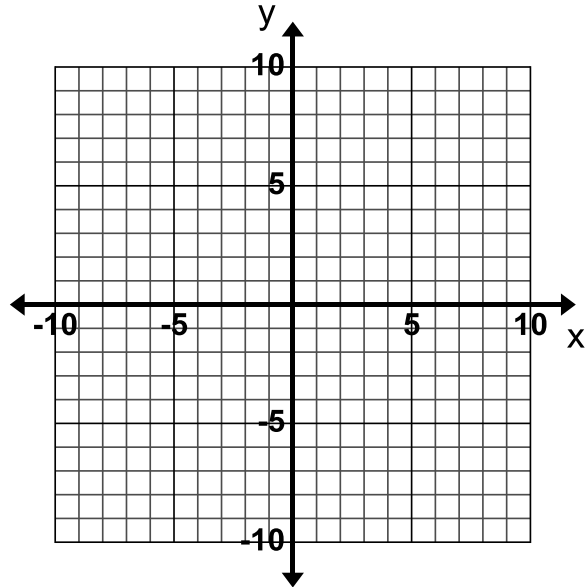
5. Graph $x - 4y = -8$



6. Graph $-\frac{2}{3}x + 3 = 3 + y$



7. Graph $-10 + 5y = 5(y + x)$

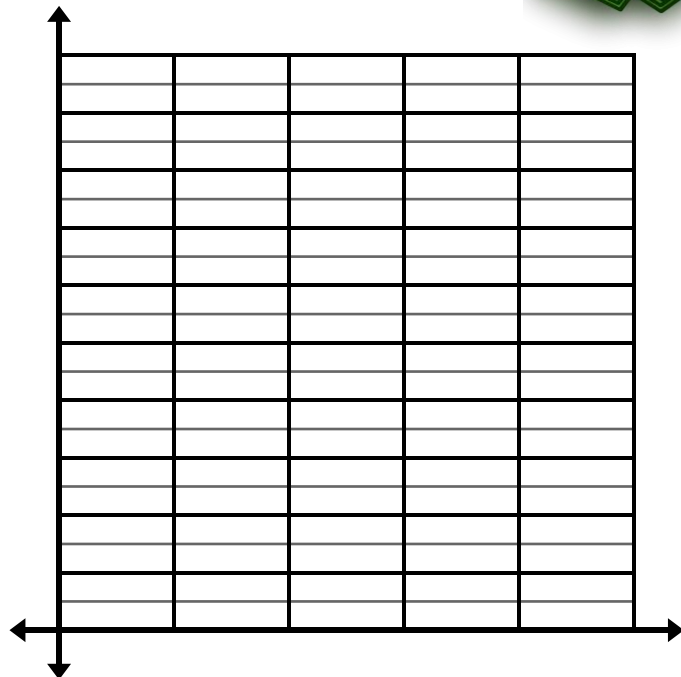


8. You have \$15 in five-dollar bills and one-dollar bills. Let x represent the number of five-dollar bills you have and y represent the number of one-dollar bills you have.

a. Write an equation that represents all the possible combinations of five-dollar bills and one-dollar bills you could have with \$15.

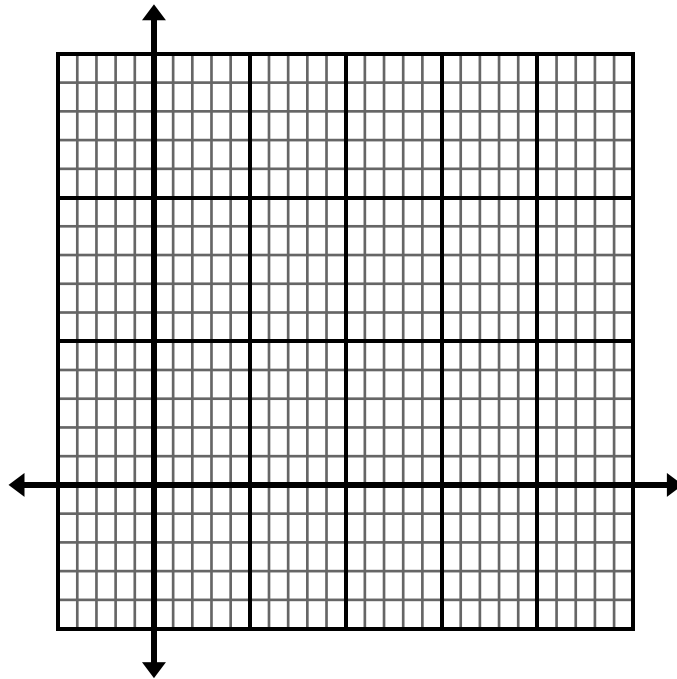


b. Graph this equation to show all the different dollar bill combinations.



c. What do the ordered pairs on the graph represent?

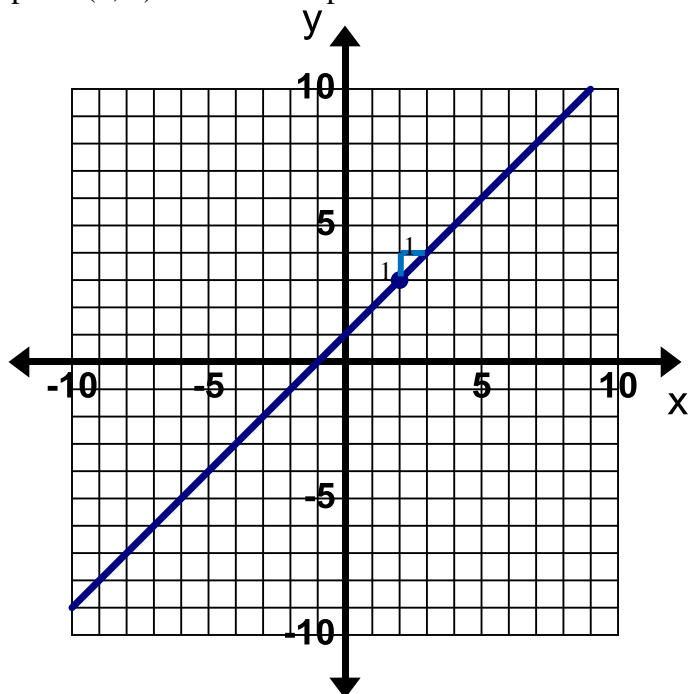
- d. How many one-dollar bills would you have if you have 2 five-dollar bills?
- e. How many five-dollar bills would you have if you have 10 one-dollar bills?
- f. Find and describe the x and y -intercepts for this context.
9. The difference between Lily's age and twice Kenny's age is 6 years. Lily is older than Kenny. Let x represent Lily's age and y represent Kenny's age.
- Write an equation that represents all the possible different ages that Lily and Kenny can be.
 - Graph this equation to show all the age combinations.



- Mark an ordered pair on the graph that represents the age of Lily and Kenny if Lily is 10.
- List at least 6 possible age combinations for Lily and Kenny?

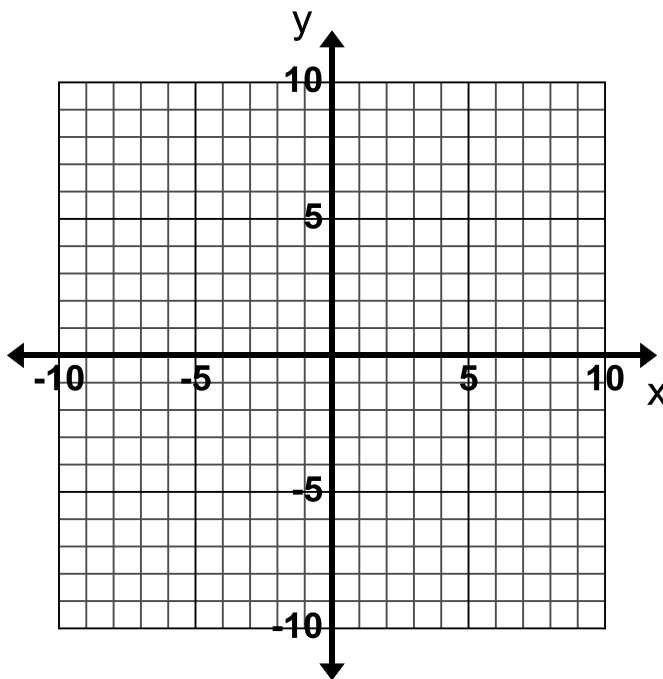
3.1d Class Activity: Graph and Write Equations for Lines Given the Slope and a Point

Example: Graph the line that passes through the point (2, 3) and has a slope of 1.



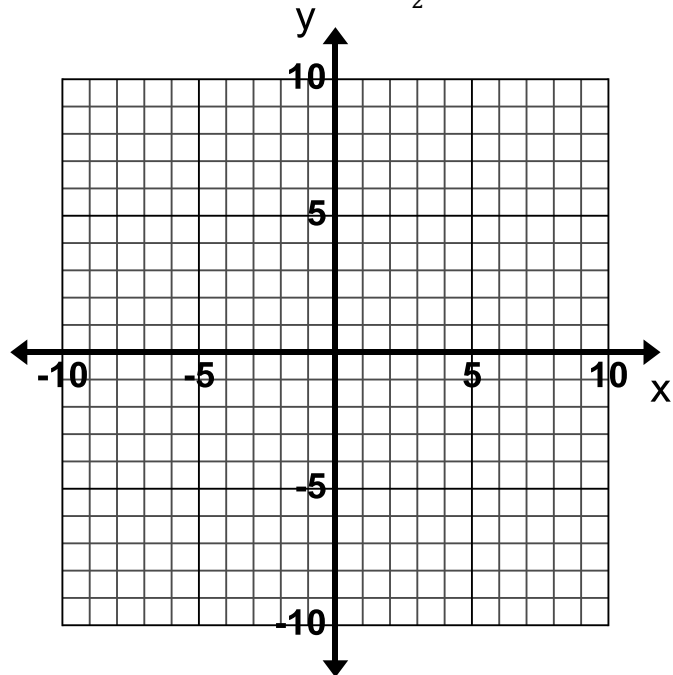
Write the equation of the line that you drew.
 $y = x + 1$

1. Graph the line that passes through the point (-1, 5) and has a slope of 2.



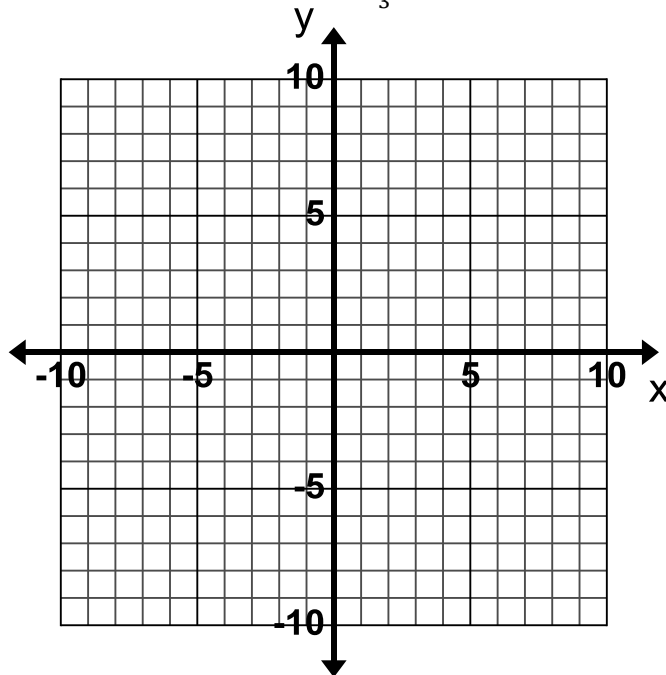
Write the equation of the line that you drew.

2. Graph the line that passes through the point (4, 1) and has a slope of $-\frac{1}{2}$.



Write the equation of the line that you drew.

3. Graph the line that passes through the point (-6, 2) and has a slope of $\frac{1}{3}$.



Write the equation of the line that you drew.

4. How did you use the graph to write the equation of the lines above?

5. Would it be practical to always graph to find the equation? Why or why not?

6. Brainstorm ideas on how you could write the equation of the line without graphing when you are given a point and the slope. Consider how you could use the equation $y = mx + b$ to find the y-intercept if you know the slope and a point on the line. Using an example from the previous page may help you work through the process.

Directions: Find the equation of the line that passes through the given point with the given slope.

7. Through $(-1, -6)$; $m = 4$

8. Through $(-3, 4)$; $m = -\frac{2}{3}$

9. Through $(4, -1)$; $m = \frac{3}{2}$

10. Through $(3, 2)$; $m = 1$

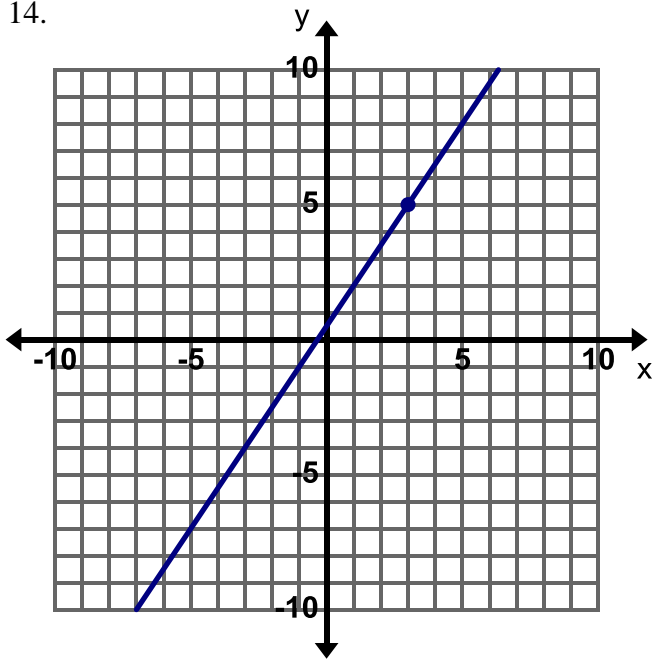
11. Through $(3, 5)$; $m = \text{undefined}$

12. Through $(3, -4)$; $m = 0$

13. **Find, Fix, and Justify:** Felipe was asked to write the equation of the line that has a slope of $\frac{1}{3}$ and passes through the point $(6, 4)$. Felipe made a common error and wrote the equation $y = \frac{1}{3}x + 4$. Describe Felipe's error and write the correct equation in the space below.

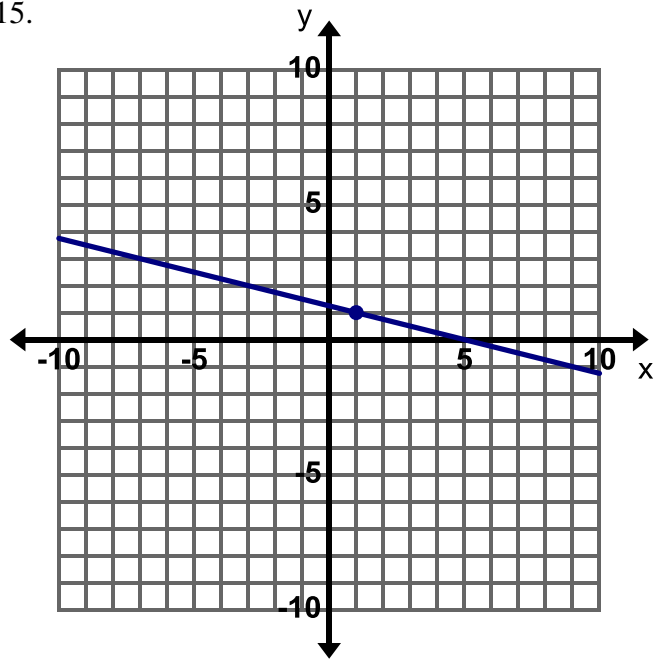
Directions: Write the equation of the line. Show your work.

14.



Equation:

15.



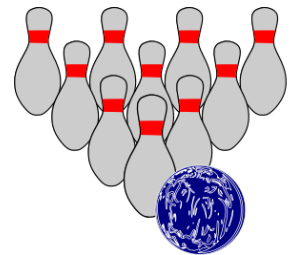
Equation:

16. Harper is at the bowling alley. She has spent \$13 so far renting bowling shoes and playing two rounds of bowling. The cost for each round is \$5 per person. Let x represent the number of rounds she has played and y represent the total cost.

a. What is the rate of change for the situation above?

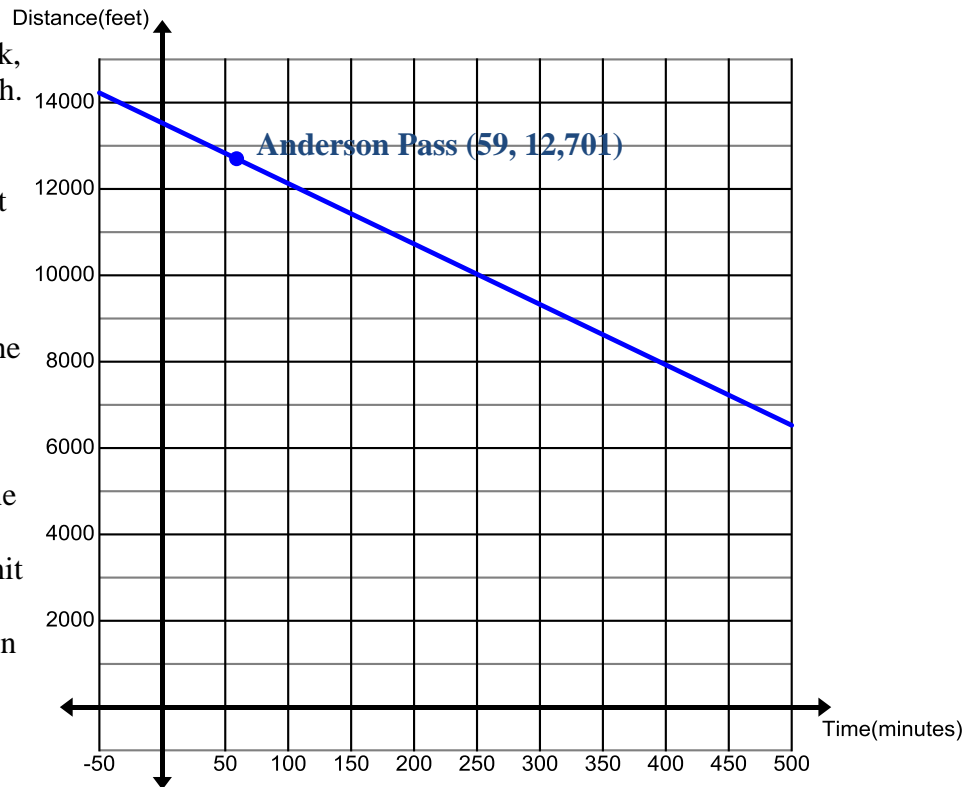
b. What point is addressed in the situation above?

c. Write an equation in Slope-Intercept form to represent the relationship between the number of rounds of bowling played and the total cost.



d. What does the y -intercept in this relationship represent?

17. Art and Sierra are descending King's Peak, the highest peak in Utah. They have been climbing down the mountain losing 14 feet of elevation every minute. They reach Anderson Pass 59 minutes after leaving the summit (the top of the peak). The graph represents the relationship between the time that has passed since leaving the summit on the x -axis and the elevation represented on the y -axis.

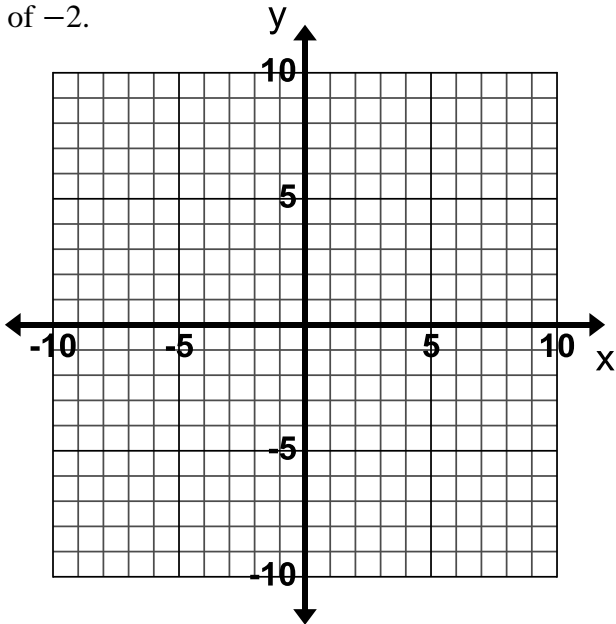


- What is the rate of change for the situation above?
- What is the elevation of Anderson Pass?
- Write an equation in Slope-Intercept form to represent the relationship between the number of minutes climbing down the peak and the current elevation.
- How high is King's peak?
- Use your equation to predict how long it will take Art and Sierra to get to Gunsight Pass which has an elevation of 11,888 feet if they continue to descend at the same rate.
- Label the summit for King's Peak, Anderson Pass, and Gunsight Pass. Also use the graph or equation to predict the elevation of Dollar Lake if Art and Sierra reach it after 3 hours and 16 min. Once you have determined the elevation for Dollar Lake label it on the graph as well.



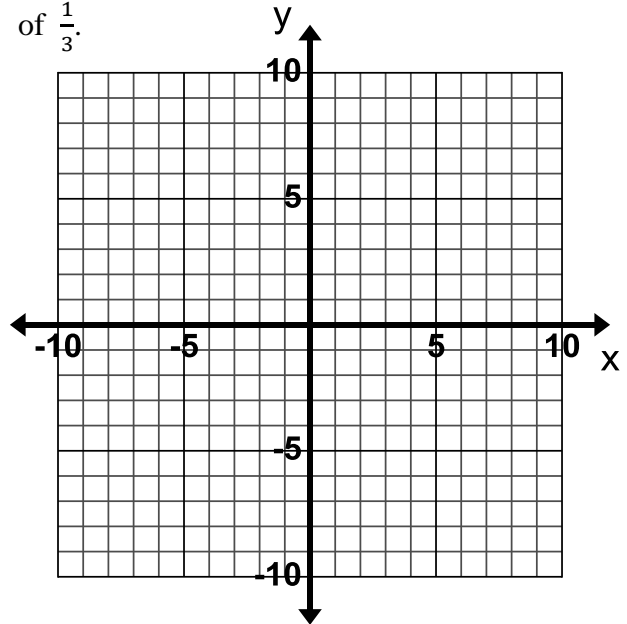
3.1d Homework: Graph and Write Equations for Lines Given the Slope and a Point

1. Graph a line that does the following:
Passes through the point $(4, -3)$ and has a slope of -2 .



Write the equation of the line that you drew.

2. Graph a line that does the following:
Passes through the point $(-6, 3)$ and has a slope of $\frac{1}{3}$.



Write the equation of the line that you drew.

Directions: Write the equation for the line that has the given slope and contains the given point.

3. slope = 1

passes through $(3, 7)$

4. slope = $\frac{2}{3}$

passes through $(3, 4)$

5. slope = 5

passes through $(6, -10)$

6. slope = -2

passes through $(3, 1)$

7. slope = 5

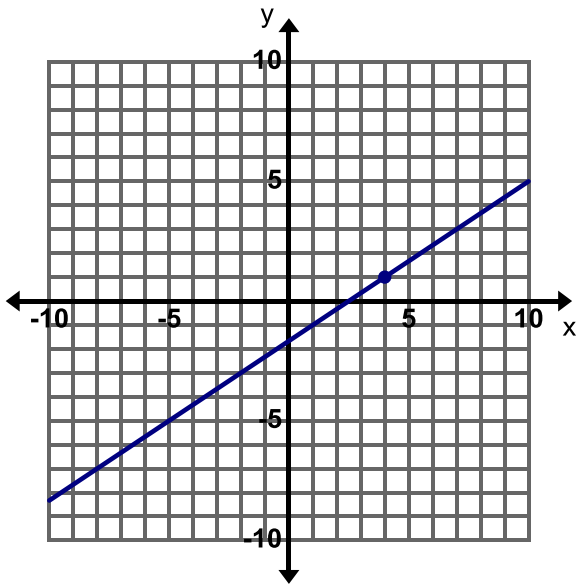
passes through $(-2, 8)$

8. slope = $\frac{1}{3}$

passes through $(0, 2)$

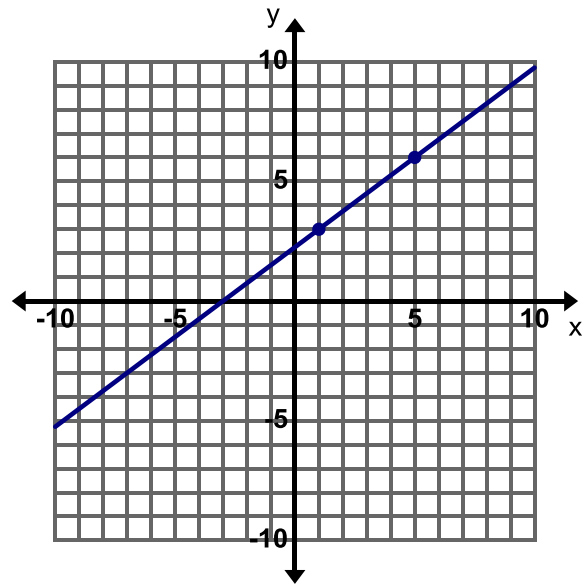
Directions: Write the equation of the line.

9.



Equation:

10.

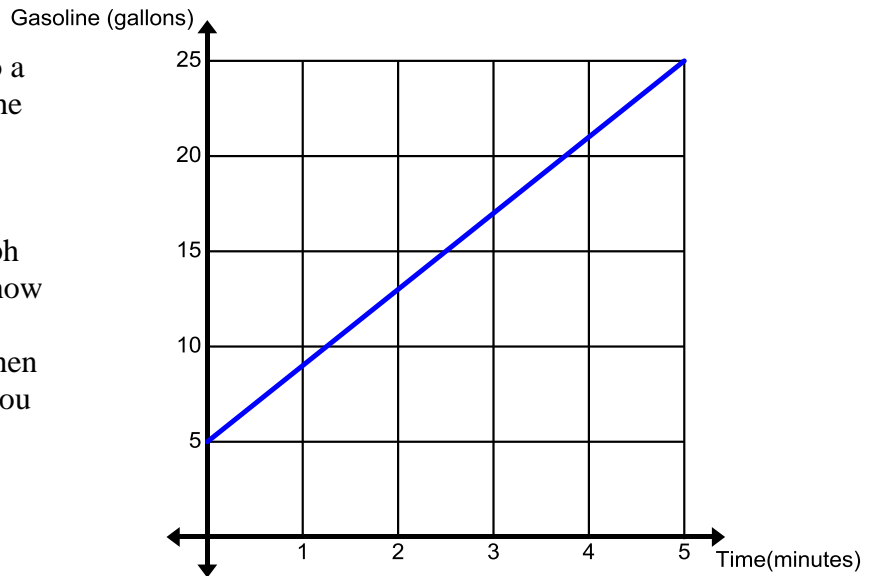


Equation:

11. In your own words, explain how to write the equation of a line in slope-intercept form when you are given the slope and a point.
12. At the beginning of the year Monica puts a set amount of money into her health benefit account. Every month she withdraws \$15 from this account for her contact lenses. After 3 months she has \$255 left in her account.
- What is the rate of change for this situation?
 - What point on the line is described in the story above?
 - Write an equation in slope-intercept form to represent the relationship between the time that has passed and the amount of money left in Monica's account. Let x represent the time in months and y represent the amount of money remaining in the account.

- d. If Monica does not use all of the money in her account by the end of the year she loses it. Monica only uses the money in the account for contact lenses; will she lose money at the end of the year?

13. The graph shown describes the amount of gasoline being put into a truck that has a 25 gallon tank. The gasoline is pumped at a rate of 4 gallons per minute.



- a. Label the point on the graph where you can determine how long it takes to fill the 25 gallon tank up with gas. Then state how the point helps you to determine the time.

- b. What is the rate of change for this story?
- c. Write an equation in slope-intercept form that describes the relationship between the time that has passed and the amount of gasoline in the tank.
- d. How much gasoline was in the tank before the tank was filled?
- e. Is there more than one method for finding the equation of this line?

14. **Think about this...**

In this lesson, you were given the slope and a point on the line and used this information to write the equation of the line in slope-intercept form. In the next lesson, you will be given 2 points and asked to write the equation in slope-intercept form. Write down your thoughts on how you might do this.

Now try it...

Write the equation of the line that passes through the points (1, 4) and (3, 10).

3.1e Class Activity: Write Equations for Lines Given Two Points

1. Describe how to write the equation of a line in slope-intercept form when you are given two points on the line.

Directions: Write the equation of the line that passes through the points given.

2. $(0, 4), (-1, 3)$	3. $(-5, 9), (-2, 0)$	4. $(0, 0), (3, -6)$
5. $(2, 2), (4, 3)$	6. $(1, 2), (1, -6)$	7. $(-2, 4), (0, 4)$

8. A box containing 20 cans of SPAM weighs 250 ounces. A box containing 28 cans of SPAM weighs 346 ounces.
 - a. Write an equation that relates the number of cans of SPAM to the weight of the box. Let x represent the number of cans of SPAM and y represent the weight of the box in ounces.
 - b. What does the y -intercept in the equation represent?
 - c. Use your equation to predict the weight of a box that contains 40 cans of SPAM.

Directions: Write an equation for a line from the information given in each table.

9.

x	y
3	8
4	9
5	10
6	11

Equation:

10.

x	y
2	3
4	6
6	9
8	12

Equation:

11.

x	y
-12	-1
-10	-1
-8	-1
-6	-1

Equation:

12. Toa takes the freeway home from work so he can use his cruise control. The table below shows the time x in minutes since he entered the freeway related to the distance y in miles he is from his exit at several points on his journey.

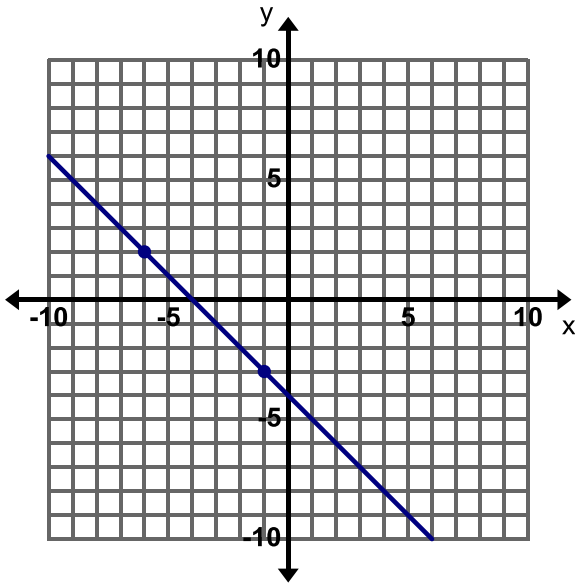
$Time(x)$	$Distance(y)$
8	34
20	25
32	16
40	10



- Write an equation that relates the time Toa has been on the freeway to the distance he is from his exit.
- What does the y -intercept represent in this equation?
- Use your equation to predict how much time will pass before Toa reaches his exit.

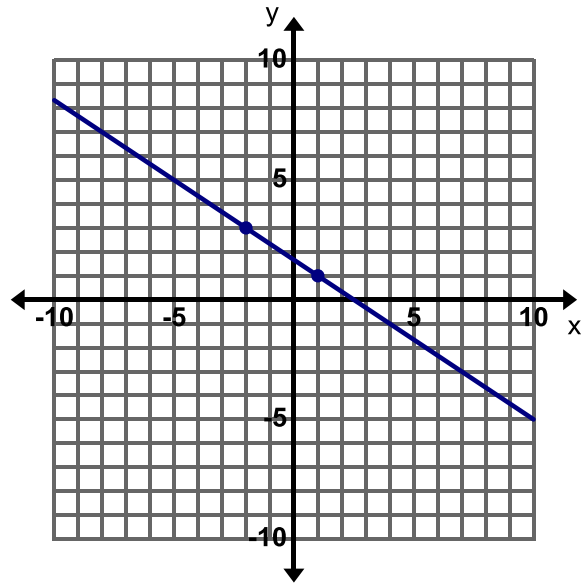
Directions: Write an equation for the line given.

13.



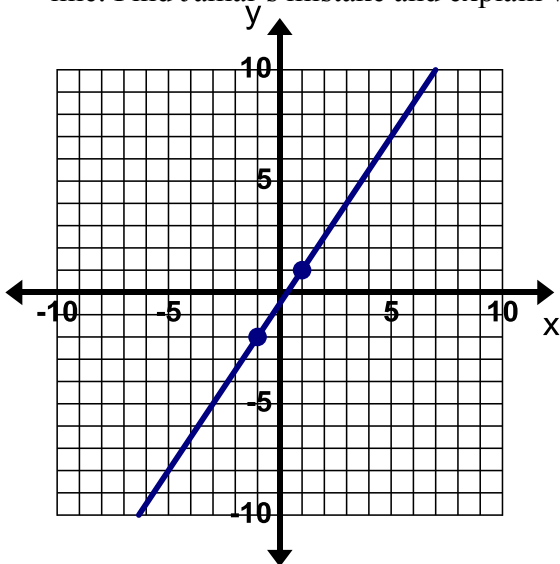
Equation:

14.



Equation:

15. **Find, Fix, and Justify:** Jamal was asked to write an equation for the line on the graph below. Jamal's work is shown to the right of the graph, he has made a common mistake in writing the equation for the line. Find Jamal's mistake and explain what he did wrong. Then write the correct equation for the line.



$$(-1, -2) \quad (1, 1)$$

$$m = \frac{-2 - 1}{-1 - 1} = \frac{-3}{-2} = \frac{3}{2}$$

$$1 = \frac{3}{2}(1) + b$$

$$1 = \frac{3}{2} + b \Rightarrow b = \frac{3}{2}$$

$$y = \frac{3}{2}x + \frac{3}{2}$$

3.1e Homework: Write Equations for Lines Given Two Points

Directions: Write the equation of the line that passes through the points given.

1. $(0, 2)$ and $(-2, 0)$	2. $(5, 0)$ and $(-10, -5)$	3. $(1, 1)$ and $(3, 3)$
4. $(4, 2)$ and $(0, -2)$	5. $(2, 3)$ and $(-2, 3)$	6. $(0, -1)$ and $(3, -2)$

7. Clarissa is saving money at a constant rate. After 2 months she has \$84 in her savings account. After 5 months she has \$210 in her account.
- Write an equation that relates the amount of money she has in her savings account to the number of months that have passed. Let x represent the number of months and y represent the total amount of money in the account.
 - Interpret the y -intercept and slope of the equation for this context.
 - Clarissa would like to purchase a plane ticket to visit her sister exactly one year after she began saving money. The plane ticket costs \$450. Will she have enough money in the account to pay for the ticket.

Directions: Write an equation for the line from the information given in each table.

8.

x	y
-1	5
0	2
1	-1
2	-4

Equation:

9.

x	y
-2	2
2	3
6	4
10	5

Equation:

10.

x	y
0	-1
0	0
0	1
0	2

Equation:

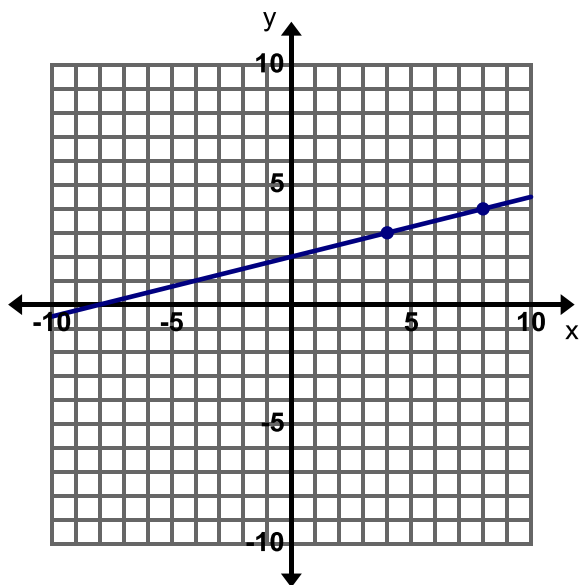
11. Create your own real-world story that matches the table below. Write an equation to represent the relationship between your variables.



x	y
5	20
12	41
20	65
22	71

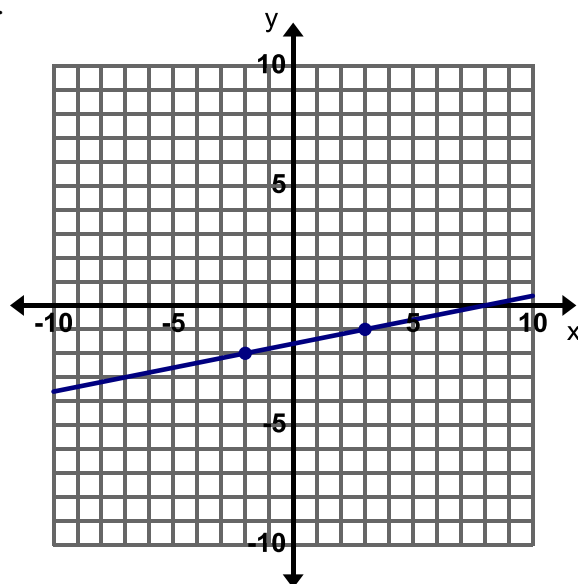
Directions: Write an equation for the line given.

12.



Equation:

13.



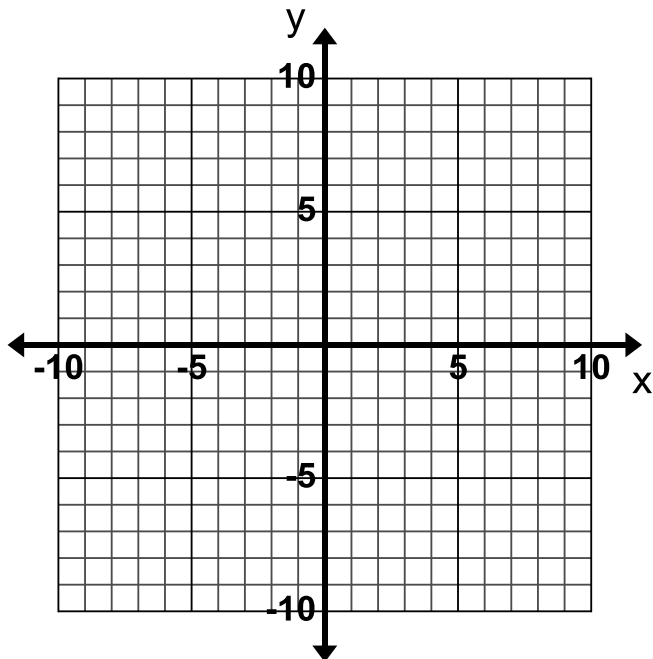
Equation:

Extra for Experts: Consider the three points $(-2, 4)$, $(1, 2)$ and $(4, r)$ on the same line. Find the value of r and explain your steps.

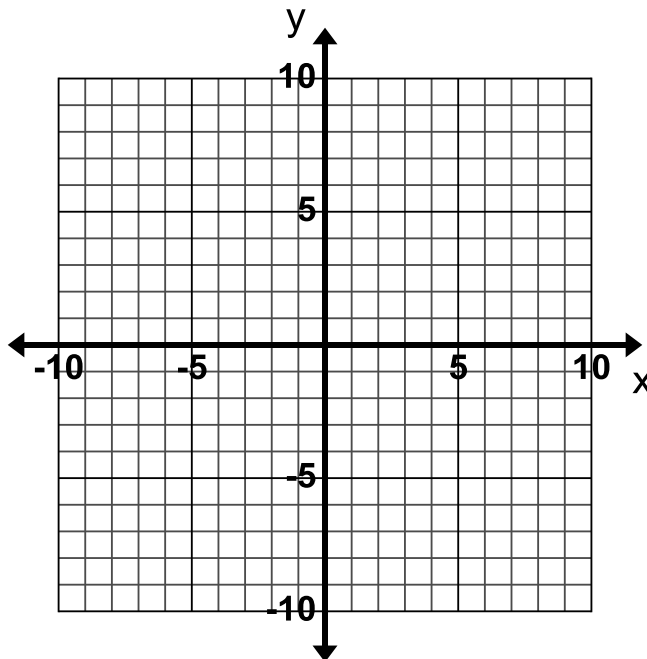
3.1f Class Activity: Graphing and Writing Equations for Lines, Mixed Review

Directions: Graph the lines for the following given information.

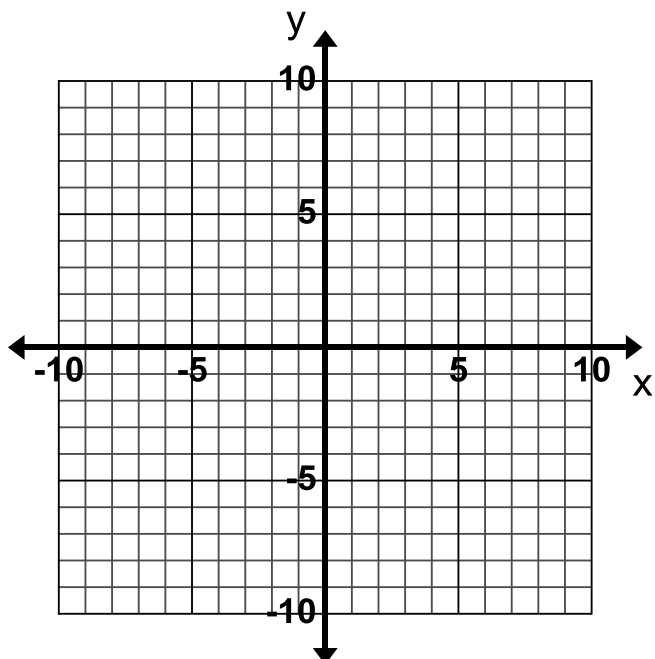
1. The equation of the line is $y = -\frac{1}{4}x$.



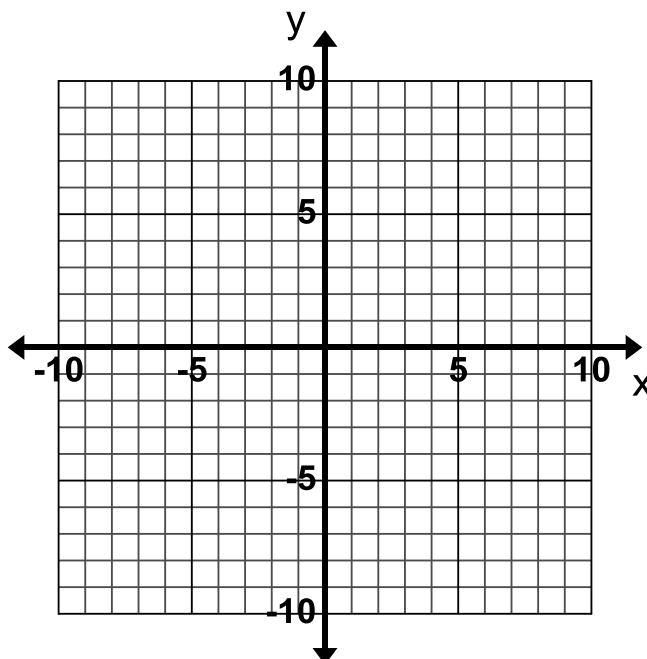
2. The equation of the line is $y = x - 8$.



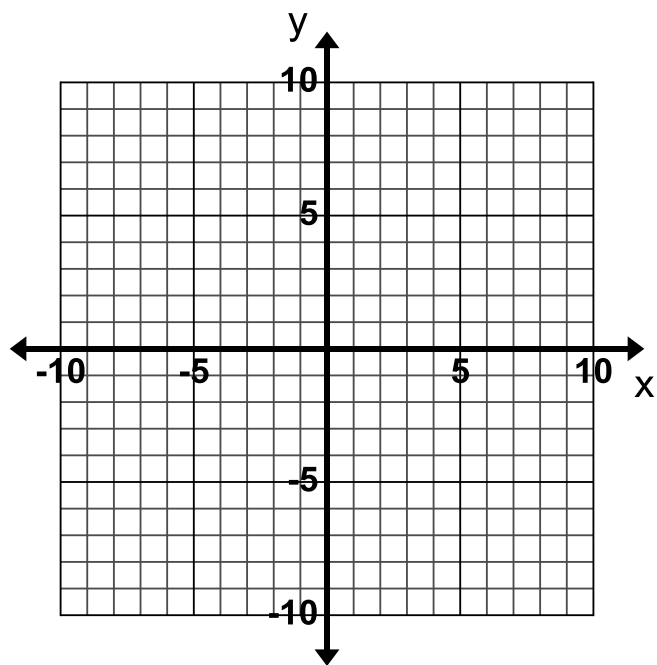
3. The equation of the line is $7x + y = 9$.



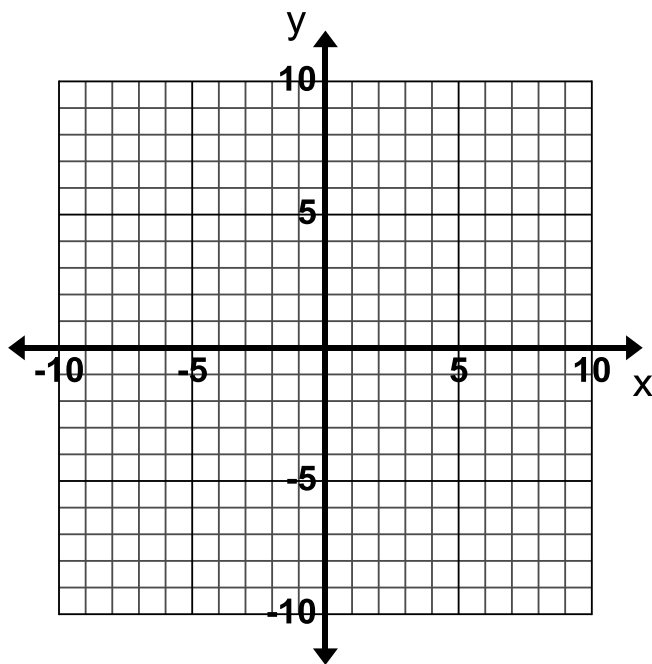
4. The equation of the line is $-3x + 5y = 30$.



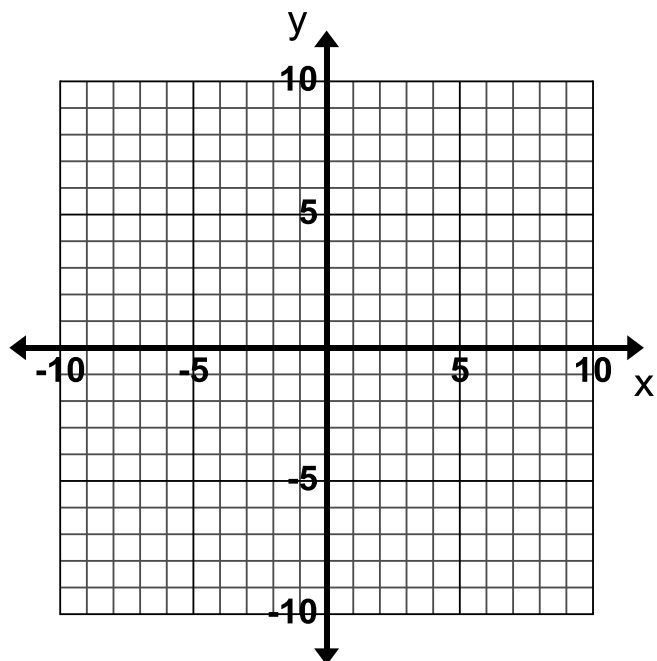
5. The equation of the line is $x + 2(y + 1) = x - 14$.



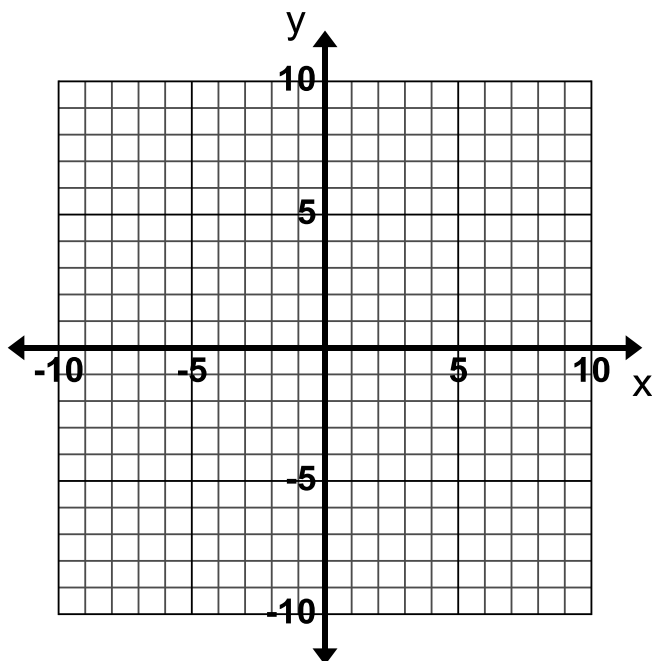
6. The equation of the line is $x = 1$.



7. The line contains the point $(-5, -5)$ and has a slope of 3.



8. The line contains the point $(-7, 3)$ and has a slope of 0.



Directions: Write the equation in slope-intercept form for each line based on the information given.

9. The slope of the line is $-\frac{1}{2}$ and the y-intercept is -5 .

10. The line has a slope of 4 and goes through the point $(6, -1)$.

11. The line contains the points $(-2, 7)$ and $(3, -3)$.

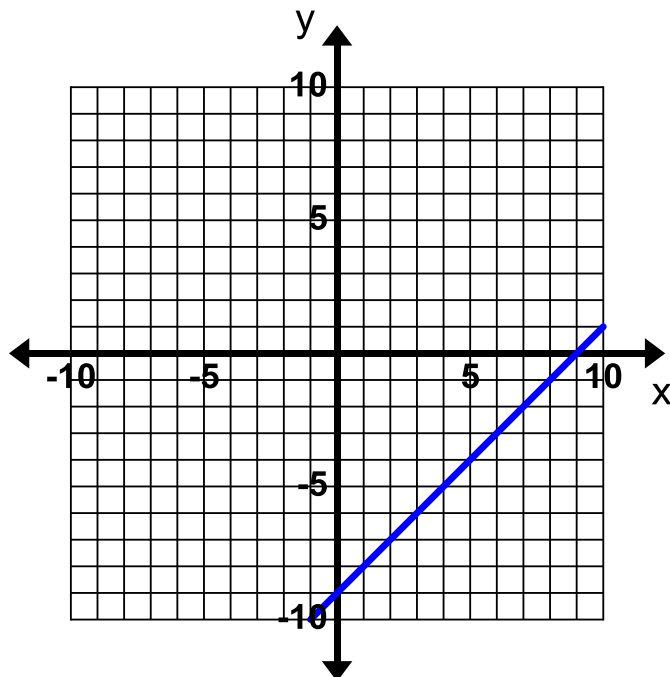
12. The line contains the points in the table.

x	y
0	-6
1	-2
2	2
3	6

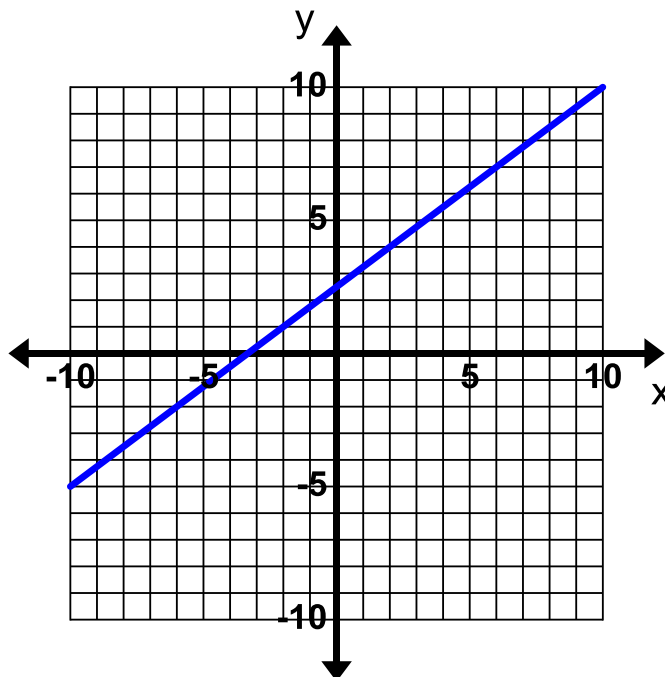
13. The line contains the points in the table.

x	y
2	2
4	1
8	-1
14	-4

14. The line graphed below.



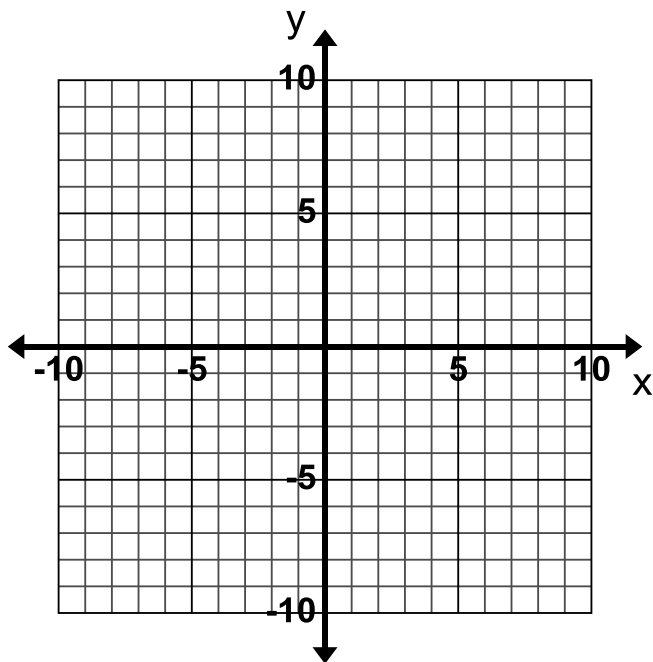
15. The line graphed below.



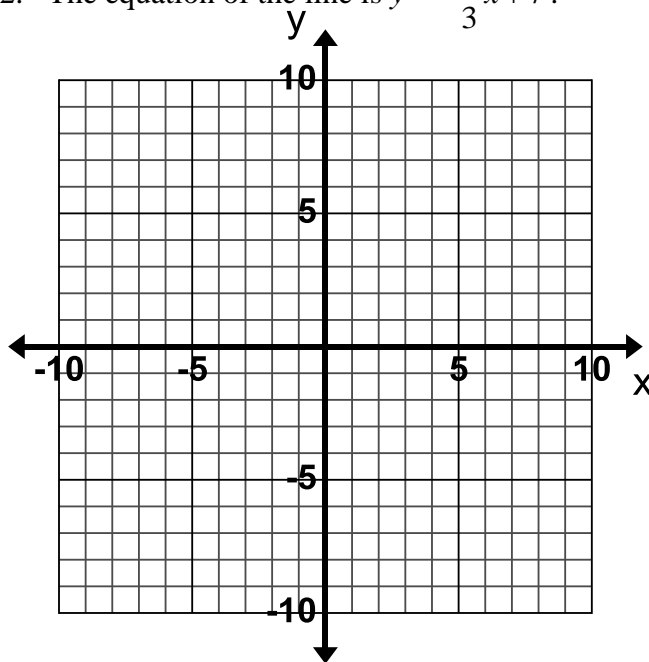
3.1f Homework: Graphing and Writing Equations for Lines, Mixed Review

Directions: Graph the lines for the following given information.

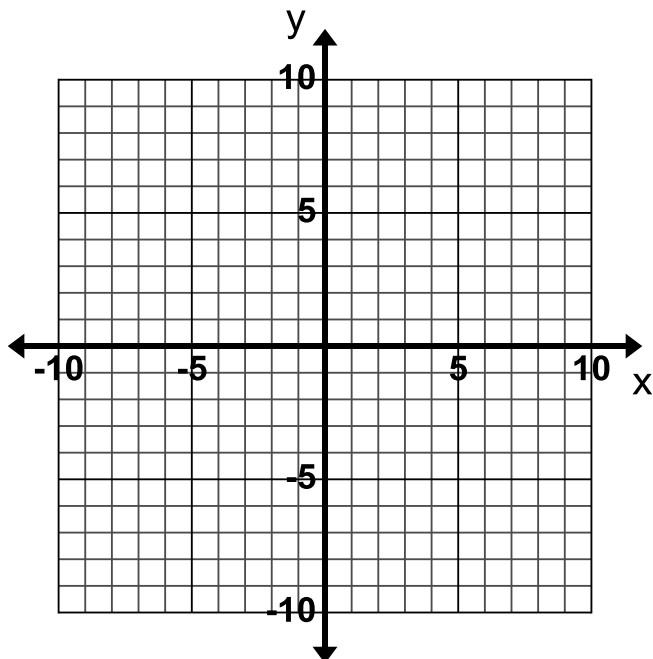
1. The equation of the line is $y = 5x - 8$.



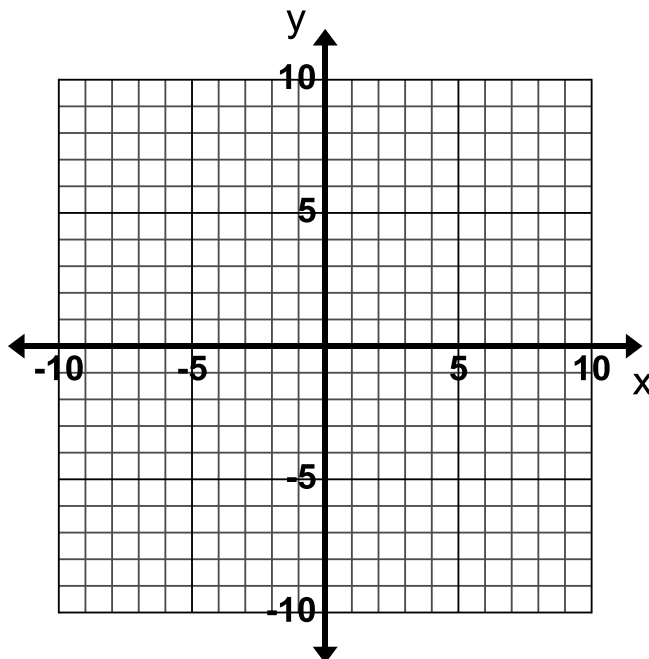
2. The equation of the line is $y = -\frac{1}{3}x + 7$.



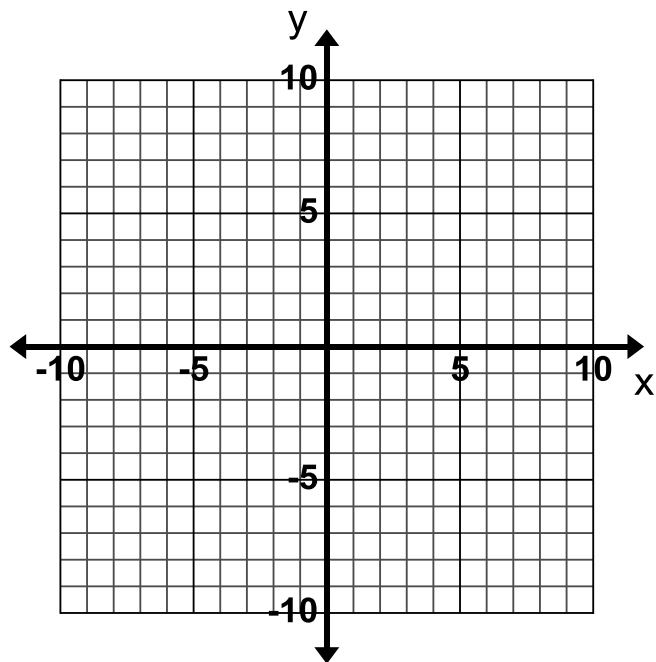
3. The equation of the line is $-4x + 2y = 8$.



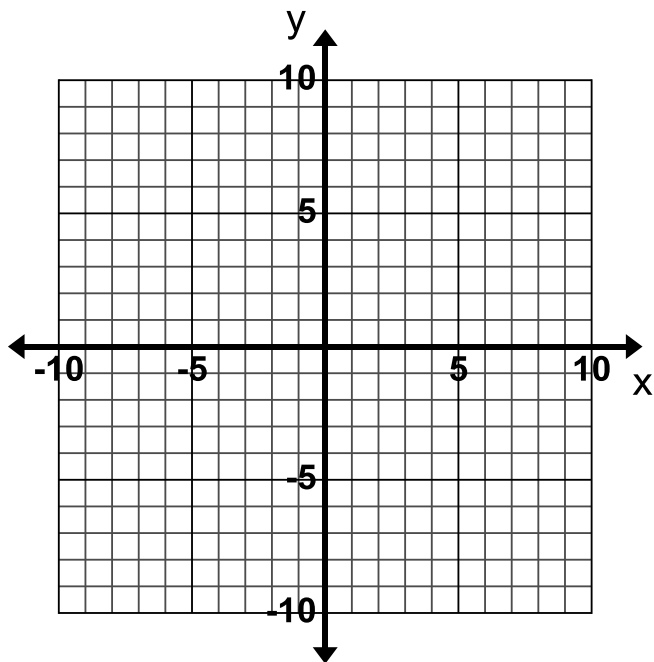
4. The equation of the line is $x - 3y = 9$.



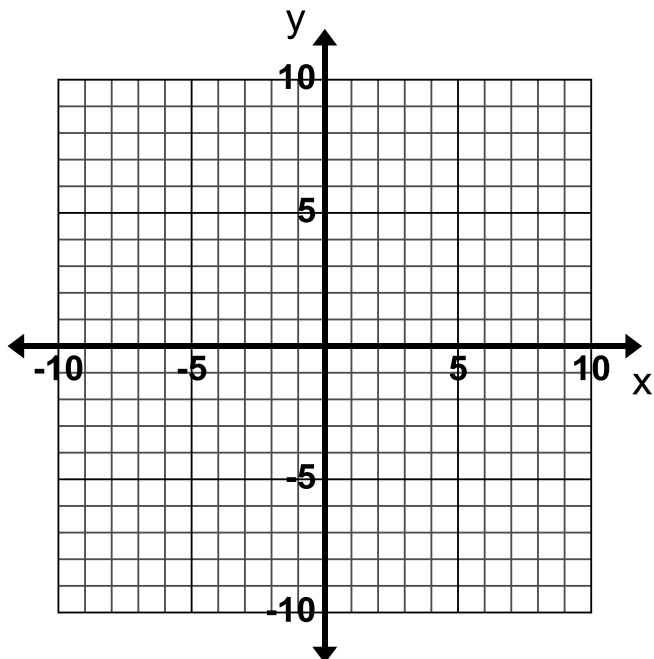
5. The equation of the line is $y + 2x = y - 2$.



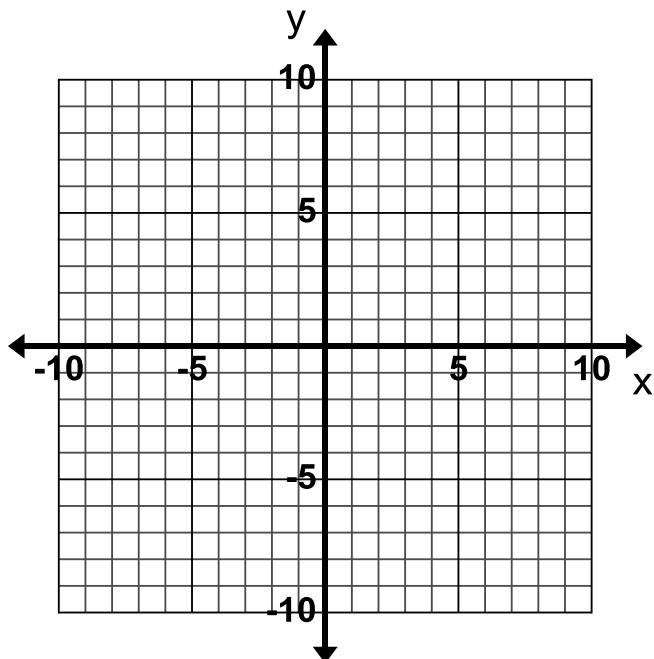
6. The equation of the line is $y = 6$.



7. The line contains the point $(1, 2)$ and has a slope of $-\frac{5}{2}$.



8. The line contains the point $(6, 3)$ and the slope is undefined.



Directions: Write the equation in slope-intercept form for each line based on the information given.

9. The slope of the line is 1 and the y-intercept is -4 .

10. The line has a slope of $-\frac{1}{4}$ and goes through the point $(-2, 4)$.

11. The line contains the points $(1, -2)$ and $(2, 4)$.

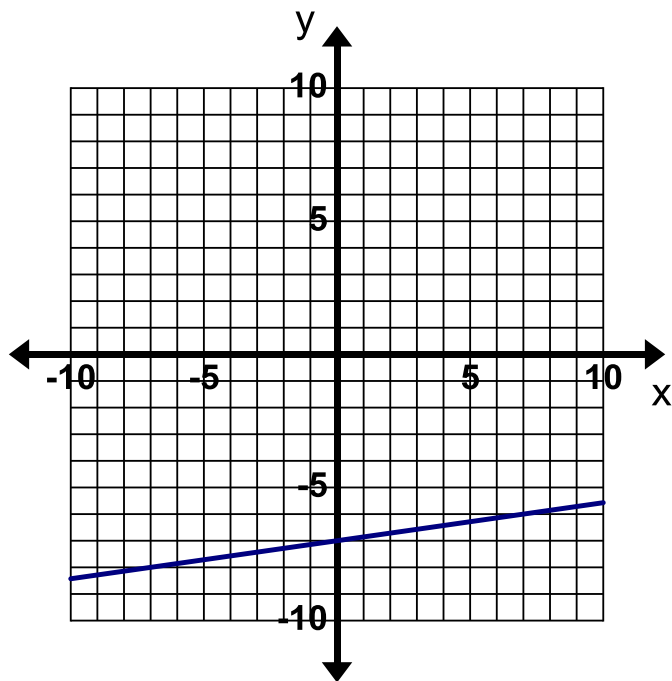
12. The line contains the points in the table.

x	y
0	0
1	-2
2	-4
3	-6

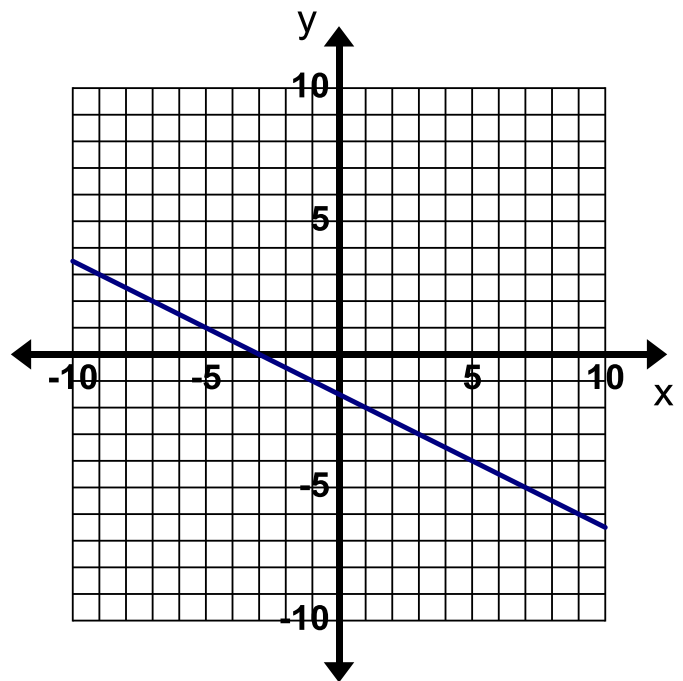
13. The line contains the points in the table.

x	y
1	5
3	-1
4	-4
6	-10

14. The line graphed below.



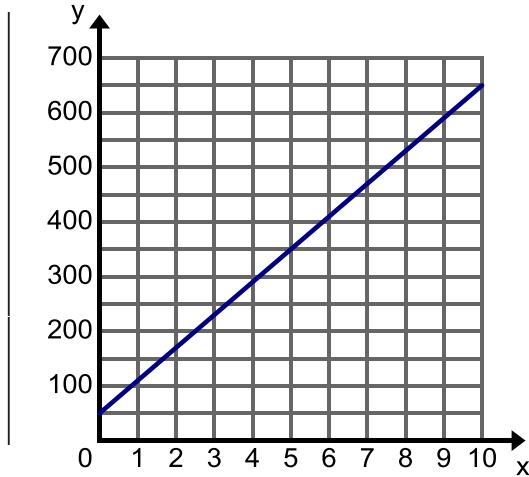
15. The line graphed below.



3.1g Classwork: Write Equations to Solve Real-world Problems

Directions: Write the equation for each of the following real-world problems.

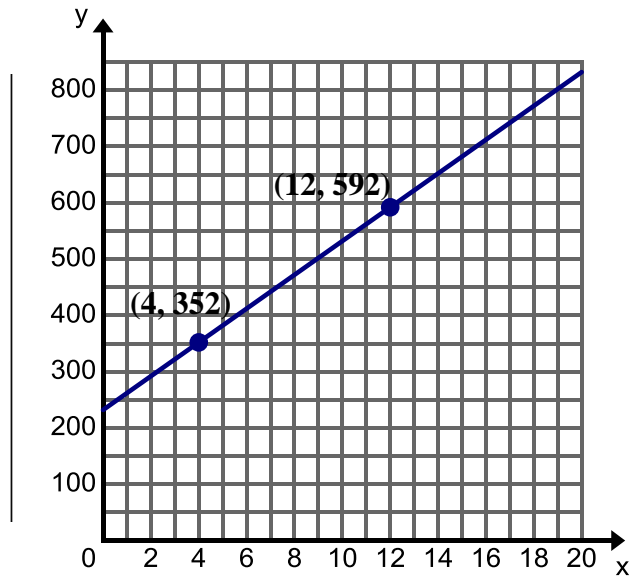
1. The graph below shows a trip taken by a car where x is time (in hours) the car has driven and y is the distance (in miles) from Salt Lake City. Label the axes of the graph.



Equation:

Use your graph and equation to tell the story of this trip taken by the car.

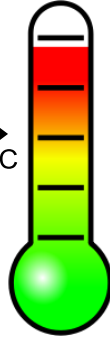
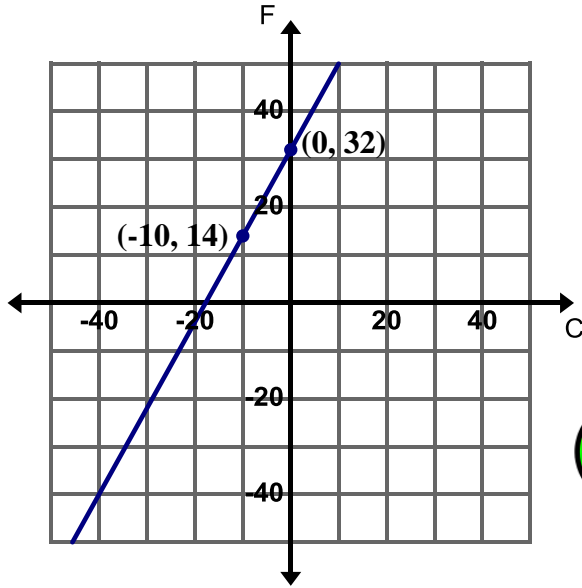
2. The graph below shows the weight of a baby elephant where x is the time (in weeks) since the elephant's birth and y is the weight (in pounds). At 4 weeks, the elephant weighed 352 lbs. and at 12 weeks, the elephant weighed 592 lbs. Label the axes of the graph.



Equation:

Use your graph and equation to tell the story of this elephant.

3. The graph below shows the relationship between temperature in degrees Celsius and temperature in degrees Fahrenheit.



Equation:

5. A handyman charges \$40 an hour plus the cost of materials. Rosanne received a bill from the handyman for \$477 for 8 hours of work.

Equation:

Use your equation to add more details to the story about the work the handyman did for Roseanne.

4. Peter is draining his hot tub so that he can clean it. He puts a hose in the hot tub to drain the water at a constant rate. After 5 minutes there are 430 gallons of water left in the hot tub. After 20 minutes there are 370 gallons of water left in the hot tub. Let x be time (in minutes) and y be water remaining (in gallons).

Equation:

Use your equation to add more details to the story of Peter draining the hot tub.

6. The table below shows the height h (in feet) of a hot air balloon t minutes after it takes off from the ground. It rises at a constant rate.

t (minutes)	h (feet)
2	300
5	750
9	1,350



Equation:

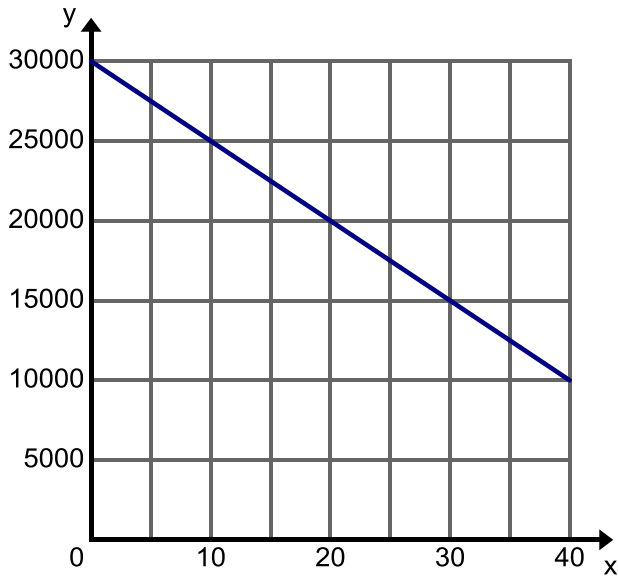
Use the table and equation to tell the story of the hot air balloon.

3.1g Homework: Write Equations to Solve Real-world Problems

Directions: Write the equation for each of the following real-world problems.



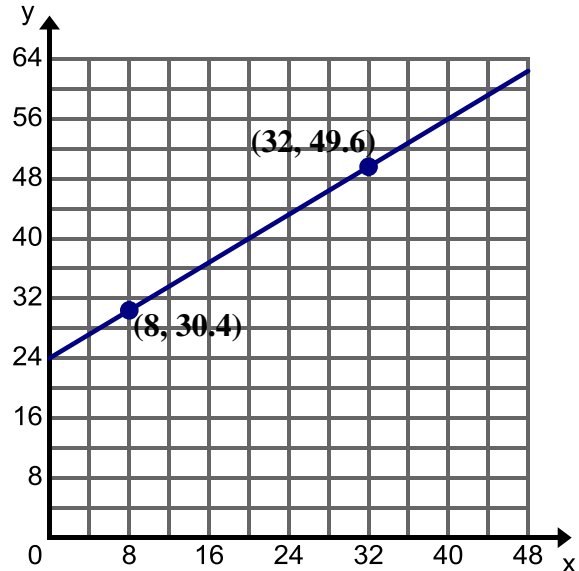
1. The graph below shows the descent of an airplane where x is time (in minutes) since the plane started its descent and y is the altitude (in feet) of the plane. Label the axes of the graph.



Equation:

Use the graph and equation to tell the story of this airplane.

2. The graph below shows the length of a boa constrictor where x is time (in weeks) since the boa constrictor's birth and y is length (in inches). The boa constrictor was 30.4 in. at 8 weeks and 49.6 in. at 32 weeks. Label the axes of the graph.



Equation:

Use the graph and equation to tell the story of this boa constrictor.

3. The table below shows the amount of money Lance has in his savings account where x is time (in months) and y is the account balance (in dollars).

x (time)	y (account balance)
0	160
3	385
6	610
9	835

Equation:

Use the table and equation to tell the story of Lance's savings.

4. The cost to rent a jet ski is \$80 per hour. In addition there also includes fee for a lesson on how to use the jet ski. Steve rented a jet ski for 3 hours and the total bill was \$265. Write the equation for the total cost of renting a jet ski y for x hours.

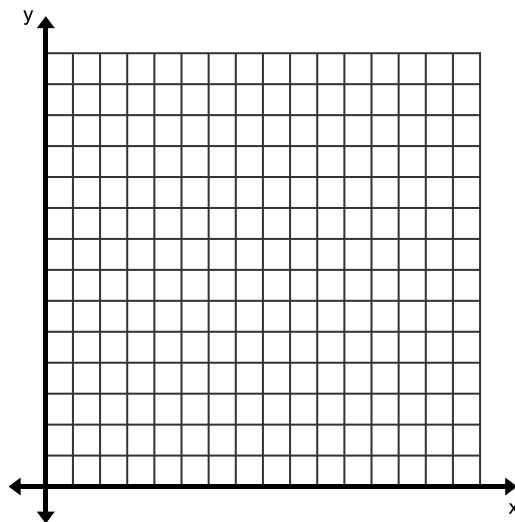
Equation:

Use your equation to add more details to the story about renting a jet ski.

5. In order to make the playoff a soccer team must get 20 points during the regular season. The team gets 2 points for a win and 1 point for a tie. A team earns just enough points to make the playoffs. Let x represent the number of wins and y represent the number of ties.

- Write an equation to relating the all the possible values of x and y that will let the team make the playoffs.
- Write the equation in Slope-Intercept Form.
- If the team wins 8 games, how many tie games will need to occur?

6. The cost of a party at The Little Gym is \$250 which includes cake, pizza, and admission for any number of children. Create the graph and equation of this situation where x is the number of children and y is the total cost.



Equation:

3.1h Self-Assessment: Section 3.1

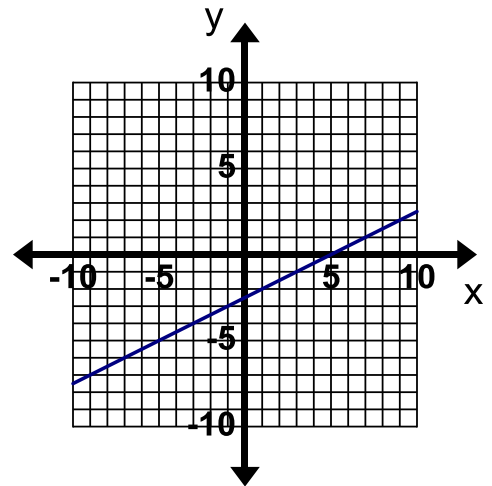
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Write a linear equation in the form $y = mx + b$ given any of the following: <ul style="list-style-type: none"> • slope and y-intercept • slope and a point • two points • a table • a graph of a linear relationship • a context of a real world situation <i>See sample problem #1</i>	I can write an equation for 1 or 2 of the given conditions.	I can write an equation for 3 or 4 of the given conditions.	I can write an equation for 5 or 6 of the given conditions.	I can write an equation for all six of the given conditions. In addition I can explain my steps in my own words.
2. Graph linear relationships given any of the following: <ul style="list-style-type: none"> • an equation • slope and a point <i>See sample problem #2</i>	I can graph the linear relationship given an equation in slope-intercept form.	I can graph the linear relationship given a slope and point.	I can graph the linear relationship from an equation in slope-intercept form. I can graph a linear relationship given a slope and point.	I can graph the linear relationship from an equation given in slope-intercept form and an equation that is not in slope intercept form. I can graph a linear relationship given a slope and point.

Sample Problem #1

For each problem below write a linear equation in the form $y = mx + b$ for the given conditions.

- a. The line has a slope of $-\frac{4}{3}$, and a y-intercept of $(0, -1)$.
- b. The line has a slope of -2 and passes through the point $(4, -3)$.
- c. The line contains the points $(0, -1)$ and $(3, -2)$.
- d.

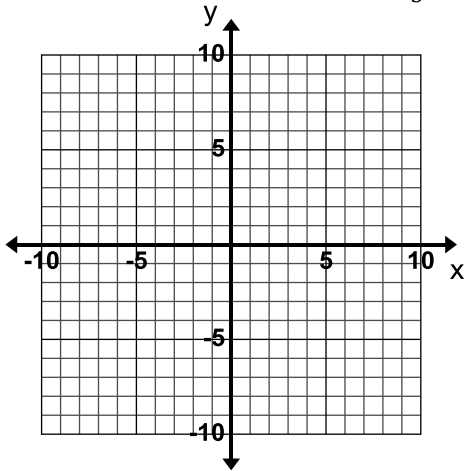


- e. A cat is running away from a dog. After 5 seconds it is 16 feet away from the dog and after 11 seconds it is 28 feet away from the dog. Let x represent the time in seconds that have passed and y represent the distance in feet that the cat is away from the dog.

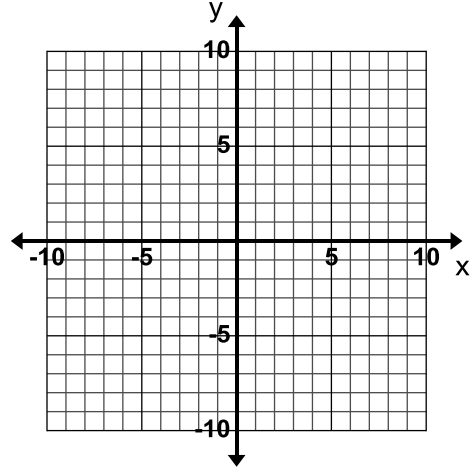
Sample Problem #2

Graph the linear relationships given the following conditions.

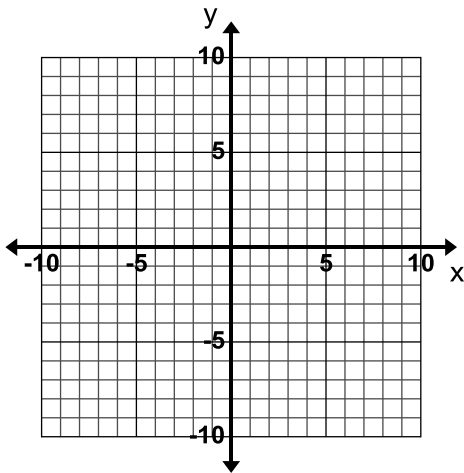
a. The equation of the line is $y = -\frac{4}{3}x - 1$.



b. The equation of the line is $2x + 3y = -6$.

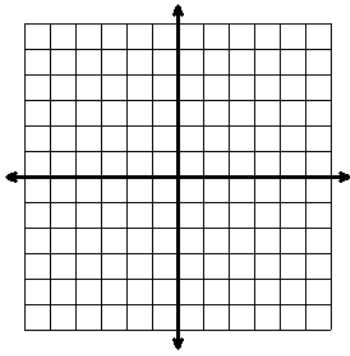


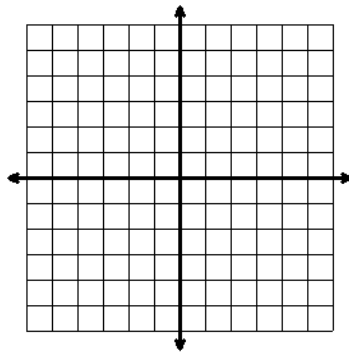
c. The line has a slope of -2 and passes through the point $(-4, -4)$.

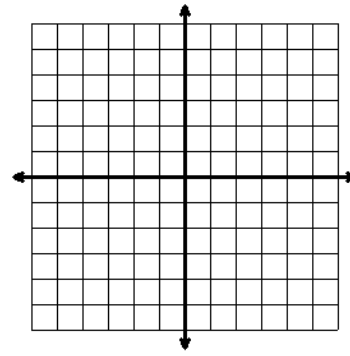


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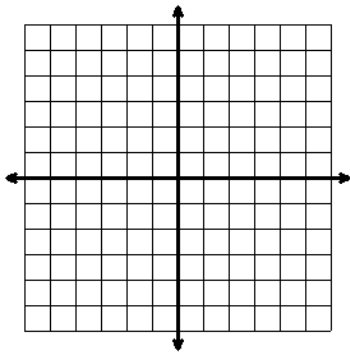
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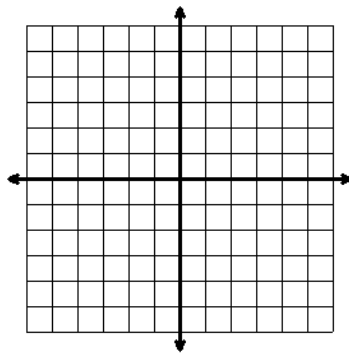


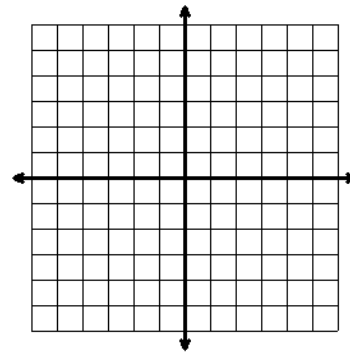




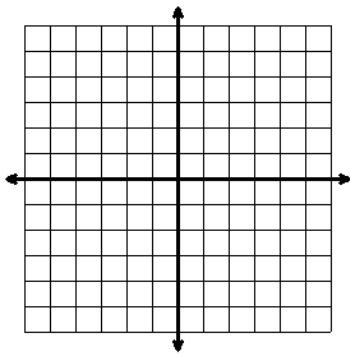
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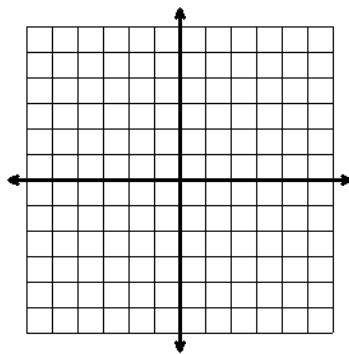


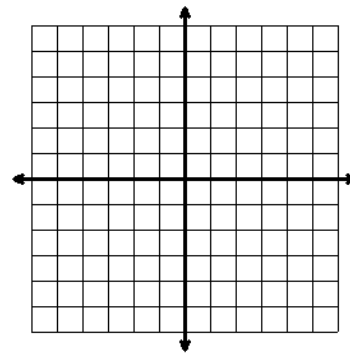




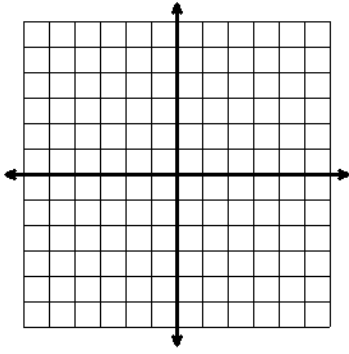
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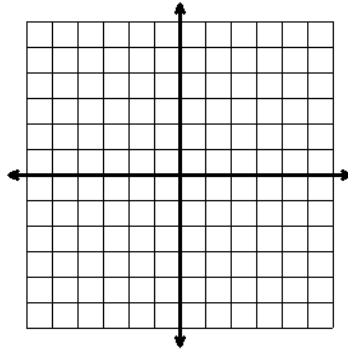


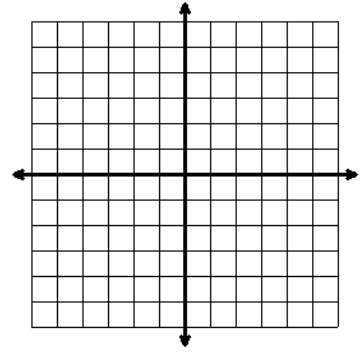




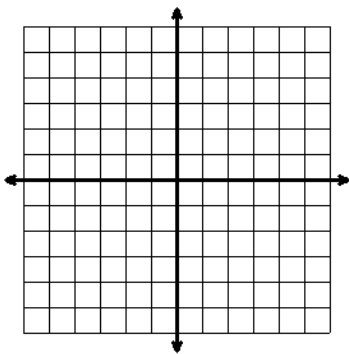
THURSDAY

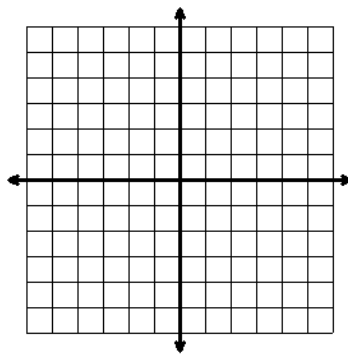


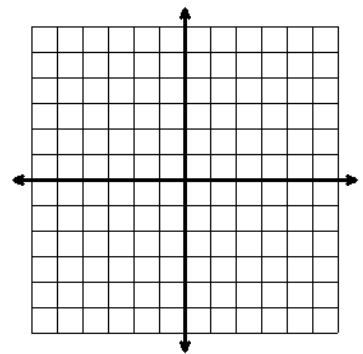




FRIDAY







Section 3.2: Relate Slopes and Write Equations for Parallel and Perpendicular Lines

Section Overview:

Students begin this section by investigating the effects of changes in the slope and y -intercept of a line, describing the transformation (translations, rotations, and reflections) that has taken place and write new equations that reflect the changes in m and b . In the next lesson students use transformations to discover how the slopes of parallel and perpendicular lines are related. Once students have an understanding of the relationship between the slopes of parallel and perpendicular lines, students write equations of lines that are parallel or perpendicular to a given line.

Concepts and Skills to Master:

By the end of this section, students should be able to:

- Write a new equation for a line given a described transformation.
- Describe a transformation of a graph given a change in its equation (change in the slope or y -intercept).
- Compare the slopes of parallel lines and explain the transformation that creates parallel lines.
- Compare the slopes of perpendicular lines and explain the transformation that creates perpendicular lines.
- Write the equation of a line parallel to a given line that passes through a given point.
- Write the equation of a line perpendicular to a given line that passes through a given point.

3.2a Class Activity: Equations for Graph Shifts



1. Graph the equation $y = x + 3$ and label the line with the equation.

a. Predict how the graph of $y = x + 1$ will compare to the graph of $y = x + 3$.

b. Predict how the graph of $y = x - 3$ will compare to the graph of $y = x + 3$.

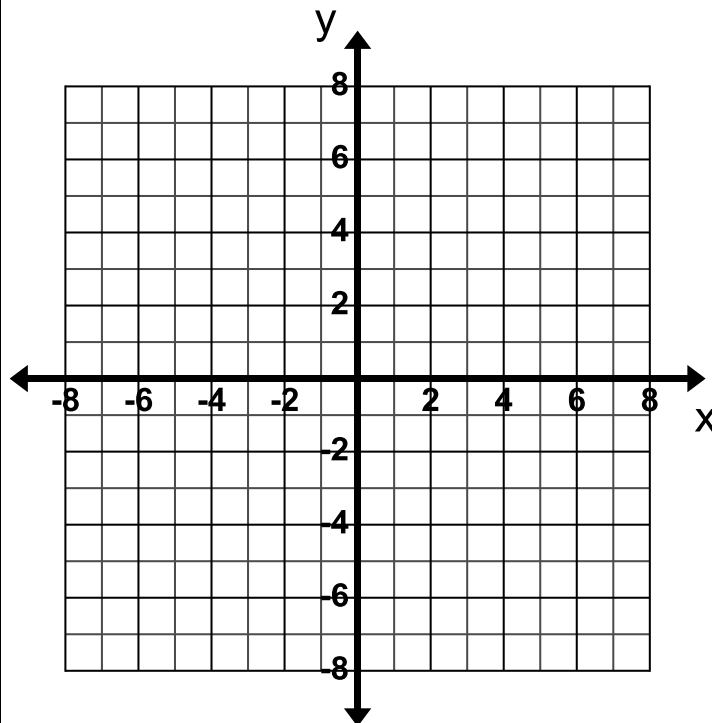
c. Graph the following equations on the same grid and label each line with its equation.

$$y = x + 1$$

$$y = x - 3$$

d. Were your predictions correct? Why or why not?

e. What is the relationship between the lines $y = x + 3$, $y = x + 1$, and $y = x - 3$?



f. Write a different equation that would be parallel to the equations in this problem.

g. Describe the movement of a line when b is increased or decreased while m is held constant.

2. Graph the equation $y = 2x - 4$ and label the line with the equation.

a. Predict how the graph of $y = x - 4$ will compare to the graph of $y = 2x - 4$.

b. Predict how the graph of $y = \frac{1}{2}x - 4$ will compare to the graph of $y = 2x - 4$.

c. Predict how the graph of $y = -2x - 4$ will compare to the graph of $y = 2x - 4$.

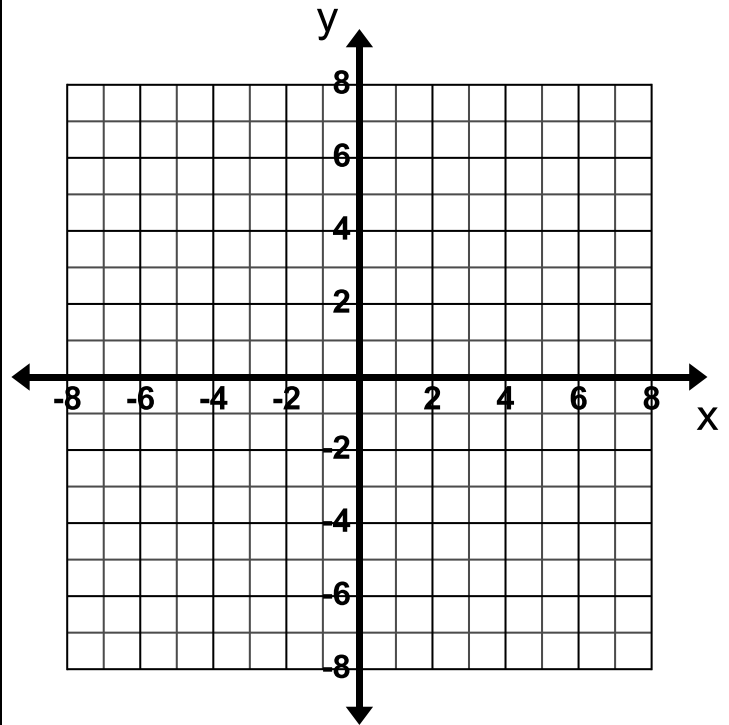
d. Graph the following equations and label each line with its equation.

$$y = x - 4$$

$$y = \frac{1}{2}x - 4$$

$$y = -2x - 4$$

e. Were your predictions correct? Why or why not?



f. Describe the movement of a line when the slope is increased or decreased while the y -intercept is held constant.

g. Describe the movement of a line when m is changed to $-m$.

h. Write the equation of a line that would be steeper than all of the equations in this problem.

3. Consider the equation $y = 2x + 4$. Write a new equation that would transform the graph of $y = 2x + 4$ in the ways described below.
- I want the slope to stay the same but I want the line to be shifted up 2 units.
 - I want the y -intercept to stay the same but I want the line to be less steep.
 - I want a line that is parallel to $y = 2x + 4$ but I want the line to be translated down 7 units.
4. Describe the relationship and transformation of the graphs of the following equations compared to the graph of the equation $y = 4x - 7$.
- $y = 2x - 7$
 - $y = 4x + 9$
 - $y = -4x - 7$
 - $y = 4x - 5$
5. Describe the relationship and transformation of the graphs of the following equations compared to the graph of the equation $y = -\frac{1}{2}x - 3$.
- $y = -\frac{1}{2}x$
 - $y = -2x - 3$
 - $y = -\frac{1}{4}x - 3$
 - $y = \frac{1}{2}x - 3$
 - $y = -\frac{1}{2}x + 5$

6. Consider the equation $y = 3x + 2$. Complete the chart below if the equation $y = 3x + 2$ is changed in the ways described below.

Change the equation $y = 3x + 2 \dots$	New Equation	Describe the Graph Shift (from the original Equation) (use the words rotation, reflection and/or translation)
Change the y -intercept to 5 while keeping the slope constant		
	$y = 3x - 3$	
Change the slope to 1 while keeping the y -intercept constant		
	$y = 4x + 2$	
		The new line is translated 2 units down from the original line $y = 3x + 2$.
Change the slope to -3 while keeping the y -intercept constant		
		The new line is a rotation of the original line $y = 3x + 2$ about the point $(0, 2)$ and the new line is steeper.

3.2a Homework: Equations for Graph Shifts

1. Consider the equation $y = x - 4$. Write a new equation that would transform the graph of $y = x - 4$ as described below.
 - a. I want the slope to stay the same but I want the line to be shifted up 3 units.
 - b. I want the y -intercept to stay the same but I want the line to be less steep.
 - c. I want a line that is parallel to $y = x - 4$ but I want the line to be translated down 6 units.
2. Describe the relationship and transformation of the graphs of the following equations compared to the graph of the equation $y = -3x$.
 - a. $y = 3x$
 - b. $y = -3x - 4$
 - c. $y = -2x$
 - d. $y = -3x + 4$
3. Describe the relationship and the transformation of the graphs of the following equations compared to the graph of the equation $y = \frac{4}{3}x + 4$.
 - a. $y = \frac{4}{3}x - 1$
 - b. $y = \frac{4}{3}x$
 - c. $y = 2x + 4$
 - d. $y = -\frac{4}{3}x + 4$
 - e. $y = \frac{1}{3}x + 4$

4. Consider the equation $y = \frac{1}{2}x + 3$. Complete the chart below if the equation $y = \frac{1}{2}x + 3$ is changed in the ways described.

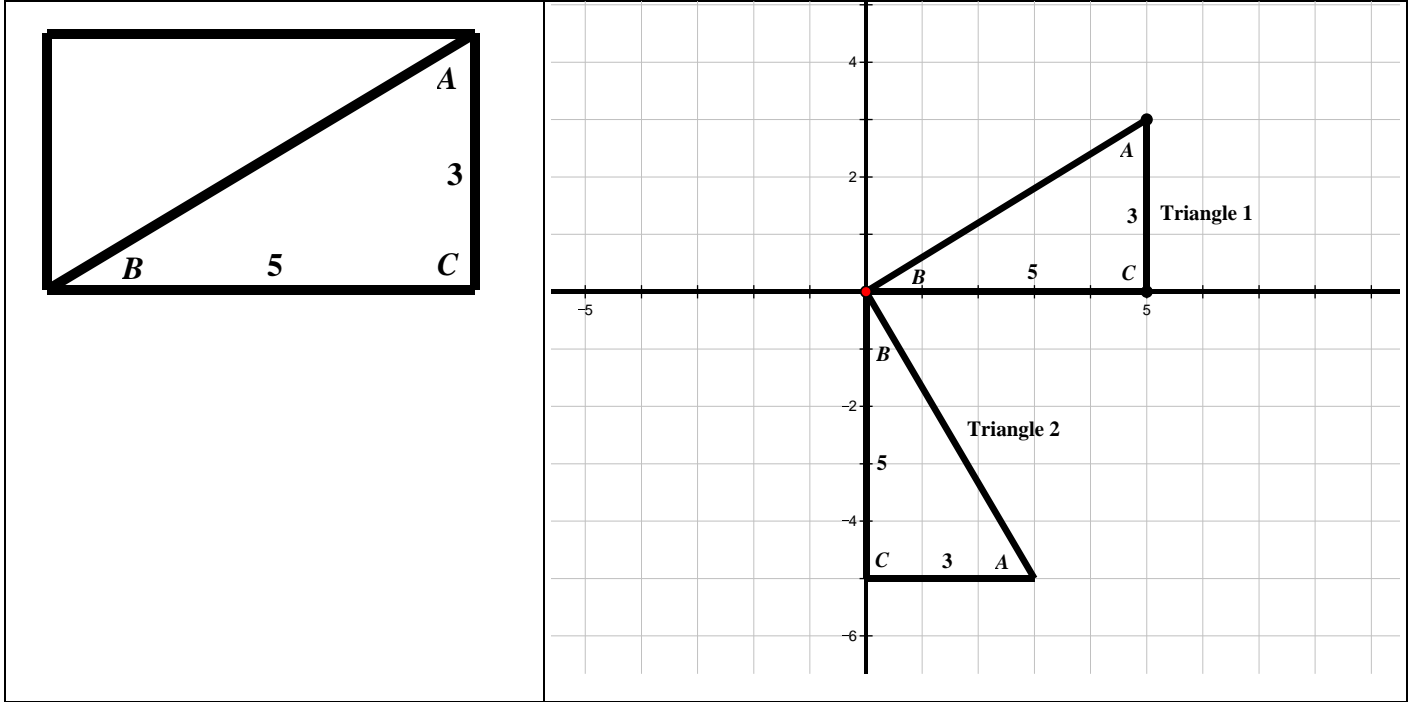
Change the equation $y = \frac{1}{2}x + 3 \dots$	New Equation	Describe the Graph Shift (from the original Equation) (use the words rotation, reflection and/or translation)
		The new line is translated 3 units up from the line $y = \frac{1}{2}x + 3$.
Change the slope to $-\frac{1}{2}$ while keeping the y-intercept constant		
	$y = \frac{1}{2}x - 2$	
Change the y-intercept to 0 while keeping the slope constant		
		The new line is a rotation of the equation $y = \frac{1}{2}x + 3$ about the point $(0, 3)$ and the new line is less steep.
Change the slope to 2 while keeping the y-intercept constant		
	$y = -2x + 3$	

5. Describe the transformation (graph shift) that occurs in each of the following situations. Use words like translation, reflection, and rotation.
- The slope is increased or decreased while the y-intercept is held constant
 - The y-intercept is decreased while the slope is held constant
 - The slope m is changed to $-m$
 - The y-intercept is increased while the slope is held constant

3.2b Class Activity: Slopes of Perpendicular Lines

Materials: Graph paper (one inch grid), 3 by 5 card, straight edge, scissors.

- On your 3 by 5 card, draw the diagonal (as shown in the 1st box below). Label as shown below. Then cut the card into two triangles.
- On your graph paper, draw the x and y axis as shown in the 2nd box below. Trace your triangle to create Triangles 1 and 2 as shown below.



- Highlight the hypotenuse \overline{AB} of each triangle. Find the slope and equation of each hypotenuse:

a. Triangle 1	b. Triangle 2
Hypotenuse Slope:	Hypotenuse Slope:
Equation of the Hypotenuse Line:	Equation of the Hypotenuse Line:

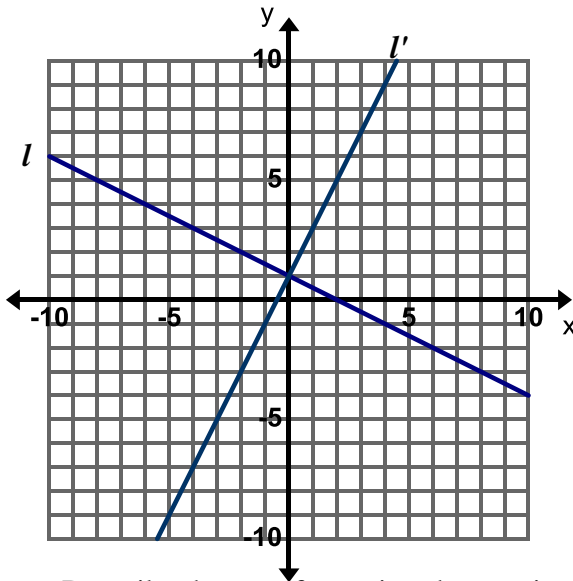
Important NOTE: For purposes of the questions below, it is given that the 3 by 5 card is a rectangle and therefore has 90 degree angles. Use the card to help view perpendicular lines or a 90 degree rotation.

- Describe the transformation(s) needed to carry Triangle 1 onto Triangle 2.
- What is the angle formed by the two hypotenuses at their y -intercept intersection? How do you know? How can you prove the measure of that angle?
- Consider the transformation that carries Triangle 1 to Triangle 2. What happens to the rise and run of the slope of the hypotenuse when you rotate the triangle 90°? Relate this to the slopes in your equations above.

7. Is there another way you can rotate Triangle 1 so that the hypotenuses of Triangle 1 and Triangle 2 are perpendicular? Observe what happens to the rise and run that form the slope of \overline{AB} .
8. What does this activity tell us about the slopes of perpendicular lines?

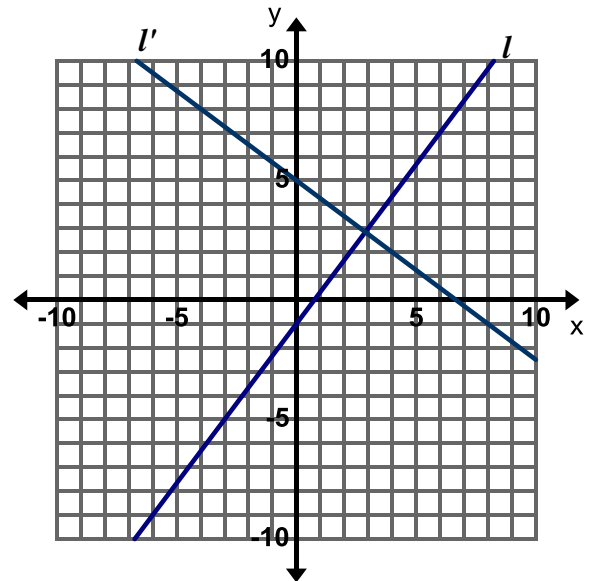
Directions: Use the pairs of lines in the graphs below to answer the questions that follow. Remember a transformation is a translation (slide), reflection (flip), or rotation (turn).

9. Pair 1



- Describe the transformation that carries line l to line l' .
- Find the slope of each of the lines.
- Describe how the lines and slopes are related.

10. Pair 2

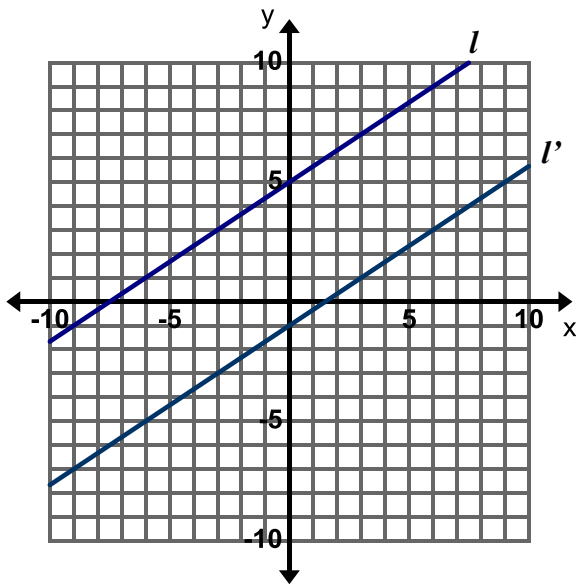


- Describe the transformation that carries line l to line l' .
- Find the slope of each of the lines.
- Describe how the lines and slopes are related.

3.2b Homework: Slopes of Parallel Lines

Directions: Use the pairs of lines in the graphs below to answer the questions that follow. Remember a transformation is a translation (slide), reflection (flip), or rotation (turn).

1. Pair 1



a. Describe the transformation that carries line l to line l' .

b. Find the slope of each line. What do you observe about the slopes?

l :

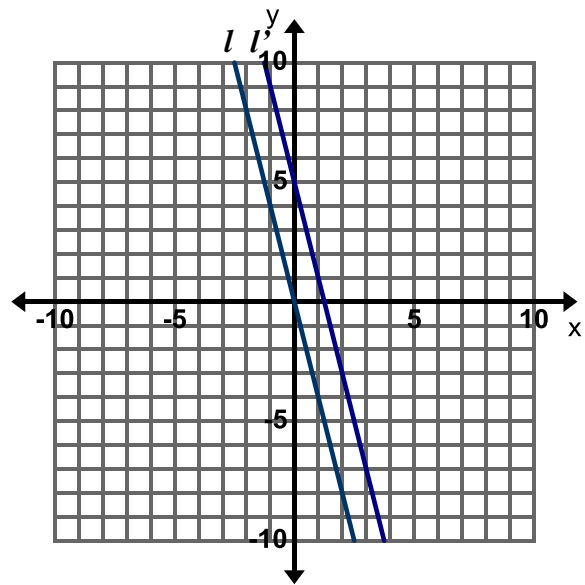
l' :

c. Write an equation for each line. How are the equations the same and how are they different?

l :

l' :

2. Pair 2



a. Describe the transformation that carries line l to line l' .

b. Find the slope of each line. What do you observe about the slopes?

l :

l' :

c. Write an equation for each line. How are the equations the same and how are they different?

l :

l' :

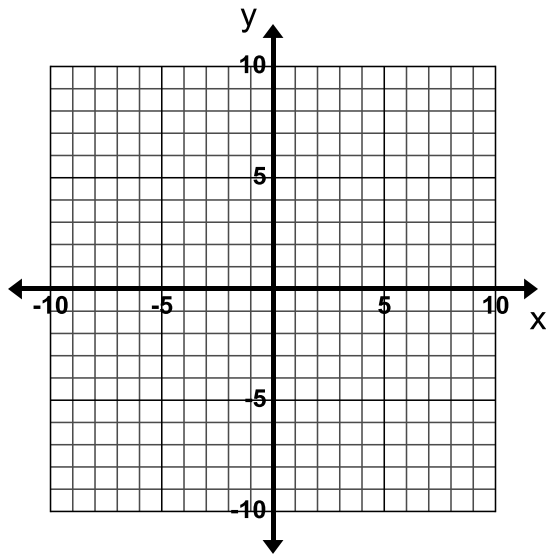
3. Given the graphs of two or more lines how can you determine if they are parallel?

3.2c Class Activity: Equations of Parallel and Perpendicular Lines

Directions: In the following problems, lines A and B are **parallel**. Graph and label both lines. Then write the equation of line B.

1. Line A: $y = 2x - 3$

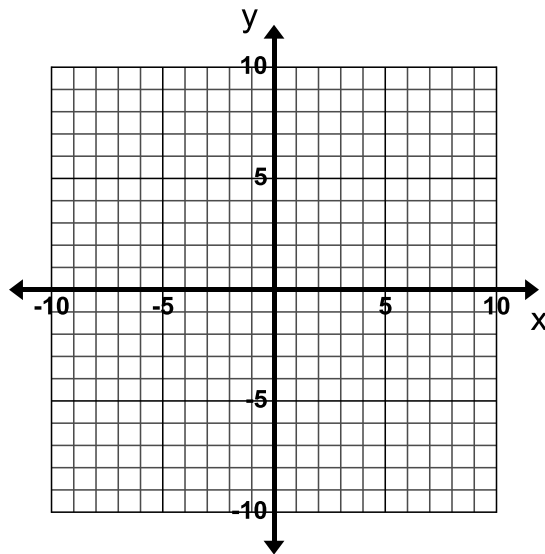
Line B: passes through $(0, 4)$



Equation of Line B:

2. Line A: $y = -4x + 1$

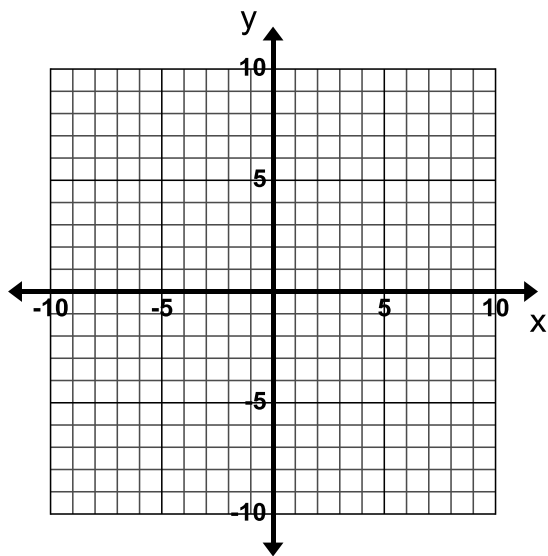
Line B: passes through $(0, -5)$



Equation of Line B:

3. Line A: $y = \frac{1}{2}x + 4$

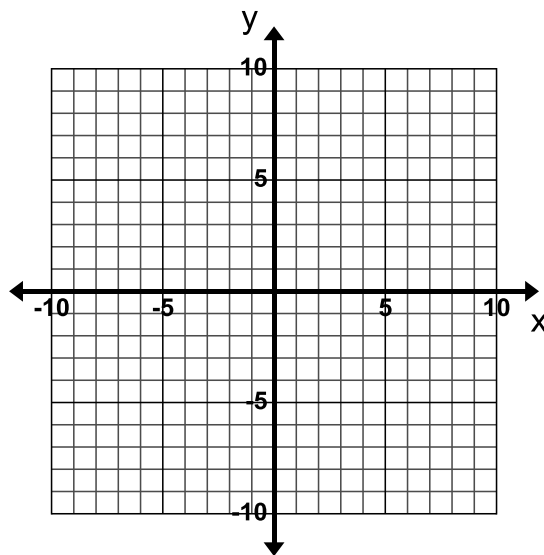
Line B: passes through $(0, 7)$



Equation of Line B:

4. Line A: $y = 4x$

Line B: passes through $(3, -3)$



Equation of Line B:

Directions: In the following problems, lines A and B are parallel. Find the equation for line B without graphing.

5. Find the equation of line B which is **parallel** to line A and passes through (2, 3).

Line A: $y = -3x + 7$

Line B:

6. Find the equation of line B which is **parallel** to line A and passes through (-6, 2).

Line A: $y = \frac{1}{3}x + 2$

Line B:

7. Given the slope of a line, how do you figure out the slope of a line **perpendicular** to it?

8. Give the slope of a line that is **perpendicular** to the following lines:

a. $y = 3x - 2$; m of perpendicular line:

b. $y = -\frac{2}{3}x$; m of perpendicular line:

c. $y = -x + 2$; m of perpendicular line:

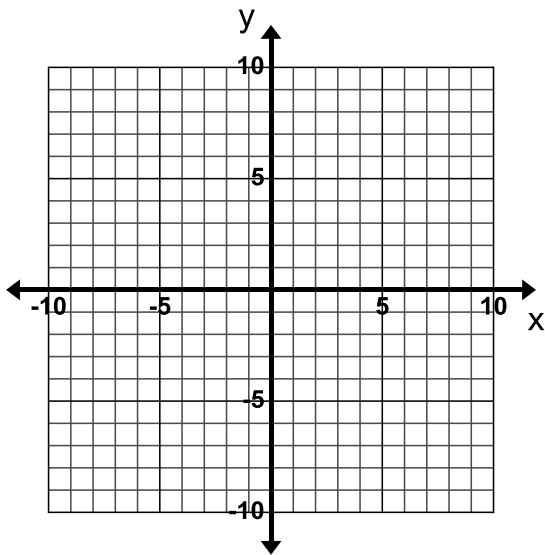
d. $y = -2x + 6$; m of perpendicular line:

Directions: In the following problems, lines A and B are **perpendicular**. Graph and label both lines. Then write the equation of line B.

9. Line A: $y = 4x + 9$

What is the slope of line B?

Line B: passes through $(4, -7)$



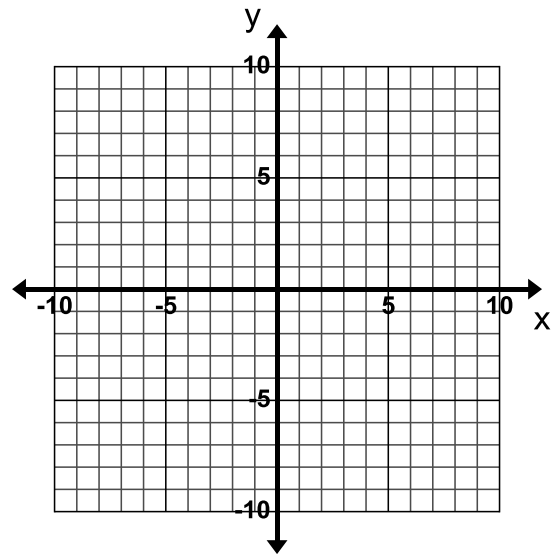
Equation of Line B:

10. Line A: $2y = 3x + 8$

Rewrite as

What is the slope of line B?

Line B: passes through $(3, 7)$



Equation of Line B:

Directions: In the following problems, lines A and B are **perpendicular**. Find the equation for line B.

<p>11. Find the equation of the line B which is perpendicular to line A and passes through (3, 7).</p> <p>Line A: $y = -3x + 7$</p> <p>Line B:</p>	<p>12. Find the equation of Line B which is perpendicular to line A and passes through (2, 4).</p> <p>Line A: $y = -\frac{1}{2}x - 2$</p> <p>Line B:</p>
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Directions: Determine if the following sets of lines are parallel, perpendicular, or neither. Justify your answer.

<p>13. Line A: $y = \frac{3}{4}x + 1$</p> <p>Line B: $y = \frac{3}{4}x - 5$</p>	<p>14. Line A: $y = \frac{3}{4}x + 1$</p> <p>Line B: $y = -\frac{3}{4}x + 1$</p>
<p>15. Line A: $y = \frac{3}{4}x + 1$</p> <p>Line B: $y = \frac{4}{3}x + 1$</p>	<p>16. Line A: $y = \frac{3}{4}x + 1$</p> <p>Line B: $y = -\frac{4}{3}x + 1$</p>
<p>17. Line A: $y = 3x + 2$</p> <p>Line B: $y = -3x + 2$</p>	<p>18. Line A: $y = 3x + 2$</p> <p>Line B: $y = 3x + 5$</p>

<p>19. Line A: $y = 3x + 2$ Line B: $y = -\frac{1}{3}x$</p>	<p>20. Line A: $y = 3x + 2$ Line B: $y = \frac{1}{3}x + 2$</p>
<p>21. Line A: $y = \frac{1}{2}x + 1$ Line B: $6x + 3y = 18$</p>	<p>22. Line A: $4x - 2y = -6$ Line B: $-6x + y = -4(x - 2)$</p>

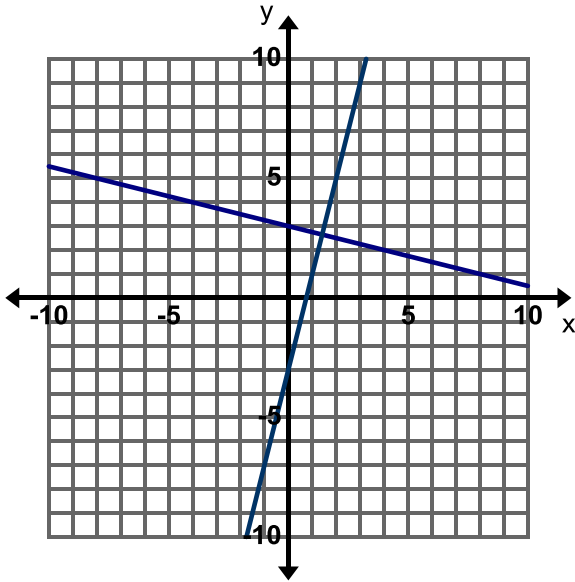
Directions: Determine whether the lines through the pairs of points are parallel, perpendicular, or neither.

<p>23. $(-3, 1)$ and $(2, 3)$ $(-3, 5)$ and $(-1, 0)$</p>	<p>24. $(-3, -1)$ and $(-1, -3)$ $(-1, 2)$ and $(-4, -1)$</p>	<p>25. $(1, 8)$ and $(-1, 1)$ $(0, 7)$ and $(2, 4)$</p>
<p>26. $(2, 0)$ and $(1, 6)$ $(1, 3)$ and $(7, 4)$</p>	<p>27. $(-3, 0)$ and $(-2, 4)$ $(2, -1)$ and $(1, -5)$</p>	<p>28. $(-3, 4)$ and $(3, 7)$ $(4, 2)$ and $(-2, 6)$</p>

3.2c Homework: Equations of Parallel and Perpendicular Lines

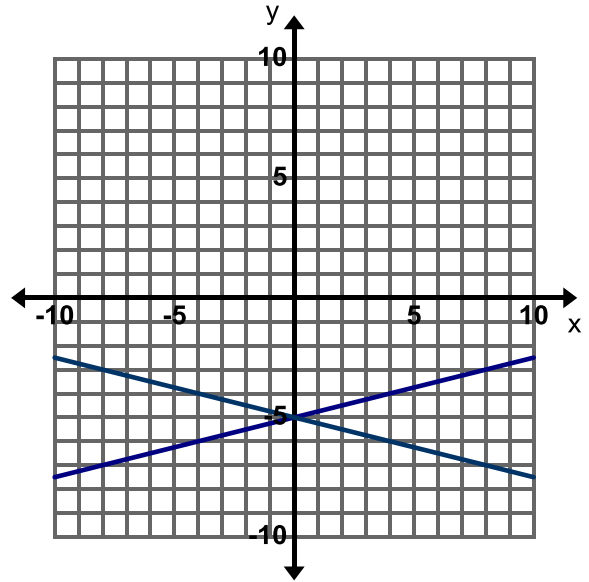
Directions: Determine if the following sets of lines in each graph are parallel or perpendicular. Justify your answer. If they are not parallel or perpendicular describe the transformation that carries one line to the other.

1. Parallel, Perpendicular, or Neither?



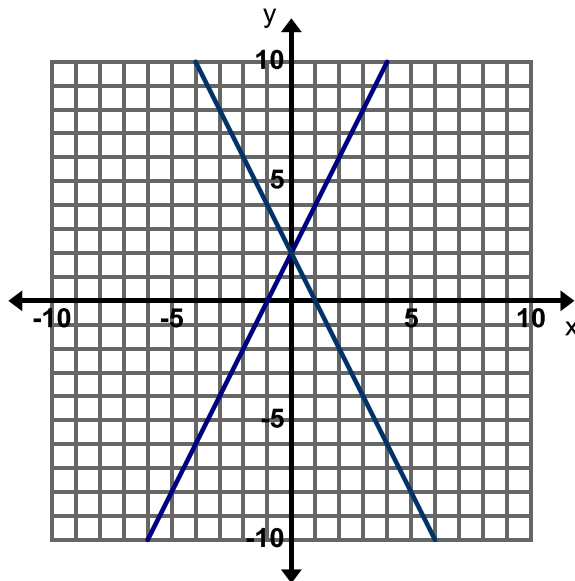
Justification:

2. Parallel, Perpendicular, or Neither?



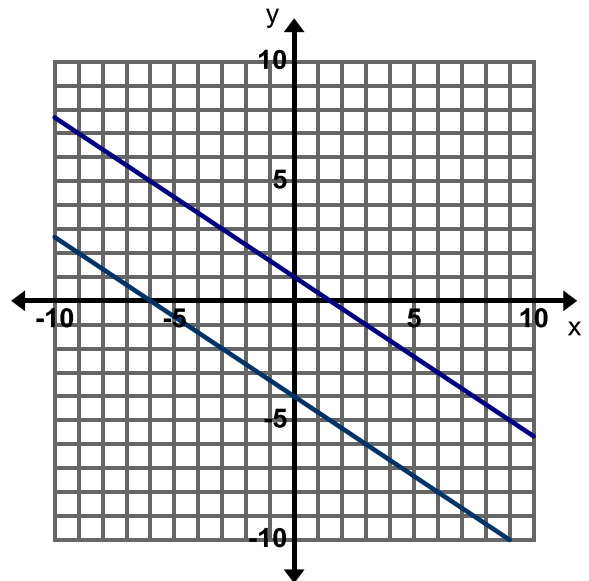
Justification:

3. Parallel, Perpendicular, or Neither?



Justification:

4. Parallel, Perpendicular, or Neither?



Justification:

Directions: Determine if the following sets of lines are parallel, perpendicular, or neither. Justify your answer.

5. Line A: $y = \frac{1}{4}x - 3$ Line B: $y = -4x + 3$	6. Line A: $y = \frac{1}{4}x - 3$ Line B: $y = \frac{1}{4}x + 3$
7. Line A: $y = \frac{1}{4}x + 2$ Line B: $y = -\frac{1}{4}x + 2$	8. Line A: $y = \frac{1}{4}x - 5$ Line B: $y = 4x + 5$
9. Line A: $y = \frac{2}{3}x + 1$ Line B: $2x - 3y = 6$	10. Line A: $2x - 3y = -9$ Line B: $3x + 2y = -8$

Directions: Determine whether the lines through the pairs of points are parallel, perpendicular, or neither.

11. $(-2, 0)$ and $(4, 3)$ $(0, 0)$ and $(1, -2)$	12. $(-2, -11)$ and $(-1, -7)$ $(2, -11)$ and $(-1, 1)$	13. $(0, 0)$ and $(3, 4)$ $(-1, -1)$ and $(2, 3)$
14. $(4, 12)$ and $(2, 6)$ $(4, -12)$ and $(2, -6)$	15. $(-1, -5)$ and $(0, -4)$ $(-1, -3)$ and $(0, -4)$	16. $(-2, 9)$ and $(0, 1)$ $(3, 13)$ and $(-1, -3)$

Directions: Write the equation of the line using algebra. (Do not graph the equations to write the equation.)

17. Write the equation of the line that is **perpendicular** to $y = \frac{2}{3}x - 5$ and passes through the point $(2, 5)$.

18. Write the equation of the line that is **perpendicular** to $y = -5x + 2$ and passes through the point $(10, -4)$.

19. Write the equation of the line that is **parallel** to $y = -3x + 2$ and passes through the point $(-3, -2)$.

20. Find the equation of the line that is **parallel** to $y = \frac{3}{5}x - 4$ and passes through the point $(5, 4)$.

3.2d Self-Assessment: Section 3.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Write a new equation for a line given a described transformation. <i>See sample problem #1</i>	I can identify the transformation but I do not know how it relates to the equation.	I can write a new equation for a line for one described transformation.	I can write a new equation for a line from both described transformations.	I can write a new equation for a line for both described transformations. I can also explain in my own words why the transformation changed the equation.
2. Describe a transformation of a graph given a change in its equation (change in the slope or y-intercept). <i>See sample problem #2</i>	I don't know how to see a transformation given a change in an equation.	I can describe one transformation.	I can accurately describe both transformations.	I can accurately describe both transformations. I can explain in my own words how the parts of the equation are related to the transformation.
3. Compare the slopes of parallel lines and explain the transformation that creates parallel lines. <i>See sample problem #3</i>	I can find the slopes of the lines but cannot describe the transformation that takes line l to line l' .	I can describe the transformation that takes line l to line l' but do not know how to find the slopes of the lines.	I can describe the transformation that takes line l to line l' and state the slope of each line.	I can describe the transformation that takes line l to line l' and state the slope of each line. I can also explain how the equations will be similar and different.

<p>4. Write the equation of a line parallel to a given line that passes through a given point.</p> <p><i>See sample problem #4</i></p>	<p>I know that parallel lines have the same slope but I do not know how to write the equation of the line that passes through the point (2,3).</p>	<p>I can graph the line given and the line that is parallel to it and goes through the point (2,3).</p>	<p>I can write the equation of the line that is parallel to $y = -3x + 7$ and passes through the point (2, 3).</p>	<p>I can write the equation of the line that is parallel to $y = -3x + 7$ and passes through the point (2, 3). I can also determine the transformation that moves the original line to the given point.</p>
<p>5. Compare the slopes of perpendicular lines and explain the transformation that creates perpendicular lines.</p> <p><i>See sample problem #5</i></p>	<p>I can find the slope of one of the lines.</p>	<p>I know how to find the slope of the lines but cannot describe the transformation that creates these lines.</p>	<p>I know how to find the slopes of perpendicular lines and can describe the transformation that creates these lines.</p>	<p>I know how to find the slopes of perpendicular lines and can describe the transformation that creates these lines. I can explain how the transformation will affect the equation of the line.</p>
<p>6. Write the equation of a line perpendicular to a given line that passes through a given point.</p> <p><i>See sample problem #6</i></p>	<p>I can find the slopes of the lines but cannot describe the transformation that takes line l to line l'.</p>	<p>I can describe the transformation that takes line l to line l' but do not know how to find the slopes of the lines.</p>	<p>I can describe the transformation that takes line l to line l' and state the slope of each line.</p>	<p>I can describe the transformation that takes line l to line l' and state the slope of each line. I can also create my own examples of lines that are perpendicular through a given point.</p>

Sample Problem #1

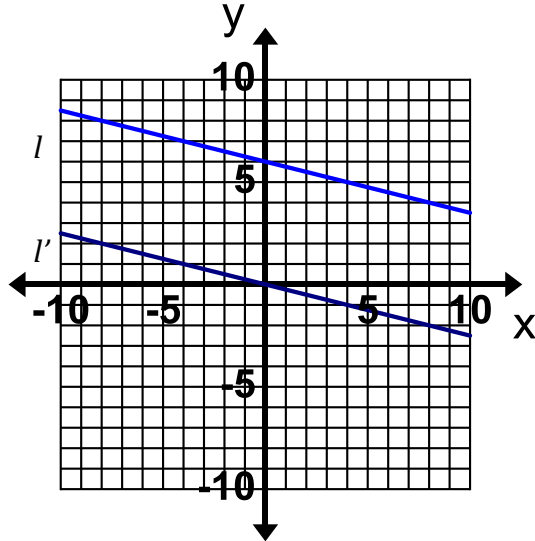
Consider the equation $y = 3x + 2$. Write a new equation that represents a line that is parallel to the original line and shifted down 3 units.

Sample Problem #2

What is the relationship and transformation of the graph of the equation $y = \frac{4}{3}x + 4$ compared to the graph of the equation $y = -\frac{4}{3}x + 5$.

Sample Problem #3

Describe the transformation that carries line l to line l' .

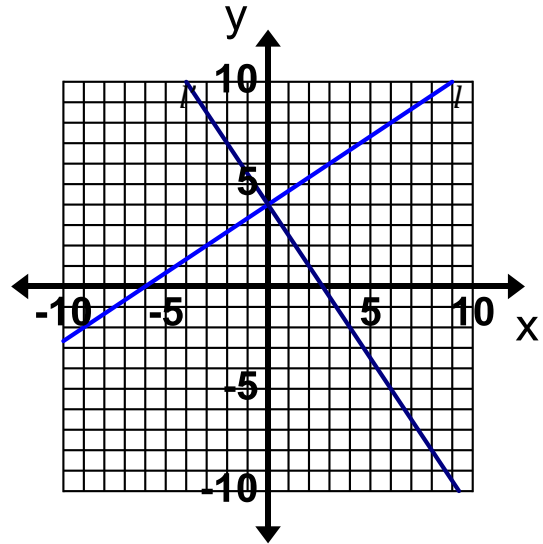


What is the slope of line l ?

What is the slope of line l' ?

Sample Problem #5

Describe the transformation that carries line l to line l' .



What is the slope of line l ?

What is the slope of line l' ?

Sample Problem #4

Write the equation of a line that is parallel to $y = -3x + 7$ and passes through $(2, 3)$.

Sample Problem #6

Write the equation of the line that is perpendicular to $y = -5x + 2$ and passes through the point $(10, -4)$.

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Chapter 4: Simultaneous Linear Equations (3 weeks)

Utah Core Standard(s):

- Analyze and solve pairs of simultaneous linear equations. (8.EE.8)
 - a) Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
 - b) Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
 - c) Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Academic Vocabulary: system of linear equations in two variables, simultaneous linear equations, solution, intersection, ordered pair, elimination, substitution, parallel, no solution, infinitely many solutions

Chapter Overview:



In this chapter we discuss intuitive, graphical, and algebraic methods of solving simultaneous linear equations; that is, finding all pairs (if any) of numbers (x, y) that are solutions of both equations. We will use these understandings and skills to solve real world problems leading to two linear equations in two variables.


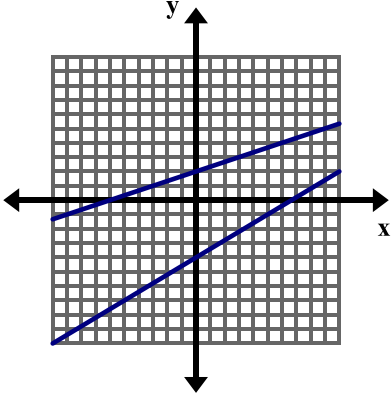


Connections to Content:




Prior Knowledge: In chapter 1, students learned to solve one-variable equations using the laws of algebra to write expressions in equivalent forms and the properties of equality to solve for an unknown. They solved equations with one, no, and infinitely many solutions and studied the structure of an equation that resulted in each of these outcomes. In chapter 3, students learned to graph and write linear equations in two-variables. Throughout, students have been creating equations to model relationships between numbers and quantities.

Future Knowledge: In subsequent coursework, students will gain a conceptual understanding of the process of elimination, examining what is happening graphically when we manipulate the equations of a linear system. They will also solve systems that include additional types of functions.

MATHEMATICAL PRACTICE STANDARDS

	<p>Make sense of problems and persevere in solving them.</p>	<p>Kevin and Nina are competing in a bike race. When Kevin is ninety miles into the race, he is in first place. Nina is in second place and is 15 miles behind Kevin.</p> <ol style="list-style-type: none"> From this point, Kevin continues the race at a constant rate of 25 mph and Nina continues the race at a constant rate of 30 mph. When will Nina catch Kevin? Solve this problem using any method you wish. If the race is 150 miles long, who will win? Assume Nina and Kevin bike at the speeds given in part a). Now suppose the following: Ninety miles into the race, Kevin is still in first place and Nina is still in second place, 15 miles behind Kevin. But now Kevin and Nina both finish out the race at a speed of 30 mph. When will Nina catch Kevin? If the race is 150 miles long, who will win? <p><i>The goal of this problem is that students will have the opportunity to explore a problem that can be solved using simultaneous linear equations from an intuitive standpoint, providing insight into graphical and algebraic methods that will be explored in the chapter. Students also gain insight into the meaning of the solution(s) to a system of linear equations. This problem requires students to analyze givens, constraints, relationships, and goals. Students may approach this problem using several different methods: picture, bar model, guess and check, table, equation, graph, etc.</i></p>
	<p>Reason abstractly and quantitatively.</p>	<p>Write a system of equations for the model below and solve the system using substitution.</p> $ \begin{array}{ccccccc} \star & + & \star & + & \star & + & 1 = \triangle \\ \star & + & \star & + & 3 = & & \triangle \end{array} $ <p><i>This chapter utilizes a pictorial approach in order to help students grasp the concepts of substitution and elimination. Students work with this concrete model and then transition into an abstract model as they begin to manipulate the equations in order to solve the system.</i></p>

	<p>Construct viable arguments and critique the reasoning of others.</p>	<p>How many solutions does the system of linear equations graphed below have? How do you know?</p>  <p><i>In order to answer this question students must understand that the graph of an equation shows all of the ordered pairs that satisfy the equation and that when we graph the equation of a line we see a limited view of that line. They must also understand what the solution to a system of linear equations is and how the solution is determined graphically. Students will use this information, along with additional supporting statements, in order to make an argument as to the number of solutions to this system of equations.</i></p>
	<p>Model with mathematics.</p>	<p>The student officers are buying packs of streamers and balloons to decorate for a school dance. Packs of balloons cost \$3.50 and packs of streamers cost \$2. If the student officers bought a total of 12 packs of decorations and spent \$31.50, how many packs of balloons did they buy? How many packs of streamers did they buy? Write the solution in a complete sentence.</p> <p><i>The ability to create and solve equations gives students the power to solve many real world problems. They will apply the strategies learned in this chapter to solve problems arising in everyday life that can be modeled and solved using simultaneous linear equations.</i></p>
	<p>Use appropriate tools strategically.</p>	<p>A farmer saw some chickens and pigs in a field. He counted 60 heads and 176 legs. Determine exactly how many chickens and pigs he saw.</p> <ol style="list-style-type: none"> Solve the problem using the methods and strategies studied in this chapter. Which method do you prefer using to solve this problem? Use your preferred method to determine the number of chickens and pigs in a field with 45 heads and 146 legs. <p><i>While solving this problem, students should be familiar with and consider all possible tools available: graphing calculator, graph paper, concrete models, tables, equations, etc. Students may gravitate toward the use of a graphing calculator given the size of the numbers. This technological tool may help them to explore this problem in greater depth.</i></p>

	<p>Attend to precision.</p>	<p>Consider the equations $-2x + y = -1$ and $y = 2x + 4$. Make sure both equations are written in slope-intercept form, then graph both equations on the coordinate plane below and solve the system of linear equations. <i>Solving systems of equations both graphically and algebraically requires students to attend to precision while executing many skills including using the properties of equality and laws of algebra in order to simplify and rearrange equations, producing graphs of equations, and simplifying and evaluating algebraic expressions in order to find and verify the solution to a system of linear equations.</i></p>
	<p>Look for and make use of structure.</p>	<p>One equation in a system of linear equations is $6x + 4y = -12$.</p> <ol style="list-style-type: none"> Write a second equation for the system so that the system has only one solution. Write a second equation for the system so that the system has no solution. Write a second equation for the system so that the system has infinitely many solutions. <p><i>In this problem, students must analyze the structure of the first equation in order to discern possible second equations that will result in one, infinitely many, or no solution.</i></p>
	<p>Look for and express regularity in repeated reasoning.</p>	<p>Gabriela and Camila like to race each other. Gabriela can run 10 feet/second while Camila can run 12 feet/second. Being a good sport, Camila gives Gabriela a 20-foot head start.</p> <p>How long will it take Camila to catch Gabriela? <i>Students can use repeated reasoning in order to solve this problem. Realizing that each second Camila closes the gap between her and Gabriela by 2 feet, students may determine that it will take 10 seconds in order for Camila to catch Gabriela.</i></p>

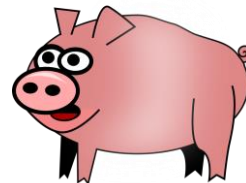
4.0 Anchor Problem: Chickens and Pigs

A farmer saw some chickens and pigs in a field. He counted 30 heads and 84 legs. Determine exactly how many chickens and pigs he saw. There are many different ways to solve this problem, and several strategies have been listed below. Solve the problem in as many different ways as you can and show your strategies below.



Strategies for Problem Solving

- **Make a List or Table**
- **Draw a Picture or Diagram**
- **Guess, Check, and Revise**
- **Write an Equation or Number Sentence**
- **Find a Pattern**
- **Work Backwards**
- **Create a Graph**
- **Use Logic and Reasoning**



Section 4.1: Understand Solutions of Simultaneous Linear Equations

Section Overview:

In this section, students are solving simultaneous linear equations that have one, no, or infinitely many solutions using intuitive and graphical methods. In order to access the problems initially students may use logic, and create pictures, bar models, and tables. They will solve simultaneous linear equations using a graphical approach, understanding that the solution is the point of intersection of the two graphs. Students will understand what it means to solve two linear equations, that is, finding all pairs (if any) of numbers (x, y) that are solutions to both equations and they will interpret the solution in a context.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Solve simultaneous linear equations by graphing.
2. Understand what it means to solve a system of equations.
3. Identify and provide examples of systems of equations that have one solution, infinitely many solutions, or no solution.
4. Interpret the solution to a system in a context.

- d. If p represents the number of pounds of peaches purchased and a represents the number of pounds of apples purchased, the situation above can be modeled by the following equations:

$$p + a = 20$$

$$2a + p = 28$$

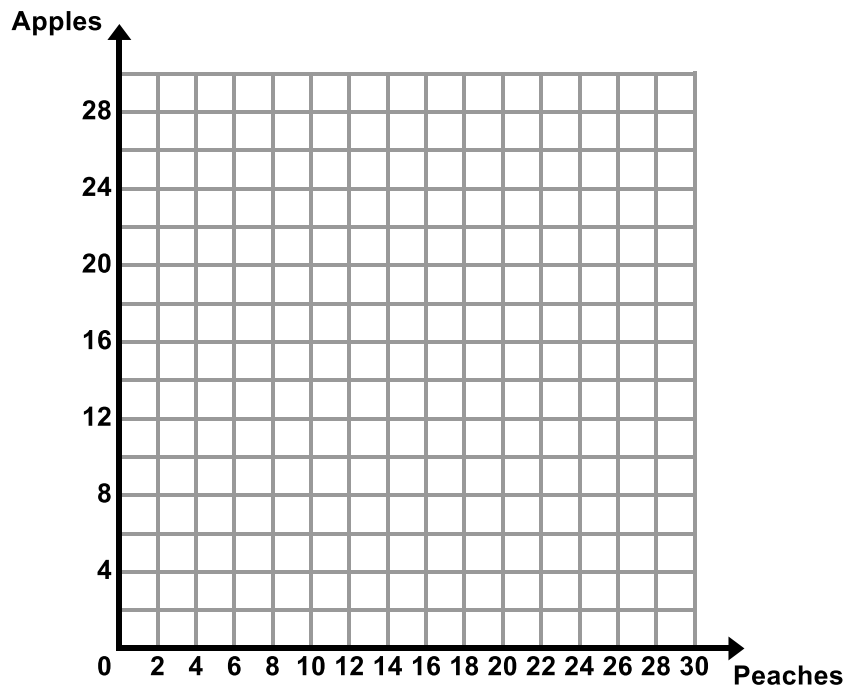
Write in words what each of these equations represents in the context.

$p + a = 20$ _____


$2a + p = 28$ _____

- e. Does the solution you found in part c) make both equations true?

- f. Graph the equations from part d) on the coordinate plane below. Label the lines according to what they represent in the context.



- g. Find the point of intersection in the graph above. What do you notice?

- h. The Bake Sale problem can be modeled and solved using a **system of linear equations**. Write in your own words what a **system of linear equations** is.
- i. Explain, in your own words, what the **solution** to a system of linear equations is. How can you find the solution in the different representations (table, graph, equation)? 
- j. Josh really likes apple pie so he wants to donate enough money so that there are an equal number of pounds of peaches and apples. How much does he need to donate?
- k. What if the students had to spend exactly \$25? Exactly \$20?
How would the equations change?

How would the graphs change?


What would the new solutions be?

- l. What if the students wanted to make 20 pies and had exactly \$64 to spend? Write the system of equations that models this problem. Find a combination that works.

4.1b Class Activity: Who Will Win the Race

1. Kevin and Nina are competing in a bike race. When Kevin is ninety miles into the race, he is in first place. Nina is in second place and is 15 miles behind Kevin.
 - a. From this point, Kevin continues the race at a constant rate of 25 mph and Nina continues the race at a constant rate of 30 mph. When will Nina catch Kevin? Solve this problem using any method you wish.

<p><u>Picture:</u></p> <div style="margin-top: 20px;"> <hr style="border: 1px solid black;"/> <p>Kevin</p> <hr style="border: 1px solid black;"/> <p>Nina</p> </div>	<p><u>Table:</u></p> <table border="1" style="width: 100%; border-collapse: collapse; margin-top: 10px;"> <tr> <td style="padding: 5px;">Time (hours)</td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> </tr> <tr> <td style="padding: 5px;">Kevin</td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td style="padding: 5px;">Nina</td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table> <table border="1" style="width: 100%; border-collapse: collapse; margin-top: 10px;"> <tr> <td style="width: 20px;"></td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table>	Time (hours)						Kevin						Nina																							
Time (hours)																																					
Kevin																																					
Nina																																					
<p><u>Graph:</u></p>	<p><u>Other Methods:</u></p> <p><i>Write an Equation, Use Logic and Reasoning, Guess, Check, and Revise, Find a Pattern, Work Backwards.</i></p>																																				

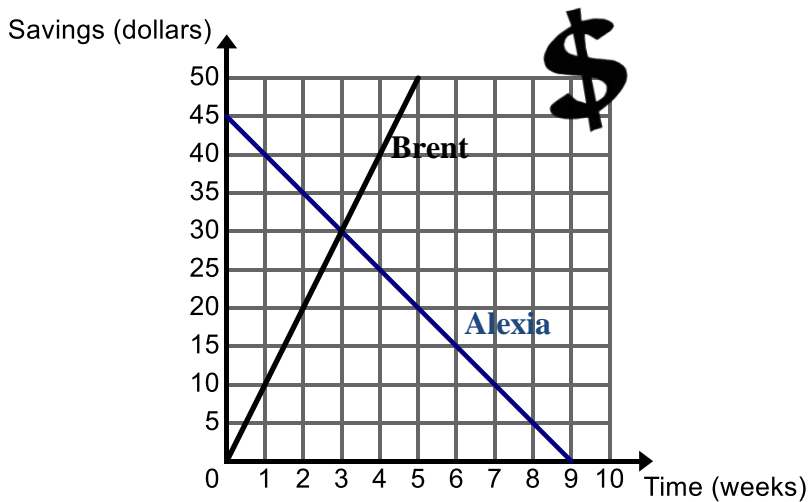
- b. If the race is 150 miles long, who will win? Assume Nina and Kevin bike at the speeds given in part a).
- c. Now suppose the following: Ninety miles into the race, Kevin is still in first place and Nina is still in second place, 15 miles behind Kevin. But now Kevin and Nina both finish out the race at a constant speed of 30 mph. When will Nina catch Kevin? If the race is 150 miles long, who will win? 

2. The graph below shows the amount of money Alexia and Brent have in savings.

- a. Write an equation to represent the amount y that each person has in savings after x weeks:


Alexia: _____


Brent: _____



- b. Tell the story of the graph. Be sure to include what the point of intersection means in the context.

4.1b Homework: Who Will Win the Race

1. Gabriela and Camila like to race each other. Gabriela can run 10 feet/second while Camila can run 12 feet/second. Being a good sport, Camila gives Gabriela a 20-foot head start. 

a. How long will it take Camila to catch Gabriela? (For ideas on how to solve this problem, see the strategies used in the classwork) 

b. If the girls are racing to a tree that is 30 yards away, who will win the race? (*Remember there are 3 feet in 1 yard*).

2. Darnell and Lance are both saving money. Darnell currently has \$40 and is saving \$5 each week. Lance has \$25 and is saving \$8 each week.

a. When will Darnell and Lance have the same amount of money?

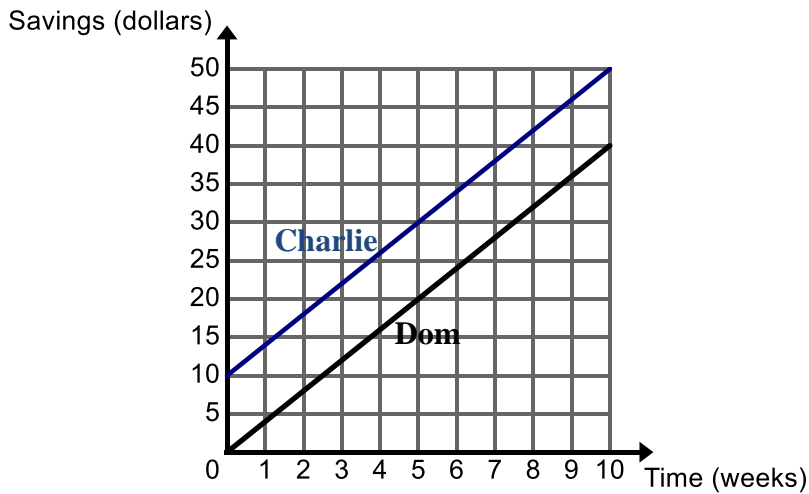
b. How much will each boy have when they have the same amount of money?

c. If both boys continue saving at this rate, who will have \$100 first?

3. The graph below shows the amount of money Charlie and Dom have in savings.

a. Write an equation to represent the amount y that each person has in savings after x weeks:

Charlie: _____ Dom: _____



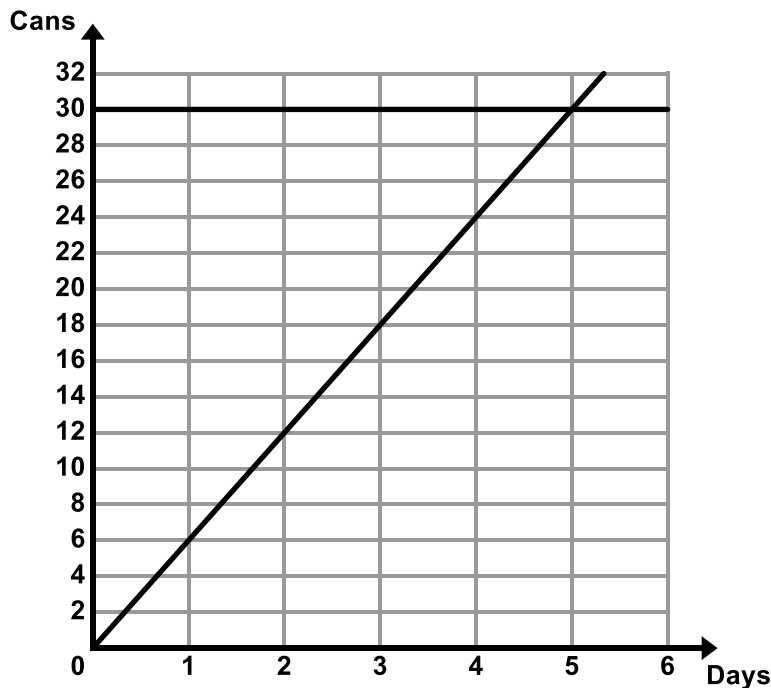
b. Tell the story of the graph.

4. Lakeview Middle School is having a food drive. The graph below shows the number of cans each class has collected for the food drive with time 0 being the start of week 3 of the food drive.



a. Write an equation to represent the number of cans y that each class has collected after x days.

Mrs. Lake's Class: _____ Mr. Luke's Class: _____



b. Tell the story of the graph.

4.1c Class Activity: Solving Simultaneous Linear Equations by Graphing

One method for solving simultaneous linear equations is graphing. In this method, both equations are graphed on the same coordinate grid, and the solution is found at the point where the two lines intersect.

Consider the simultaneous linear equations shown below and answer the questions that follow:

$$2x + y = 4$$

$$y = 4x - 2$$

1. What problems might you encounter as you try to graph these two equations?
2. What form of linear equations do we typically use when graphing?

As we have seen, it is possible to rearrange an equation that is not in slope-intercept form using the same rules we used when solving equations. We can rearrange this equation to put it in slope-intercept form. Remember, slope-intercept form is the form $y = mx + b$, so our goal here will be to isolate y on the left side of the equation, then arrange the right side so that our slope comes first, followed by the y -intercept.

$$2x + y = 4$$

Subtract $2x$ from both sides to isolate y

$$y = 4 - 2x$$

(Remember that 4 and $-2x$ are not like terms and cannot be combined)

$$y = -2x + 4$$

Rearrange the right side so that the equation is truly in slope-intercept form

3. Let's look at an example that is a little more challenging. With your teacher's help, write in the steps you complete as you go.

$$4x - 8y = 16$$

$$-8y = 16 - 4x$$

$$y = -2 + \frac{1}{2}x$$

$$y = \frac{1}{2}x - 2$$

4. **Skill Review:** Put the following equations into slope-intercept form.



a. $5x + y = 9$

c. $4y - x = 16$

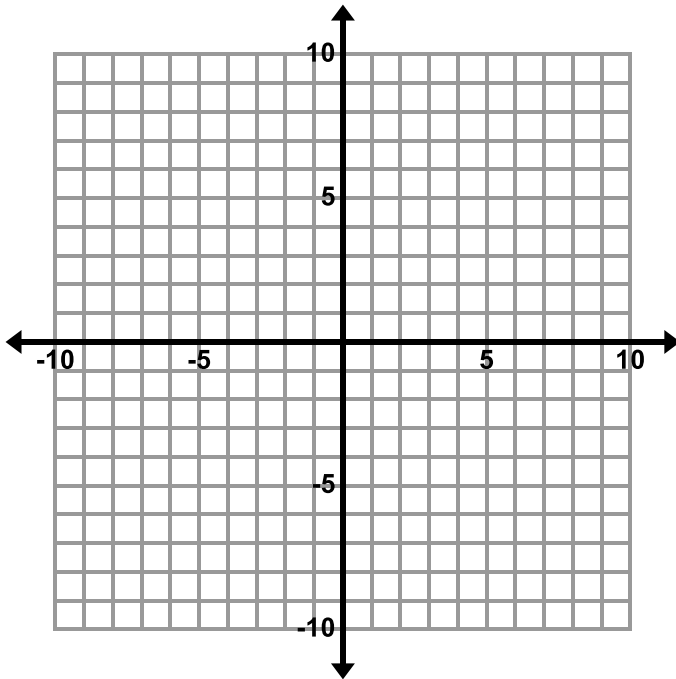
e. $-y = x - 2$

b. $4x + 2y = -12$

d. $4x - 2y = -24$

f. $-2x + 5y = 3$

5. Consider the linear equations $2x + y = 4$ and $y = 4x - 2$ from the previous page. Graph both equations on the coordinate plane below.



a. Find the coordinates (x, y) of the point of intersection.

b. Verify that the point of intersection you found satisfies both equations.

The **solution(s) to a pair of simultaneous linear equations** is all pairs (if any) of numbers (x, y) that are solutions of both equations, that is (x, y) satisfy both equations. When solved graphically, the solution is the point or points of intersection (if there is one).

6. Determine whether $(3, 8)$ is a solution to the following system of linear equations:

$$2x + y = 14$$

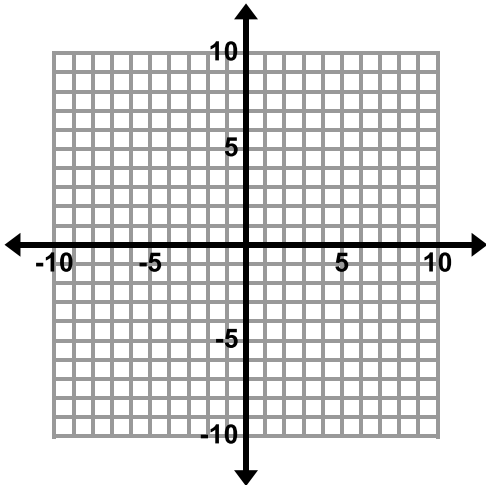
$$x + y = 11$$


7. Determine whether $(0, -5)$ is a solution to the following system of linear equations:

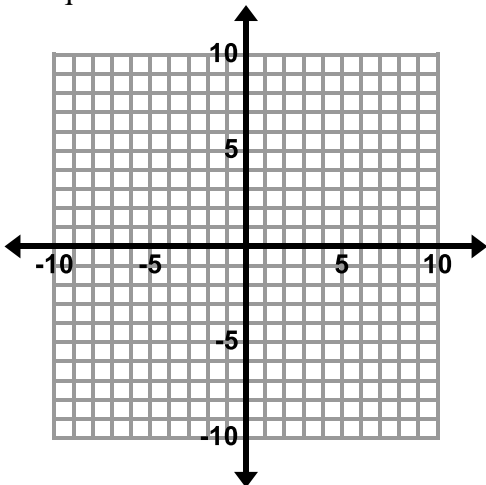
$$y = 2x - 5$$

$$4x + 5y = 25$$

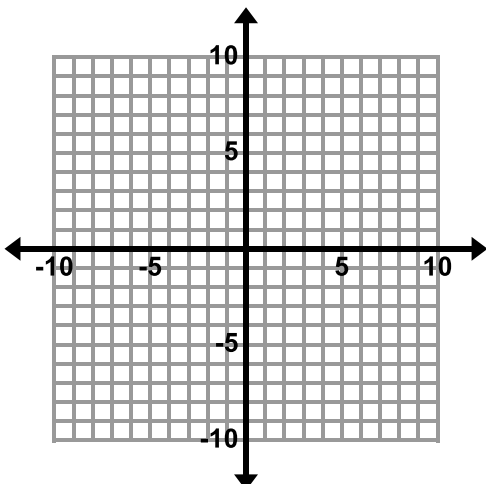
8. Consider the equations $y = -2x$ and $y = -\frac{1}{2}x - 3$. Make sure both equations are written in slope-intercept form, then graph both equations on the coordinate plane below and find the solution. Verify that the solution satisfies both equations.



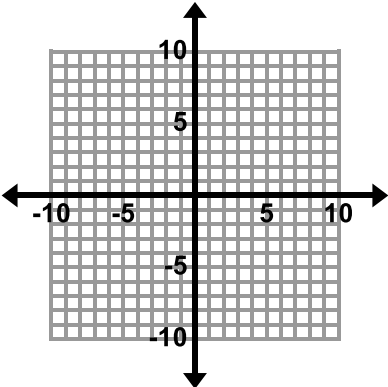
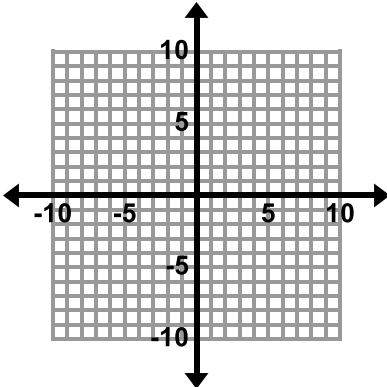
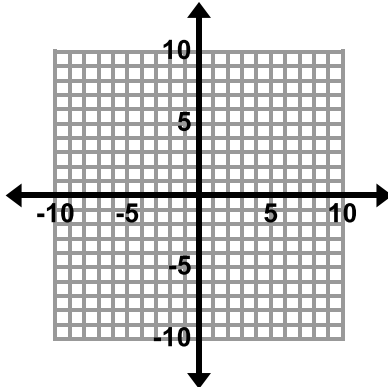
9. Consider the equations $-2x + y = -1$ and $y = 2x + 4$. Make sure both equations are written in slope-intercept form, then graph both equations on the coordinate plane below and solve the system of linear equations. 




10. Consider the equations $x + y = 3$ and $3x + 3y = 9$. Graph both equations on the coordinate plane below and solve the system of linear equations.



11. In the table below, draw an example of a graph that represents the different solving outcomes of a system of linear equations:

One Solution	No Solution	Infinitely Many Solutions
		

12. Without graphing, determine whether the following systems of linear equations will have one solution, no solution, or infinitely many solutions. 

a. $y = 8x + 2$ and $y = -4x$	b. $y = -\frac{2}{3}x - 5$ and $y + \frac{2}{3}x = 1$
c. $2x + y = 8$ and $y = 2x - 2$	d. $x + y = 5$ and $-2x - 2y = -10$
e. $3x + 2y = 5$ and $3x + 2y = 6$	f. $y = 2x + 5$ and $4x - 2y = -10$

13. One equation in a system of linear equations is $6x + 4y = -12$.

a. Write a second equation for the system so that the system has only **one solution**.

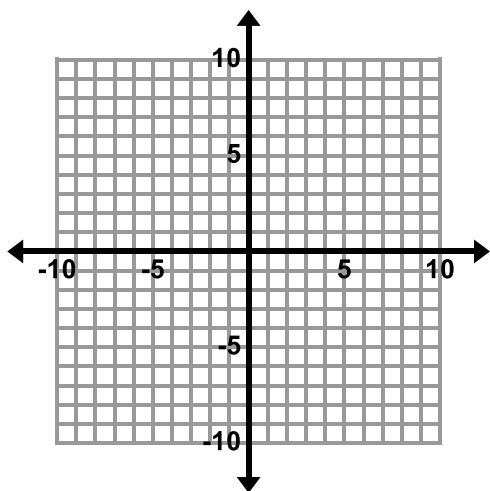
b. Write a second equation for the system so that the system has **no solution**.

c. Write a second equation for the system so that the system has **infinitely many solutions**.

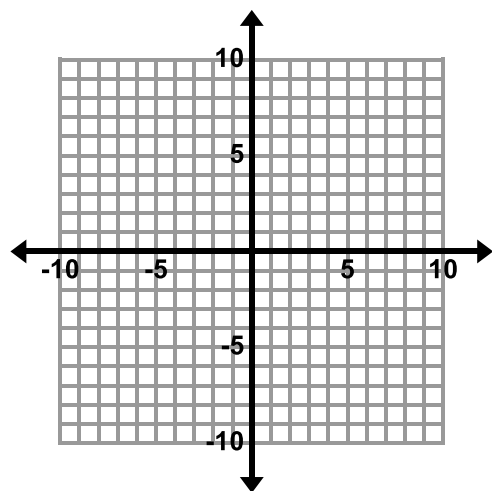
4.1c Homework: Solving Simultaneous Linear Equations by Graphing

1. Solve the system of linear equations graphically. If there is one solution, verify that your solution satisfies both equations.

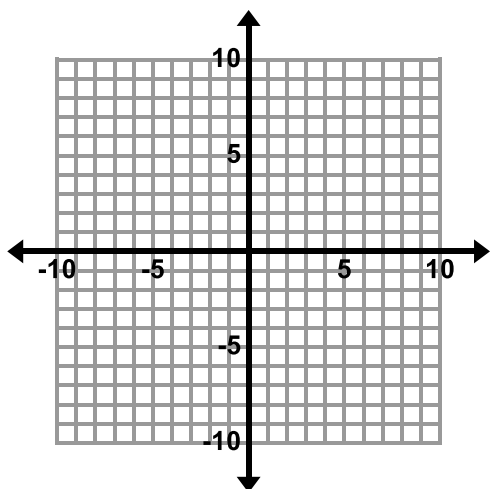
a. $y = 3x + 1$ and $x + y = 5$



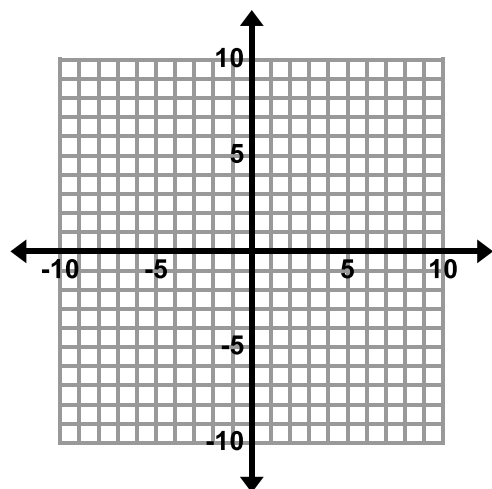
b. $y = -5$ and $2x + y = -3$



c. $y = -3x + 4$ and $y = \frac{1}{2}x - 3$

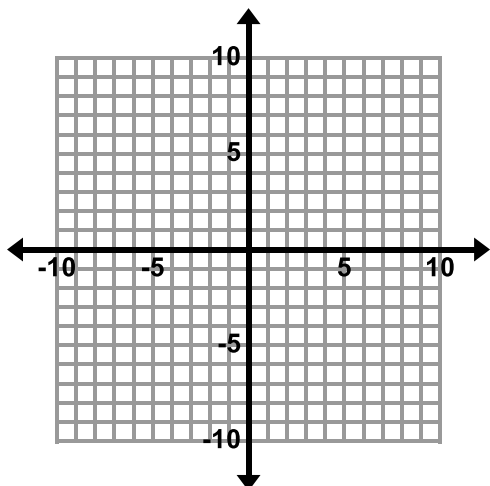


d. $x - y = -2$ and $-x + y = 2$

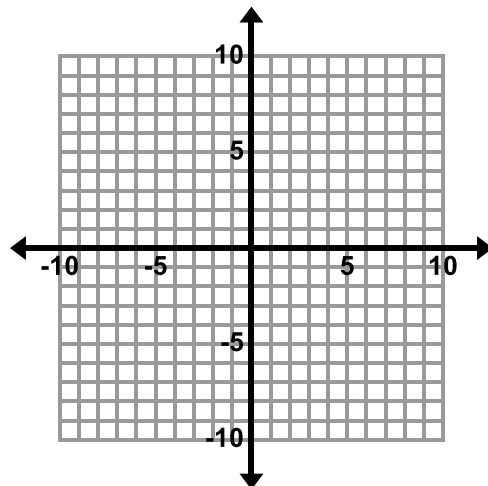


List 2 points that are solutions to this system.

e. $y = \frac{1}{2}x - 2$ and $y = \frac{1}{2}x + 4$



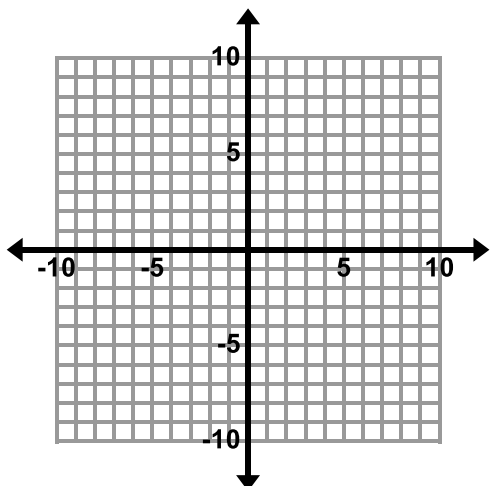
f. $2x - 8y = 6$ and $x - 4y = 3$



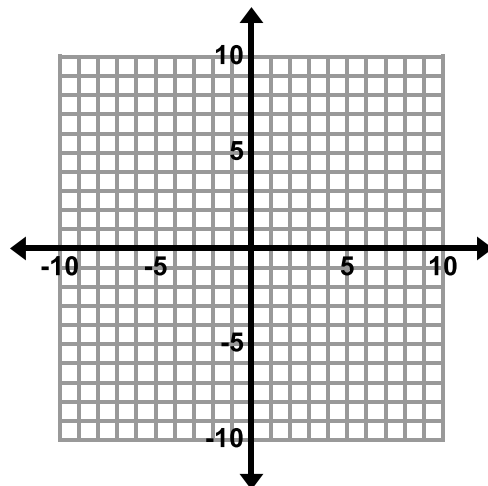
Circle the ordered pair(s) that are solutions to this system.

- (0, 0) (0, -1) (3, 0) (9, 3)

g. $y = 6x - 6$ and $y = 3x - 6$




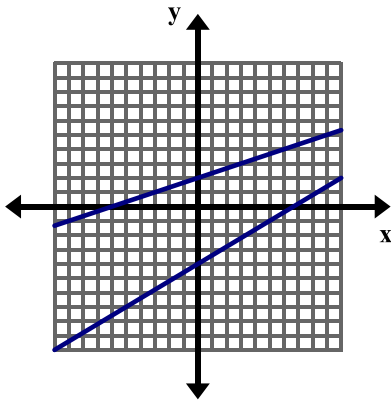
h. $2x + y = -4$ and $y + 2x = 3$



2. Without graphing, determine whether the following systems of linear equations will have one solution, no solution, or infinitely many solutions.

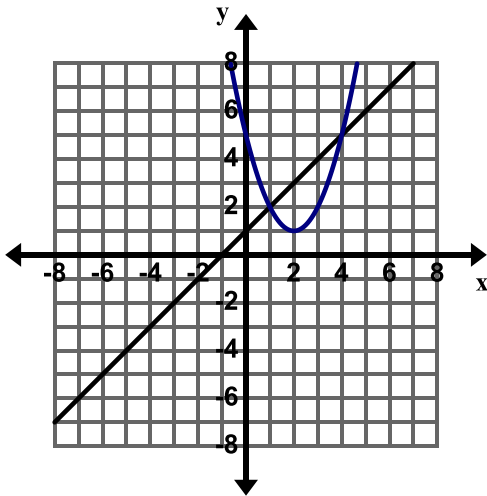
a. $x + y = 5$ and $x + y = 6$	b. $-3x + 9y = 15$ and $y = \frac{1}{3}x + \frac{5}{3}$
c. $y = 6$ and $y = 2x + 1$	d. $x - y = 5$ and $x + y = 5$

3. How many solutions does the system of linear equations graphed below have? How do you know? 



4. One equation in a system of linear equations is $y = x - 4$.
- Write a second equation for the system so that the system has only **one solution**.
 - Write a second equation for the system so that the system has **no solution**.
 - Write a second equation for the system so that the system has **infinitely many solutions**.

5. The grid below shows the graph of a line and a parabola (the curved graph).



- How many solutions do you think there are to this system of equations? Explain your answer.
- Estimate the solution(s) to this system of equations.
- The following is the system of equations graphed above.

$$y = x + 1$$

$$y = (x - 2)^2 + 1$$

How can you verify whether the solution(s) you estimated in part b) are correct?

- Verify the solution(s) from part b).

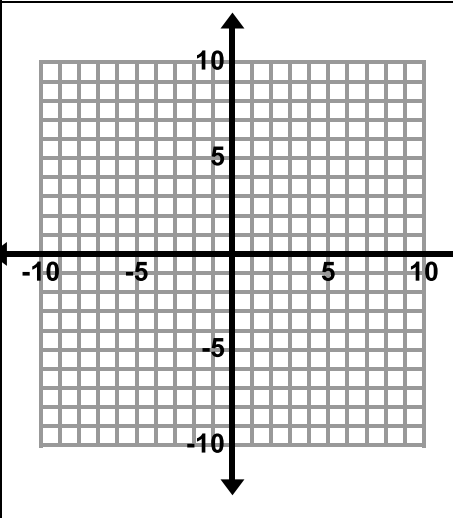
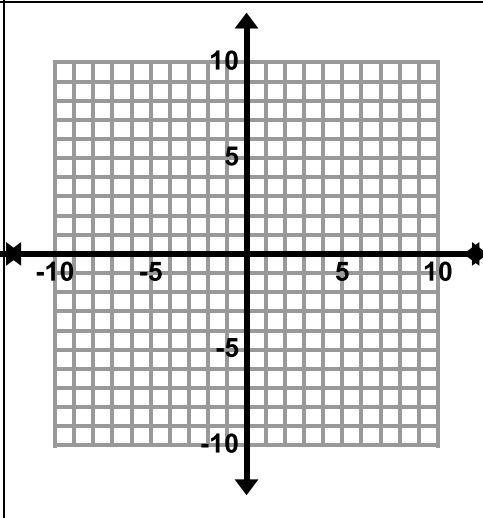
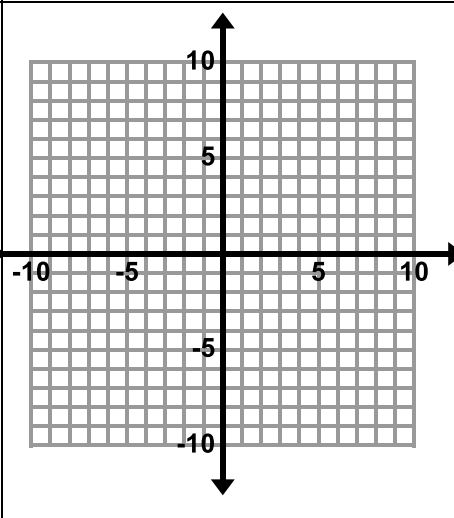
4.1d Self-Assessment: Section 4.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

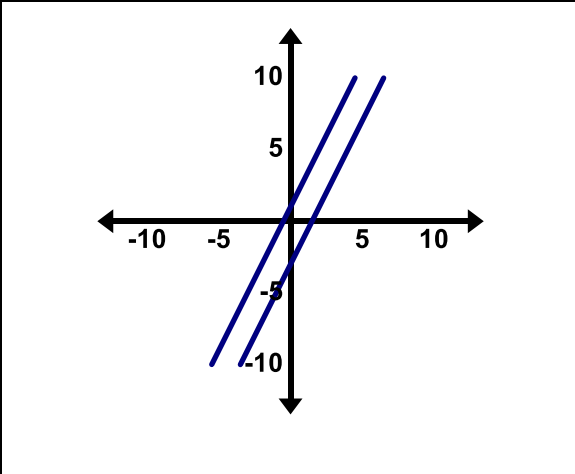
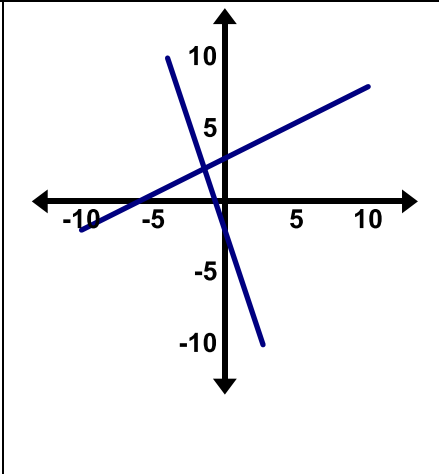
Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Solve simultaneous linear equations by graphing. Sample Problem #1	I can identify the solution to a system of linear equations when given the graphs of both equations.	I know that I can find a solution to a system of equations by graphing, but I often mess up with the graphing or getting the equations in slope-intercept form.	I can graph to find the solution to a system of equations, but I am not sure how to verify using algebra that the solution is correct.	I can re-write equations in slope-intercept form, graph them to find the solution, and plug the solution back in to verify my answer with very few mistakes.
2. Understand what it means to solve a system of equations. Sample Problem #1	I know that when I graph a system of equations, the answer is where the lines cross.	I know that when I graph a system of equations, the answer is the point where the two lines intersect, and is written as an ordered pair (x, y) .	I know that the solution to a system of equations is the point where the lines intersect, and that if you plug this point into the equations they should both be true.	I understand that the solution to a system of equations is the point on the coordinate plane where two lines intersect and because of this, it is also an ordered pair that satisfies both equations at the same time.
3. Identify and provide examples of systems of equations that have one solution, infinitely many solutions, or no solution. Sample Problems #2, #3	I can look at the graph of a system of equations and tell if it has one, no, or infinitely many solutions, but I sometimes get them mixed up.	I can look at the graph of a system of equations and tell if it has one, no, or infinitely many solutions. I can sometimes tell just by looking at the equations as well.	I know how to tell how many solutions a system of equations has by looking at a graph. I can also tell how many solutions a system of equations has by looking at the equations. When given an equation, I can write another equation that would give the system of equations one, no, or infinitely many solutions.	I know how to tell how many solutions a system of equations has by looking at a graph and by looking at just the equations. I understand what it is about the structure of the equations that makes the graphs look the way they do. I can write a system of equations that would have one solution, no solution, or infinitely many solutions.
4. Interpret the solution to a system in a context. Sample Problem #4	When I solve a story problem involving a system of equations I struggle to explain what the solution represents in the context.	When I solve a story problem involving a system of equations, I understand what the solution means, and I can explain to someone what my answer means most of the time.	When given a story problem involving a system of equations, I can write a sentence explaining what the answer means in the context.	When given a story problem involving a system of equations, I can write a sentence describing what the answer means in the context. I can also answer additional questions about the situation.

Section 4.1 Sample Problems (For use with self-assessment)

1. Graph the following systems of equations to find the solution. After you have found your solution, verify that it is correct.

$\begin{cases} y = 3x - 5 \\ y = \frac{1}{2}x \end{cases}$	$\begin{cases} y = -2x + 7 \\ x + 3y = -9 \end{cases}$	$\begin{cases} -4x + 6y = 6 \\ x + y = 6 \end{cases}$
		
<p>Verify:</p>	<p>Verify:</p>	<p>Verify:</p>

2. Tell whether the system of equations has one solution, infinitely many solutions, or no solutions.

		$\begin{cases} y = 3x + 4 \\ 2y = 6x + 8 \end{cases}$ $\begin{cases} y = -\frac{1}{4}x + 6 \\ y = -\frac{1}{4}x - 4 \end{cases}$
---	--	---

3. One equation in a system of linear equations is $y = -2x + 4$.
- Write a second equation for the system so that the system has only **one solution**.

 - Write a second equation for the system so that the system has **no solution**.

 - Write a second equation for the system so that the system has **infinitely many solutions**.
4. At the county fair, you and your little sister play a game called Honey Money. In this game she covers herself in honey and you dig through some sawdust to find hidden money and stick as much of it to her as you can in 30 seconds. The fair directors have hid only \$1 bills and \$5 bills in the sawdust. During the game your little sister counts as you put the bills on her. She doesn't know the difference between \$1 bills and \$5 bills, but she knows that you put 16 bills on her total. You were busy counting up how much money you were going to make, and you came up with a total of \$40. After the activity you put the all the money into a bag and your little sister takes it to show her friends and loses it. The fair directors find a bag of money, but say they can only give it to you if you can tell them how many \$1 bills you had, and how many \$5 bills you had. What will you tell the fair directors so you can get your money back?
- Solve this problem using any method you wish. Show your work in the space below.

- Write your response to the fair directors in a complete sentence on the lines provided.

Section 4.2: Solve Simultaneous Linear Equations Algebraically

Section Overview:

In this section, students are solving simultaneous linear equations that have one, no, or infinitely many solutions using algebraic methods. The section utilizes concrete models and real world problems in order to help students grasp the concepts of substitution and elimination. Students then solve systems of linear equations abstractly by manipulating the equations. Students then apply the skills they have learned in order to solve real world problems that can be modeled and solved using simultaneous linear equations.

Concepts and Skills to Master:

By the end of this section, students should be able to:

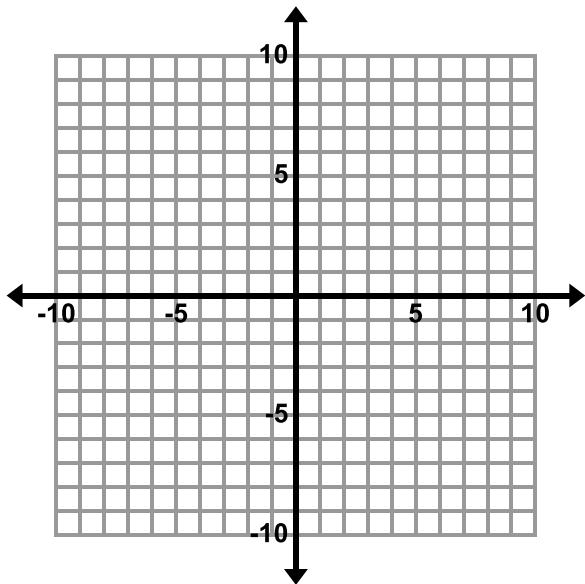
1. Determine which method of solving a system of linear equations may be easier depending on the problem.
2. Solve simultaneous linear equations algebraically.
3. Create a system of linear equations to model a real world problem, solve the system, and interpret the solution in the context.

4.2a Class Activity: Introduction to Substitution

In the previous section, you learned how to solve a system of linear equations by graphing. In this section, we will learn another way to solve a system of linear equations. Solve the following system by graphing.

$$y = 3x + 2$$

$$y = -5x$$



Give some reasons as to why graphing is not always the best method for solving a system of linear equations.

In this section, we will learn about algebraic methods for solving systems of linear equations. These methods are called substitution and elimination.

Directions: Find the value of each shape. Verify your answers.

1.

$$\bigcirc + \bigcirc + \square = 25$$

$$\square = 5$$

$$\bigcirc = \underline{\hspace{2cm}}$$

How did you determine the circle's value?

2.

$$\begin{aligned} \bigcirc + \bigcirc + \square &= 18 \\ \bigcirc + \square &= 10 \end{aligned}$$

$$\square = \underline{\hspace{2cm}}$$

$$\bigcirc = \underline{\hspace{2cm}}$$

How did you determine the value of each shape?

3.

$$\begin{aligned} \bigcirc + \bigcirc + \square + \square &= 20 \\ \bigcirc + \bigcirc + \square &= 17 \end{aligned}$$

$$\square = \underline{\hspace{2cm}}$$

$$\bigcirc = \underline{\hspace{2cm}}$$

How did you determine the value of each shape?

4.

$$\begin{aligned} \triangle + \triangle + \triangle &= 27 \\ \triangle + \square &= 8 \end{aligned}$$

$$\triangle = \underline{\hspace{2cm}}$$

$$\square = \underline{\hspace{2cm}}$$

How did you determine the value of each shape?

5.

$$\begin{aligned} \star + \star &= 16 \\ \star + \star + \triangle + \triangle &= 26 \end{aligned}$$

$$\star = \underline{\hspace{2cm}}$$

$$\triangle = \underline{\hspace{2cm}}$$

How did you determine the value of each shape?

6.

$$\begin{array}{ccccccc} \star & + & \star & + & \circ & + & \circ & + & \circ & = & 19 \\ \star & + & \star & + & \circ & = & 13 \end{array}$$

$$\begin{array}{l} \star = \underline{\hspace{2cm}} \\ \circ = \underline{\hspace{2cm}} \end{array}$$

How did you determine the value of each shape?

7.

$$\begin{array}{ccccccc} \circ & + & \square & + & \square & = & 30 \\ \square & = & \circ & + & \circ \end{array}$$

$$\begin{array}{l} \square = \underline{\hspace{2cm}} \\ \circ = \underline{\hspace{2cm}} \end{array}$$

How did you determine the value of each shape?

8.

$$\begin{array}{ccccccc} \circ & + & \circ & + & \triangle & + & \triangle & = & 16 \\ \circ & + & \circ & = & \triangle & + & \triangle & + & 4 \end{array}$$

$$\begin{array}{l} \circ = \underline{\hspace{2cm}} \\ \triangle = \underline{\hspace{2cm}} \end{array}$$

How did you determine the value of each shape?

9.

$$\begin{array}{ccccccc} \square & + & \square & + & \star & = & 21 \\ \square & + & \square & + & \square & + & \square & + & \star & + & \star & = & 42 \end{array}$$

$$\begin{array}{l} \square = \underline{\hspace{2cm}} \\ \star = \underline{\hspace{2cm}} \end{array}$$

Directions: Draw a picture of each equation with shapes and then find the value of each shape.

10. $3x + 2y = 41$

$$2y = 8$$

11. $2x + y = 9$

$$x + y = 5$$

12. $x + 3y = 41$

$$x + 2y = 32$$

13. $2x + 2y = 18$

$$2x = y$$

Challenge Questions: Find the value of each variable using shapes.

$$14. x + 2y = 46$$

$$y + 3z = 41$$

$$3z = 27$$

$$15. 2x + z = 46$$

$$3z = 18$$

$$2y + z = 40$$

$$16. 2x + 2y = 50$$

$$2x + y = 42$$

$$y + 2z = 18$$

4.2a Homework: Introduction to Substitution

Directions: Find the value of each shape. Explain how you determined each. Verify your answers.

1.

$$\bigcirc + \bigcirc + \bigcirc + \bigcirc + \square + \square = 34$$

$$\bigcirc + \bigcirc + \square + \square = 26$$

$$\bigcirc = \underline{\hspace{2cm}}$$

$$\square = \underline{\hspace{2cm}}$$

2.

$$\triangle + \triangle + \triangle + \hexagon = 27$$

$$\triangle + \triangle + \hexagon = 20$$

$$\triangle = \underline{\hspace{2cm}}$$

$$\hexagon = \underline{\hspace{2cm}}$$

3.

$$\odot + \odot + \odot + \odot + \smile + \smile = 42$$

$$\smile + \smile = \odot + \odot + \odot$$

$$\odot = \underline{\hspace{2cm}}$$

$$\smile = \underline{\hspace{2cm}}$$

4.

$$\text{drop} + \text{drop} + \text{drop} + \text{drop} + \text{drop} + \text{drop} = \text{cloud}$$

$$\text{drop} + \text{drop} + \text{drop} + \text{drop} + 8 = \text{cloud}$$

$$\text{drop} = \underline{\hspace{2cm}}$$

$$\text{cloud} = \underline{\hspace{2cm}}$$

Directions: Draw a picture of each equation with shapes and then find the value of each shape.

5. $x + y = 15$

$$y = x + 10$$

6. $y + x = 5$

$$x = y - 3$$

7. $y = 4x$

$$x + y = 5$$

8. $2x + y = 7$

$$x + y = 1$$

9. $3x + 4y = 19$

$$3x + 6y = 33$$

10. $5x + 6y = 100$

$$4x + 6y = 92$$

4.2b Class Activity: Substitution Method for Solving Systems of Equations

Directions: Write a system of equations from the shapes. Find the value of each shape.

1.

$$\begin{array}{l} \bigcirc + \square + \square = 30 \\ \square = \bigcirc + \bigcirc \end{array}$$

System of Equations:

$$\begin{array}{l} \bigcirc = \underline{\hspace{2cm}} \\ \square = \underline{\hspace{2cm}} \end{array}$$

How did you determine the value of each shape?

2.

$$\begin{array}{l} \triangle + \triangle + \triangle + \bigcirc = 12 \\ \triangle + \triangle + \triangle + \triangle + \bigcirc + \bigcirc + \bigcirc = 26 \end{array}$$

$$\begin{array}{l} \triangle = \underline{\hspace{2cm}} \\ \bigcirc = \underline{\hspace{2cm}} \end{array}$$

System of Equations:

How did you determine the value of each shape?

To solve any system of linear equations using substitution, do the following:

1. Rewrite one of the equations so that one variable is expressed in terms of the other (solve one of the equations for one of its variables).
2. Substitute the expression from step 1 into the other equation and solve for the remaining variable.
3. Substitute the value from step 2 into the equation from step 1 and solve for the remaining variable.
4. Check the solution in each of the original equations.

Revisit problem #2 from above and use these steps to solve.

3.

$$\begin{array}{l} \square + \square + 7 = \bigcirc \\ \square + \square + \square + \bigcirc + \bigcirc = 35 \end{array}$$

a. Write a system of equations for the picture above.

b. Solve this system of equations using substitution showing all steps. Check your solution.

4.

$$\begin{array}{l} \star + \star + \star + 1 = \triangle \\ \star + \star + 3 = \triangle \end{array}$$

a. Write a system of equations for the picture above.

b. Solve this system of equations using substitution showing all steps. Check your solution.

5.

$$\begin{array}{l} \bigcirc + \bigcirc + \bigcirc + 5 = \triangle \\ \bigcirc + \bigcirc + \bigcirc + 3 = \triangle \end{array}$$

$$\begin{array}{l} \bigcirc = \underline{\hspace{2cm}} \\ \triangle = \underline{\hspace{2cm}} \end{array}$$


- Write a system of equations for the pictures above.
- Solve this system of equations using substitution showing all steps. Check your solution.
- Describe what you would see in a graph of this system.

6.

$$\begin{array}{l} \square + \square + \star = 21 \\ \square + \square + \square + \square + \star + \star = 42 \end{array}$$

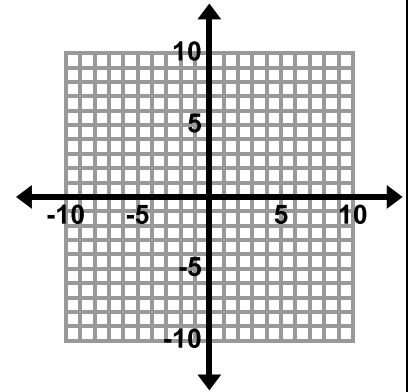
$$\begin{array}{l} \square = \underline{\hspace{2cm}} \\ \star = \underline{\hspace{2cm}} \end{array}$$

- Write a system of equations for the pictures above.
- Solve this system of equations using substitution showing all steps. Check your solution.
- Describe what you would see in a graph of this system.

Directions: Solve each system using the substitution method. When asked, solve the system by graphing in addition to using the substitution method. 

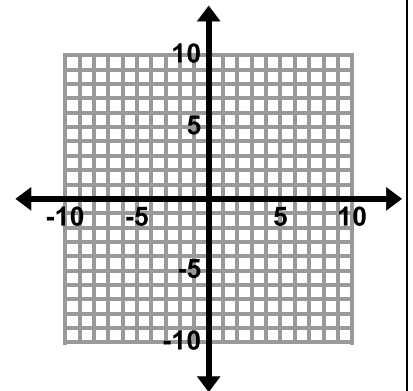
7. $y = 5x + 4$
 $y = -3x - 12$

8. $y = 6x + 4$
 $y = 6x - 10$



9. $y = x + 2$
 $x + 3y = -2$

10. $x = 2y - 4$
 $x + y = 2$



11. $y - x = 5$
 $2x + y = -10$

12. $2x + y = 5$
 $y = -5 - 2x$

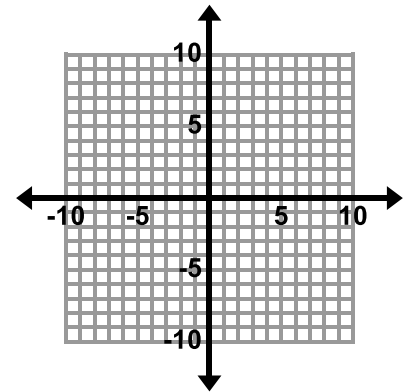
$$\begin{aligned} 13. \quad y + x &= 5 \\ x &= y - 3 \end{aligned}$$

$$\begin{aligned} 14. \quad x + y &= 4 \\ y &= -x + 4 \end{aligned}$$

List 2 points that are solutions to this system.

$$\begin{aligned} 15. \quad x + 2y &= 7 \\ 2x + 3y &= 12 \end{aligned}$$

$$\begin{aligned} 16. \quad 6x + y &= -5 \\ -12x - 2y &= 10 \end{aligned}$$



Solve by graphing.

$$\begin{aligned} 17. \quad x &= 2y + 4 \\ 4y + x &= 2 \end{aligned}$$

$$\begin{aligned} 18. \quad y &= 3x + 2 \\ y &= -5x \end{aligned}$$

Directions: The following are examples of real-world problems that can be modeled and solved with systems of linear equations. Answer the questions for each problem.



19. Nettie's Bargain Clothing is having a huge sale. All shirts are \$3 each and all pants are \$5 each. You go to the sale and buy twice as many shirts as pants and spend \$66.

The following system of equations models this situation where s = number of shirts and p = number of pants:

$$s = 2p$$

$$3s + 5p = 66$$

- a. Write in words what each of the equations in the system represents in the context.

$$s = 2p \underline{\hspace{15em}}$$

$$3s + 5p = 66 \underline{\hspace{15em}}$$

- b. Solve this system using substitution to determine how many of each item you bought. Write your answer in a complete sentence.

20. Xavier and Carlos have a bet to see who can get more "friends" on a social media site after 1 month. Carlos has 5 more friends than Xavier when they start the competition. After much work, Carlos doubles his amount of friends and Xavier triples his. In the end they have a total of 160 friends together.

The following system of equations models this situation where c = the number of friends Carlos starts with and x = the number of friends Xavier starts with.

$$c = x + 5$$

$$2c + 3x = 160$$

- a. Write in words what each of the equations in the system represents in the context.

$$c = x + 5 \underline{\hspace{15em}}$$

$$2c + 3x = 160 \underline{\hspace{15em}}$$

- b. Solve this system using substitution to determine how many friends each boy started with. Write your answer in a complete sentence.

4.2b Homework: Substitution Method for Solving Systems of Equations

Directions: Solve each system of linear equations using substitution.

1. $y = 4x$ $x + y = 5$	2. $x = -4y$ $3x + 2y = 20$	3. $y = x - 1$ $x + y = 3$
4. $3x - y = 4$ $2x - 3y = -9$	5. $x - y = 6$ $2x = 12 + 2y$	6. $2x + 2y = 6$ $x + y = 0$
7. $2x + y = -15$ $y - 5x = 6$	8. $y = 3x + 4$ $y = x - 7$	9. $y = -3x + 6$ $9x + 3y = 18$

10. Niona needs \$50 to go on a school trip. She sells necklaces for \$15 each and bracelets for \$5 each. If she raises the money by selling half as many necklaces as bracelets, how many necklaces and bracelets does she sell? Write a system of linear equations that represent this problem.

Define your Unknowns:



System of Equations:

Next to each equation, write in words what the equation represents in the context.

Solve. Write your answer in a complete sentence.

11. A restaurant needs to order stools and chairs. Each stool has 3 legs and each chair has 4 legs. The manager wants to be able to seat 36 people. The restaurant has hard wood floors and the manager doesn't want to scratch them. Therefore, they have ordered 129 plastic feet covers for the bottom of the legs to ensure the stools and chairs don't scratch the floor. How many chairs and how many stools did the restaurant order?
- Write a system of equations that matches the verbal descriptions given below if s = number of stools and c = number of chairs.

System of Equations:

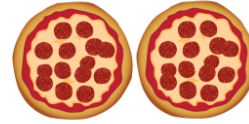
Equation 1: _____ Each stool needs 3 plastic feet covers. Each chair needs 4 plastic feet covers. One hundred twenty-nine plastic feet covers are needed.

Equation 2: _____ There are a total of 36 chairs and stools needed.

- Solve the system. Write your answer in a complete sentence.

4.2c Class Activity: Elimination Method for Solving Systems of Linear Equations

1. Ariana and Emily are both standing in line at Papa Joe's Pizza. Ariana orders 4 large cheese pizzas and 1 order of breadsticks. Her total before tax is \$34.46. Emily orders 2 large cheese pizzas and 1 order of breadsticks. Her total before tax is \$18.48. Determine the cost of 1 large cheese pizza and 1 order of breadsticks. **Explain** the method you used for solving this problem.



2. Carter and Sani each have the same number of marbles. Sani's little sister comes in and takes some of Carter's marbles and gives them to Sani. After she has done this, Sani has 18 marbles and Carter has 10 marbles. How many marbles did each of the boys start with? How many marbles did Sani's sister take from Carter and give to Sani?

3.

$$\bigcirc + \bigcirc + \bigcirc + \bigcirc + \square = 18$$

$$\bigcirc = \underline{\quad}$$

$$\bigcirc + \bigcirc + \square = 10$$

$$\square = \underline{\quad}$$

a. Find the value of each shape.

b. Which of the problems from the previous page is this similar to? Compare the strategies you used to solve these problems.

4.

$$\bigcirc + \square = 15$$

$$\bigcirc = \underline{\quad}$$

$$\bigcirc - \square = 7$$

$$\square = \underline{\quad}$$

a. Find the value of each shape.

b. Which of the problems from the previous page is this similar to? Compare the strategies you used to solve these problems.

Directions: Write a system of equations from the shapes. Find the value of each shape. Show the solving actions algebraically.

5. $\bigcirc + \bigcirc + \square = 14$

$\bigcirc + \bigcirc - \square = 10$

$\bigcirc = \underline{\hspace{2cm}}$

$\square = \underline{\hspace{2cm}}$

Equations:

$2c + s = 14$

$2c - s = 10$

c (*circle*) =

s (*square*) =

6. $\square + \bigcirc + \bigcirc = 19$

$\bigcirc - \square = 11$

$\bigcirc = \underline{\hspace{2cm}}$

$\square = \underline{\hspace{2cm}}$

Equations:

7. $\square + \square + \bigcirc = 27$

$\bigcirc - \square - \square = 15$

$\bigcirc = \underline{\hspace{2cm}}$

$\square = \underline{\hspace{2cm}}$

Equations:

8. The name of the method you are using to solve the systems of linear equations above is **elimination**. Why do you think this method is called **elimination**?

Directions: Write a system of equations from the shapes. Find the value of each shape. Show the solving actions algebraically.

9.

$$\star + \star - \bigcirc = 8$$

$$\star = \underline{\hspace{2cm}}$$

$$\star + \star - \bigcirc - \bigcirc = 4$$

$$\bigcirc = \underline{\hspace{2cm}}$$

10.


$$\triangle + \triangle + \triangle + \triangle + \star + \star = 8$$

$$\triangle = \underline{\hspace{2cm}}$$

$$\triangle + \triangle + \star + \star = -6$$

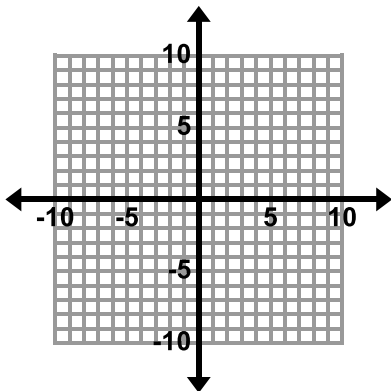
$$\star = \underline{\hspace{2cm}}$$

11. How are problems 9 and 10 different from #5 – 7. Describe in your own words how you solved the problems in this lesson.

Directions: Solve each system of linear equations using **elimination**. Make sure the equations are in the same form first. Graph the first three problems as well as using elimination. 

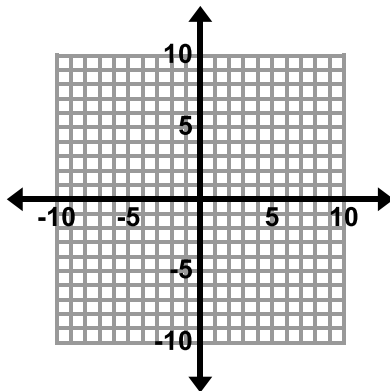
$$\begin{aligned} 12. \quad x + y &= -3 \\ 2x - y &= -3 \end{aligned}$$

Solve by graphing.



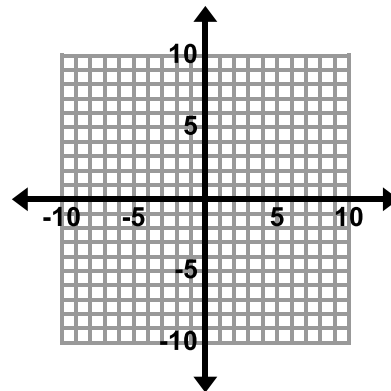
$$\begin{aligned} 13. \quad x + y &= 5 \\ -x - y &= -5 \end{aligned}$$

Solve by graphing.



$$\begin{aligned} 14. \quad x + y &= 3 \\ 2x - y &= -3 \end{aligned}$$

Solve by graphing.



$$\begin{aligned} 15. \quad 2x - y &= -3 \\ 3x - y &= 1 \end{aligned}$$

$$\begin{aligned} 16. \quad 2x - y &= 9 \\ x + y &= 3 \end{aligned}$$

$$\begin{aligned} 17. \quad 7x - 4y &= -30 \\ 3x + 4y &= 10 \end{aligned}$$

<p>18. $2x + y = 6$ $2x + y = -7$</p>	<p>19. $3x - y = 1$ $x = -y + 3$</p>	<p>20. $x = y + 3$ $x - 2y = 3$</p>
---	--	---

21. Complete the story for the system of equations shown below if s is number of shirts and p is number of pants. Solve the system and write your solution in a complete sentence.

$$s + p = 18$$

$$5s + 12p = 160$$

Solve:

Story

Jennifer is buying shirts and pants at a sale.

She buys 18...

Shirts cost \$5 each and pants cost...

Jennifer spends...

How many shirts and how many pants did Jennifer purchase?

Solution (in a complete sentence):

4.2c Homework: Elimination Method of Solving Linear Systems

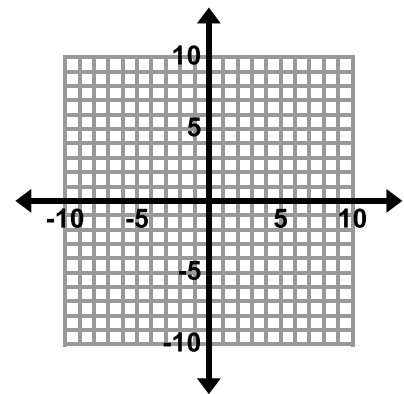
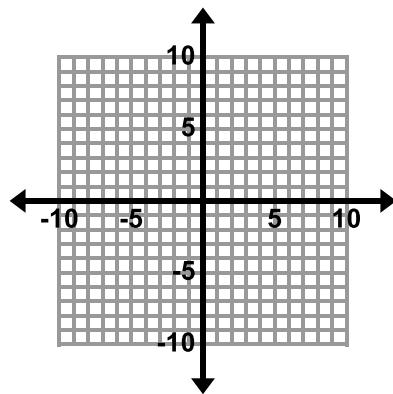
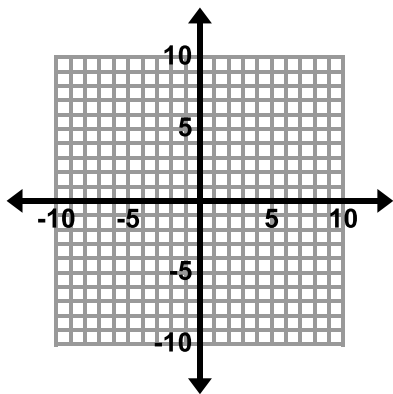
Directions: Solve each system of linear equations using **elimination**. Make sure the equations are in the same form first. Choose three problems to solve by graphing as well as using elimination to solve the system. The graphs are located after problem #9.

1. $6x - y = 5$ $3x + y = 4$	2. $x + 4y = 9$ $-x - 2y = 3$	3. $x + 5y = -8$ $-x - 2y = -13$
4. $2x + y = 7$ $x + y = 1$	5. $4x + 3y = 18$ $4x = 8 + 2y$	6. $-5x + 2y = 22$ $3x + 2y = -10$

$$7. \begin{aligned} 6x - 3y &= 36 \\ 5x &= 3y + 30 \end{aligned}$$

$$8. \begin{aligned} -4x + y &= -12 \\ -y + 6x &= 8 \end{aligned}$$

$$9. \begin{aligned} x + y &= 7 \\ -x - y &= 12 \end{aligned}$$



10. An exam worth 145 points contains 50 questions. Some of the questions are worth two points and some are worth five points. How many two-point questions are on the test? How many five-point questions are on the test?

4.2d Class Activity: Elimination Method Multiply First

1. Solve the following system of linear equations using **elimination**:

$$4x + y = 7$$

$$-2x - 3y = -1$$

To solve any system of linear equations using elimination, do the following:

1. Write both equations in the same form.
2. Multiply the equations by nonzero numbers so that one of the variables will be eliminated if you take the sum or difference of the equations.
3. Take the sum or difference of the equations to obtain a new equation in just one unknown.
4. Solve for the remaining variable.
5. Substitute the value from step 4 back into one of the original equations to solve for the other unknown.
6. Check the solution in each of the original equations.

Directions: Solve each system of linear equations using **elimination**.

2. $x + 2y = 15$ $5x + y = 21$	3. $-3x + 2y = -8$ $6x - 4y = -20$
4. $2x - 3y = 5$ $-3x + 4y = -8$	5. $3x - 2y = 2$ $5x - 5y = 10$
6. $9x + 13y = 10$ $-9x - 13y = 8$	7. $-16x + 2y = -2$ $y = 8x - 1$

$$8. \begin{cases} \frac{1}{2}x + y = 0 \\ x - y = -15 \end{cases}$$

$$9. \begin{cases} \frac{2}{3}x + \frac{1}{3}y = 13 \\ -2x + y = -21 \end{cases}$$

Directions: Write a system of equations for each word problem below and then solve the system of equations

using **elimination**. Write your answer in a complete sentence.



10. The student officers are buying packs of streamers and balloons to decorate for a school dance. Packs of balloons cost \$3.50 and packs of streamers cost \$2. If the student officers bought a total of 12 packs of decorations and spent \$31.50, how many packs of balloons did they buy? How many packs of streamers did they buy? Write the solution in a complete sentence.



Define your Unknowns:

Equation for Number of Packs of Decorations:

Equation for Cost of Decorations:

Solve:

Solution (in a complete sentence):

11. Jayda has a coin collection consisting of nickels and dimes. Write a story that matches the system of equations shown below that describes the coins in Jayda's collection where n is the number of nickels Jayda has and d is the number of dimes Jayda has.

$$\begin{cases} n + d = 28 \\ 0.05n + .1d = 2.25 \end{cases}$$

Story

Solve:

Solution (in a complete sentence):

4.2d Homework: Elimination Method Multiply First

Directions: Solve each system using **elimination**.



1. $x + y = 4.5$ $-2x + 4y = 6$	2. $4x + y = -8$ $3x + 3y = 3$
3. $2x + y = 7$ $4x + 2y = 14$	4. $2x + 3y = -10$ $-4x + 5y = -2$
5. $x - 2y = \frac{2}{3}$ $-3x + 5y = -2$	6. $-3x - y = -15$ $8x + 4y = 48$
7. $3x - y = 10$ $2x + 5y = 35$	8. $x + y = 15$ $-2x - 2y = 30$

Directions: Write a system of equations for each word problem below and then solve the system of equations using **elimination**. Write your answer in a complete sentence.

9. Tickets for a matinee are \$5 for children and \$8 for adults. The theater sold a total of 142 tickets for one matinee. Ticket sales were \$890. How many of each type of ticket did the theater sell? Write the solution in a complete sentence.

Define your Unknowns:

Equation for Number of Tickets Sold:

Equation for Ticket Sales:

Solve:

Solution (in a complete sentence):

10. Jasper has a coin collection consisting of quarters and dimes. He has 50 coins worth \$8.60. How many of each coin does he have? Write the solution in a complete sentence.

Define your Unknowns:

Equation for Number of Coins:

Equation for Value of Coins:

Solve:

Solution (in a complete sentence):

4. Sarah has \$400 in her savings account and she has to pay \$15 each month to her parents for her cell phone. Darius has \$50 and he saves \$20 each month from his job walking dogs for his neighbor. At this rate, when will Sarah and Darius have the same amount of money? How much money will they each have?

5. The admission fee at a local zoo is \$1.50 for children and \$4.00 for adults. On a certain day, 2,200 people enter the zoo and \$5,050 is collected. How many children and how many adults attended?

6. Dane goes to a fast food restaurant and orders some tacos t and burritos b . Write a story that matches the system of equations shown below that describes the number of items Dane ordered and how many calories he consumed. Solve the system to determine how many tacos and how many burritos Dane ordered and ate.

$$t + b = 5$$

$$170t + 370b = 1250$$



Story

Solve:

Solution (in a complete sentence):

4.2f Class Activity: Solving Systems of Equations Mixed Strategies

Directions: Choose the method you feel is easiest for a given problem (graphing, substitution, or elimination). Place a letter in the box (g, s, or e) for each problem to identify the method you will be using before you solve the system. Solve each system of linear equations. There are blank graphs for you to use after #12.

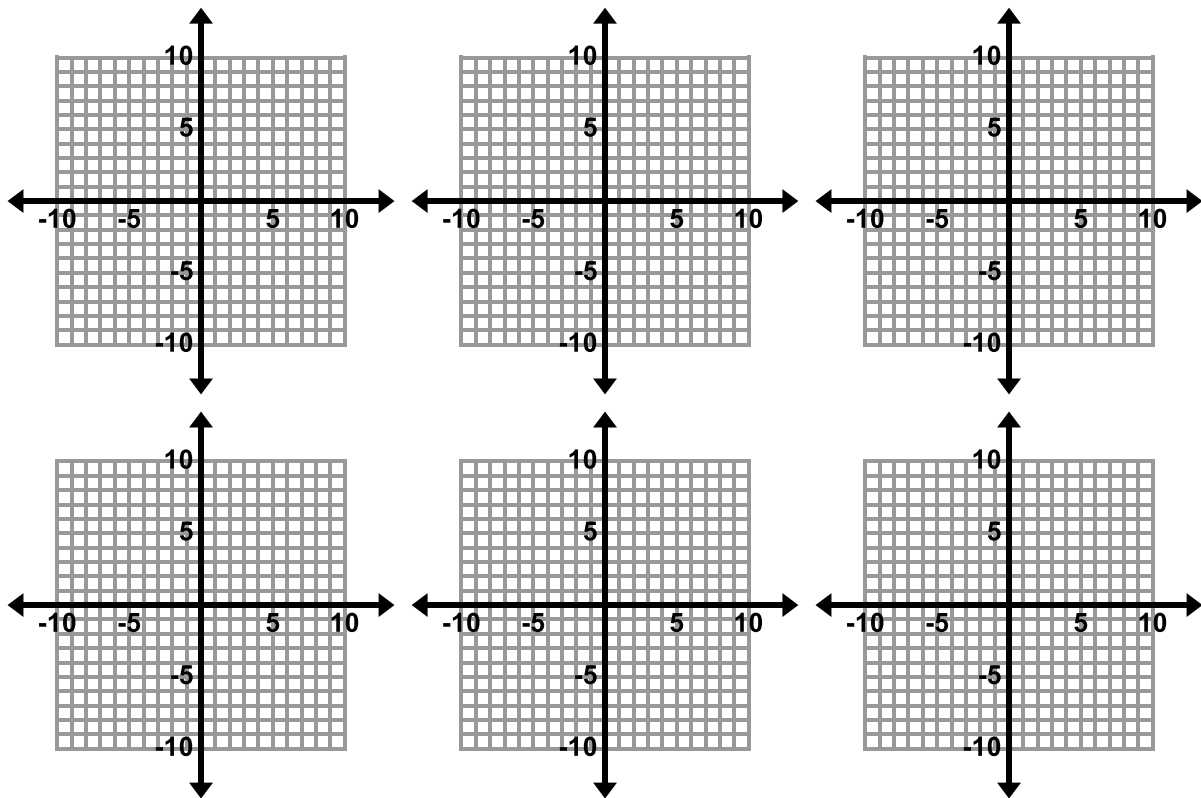
1. $2x - 3y = 12$ $x = 4y + 1$ <input type="checkbox"/>	2. $x + y = 3$ $3x - 4y = -19$ <input type="checkbox"/>
3. $y = x - 6$ $y = x + 2$ <input type="checkbox"/>	4. $y - 2x = 1$ $2x + y = 5$ <input type="checkbox"/>
5. $y = 4x - 3$ $y = x + 6$ <input type="checkbox"/>	6. $x - y = 0$ $2x + 4y = 18$ <input type="checkbox"/>
7. $3y - 9x = 1$ $y = 3x + \frac{1}{3}$ <input type="checkbox"/>	8. $x + 2y = 6$ $-7x + 3y = -8$ <input type="checkbox"/>

9. $y = -x + 5$
 $x - 4y = 10$

10. $y = x + 5$
 $y = 2x - 10$

11. $3x + 2y = -5$
 $x - y = 10$

12. $2x - 5y = 6$
 $2x + 3y = -2$



4.2f Homework: Solving Systems of Equations Mixed Strategies

Directions: Choose the method you feel is easiest for a given problem (graphing, substitution, or elimination). Place a letter in the box (g, s, or e) for each problem to identify the method you will be using before you solve the system. Solve each system of linear equations. There are blank graphs for you to use after #8.

1. $y = 4x$
 $x + y = 5$

2. $x = -4y$
 $3x + 2y = 20$

3. $y = x - 1$
 $y = -x + 3$

4. $3x - y = 4$
 $2x - 3y = -9$

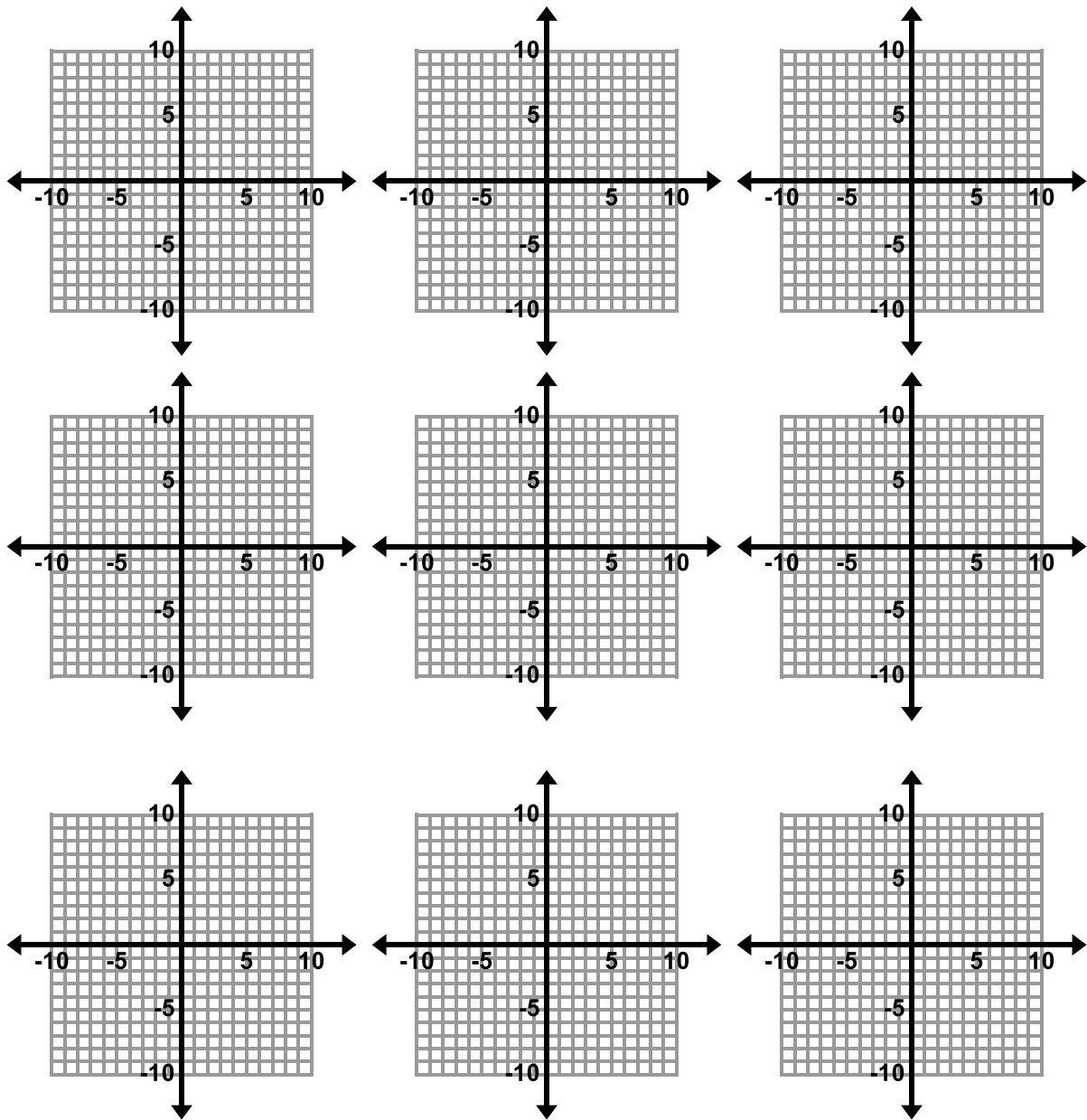
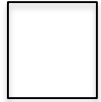
5. $x + 5y = 4$
 $3x + y = -2$

6. $y = -x + 10$
 $y = 10 - x$

7. $y = 2x$
 $x + y = 12$



8. $y = 2x - 5$
 $4x - y = 7$



4.2g Self-Assessment: Section 4.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
<p>1. Determine which method of solving a system of linear equations may be easier depending on the problem.</p> <p>Sample Problems #1, #2.</p>	I am struggling to determine the best method to use to solve a system of linear equations.	I can determine the best method to use to solve all of the equations in Set A on the following page.	I can determine the best method to use to solve all of the equations in Set A and most of the equations in Set B.	I can determine which method of solving a system of linear equations will be easier depending on the problem.
<p>2. Solve simultaneous linear equations algebraically.</p> <p>Sample Problems #1, #2</p>	I can solve the equations in Set A on the following page.	I can solve all of the equations in Set A and most of the equations in Set B.	I can solve simultaneous linear equations algebraically by using both substitution and elimination methods.	I can solve simultaneous linear equations algebraically by using both substitution and elimination methods. I can also explain why I chose a particular solution method.
<p>3. Create a system of linear equations to model a real world problem, solve the system, and interpret the solution in the context.</p> <p>Sample Problem #3</p>	When faced with word problems similar to those in this chapter, I can match the different pieces of the equations that have been given to me to the story and solve the system of equations.	When faced with word problems similar to those in this chapter, I can identify the important quantities in a practical situation, complete partial expressions and equations <i>that have been given to me</i> , solve the system, and interpret the solution in the context.	When faced with word problems similar to those in this chapter, I can identify the important quantities in a practical situation, write expressions and equations showing the relationship between the quantities, solve the equations, and interpret the solution in the context.	When faced with word problems similar to those in this chapter, I can identify the important quantities in a practical situation, <i>write</i> expressions and equations showing the relationship between the quantities, solve the equations, and interpret the solution in the context. I can explain the reasoning behind each step in the process of arriving at my answer.

Section 4.2 Sample Problems (For use with self-assessment)

1. Determine which method will be easier to use for each of the following problems by placing S (substitution), G (graphing) or E (elimination) in the small box in the corner of each problem.
2. Solve the following systems of equations algebraically.

Set A

1. $y = 2$ $y = 3x + 2$ <input data-bbox="505 394 581 472" type="checkbox"/>	2. $x = y + 2$ $4x - 3y = 11$ <input data-bbox="971 394 1047 472" type="checkbox"/>	3. $x + 2y = 13$ $-x + 4y = 11$ <input data-bbox="1430 394 1507 472" type="checkbox"/>
---	--	---

Set B

1. $y = -4x + 8$ $5x + 2y = 13$ <input data-bbox="509 835 586 913" type="checkbox"/>	2. $2y = x - 5$ $2y = x + 5$ <input data-bbox="971 835 1047 913" type="checkbox"/>	3. $3x = y - 20$ $-7x + y = 40$ <input data-bbox="1430 835 1507 913" type="checkbox"/>
---	---	---

3. Tickets to the local basketball arena cost \$54 for lower bowl seats and \$20 for upper bowl seats. A large group purchased 123 tickets at a cost of \$4,262. How many of each type of ticket did they purchase?

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Chapter 5: Functions (3 weeks)

Utah Core Standard(s):

- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (8.F.1)
- Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line. (8.F.3)
- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (8.F.4)
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. (8.F.2)
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (8.F.5)

Academic Vocabulary: function, input, output, relation, mapping, independent variable, dependent variable, linear, nonlinear, increasing, decreasing, constant, discrete, continuous, intercepts



Chapter Overview: In this chapter, the theme changes from that of solving an equation for an unknown number, to that of “function” that describes a relationship between two variables. Students have been working with many functional relationships in previous chapters; in this chapter we take the opportunity to formally define function. In a function, the emphasis is on the relationship between two varying quantities where one value (the output) depends on another value (the input). We start the chapter with an introduction to the concept of function and provide students with the opportunity to explore functional relationships algebraically, graphically, numerically in tables, and through verbal descriptions. We then make the distinction between linear and nonlinear functions. Students analyze the characteristics of the graphs, tables, equations, and contexts of linear and nonlinear functions, solidifying the understanding that linear functions grow by equal differences over equal intervals. Finally, students use functions to model relationships between quantities that are linearly related. Students will also describe attributes of a function by analyzing a graph and create a graphical representation given the description of the relationship between two quantities.

Connections to Content:

Prior Knowledge: Up to this point, students have been working with linear equations. They know how to solve, write, and graph equations. In this chapter, students make the transition to function. In the realm of functions, we begin to interpret symbols as variables that range over a whole set of numbers. Functions describe situations where one quantity determines another. In this chapter, we seek to understand the relationship between the two quantities and to construct a function to model the relationship between two quantities that are linearly related.

Future Knowledge: This chapter builds an understanding of what a function is and gives students the opportunity to interpret functions represented in different ways, identify the key features of functions, and construct functions for quantities that are linearly related. This work is fundamental to future coursework where students will apply these concepts, skills, and understandings to additional families of functions.

MATHEMATICAL PRACTICE STANDARDS

	<p>Make sense of problems and persevere in solving them.</p>	<p>On Tamara’s first day of math class, her teacher asked the students to shake hands with everyone in the room to introduce themselves. There are 26 students total in the math class. Can you determine the number of handshakes that took place in Tamara’s math class on the first day of class? Can the relationship between number of students and the number of handshakes exchanged be modeled by a linear function? Justify your answer.</p> <p><i>As students grapple with this problem, they will start to look for entry points to its solution. They may consider a similar situation with fewer students. They may construct a picture, table, graph, or equation. They may even act it out, investigating the solution with a concrete model. Once they have gained entry into the problem, students may look for patterns and shortcuts that will help them to arrive at a solution either numerically or algebraically.</i></p>
	<p>Reason abstractly and quantitatively.</p>	<p>Nazhoni has completed her Driver’s Education Training and is at the DMV (Division of Motor Vehicles) waiting in line to get her license to drive. She entered the DMV at 12:50 and pulled a number 17 to reserve a spot in line. Nazhoni notices that all of the employees at the DMV are still at lunch when she arrives. Once the employees return they start with number 1. There is digital sign showing the number for the person who is at the counter being helped. Nazhoni jots down some information on a piece of scratch paper as she is waiting in line.</p> <p><i>#5 was called to the counter at 1:25 pm</i></p> <p><i>#10 was called to the counter at 2:00 pm</i></p> <p><i>I have to leave by 2:45 pm in order to pick up my sister from school on time.</i></p> <p>Will Nazhoni make it to the front of the line in time to pick up her sister from school?</p> <p><i>In order to solve this problem, students must make sense of the quantities involved in this situation and the relationship between the quantities. Students may first investigate this problem numerically, determining the average wait time between each person called to the counter. Students may also abstract this situation and construct a function to model the amount of time Nazhoni will have to wait based on the number she draws.</i></p>



Construct viable arguments and critique the reasoning of others.

Compare and contrast the relationship of the gumball machines at Vincent Drug and Marley's Drug Store. If needed revise your conjecture about what kind of relationship makes a function and what disqualifies a relationship from being a function.

As students create, modify, and formulate their definition of a function they are constructing a viable argument that describes their thoughts on what a function is and what it is not. They make conjectures and build a logical progression of statements to explore the truth about their conjectures. They can share their definitions with others and decide whether they make sense and compare others' thoughts and ideas to their own.



Model with mathematics.

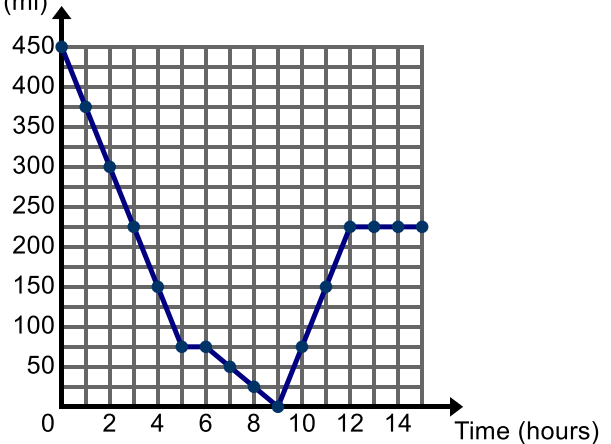
Throughout this chapter, students will apply the mathematics they have learned to solve problems arising in everyday life, society, and workplace. The following problems give students the opportunity to use functions to model relationships between two quantities.

Steve is a lifeguard at a local community pool. Each day at noon, he records the temperature and the number of people in the pool. Do you think the relationship between temperature and number of people in the pool is linear? Why or why not?

Two thousand, five hundred students attend a local high school. School starts at 8 am and ends at 2:30 pm. Many students stay after school for clubs, sports, etc. The school has a one-hour lunch at noon and seniors are allowed to leave campus for lunch. Sketch a graph of the number of cars in the student parking lot from 6 am to 4 pm.

Ben and his family took a road trip to visit their cousins. The graph below shows their journey. Label the key features of the graph. Write a story about the graph.

Distance from Las Vegas (mi)





Use appropriate tools strategically.

Directions: Determine whether the situations you choose can be modeled by a linear function or not. Provide evidence to support your claim. Show your work in the space below.

Mr. Cortez drove at a constant rate for 5 hours. At the end of 2 hours he had driven 90 miles. After 5 hours, he had driven 225 miles. Can the relationship between time and distance driven be modeled by a linear function? Provide evidence to support your claim.

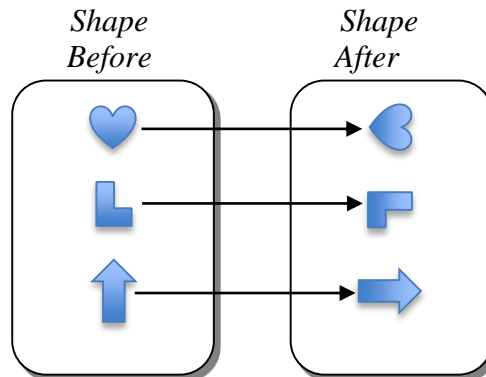
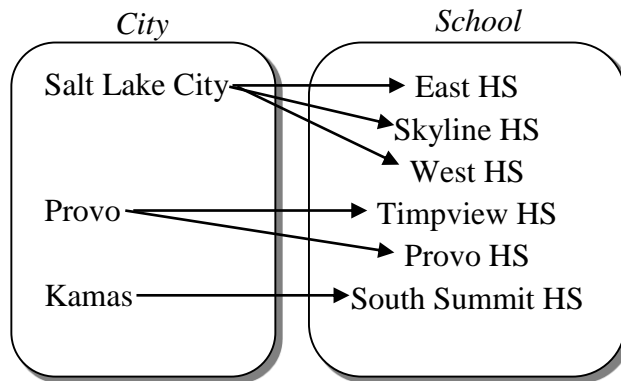
Round 1 of a tennis tournament starts with 64 players. After each round, half the players have lost and are eliminated from the tournament. Therefore, in round 2 there are 32 players, in round 3 there are 16 players and so on. Can the relationship between round number and number of players remaining be modeled by a linear function? Provide evidence to support your claim.

The first step in constructing a function to model the relationship between two quantities is to determine what type of model is a potential fit for the data. At this point, student knowledge of the rate of change of a linear function is a tool the students rely on to determine whether the relationship between two quantities can be modeled by a linear function.



Attend to precision.

Determine whether each representation describes a function.



Is letter grade a function of percentage scored on a test?

In order to determine whether or not a given representation describes a function, students must be precise in their understanding of what a function is.

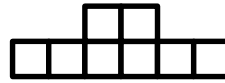


Look for and make use of structure.

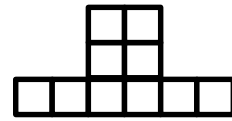
Examine the patterns below. Can the relationship between stage number and number of blocks in a stage be modeled by a linear function? Provide at least 2 pieces of evidence to support your answer.



Current Display



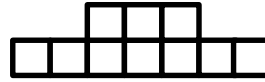
Stage 2



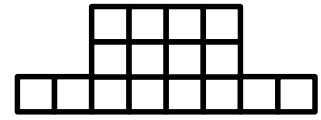
Stage 3



Current Display



Stage 2



Stage 3

While examining the patterns above, students may see that a linear pattern exhibits growth in one direction while the second pattern shown exhibits growth in two directions. These geometric representations give insight into the structure of a linear equation (and a quadratic equation which will be studied in subsequent courses).

Circle the letter next to each equation if it represents a linear function.

$$2x + 4y = 16$$

$$y = x^2 + 5$$

$$y = x(x + 2)$$


$$xy = 24$$

The equations above are a sampling of the types of functions students will encounter in this chapter. By the end of the chapter, students will solidify their understanding of the structure of a linear function and will surface ideas about the structure of additional types of functions that will be studied in subsequent courses.



Look for and express regularity in repeated reasoning.

Emily's little brother painted on her math homework. She knows the data in each of the tables below represents a linear function. Help Emily determine what number is hidden behind the blob of paint.

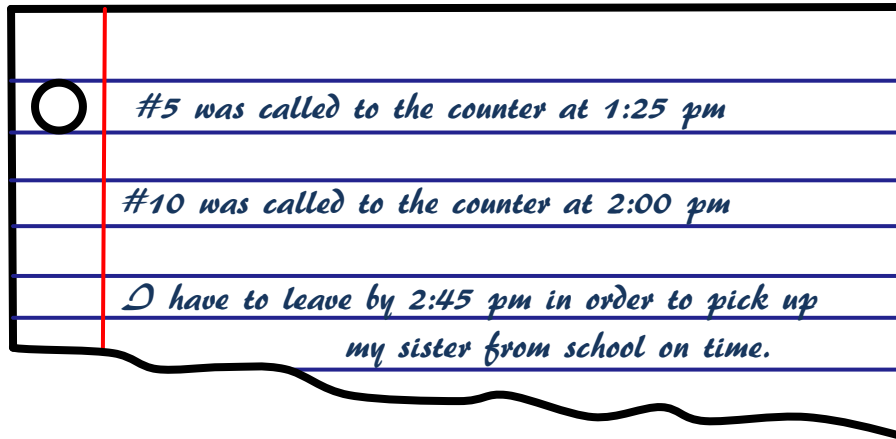
x	10	20	30	40
y	8	13		23

Slope is a calculation that is repeated in a linear relationship. In order to solve this problem and similar problems, students must understand that linear functions grow by equal differences over equal intervals and apply this knowledge in order to complete the table.

5.0: Anchor Problem: Waiting at the DMV



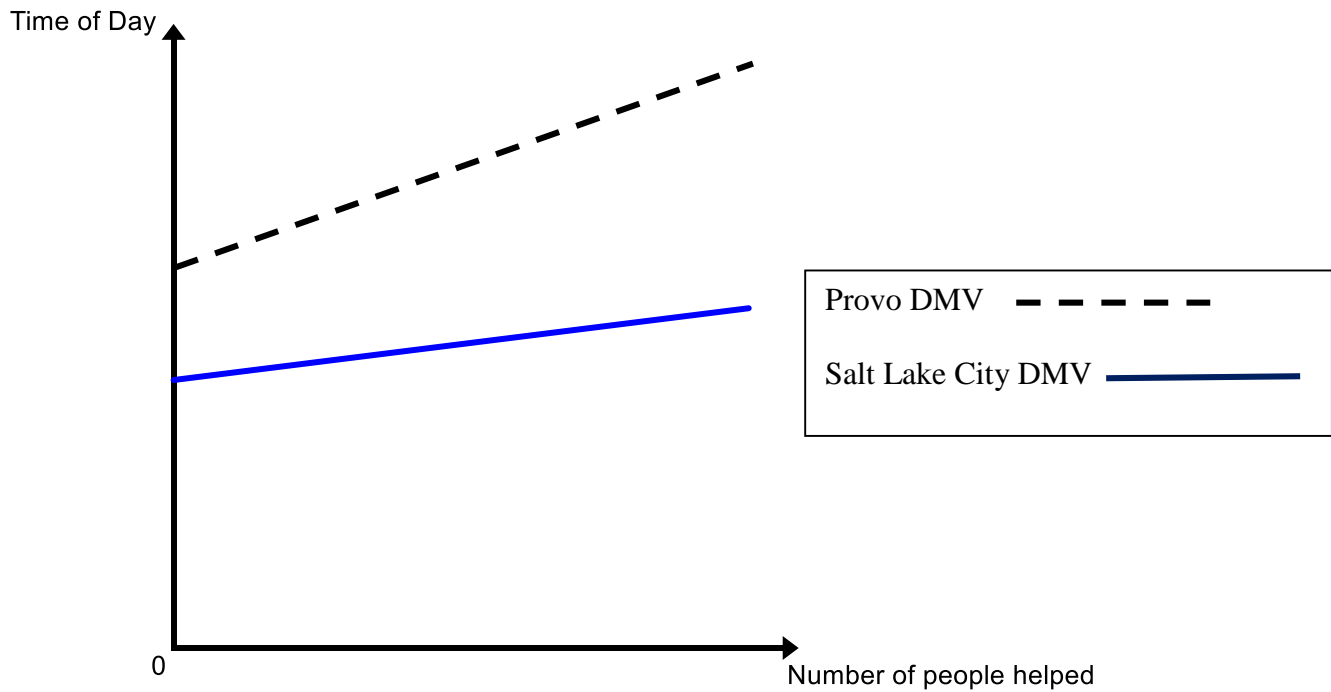
1. Nazhoni has completed her Driver's Education Training and is at the DMV (Division of Motor Vehicles) waiting in line to get her license to drive. She entered the DMV at 12:50 and pulled a number 17 to reserve a spot in line. Nazhoni notices that all of the employees at the DMV are still at lunch when she arrives. Once the employees return they start with number 1. There is digital sign showing the number for the person who is at the counter being helped. Nazhoni jots down some information on a piece of scratch paper as she is waiting in line.



- a. Use the picture of the scratch paper above to estimate what time it will be when Nazhomi will make it to the front of the line. (**Note: Assume that each person takes the same amount of time while being helped at the counter**)
- b. Will Nazhoni make it to the front of the line in time to pick up her sister from school?
- c. What time did the employees return from lunch and begin working.
- d. Write an equation that represents the amount of time Nazhoni would have to wait dependent on the number she draws when she enters the DMV at 12:50.

This problem was adapted from a task on Illustrative Mathematics.

2. The DMV in Provo and Salt Lake opened their doors for the day at the same time. The graphs below show the time of day as a function of the number of people called to the counter. Write down as many differences between the two DMVs as you can based upon the graphs.



3. Do you think it is realistic that it takes the exact same amount of time for each person at the DMV? Explain.
4. The following table shows more realistic data for the waiting time at the DMV: Is there a constant rate of change for this data? If not, is the data still useful? What can be inferred about the information given from the table?

Time	# Being Helped
12:58	30
1:25	33
2:00	37
2:08	38
2:50	44
3:30	49

Section 5.1: Define Functions

Section Overview:

This section begins by using a context to introduce a relation that represents a function and one that is not a function. By analyzing several situations students derive their own definition of a function. They also create their own representations of relations that are functions and those that are not functions. In the next lesson a candy machine analogy is used to help students further their understanding of a function as a rule that assigns to each input exactly one output. Students then play the function machine game and discover the rule that generates the output for a given input. As the section progresses, students are given different representations of relationships (i.e. table, graph, mapping, story, patterns, equations, and ordered pairs) and must determine if the representation describes a function. In the last lesson, students determine the dependent and independent variables in a functional relationship, understanding that the roles of the variables are often interchangeable depending on what one is interested in finding.

Concepts and Skills to be Mastered:

By the end of this section students should be able to:

1. Understand that a function is a rule that assigns to each input exactly one output.
2. Determine whether a given relation defines a function given different representations (i.e., table, graph, mapping, story, patterns, equations, and ordered pairs).
3. Determine the independent and dependent variable in a functional relationship.

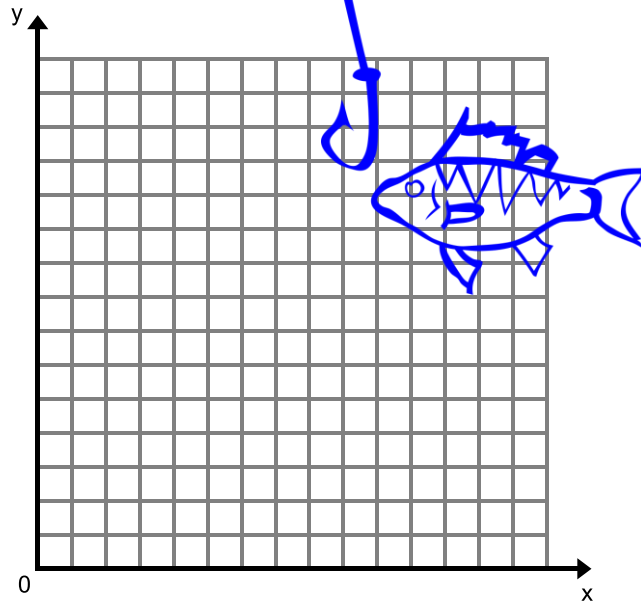
5.1a Class Activity: Introduction to Functions

1. Jason is spending the week fishing at the Springville Fish Hatchery. Each day he catches 3 fish for each hour he spends fishing. This relationship can be modeled by the equation $y = 3x$, where x = number of hours spent fishing and y = the number of fish caught.



- a. Complete the graph and table below for this relationship.

Number of hours spent fishing x	Number of fish caught y
3	
2	
1	
2	
0	
4	

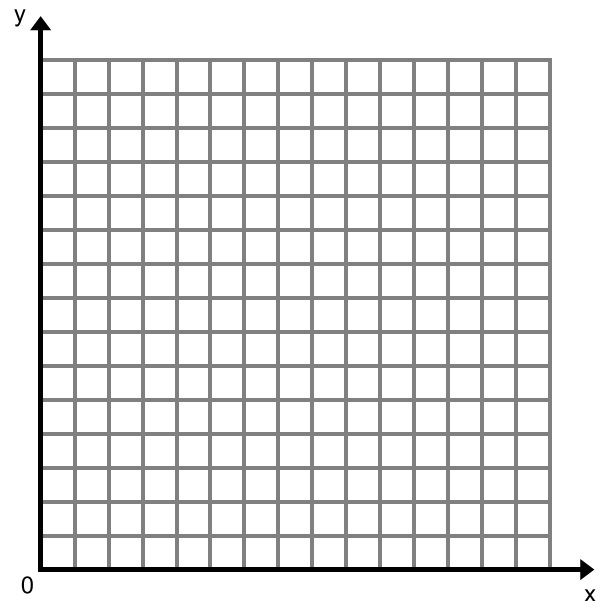


The situation above is an example of a **function**. We would say that *the number of fish caught is a function of the number of hours Jason spends fishing*.

2. Sean is also spending the week fishing; however he is fishing in the Bear River. Each day he records how many hours he spends fishing and how many fish that he caught. The table of values below shows this relationship.

- a. Complete the graph for this relationship.

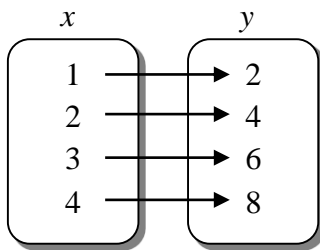
Number of hours spent fishing x	Number of fish caught y
1	4
0	0
2	5
3	1
3	8
5	5



This situation is an example of a relation that is **not a function**. The number of fish that Sean catches is **not** a function of the number of hours he spends fishing.

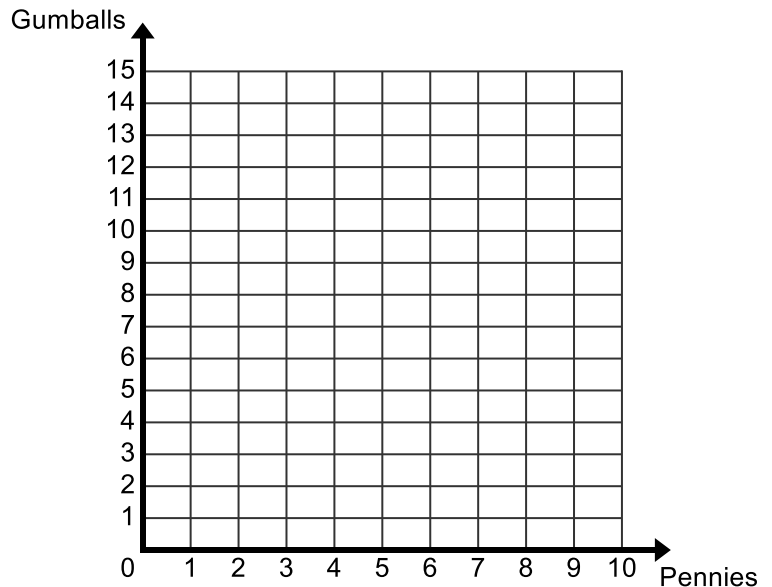
- Compare and contrast the relationship for Jason’s week spent fishing and Sean’s week spent fishing. Make a conjecture (an educated guess) about what kind of relationship makes a function and what disqualifies a relation from being a function. 

- Vanessa is buying gumballs at Vincent’s Drug Store. The mapping below shows the relationship between number of pennies, or x , she puts into the machine and the number of gumballs she gets out, or y .



- Complete the graph and table below for this relationship.

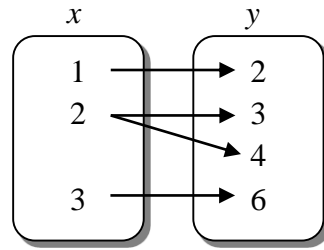
Number of pennies x	Number of gumballs y



- Write an equation that models this relationship.

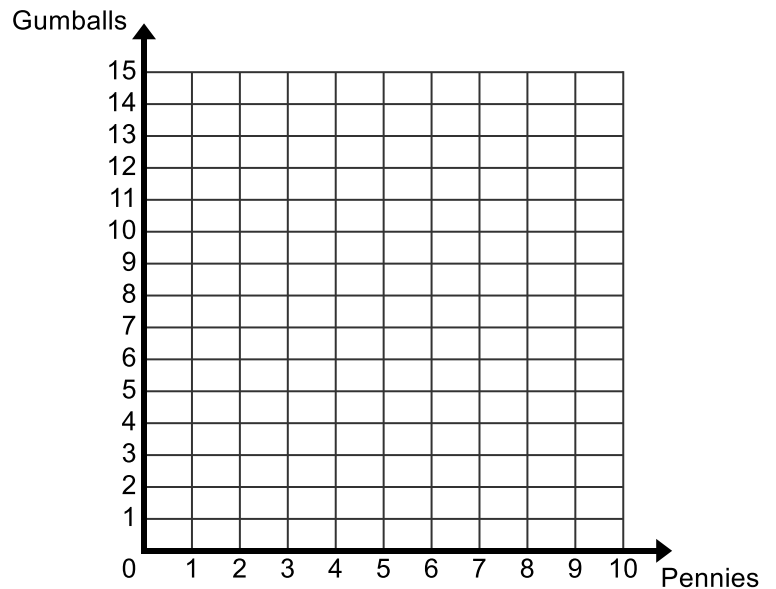
This is also an example of a **function**. We would say that *the number of gumballs received is a function of the number of pennies put in the machine*.

5. Kevin is across town at Marley's Drug Store. The mapping below relates the number of pennies he puts into the machine and how many gumballs he gets out.



- a. Complete the graph and table below for this relationship.

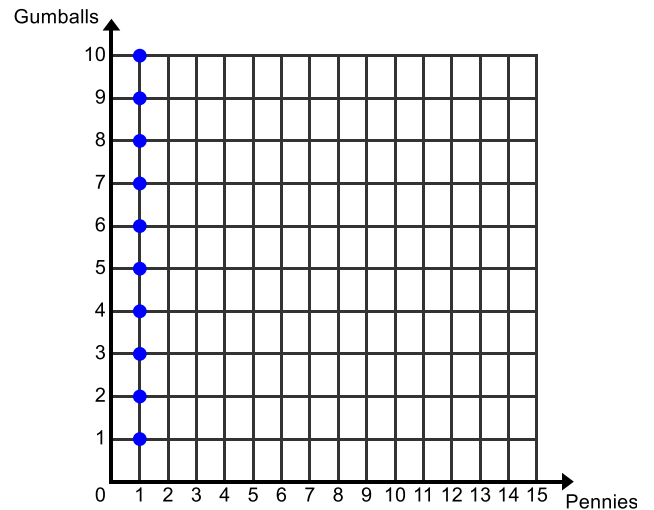
Number of pennies x	Number of gumballs y



This situation is an example of a relation that is **not a function**.

6. Cody is at Ted's Drug Store. The graph below relates the number of pennies he puts into the machine on different occasions and how many gumballs he gets out.

- a. Explain how this gumball machine works.



- b. In this example, is the number of gumballs received a function of the amount of money put in? Explain your answer.

7. Compare and contrast the relationship of the gumball machines at the different drugstores. If needed revise your conjecture about what kind of relationship makes a function and what disqualifies a relationship from being a function.

Below is a formal definition of a function. As you read it compare it to the conjecture you made about what makes a relation a function.

Given two variables, x and y , y is a function of x if there is a rule that determines one unique y value for a given x value.

Refer back to the first two examples. When Jason went fishing, he caught a unique number of fish based on the number of hours he spent fishing. If you know the number of hours Jason fishes for, you can determine the number of fish he will catch; therefore the number of fish he catches is a function of the number of hours he spends fishing. On the other hand, when Sean is fishing, it is not possible to determine the number of fish he catches based on the number of hours he fishes. On one day, he fished for three hours and caught one fish and on another day he fished for three hours and caught eight fish. There are two different y values assigned to the x value of 3 hours. In Sean's situation, the number of fish he catches is **not** a function of the number of hours he spends fishing.

Likewise, the gumball machine at Vincent's Drug Store represents a function because each penny inserted into the gumball machine generates a unique amount of gumballs. If you know how many pennies are inserted into the gumball machine at Vincent's, you can determine how many gumballs will come out. However, the gumball machine at Marley's Drug Store is **not** a function because there is not a unique number of gumballs generated based on the number of pennies you put in. One time 2 pennies were inserted and 4 gumballs came out and at another time 2 pennies were inserted and 3 gumballs came out. You are unable to determine the number of gumballs that will come out based on how many pennies are put into the machine.

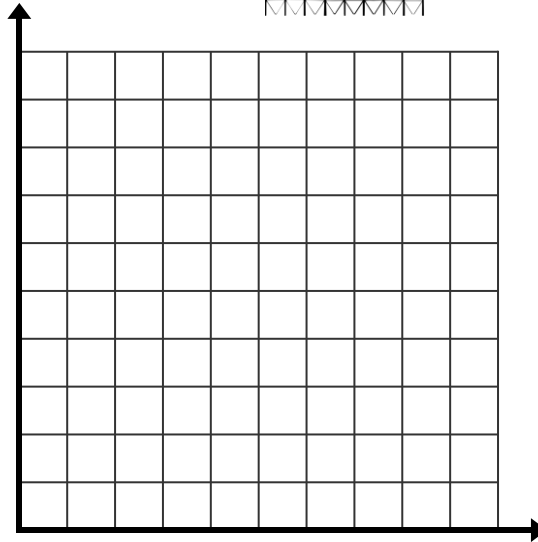
8. Explain in your own words why the number of gumballs received at Ted's Drug store is not a function of the amount of money put in. Be specific and give examples to support your reasoning.

9. The cost for entry into a local amusement park is \$45. Once inside, you can ride an unlimited number of rides.

- a. Complete the graph and table below for this relationship.



Number of rides x	Amount spent (dollars) y



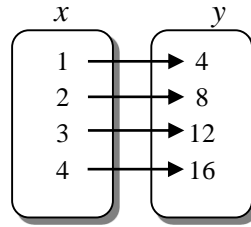
- b. Is the amount one spends a function of the number of rides he/she goes on? Why or why not?

10. The table below show the number of hours Owen plays his favorite video game and the number of points he scores.

Time Spent Playing (hours)	Number of Points Scored
1	5,000
1	5,550
1	6,500
2	11,300
2	12,400
3	15,000

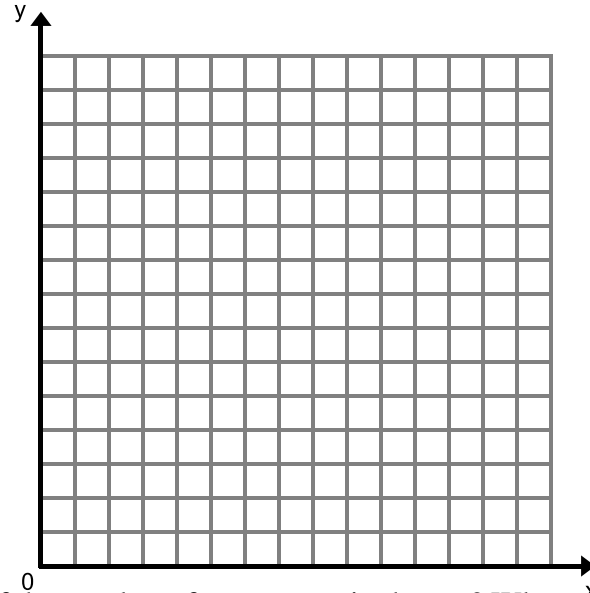
- a. Is the number of points Owen scores a function of the amount of time he spends playing? Why or why not?

3. The cost for cars entering a scenic by-way toll road in Wyoming is given by the mapping below. In this relation y is the dollar amount to enter the by-way and x is the number of passengers in the car.



- a. Complete the graph and table below for this relationship.

Number of passengers x	Amount per car (dollars) y

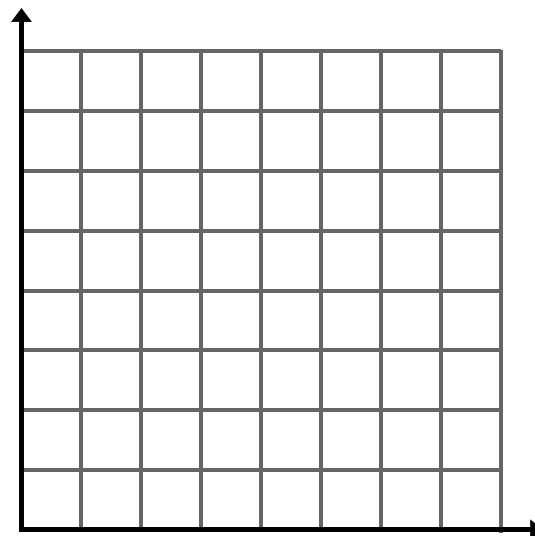


- b. Is the amount spent per car a function of the number of passengers in the car? Why or why not?

4. The cost for cars entering a scenic by-way toll road in Utah is \$5 regardless of the number of passengers in the car.

- a. Complete the graph and table below for this relationship.

Number of passengers x	Amount per car y

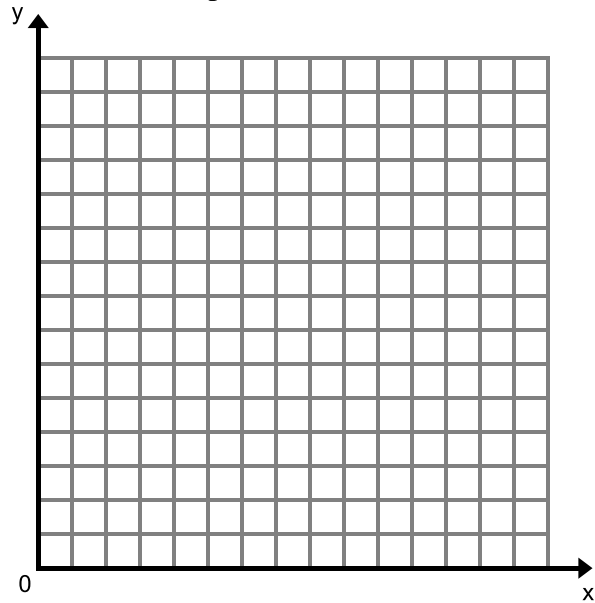


- b. Is the amount spent per car a function of the number of passengers in the car? Why or why not?

5. Create your own context or story that represents a relation that is a function.

a. Story:

b. Complete the graph and table below for this relationship.

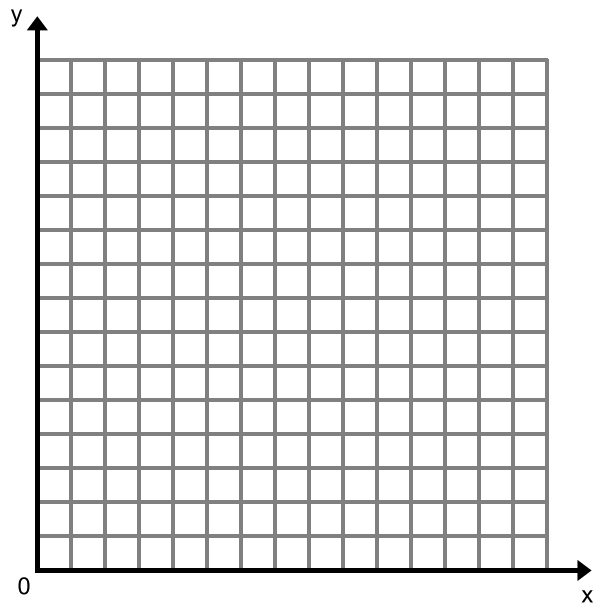


c. Explain why this relation is a function.

6. Create your own context or story that represents a relation that is **not** a function.

a. Story:

b. Complete the graph and table below for this relationship.



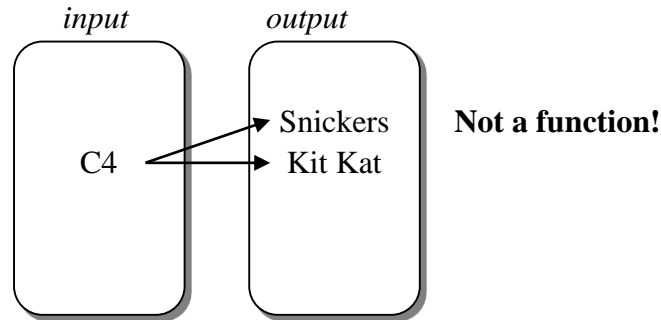
c. Explain why this relation is not a function.

5.1b Class Activity: Function Machine

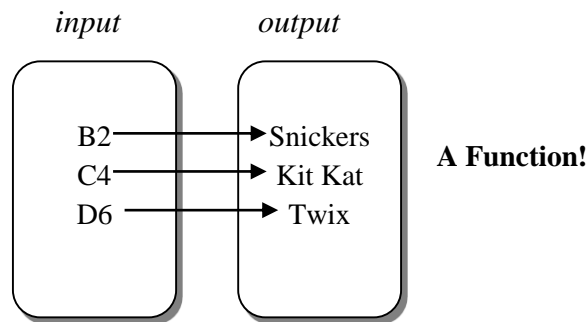
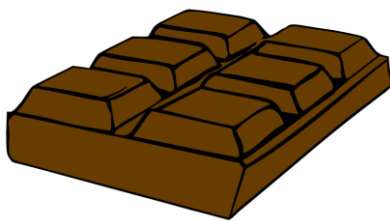


One way to think about the x and y variables in a functional relationship are as input (x) and output (y) values. To better understand how input and output values are related in a function consider the following analogy.

The Candy Machine Analogy: When you buy candy from a vending machine, you push a button (your input) and out comes your candy (your output). Let's pretend that C4 corresponds to a Snickers bar. If you input C4, you would expect to get a Snickers bar as your output. If you entered C4 and sometimes the machine spits out a Snickers and other times it spits out a Kit Kat bar, you would say the machine is “not functioning” – one input (C4) corresponds to two different outputs (Snickers and Kit Kat).

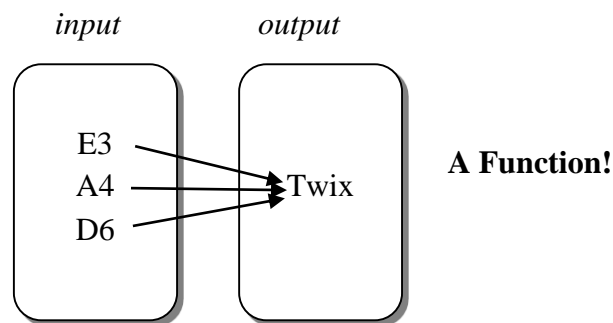


Let's look at what a diagram might look like for a machine that is “functioning” properly:



In this situation, each input corresponds to exactly one output. The candy bar that comes out of the machine is dependent on the button you push. We call this variable the **dependent variable**. The button you push is the **independent variable**.

Let's look at one more scenario with the candy machine. There are times that different inputs will lead to the same output. In the case of the candy machines, companies often stock popular items in multiple locations in the machine. This can be represented by the following diagram:

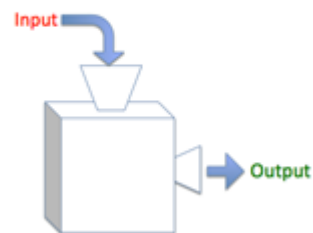


Even though the different inputs correspond to the same output, our machine is still “functioning” properly. This still fits the special requirement of a function – each input corresponds to exactly one output.

THE FUNCTION MACHINE:

In this activity, you will give your teacher a number. He/she will perform some operations on the number, changing it to a new number. Your goal is to figure out what rule is being applied to the number. Use the tables below to keep track of the numbers you give your teacher (inputs) and the numbers your teacher gives you back (outputs). Once you figure out the function, write it in the space below the table.

Function Machine



INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

Now try it on your own or with a partner. Write the function for each of the following relations. Use the words input and output in your written function equation. Then write the function as an equation using x and y . The first one has been done for you. There is space for you to verify your function is correct.

1.	Double the input increased by one will get the output.	
Input x	Function: $y = 2x + 1$	Output y
8	$2(8) + 1 = 17$	17
0	$2(0) + 1 = 1$	1
-3	$2(-3) + 1 = 1$	-5
2		5
1		3
-1		-1
7		15

2.		
Input x	Function:	Output y
3		-2
0		-5
10		5
2		-7
5		0
1		-4
-4		-9

3.		
Input x	Function:	Output y
5		25
-2		-10
0		0
1		5
-4		-20
-9		-45
3		15

4.		
Input x	Function:	Output y
-3		-4
-2		-2
-1		0
0		2
1		4
2		6
3		8

5.		
Input x	Function:	Output y
4		-2
-3		-16
0		-10
1		-8
7		4
9		8
5		0

6.		
Input x	Function:	Output y
2		3
5		-9
2		7
5		10
4		9
2		0
-1		-3

5.1b Homework: Function Machine

Directions: Write the function for each of the following relations.

1.		
Input x	Function:	Output y
5		2.5
-2		-4.5
0		-2.5
1		-1.5
-4		-6.5
-9		-11.5
3		0.5

2.		
Input x	Function:	Output y
2		$\frac{1}{2}$
-4		-1
0		0
1		$\frac{1}{4}$
-9		$-\frac{9}{4}$
24		6
8		2

3.		
Input x	Function:	Output y
-4		-9
-3		-6
-2		-3
0		3
1		6
2		9
3		12

4.		
Input x	Function:	Output y
2		3
-4		-3
0		1
1		2
-9		-8
-17		-16
10		11

5.		
Input x	Function:	Output y
-4		16
-3		9
-2		4
-1		1
0		0
1		1
2		4

6.		
Input x	Function:	Output y
1		3
6		-8
1		7
5		10
6		8
2		0
-1		-3

7. Were you able to find a function for number 6? If so, write it down. If not, explain why.

Directions: Create your own function machines, fill in the values for each input and its corresponding output.

8.		
Input	Function: _____	Output

9.		
Input	Function: _____	Output

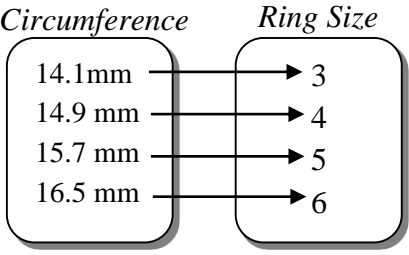
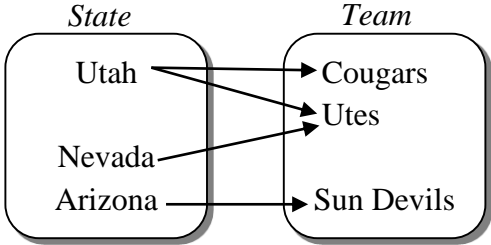
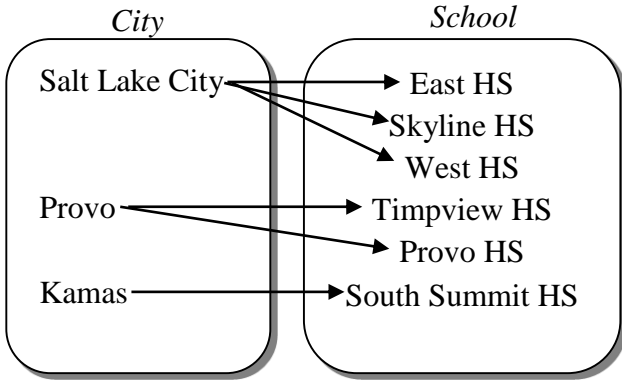
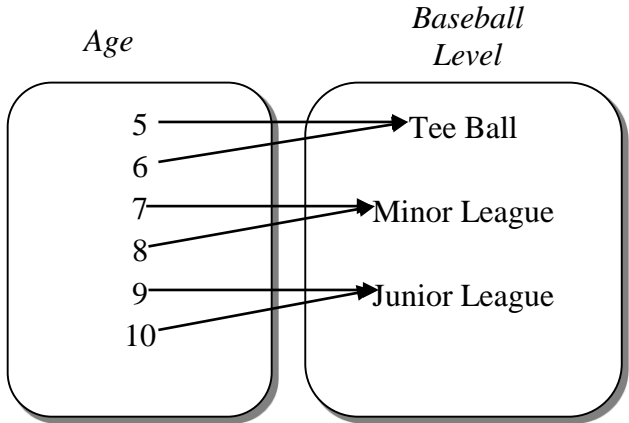
10. Create a machine that is **not** a function. Explain why your machine is “dysfunctional”.

Input	Function: _____ =	Output

5.1c Class Activity: Representations of a Function


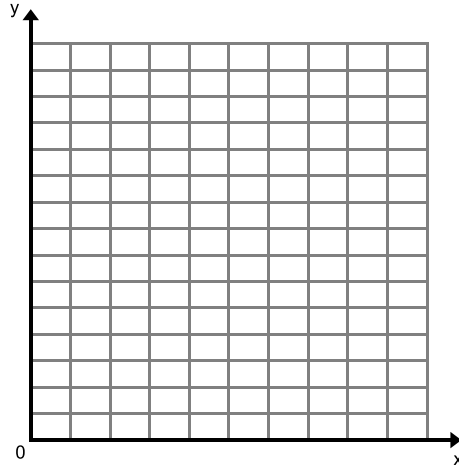


Functions can also be described by non-numeric relations. A **mapping** is a representation of a function that helps to better understand non-numeric relations. Study each relation and its mapping below. Then decide if the relation represents a function. Explain your answer.

<p>1. Input: circumference of finger Output: ring size</p>  <p>Function? Explain.</p>	<p>2. Input: state a person lives in Output: the team they root for in college football</p>  <p>Function? Explain.</p>
<p>3. Write the ordered pairs (circumference, ring size) that correspond to problem #1.</p>	<p>4. Write the ordered pairs (state a person lives in, team they root for) that correspond to problem #2.</p>
<p>5. Input: city student lives in Output: high school they go to</p>  <p>Function? Explain.</p>	<p>6. Input: Age Output: Level of Baseball Team</p>  <p>Function? Explain.</p>

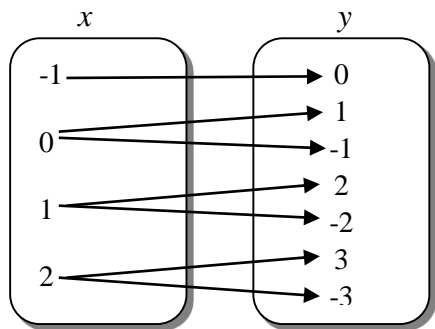
As we have seen, there are many ways to represent a relation or function. In the following problems, you will be given one representation of a relation and asked to create additional representations. Then, you will be asked to determine whether the relation represents a function or not.

7. **Story:** A candle is 27 centimeters high and burns 3 centimeters per hour. An equation that models this relation is $c = 27 - 3h$ where c is the height of the candle in centimeters and h is the number of hours the candle has been burning.
- a. Express this relation as a table, mapping, graph, and set of ordered pairs.

<p>Table</p> <table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 20px;"> <thead> <tr style="background-color: #f2f2f2;"> <th style="padding: 5px;">Time (hours) <i>h</i></th> <th style="padding: 5px;">Height (cm) <i>c</i></th> </tr> </thead> <tbody> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> </tbody> </table> <div style="text-align: center;">  </div>	Time (hours) <i>h</i>	Height (cm) <i>c</i>											<p>Mapping</p> <div style="display: flex; justify-content: space-around; text-align: center; margin-bottom: 10px;"> <div style="margin: 0 10px;"><i>Hours</i></div> <div style="margin: 0 10px;"><i>Height</i></div> </div> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; border-radius: 15px; width: 100px; height: 150px; margin: 10px;"></div> <div style="border: 1px solid black; border-radius: 15px; width: 100px; height: 150px; margin: 10px;"></div> </div>
Time (hours) <i>h</i>	Height (cm) <i>c</i>												
<p>Graph</p> 	<p>Set of Ordered Pairs</p>												

- b. Is the height of the candle a function of the amount of time it has been burning? Explain.

8. Mapping:

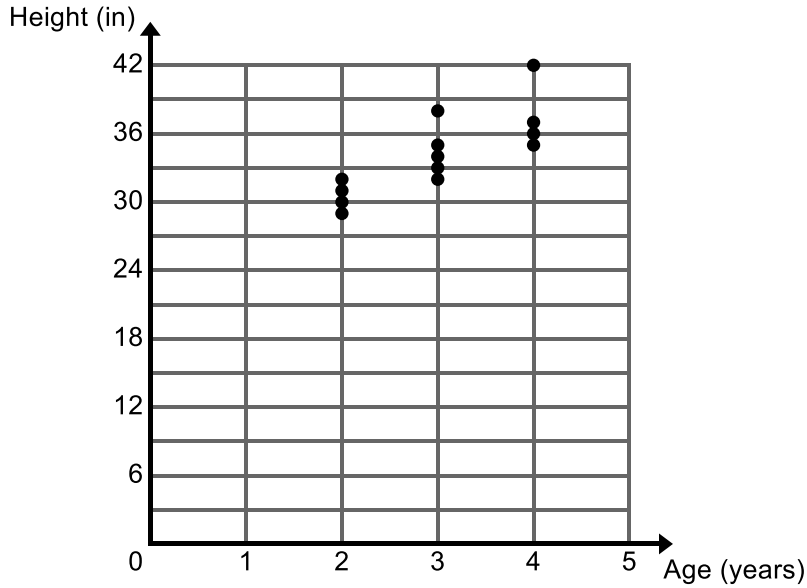


a. Express this relation as a table, graph, and set of ordered pairs.

<p>Table</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr style="background-color: #e0e0e0;"> <th style="padding: 5px;">x</th> <th style="padding: 5px;">y</th> </tr> </thead> <tbody> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> </tbody> </table>	x	y																	<p>Graph</p>
x	y																		
<p>Set of Ordered Pairs</p>																			

b. Is this relation a function? Explain.

9. **Graph:** Discovery Place Preschool gathered data on the age of each student (in years) and the child's height (in inches). The graph below displays the data they gathered.

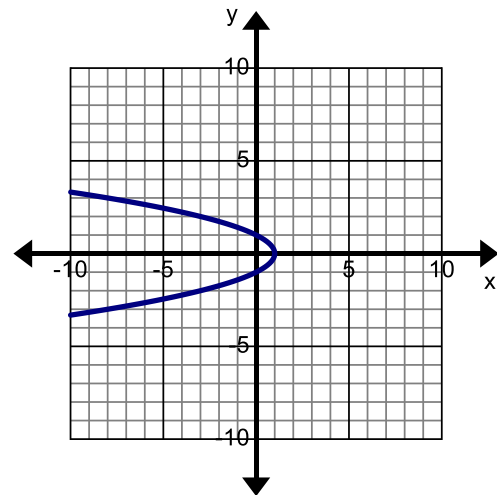


- a. Is a child's height a function of the child's age? Explain.

Directions: Determine if each relation or situation defines a function. Justify your answer. It may help to make an additional representation of the relation.

10. $\{(30, 2), (45, 3), (32, 1.5), (30, 4), (41, 3.4)\}$

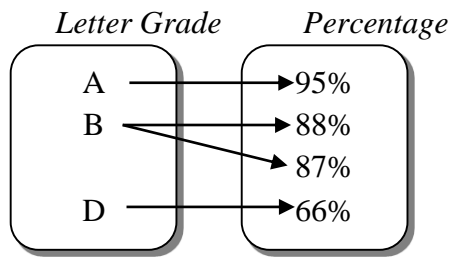
11.



12. $x = 2$

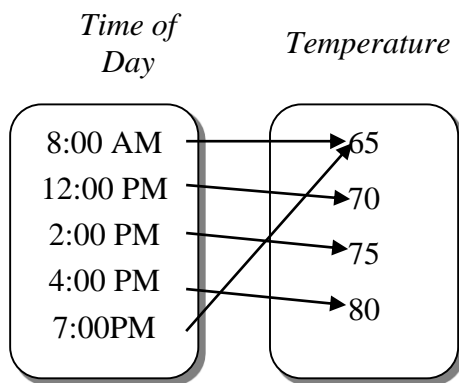
13. $3x + 6y = 18$

14.



15. Is letter grade a function of percentage scored on a test?

16.



17. Is time of day a function of the temperature?

18.

Length of Radius (cm)	Length of Diameter (cm)
0.5	1
1	2
1.5	3
2	4

19. **Input:** name of city in the U.S.

Output: state city is in

Hint: There are 16 states in the United States that have a city called Independence.

5.1c Homework: Representations of a Function



1. Use the pattern below to answer the questions that follow.

Pattern:



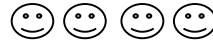
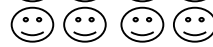
Stage 1



Stage 2



Stage 3



Stage 4

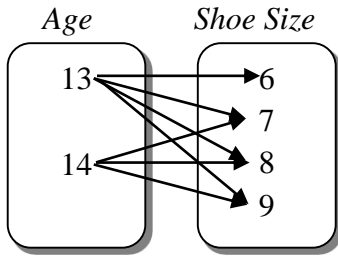
a. Express this relation as a table, mapping, and graph.

<p>Table</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr style="background-color: #e0e0e0;"> <th style="padding: 5px;">Stage number</th> <th style="padding: 5px;">Number of Smiles</th> </tr> </thead> <tbody> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> <tr><td style="height: 20px;"> </td><td> </td></tr> </tbody> </table>	Stage number	Number of Smiles									<p>Mapping</p> <div style="display: flex; justify-content: space-around; align-items: center; margin-bottom: 10px;"> <div style="text-align: center;"><i>Stage</i></div> <div style="text-align: center;"><i>Smiles</i></div> </div> <div style="display: flex; justify-content: space-around;"> <div style="border: 2px solid black; border-radius: 15px; width: 100px; height: 150px; margin: 10px;"></div> <div style="border: 2px solid black; border-radius: 15px; width: 100px; height: 150px; margin: 10px;"></div> </div>
Stage number	Number of Smiles										
<p>Graph</p>	<p>Set of Ordered Pairs</p>										

b. Is this relation a function? Explain how you know.

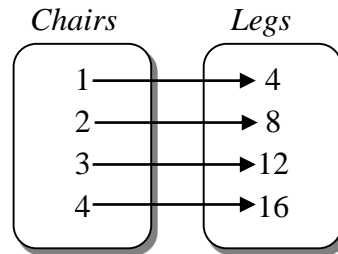
Directions: Determine if each relation or situation defines a function. Justify your answer.

2. **Input:** age
Output: shoe size



Function? Explain.

3. **Input:** number of chairs
Output: number of legs



Function? Explain.

4. List the ordered pairs that correspond to #2: (age, shoe size).

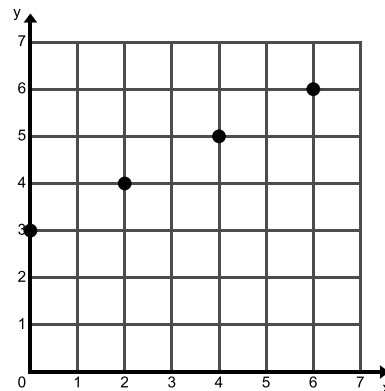
5. List the ordered pairs that correspond to #3: (number of chairs, number of legs)

6.

x	y
0.2	1.5
0.4	1.25
0.6	1.5
0.8	1.25

Function? Explain.

7.

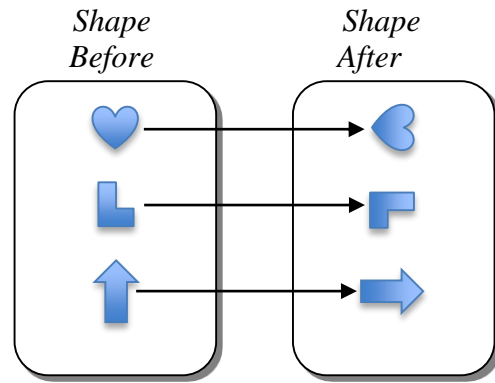


Function? Explain.

8. A car is traveling at a constant rate of 60 mph. Is the car's distance traveled a function of the number of hours the car has been driving?

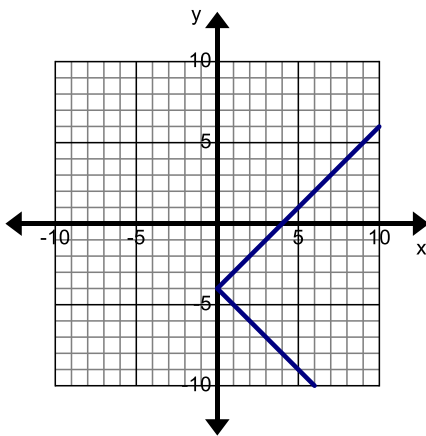
Function? Explain.

9.



Function? Explain.

10.



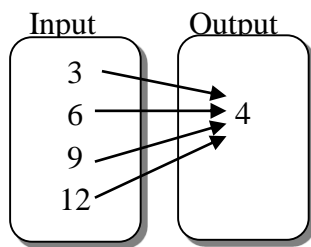
Function? Explain.

11.

Number of People	1	3	4	7
Cost	4	8	10	16

Function? Explain.

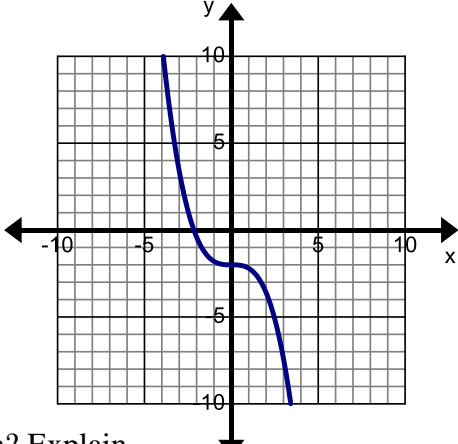
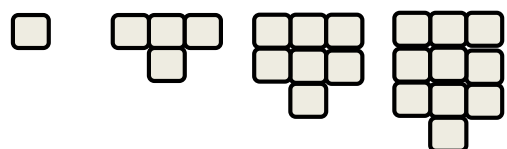
12.



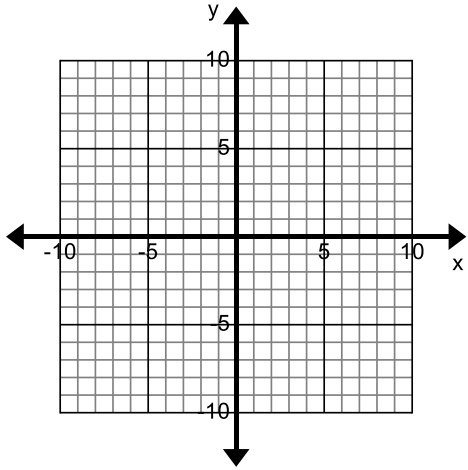
Function? Explain.

$$13. y = \frac{1}{3}x + 4$$

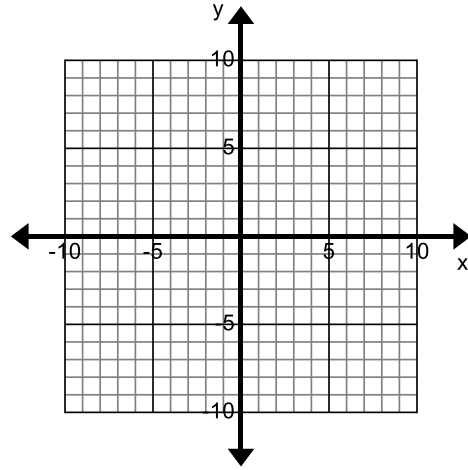
Function? Explain.

<p>14. You know your cousin lives at the zip code 12345 so you type it in Google to find your cousin's full address. Is address a function of zip code?</p> <p>Function? Explain.</p>	<p>15. You know your cousin's cellular phone number is (123) 456-7890 so you dial that number to call him. Is person being called a function of phone number dialed?</p> <p>Function? Explain.</p>
<p>16. You call the post office and ask for the zip code for the city of Salt Lake City, UT. Is zip code a function of the name of the city?</p> <p>Function? Explain.</p>	<p>17. $y = -5$</p> <p>Function? Explain.</p>
<p>18.</p>  <p>Function? Explain.</p>	<p>19.</p>  <p>Stage 1 Stage 2 Stage 3 Stage 4</p> <p>Function? Explain.</p>
<p>20. $\{(2, -10), (5, -25), (8, -40), (-5, 25)\}$</p> <p>Function? Explain.</p>	<p>21. $\{(-1, 0), (1, 2), (1, 4), (5, 2)\}$</p> <p>Function? Explain.</p>

22. Draw a graph of a relation that is a function. Explain how you know.



23. Draw a graph of a relation that is **not** a function. Explain how you know.




24. Make a mapping of a relation that is a function. Explain how you know.

25. Create a set of ordered pairs that do **not** represent a function. Explain how you know.

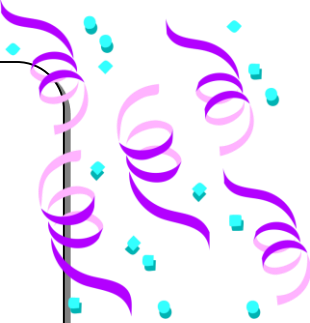
5.1d Class Activity: Birthdays

1. Make a mapping that shows the students in your class and their birthdays.

Student	Birthday
	

- a. Is the birth date of a student a function of the individual student? Justify your answer.

2. Make a mapping that shows the first name of the students in your class and their birthdays.

First Name	Birthday
	

b. Is the birth date of a student a function of the student's first name? Justify your answer.

3. Make a mapping, switching the input to be birthday and the output to be student.



Birthday

Student

A large, empty, rounded rectangular box intended for writing a mapping from Birthday to Student.

A large, empty, rounded rectangular box intended for writing a mapping from Student to Birthday.

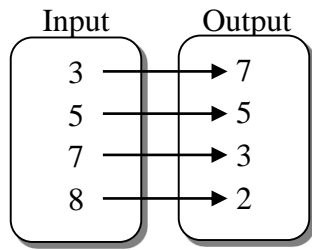
a. Is student a function of birth date in your class? Justify your answer.

5.1d Homework: Birthdays

Directions: Determine if each relation or situation defines a function. Make an additional representation of the relation to help you. Justify your answer.

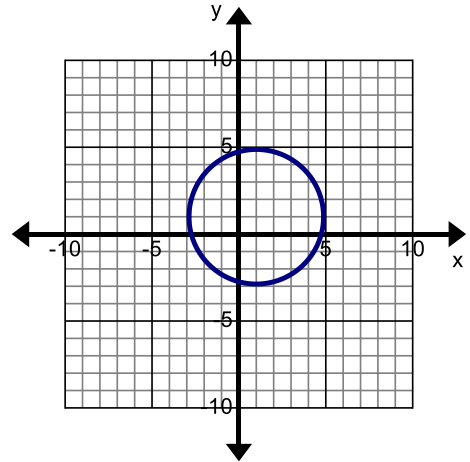
<p>a. Is a student's ID number a function of his/her first name? Consider all students in your school.</p> <p>Function? Explain.</p>	<p>b. Is a student's first name a function of his/her student ID number? Consider all students in your school.</p> <p>Function? Explain.</p>
<p>c.</p> <div style="display: flex; justify-content: center; align-items: center; gap: 20px;"> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; text-align: center;"> <i>Student</i> Raul Tony Xao Jamal </div> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; text-align: center;"> <i>Order</i> Pasta Salad Steak Pizza </div> </div> <p>Function? Explain.</p>	<p>d.</p> <div style="display: flex; justify-content: center; align-items: center; gap: 20px;"> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; text-align: center;"> <i>Student</i> Sam Joe Luis Mia </div> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; text-align: center;"> <i>Color</i> Red Blue Green </div> </div> <p>Function? Explain.</p>
<p>e. Les surveys the students in his class to determine if shoe size is a function of last name of the student? What would have to be true about the names of the students in the class if Les found that shoe size is not a function of the last name of the student?</p>	<p>f.</p> <div style="display: flex; justify-content: center; align-items: center; gap: 20px;"> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; text-align: center;"> <i>Ordered Pair Before</i> (2, 3) (1, 2) (-4, 3) (-3, -5) </div> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; text-align: center;"> <i>Ordered Pair After</i> (-2, 3) (-1, 2) (4, 3) (3, -5) </div> </div> <p>Function? Explain.</p>

g.



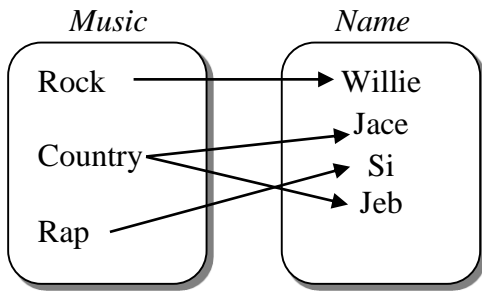
Function? Explain.

h.



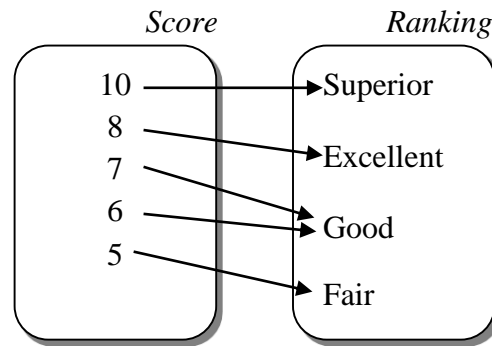
Function? Explain.

i. **Input:** favorite type of music
Output: name



Function? Explain.

j. **Input:** a pianist's overall score in a music competition
Output: ranking



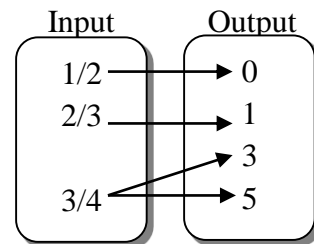
Function? Explain.

k.

Input	Output
25	14
30	13
30	12
35	11

Function? Explain.

l.



Function? Explain.

5.1e Class Activity: More About Functions

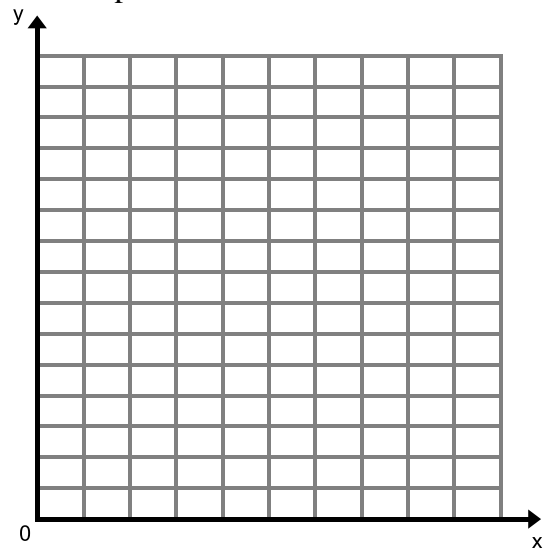
- Paradise Valley Orchards has the banner shown hanging from their store window. Sally is trying to determine how much she will spend depending on how many bushels of apples she purchases.



- Write an equation that gives the amount Sally will spend y depending on how many bushels of apples x she purchases.
- Complete the graph and table below for this relationship.

Number of Bushels x	Amount Spent (dollars) y

c.



We know from the previous lessons, that the relationship between number of bushels purchased and amount spent is an example of a function. The equation above gives us a rule for how to determine the amount of money spent based on the number of bushels purchased.

In a functional relationship represented with an equation, the **independent variable** represents the input or x -value of the function and the **dependent variable** represents the output or y -value of the function. In a function, the **dependent variable** is determined by or depends on the **independent variable**. In our example above the **independent variable** is the number of bushels purchased and the **dependent variable** is the amount of money spent. The amount of money one spends **depends** on the number of bushels one purchases. Another way to say this is that the amount of money spent is a function of the number of bushels purchased.

If we think of our input machine, we are inputting the number of bushels purchased and the machine takes that number and multiplies it by 15 to give us our output which is the amount of money we will spend.

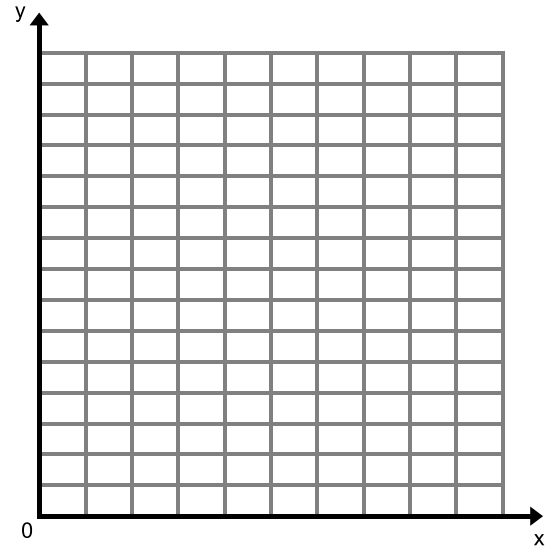
2. Miguel is taking a road trip and is driving at a constant speed of 65 mph. He is trying to determine how many miles he can drive based on how many hours he drives.

a. Identify the **independent variable** in this situation: _____

b. Identify the **dependent variable** in this situation: _____

c. Complete the graph and table below for this relationship. Make sure you label the columns and axes in your table and graph.

x	y

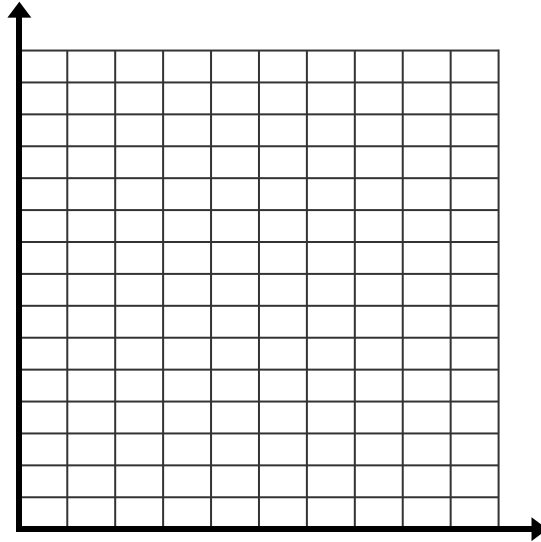


d. Write an equation that represents this situation: _____

e. In this situation _____ is a function of _____.

3. The drama club is selling tickets to the Fall Ball. They use \$2 from each ticket sale for food and decorations.
- Identify the **independent variable** in this situation: _____
 - Identify the **dependent variable** in this situation: _____
 - Create a table, graph, and equation for this function.

x	y

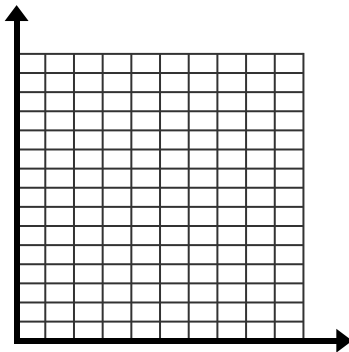


Equation: _____

- Complete the following sentence for this situation.

_____ is a function of _____.

4. The average cost of a movie ticket has steadily increased over time.
- Identify the dependent and independent variables in this functional relationship.
 - Sketch a possible graph of this situation.



5. Susan is reading her history text book for an upcoming test. She can read 5 pages in 10 minutes. Susan is interested in determining how many pages she can read based on how long she reads for.
- Identify the **independent variable** in this situation: _____
 - Identify the **dependent variable** in this situation: _____
 - Create a representation (table, graph, equation) of this function in the space below.
6. Chris is also reading his history text book for an upcoming test and can also read 5 pages in 10 minutes. However, Chris is interested in determining how long it will take him to read based on how many pages he has to read.
- Identify the **independent variable** in this situation: _____
 - Identify the **dependent variable** in this situation: _____
 - Create a representation (table, graph, equation) of this function in the space below.

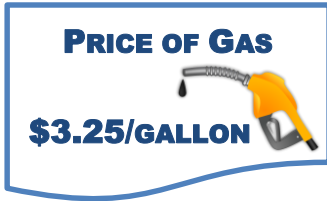
Directions: Each of the following situations represents a functional relationship between two quantities. Determine the dependent variable and the independent variable. The first one has been done for you.



7. In warm climates, the average amount of electricity used rises as the daily average temperature increases and falls as the daily average temperature decreases.
8. The number of calories you burn increases as the number of minutes that you walk increases.
9. The air pressure inside a tire increases with the temperature.
10. As the amount of rain decreases, so does the water level of the river.
11. The total number of jars of pickles that a factory can produce depends on the number of pickles they receive.
12. The weight of the box increases as the number of books placed inside the box increases.

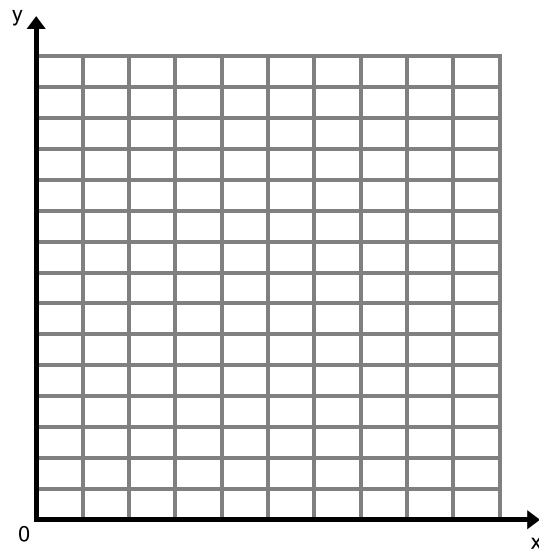
5.1e Homework: More About Functions

1. Shari is filling up her gas tank. She wants to know how much it will cost to put gas in her car. The sign below shows the cost for gas at Grizzly's Gas-n-Go.



- Identify the **independent variable** in this situation: _____
- Identify the **dependent variable** in this situation: _____
- Complete the graph and table below for this relationship. Make sure you label the columns and axes in your table and graph.

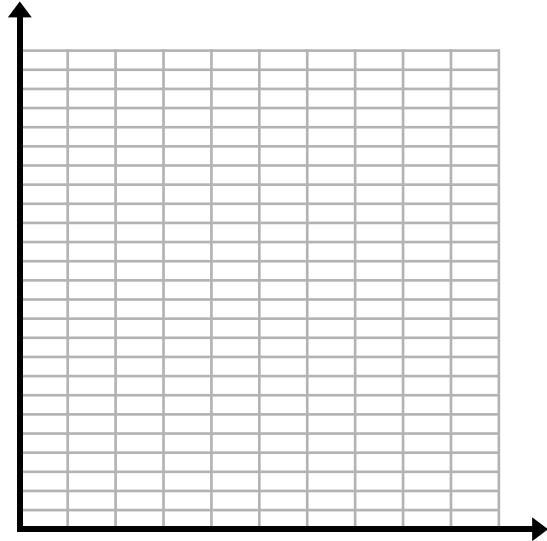
x	y



- Write an equation that represents this situation: _____
- In this situation _____ is a function of _____.

2. Peter is the event planner for a team race taking place in Park City, UT. He needs to determine how many bottles of water to have ready at the finish line of the race so that each participant in the race receives a bottle of water. There are 4 people on a team.
- Identify the **independent variable** in this situation: _____
 - Identify the **dependent variable** in this situation: _____
 - Create a table, graph, and equation of this situation.

x	y



Equation: _____

- Complete the following sentence:
 _____ is a function of _____.

Directions: Each of the following situations represents a functional relationship between two quantities. Underline the two quantities. Put an I above the independent variable and a D above the dependent variable.

- As the size of your family increases so does the cost of groceries.
- The value of your car decreases with age.
- The greater the distance a sprinter has to run the more time it takes to finish the race.
- A car has more gas in its tank can drive a farther distance.
- A child's wading pool is being inflated. The pool's size increases at a rate of 2 cubic feet per minute.
- The total number of laps run depends on the length of each workout.

9. A tree grows 15 feet in 10 years.
10. There are 5 inches of water in a bucket after a $2\frac{1}{2}$ hour rain storm.
11. Jenny has 30 coins she has collected over 6 years.
12. Sally's track coach wants to know how far she can run based on the amount of time she runs for.
13. Whitney is training for a half marathon. She wants to know how long it will take her to run based on how far she has to run for.
14. Write your own relationship that contains an independent and dependent variable.


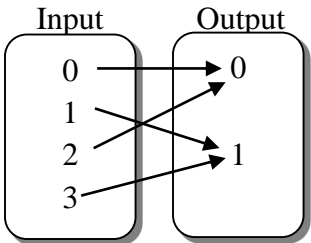
5.1f Self-Assessment: Section 5.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Understand that a function is a rule that assigns to each input exactly one output.				
2. Determine whether a given relation defines a function given different representations (i.e., table, graph, mapping, story, patterns, equations, and ordered pairs).				
3. Determine the independent and dependent variables in a functional relationship.				

1. Define function in your own words. Provide examples to support your definition.

2. Do the representations below define a function? Why or why not?

<p>a.</p>  <p>Stage 1 Stage 2 Stage 3 Stage 4 Stage 5</p> <p>Is the number of hearts in a stage a function of the stage number?</p> <p>Is the stage number a function of the number of hearts in a stage?</p>	<p>b. Maria is draining her hot tub at a rate of 5.5 gallons per minute. Is the amount of water left in the pool a function of the amount of time she has been draining it?</p>										
<p>c.</p> <table border="1" data-bbox="354 892 587 1144"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>10</td> <td>1</td> </tr> <tr> <td>10</td> <td>2</td> </tr> <tr> <td>10</td> <td>3</td> </tr> <tr> <td>10</td> <td>4</td> </tr> </tbody> </table>	x	y	10	1	10	2	10	3	10	4	<p>d.</p> 
x	y										
10	1										
10	2										
10	3										
10	4										
<p>e. Is state capitol a function of state name? Consider states in the United States.</p>	<p>f. Is a person's weight a function of the person's age?</p>										
<p>g. Is the amount of time it takes a person to run a marathon a function of the person's age?</p>	<p>h. $\{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1)\}$</p>										

<p>i.</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; border-radius: 15px; padding: 5px; width: 150px;"> <p style="text-align: center;"><i>Student</i></p> <p>Katniss Peta Finnick</p> </div> <div style="border: 1px solid black; border-radius: 15px; padding: 5px; width: 150px;"> <p style="text-align: center;"><i>Sport</i></p> <p>Archery Fishing Swimming Running</p> </div> </div>	<p>j. $y = \frac{1}{4}x - 2$</p>
<p>k.</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; border-radius: 15px; padding: 5px; width: 150px;"> <p style="text-align: center;"><i>Shape Before</i></p> </div> <div style="border: 1px solid black; border-radius: 15px; padding: 5px; width: 150px;"> <p style="text-align: center;"><i>Shape After</i></p> </div> </div>	<p>l.</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; border-radius: 15px; padding: 5px; width: 150px;"> <p style="text-align: center;"><i>Ordered Pair Before</i></p> <p>(2, 4) (4, 6) (-2, -8) (-4, -6)</p> </div> <div style="border: 1px solid black; border-radius: 15px; padding: 5px; width: 150px;"> <p style="text-align: center;"><i>Ordered Pair After</i></p> <p>(1, 2) (2, 3) (-1, -4) (-2, -3)</p> </div> </div>

3. In each of the following situations an independent variable is given. Determine a possible dependent variable that would create a functional relationship.
- The amount of gas remaining in a tank
 - Time
 - Number of people
 - Number of t-shirts
 - Circumference of head

Section 5.2: Explore Linear and Nonlinear Functions

Section Overview:

This section focuses on the characteristics that separate linear from nonlinear functions. Students will analyze the different representations of a function (graph, table, equation, and context) to determine whether or not the representations suggest a linear relationship between the two variables. In the process of studying non-linear functions, students will solidify their understanding of how a linear function grows (changes).

Concepts and Skills to Master:

By the end of this section, students should be able to:

- Distinguish between linear and nonlinear functions given a context, table, graph, or equation.

5.2a Class Activity: Display Designs



Complete Foods, a local grocery store, has hired three different companies to come up with a display for food items that are on sale each week. They currently have a display that is 6 boxes wide as shown below.

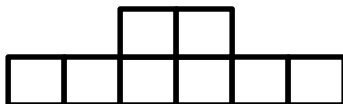


They would like the center part of the display to be taller than the outside pieces of the display to showcase their “mega deal of the week”. The following are the designs that two different companies submitted to Complete Foods, using the current display as their starting point.

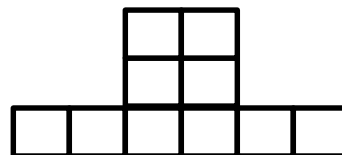
Design Team 1:



Current Display (Stage 1)



Stage 2



Stage 3

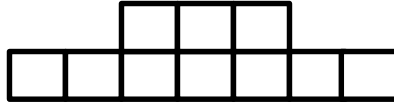
Stage 4

1. Draw Stage 4 of this design. Describe how you went about drawing stage 4.
2. Can the relationship between stage number and number of blocks in a stage in this pattern be modeled by a linear function? Provide at least 2 pieces of evidence to support your answer.

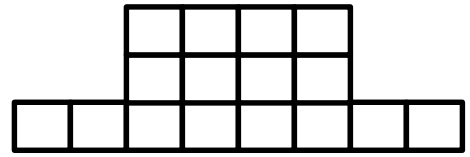
Design Team 2:



Current Display (Stage 1)



Stage 2



Stage 3

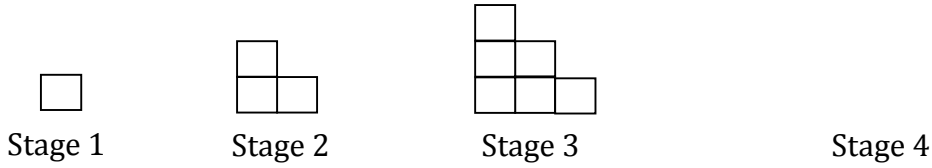
Stage 4

3. Draw Stage 4 of this design. Describe how you went about drawing stage 4.
4. Can the relationship between stage number and number of blocks in a stage in this pattern be modeled by a linear function? Provide at least **2** pieces of evidence to support your answer.

5.2a Homework: More Patterns – Are They Linear?

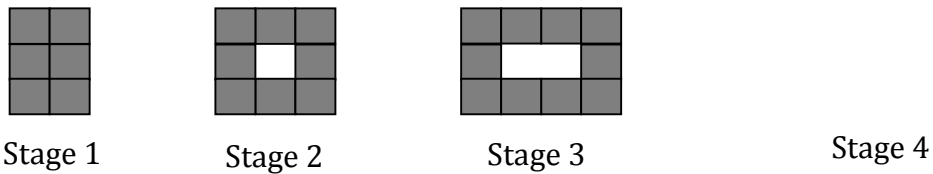
Directions: For each of the following patterns, draw the next stage and determine whether relationship between the stage number and the number of blocks in a stage can be represented by a linear function. Justify your answer.

1.



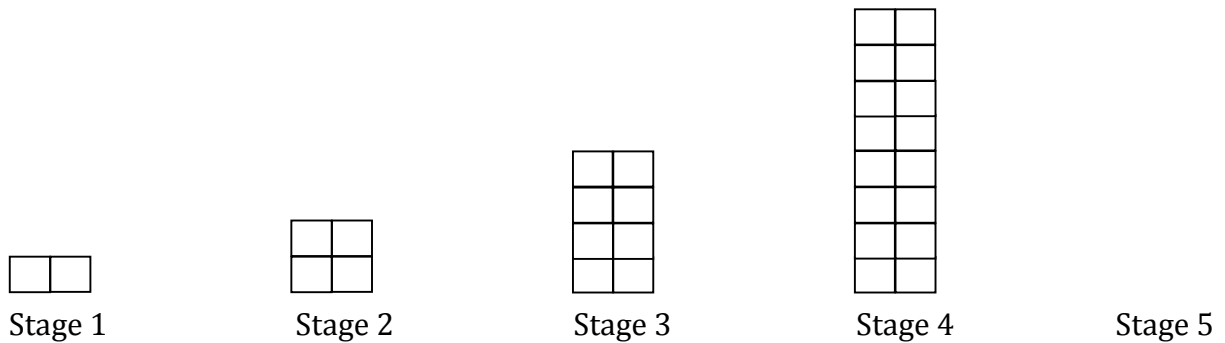
- a. Is this pattern linear? _____
Justification:

2. Consider the gray tiles only



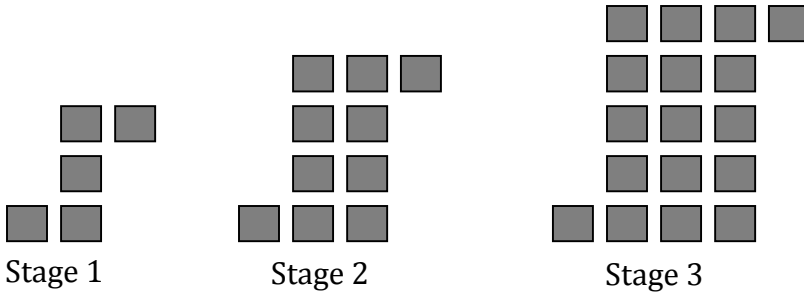
- a. Is this pattern linear? _____
b. Justification:

3.



- a. Is this pattern linear? _____
b. Justification:

4.



- a. Is this pattern linear? _____
- b. Justification:

5. Make up your own pattern that is **not** linear. Prove that your pattern is not linear with at least **2** pieces of evidence.

5.2b Class Activity: Linear and NonLinear Functions in Context



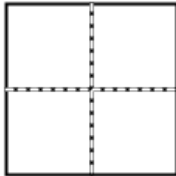
1. Consider the **area** of a square as a function of the side length of the square.
 - a. Draw pictures to represent these squares. The first two have been drawn for you.

$$A = 1$$



Side length = 1

$$A = 4$$

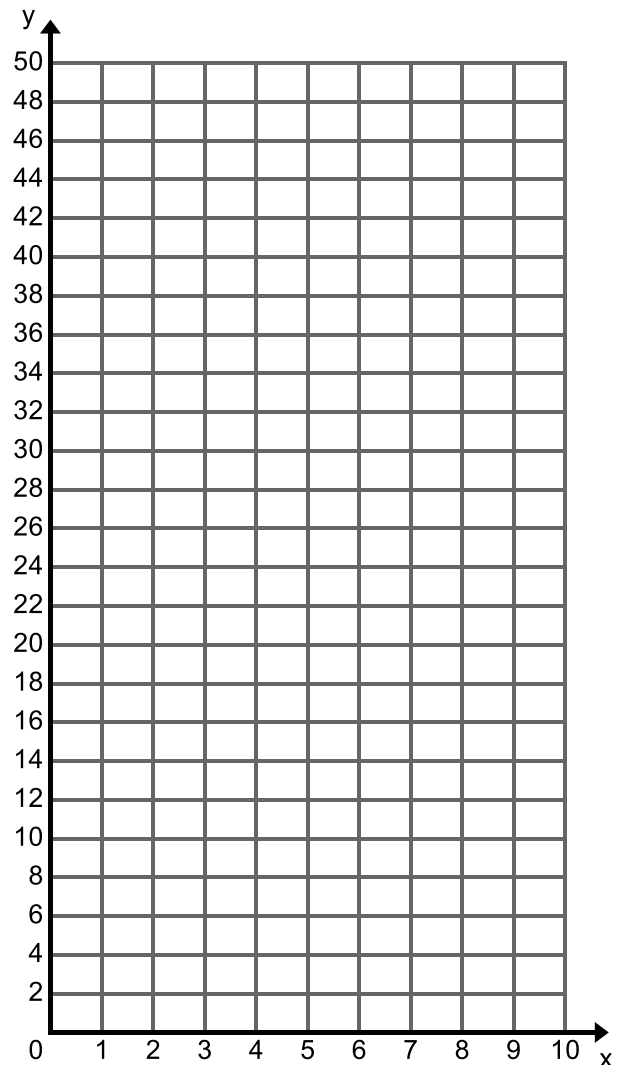


Side length = 2

- b. Complete the graph and table for this function.

side length	area
1	
2	
3	
4	
5	

- c. What is the dependent variable? The independent variable?
- d. Write an equation to model A as a function of s .
- e. Does this graph pass through the point $(8, 64)$? Explain how you know.
- f. What does the point $(8, 64)$ represent in this context?
- g. List three more ordered pairs that this graph passes through.
- h. Is this function linear? Explain or show on the graph, table, and equation why or why not?



2. Consider the **perimeter** of a square as a function of the side length of the square.

a. Complete the graph and table for this function.

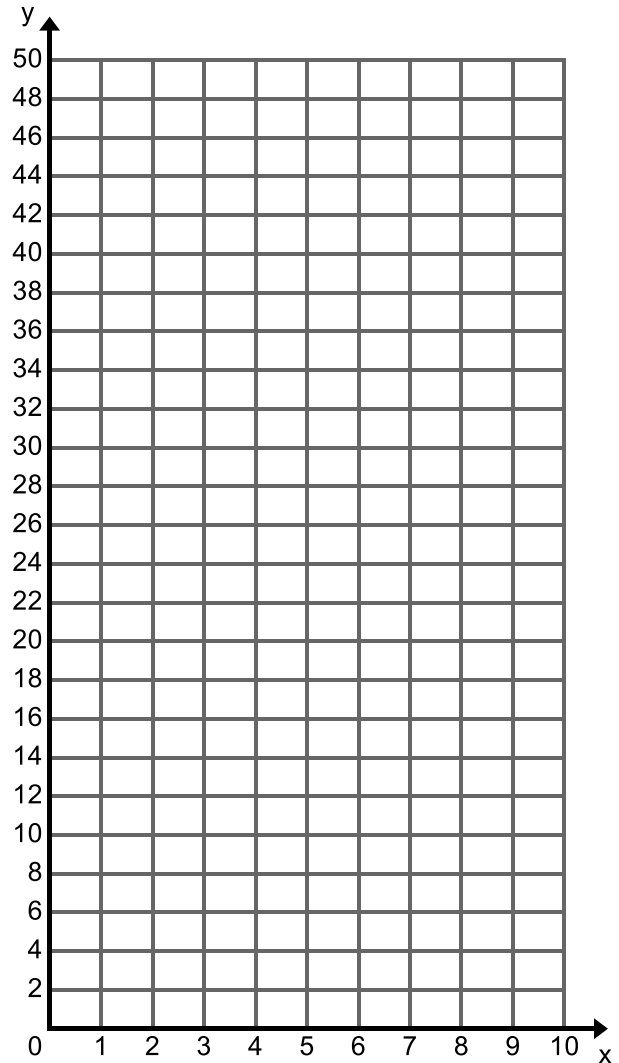
side length	perimeter
1	
2	
3	
4	
5	

b. What is the dependent variable? The independent variable?

c. Write an equation to model P as a function of s .

d. Find another ordered pair that the graph passes through.

e. What does the point $(10, 40)$ represent in this context?



f. Is this function linear? Explain or show on the graph, table, and equation why or why not?

5.2b Homework: Linear and NonLinear Functions in Context


1. The following tables show the distance traveled by three different cars over five seconds.

Car 1	
Time (s)	Distance (ft.)
1	4
2	7
3	10
4	13
5	16

Car 2	
Time (s)	Distance (ft.)
1	2
2	5
3	10
4	17
5	26

Car 3	
Time (s)	Distance (ft.)
1	3
2	5
3	9
4	17
5	33



- Consider the relationship between time and distance traveled for each car. Which of the tables of data can be modeled by a linear function? Which ones cannot be modeled by a linear function? Justify your answer.
- For any of the data sets that can be modeled by a linear function, write a function that models the distance traveled D as a function of time t .
- What is the dependent variable in this situation? The independent variable?
- Which car is traveling fastest? Justify your answer. 


2. Hermione argues that the table below represents a linear function. Is she correct? How do you know?

x	2	4	8	16
y	1	3	5	7



3. Emily's little brother painted on her math homework. She knows the data in each of the tables below represents a linear function. Help Emily determine what number is hidden behind the blob of paint.

a.


x	10	20	30	40
y	8	13		23



b.

x	-2	0	2	3
y	-5		7	10

c.

x	0	1		6
y	0	3	9	18

Directions: Choose 3 of the following situations. Determine whether the situations you choose can be modeled by a linear function or not. Provide evidence to support your claim. Show your work in the space below.



4. Mr. Cortez drove at a constant rate for 5 hours. At the end of 2 hours he had driven 90 miles. After 5 hours, he had driven 225 miles.
- What is the dependent variable in this relationship? _____
 - What is the independent variable in this relationship? _____
 - Can this relationship be modeled by a linear function? Provide evidence to support your claim.

5. Round 1 of a tennis tournament starts with 64 players. After each round, half the players have lost and are eliminated from the tournament. Therefore, in round 2 there are 32 players, in round 3 there are 16 players and so on.



- What is the dependent variable in this relationship? _____
- What is the independent variable in this relationship? _____
- Can this relationship be modeled by a linear function? Provide evidence to support your claim.

6. A rock is dropped from a cliff that is 200 feet above the ground. The table below represents the height of the rock (in feet) with respect to time (in seconds).

Time (s)	Height (ft.)
0	200
1	184
2	136
3	56

- What is the dependent variable in this relationship? _____
- What is the independent variable in this relationship? _____
- Can this relationship be modeled by a linear function? Provide evidence to support your claim.

7. A student comes to school with the flu and infects three other students within an hour before going home. Each newly infected student passes the virus to three new students in the next hour. This pattern continues until all students in the school are infected with the virus.
- What is the dependent variable in this relationship? _____
 - What is the independent variable in this relationship? _____
 - Can this relationship be modeled by a linear function? Provide evidence to support your claim.
8. A piece of paper is cut into two equal sections. Each new piece is cut into two additional pieces of equal size. This pattern continues until it is no longer possible to cut the paper any more.
- What is the dependent variable in this relationship? _____
 - What is the independent variable in this relationship? _____
 - Can this relationship be modeled by a linear function? Provide evidence to support your claim.

5.2d Class Activity: Different Types of Functions

- Sketch the general appearance of the graph of the equation $y = mx + b$.
 - What do m and b represent?
 - What makes the graph linear?
- Complete the table of values for the functions shown in the table below. Using the table of values, predict what the graphs of the equations will look like. Compare the tables to the table for $y = x$.



$y = x$	
x	y
-2	
-1	
0	
1	
2	

$y = x $	
x	y
-2	
-1	
0	
1	
2	

$y = x^2$	
x	y
-2	
-1	
0	
1	
2	

$y = \frac{1}{x}$	
x	y
-2	
-1	
0	
1	
2	

$y = \sqrt{x}$	
x	y
0	
1	
2	
3	
4	

$y = 2^x$	
x	y
0	
1	
2	
3	
4	

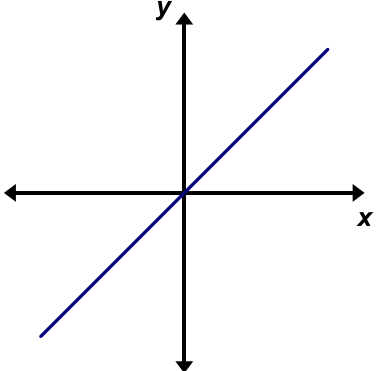
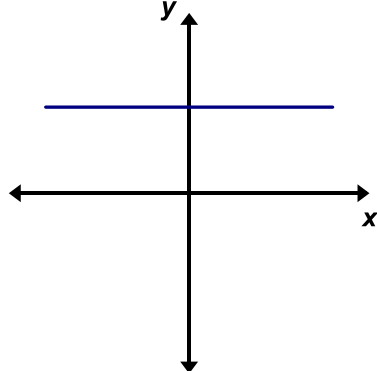
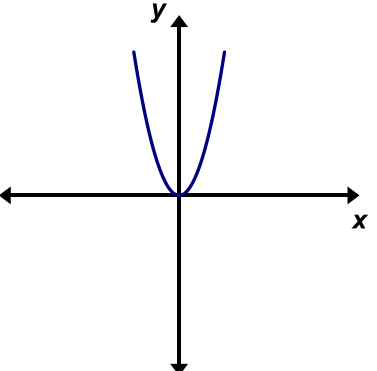
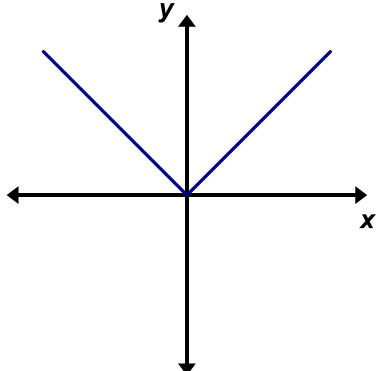
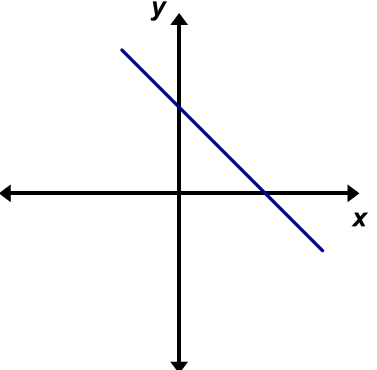
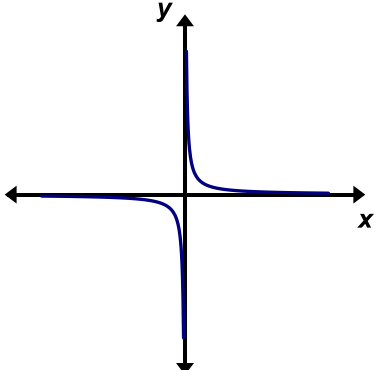
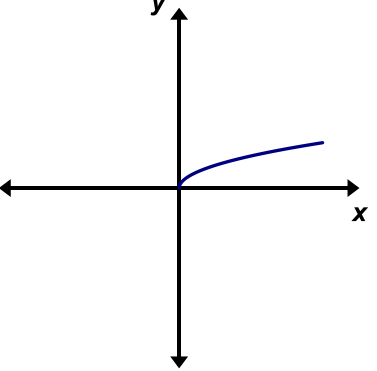
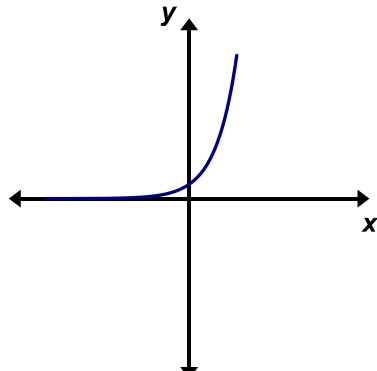
$x + y = 6$	
x	y
-2	
-1	
0	
1	
2	

$y = 6$	
x	y
-2	
-1	
0	
1	
2	

3. Use the tables of values from the previous page to match each equation to its graph below. Write the

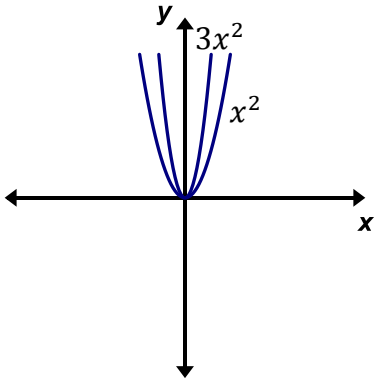
equation to the left of the graph. The first one has been done for you.



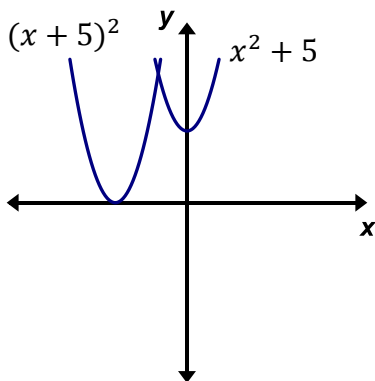
$y = x$ 	
	
	
	

4. Compare each of the graphs on the previous page to the graph of $y = x$. What is the same? What is different? Discuss with a class mate.

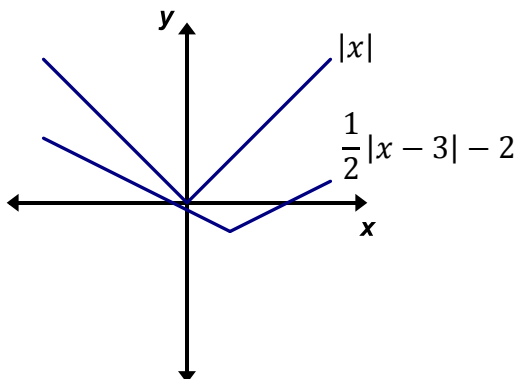
5. The graphs of x^2 and $3x^2$ are shown below. Compare these graphs. What is the same? What is different?



6. The graphs of $y = x^2 + 5$ and $y = (x + 5)^2$ are shown below. Compare these graphs to the graph of $y = x^2$. What is the same? What is different?



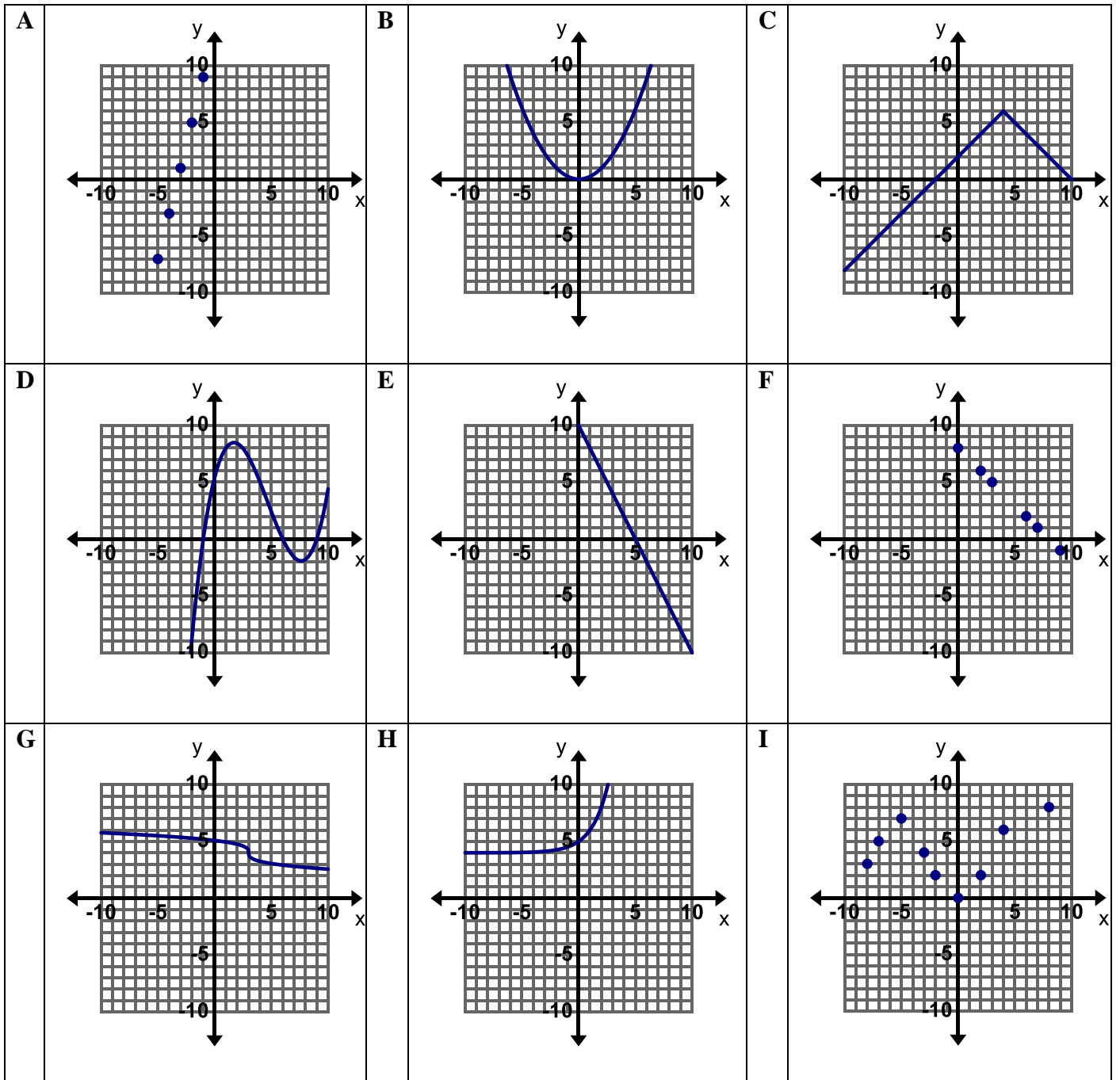
7. The graphs of $y = |x|$ and $y = \frac{1}{2}|x - 3| - 2$ are shown below. Compare these graphs. What is the same? What is different?



8. Describe the basic structure of an equation that defines a **linear function**. Think about the different forms a linear equation might take. Provide examples of the different forms.
9. Describe attributes you see in the equations that define **nonlinear functions**. Provide examples.

5.2d Homework: Different Types of Functions

1. Circle the letter next to the graph if it represents a linear function.



Bonus: Do any of the graphs of nonlinear functions have a shape similar to the ones studied in class? Make a prediction about the basic structure of the equations of these functions.

2. Circle the letter next to the table if the data represents a linear function.

A	x	y	B	x	y	C	x	y	D	x	y
	0	-5		0	15		0	4		0	98
	1	0		1	12.5		1	8		1	98
	2	5		2	10		2	16		2	98
	3	10		3	7.5		3	32		3	98
E	x	y	F	x	y	G	x	y	H	x	y
	-3	3		1	2		2.1	4		0	2
	-2	2		3	4		2.2	5		4	6
	2	2		6	6		2.3	6		8	18
	3	3		10	8		2.4	7		12	54
I	x	y	J	x	y	K	x	y	L	x	y
	3	20		10	-20		5	0		15.1	4.2
	6	24		30	-40		10	-1		16.7	12.2
	12	32		50	-60		15	-2		18.3	20.2
	24	48		70	80		20	-3		19.9	28.2

3. Circle the letter next to each equation if it represents a linear function.

A	$2x + 4y = 16$	B	$y = 2x + 5$	C	$y = x^2 + 5$	D	$y = 5 \cdot 3^x$
E	$y = \frac{4}{x} + 3$	F	$y = \frac{x}{4} + 3$	G	$y = \sqrt{4x}$	H	$x^2 + y^2 = 25$
I	$xy = 24$	J	$2x + y = 6$	K	$y = -\frac{2}{3}x$	L	$y = 8$
M	$y = \frac{2}{3}x$	N	$y = x^3$	O	$3x - y = 2$	P	$y = x(x + 2)$

Bonus: Can you predict the basic shape of any of the graphs of the nonlinear equations in #3?

5.2e Class Activity: Matching Representations of Functions

Matching Activity: Match the following representations together. Each representation will have a

- 1) a story,
- 2) an equation,
- 3) a table of values, and
- 4) a graph.



After you have matched the representations, **label** the axes of the graphs on the graph cards, **answer** the questions asked in the word problems on the story cards, and **identify** the dependent and independent variable in each story.

Story	Equation	Table	Graph
DD			
Z			
AA			
FF			
EE			
Y			
BB			
CC			

A

x	0	3	6	9	12	15
y	6	7	8	9	10	11

B

x	0	1	2	3	4	5
y	6	100	162	192	190	156

C

x	0	4	8	12	16	20
y	0	200	200	200	224	248

D

x	0	1	2	3	4	5
y	6	12	24	48	96	192

E

x	0	5	10	15	20	25
y	6	7	8	9	10	11

F

x	0	1	2	3	4	5
y	200	196	192	188	184	180

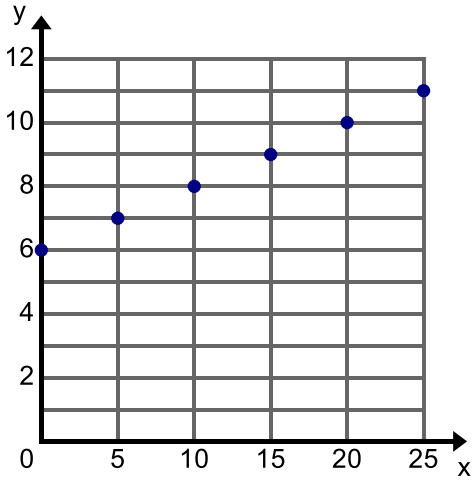
G

x	0	1	2	3	4	5
y	6	206	406	606	806	1006

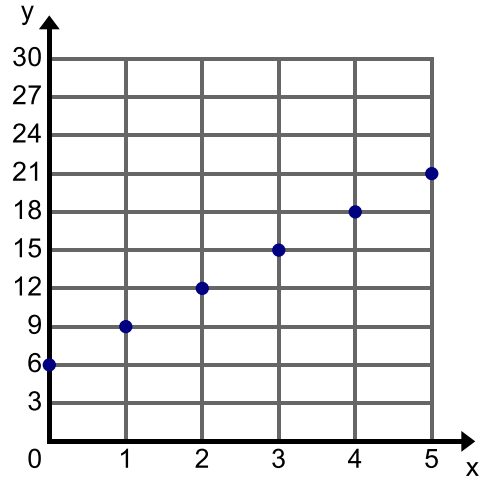
H

x	0	1	2	3	4	5
y	6	9	12	15	18	21

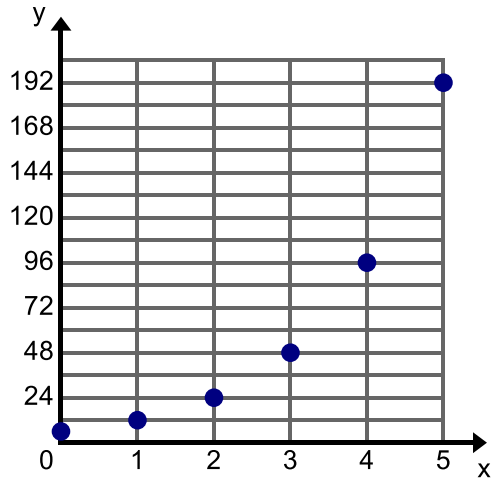
I



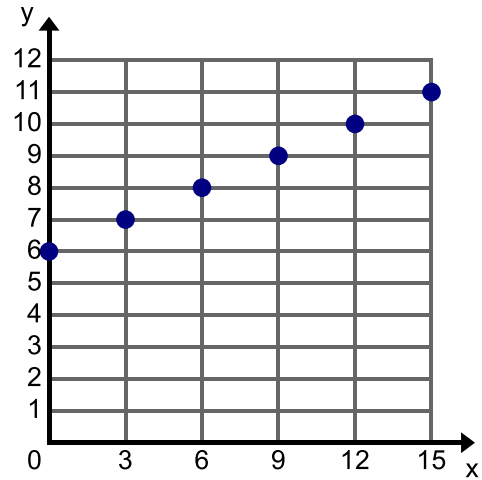
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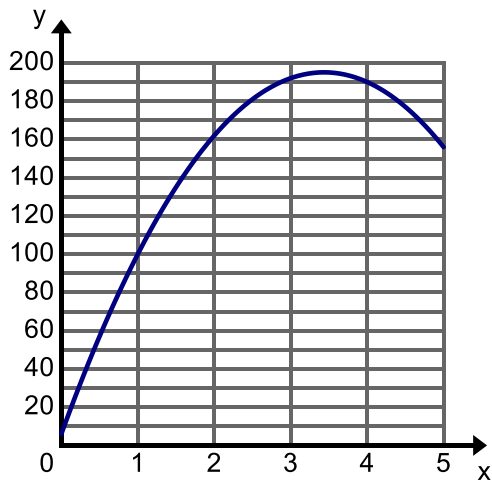
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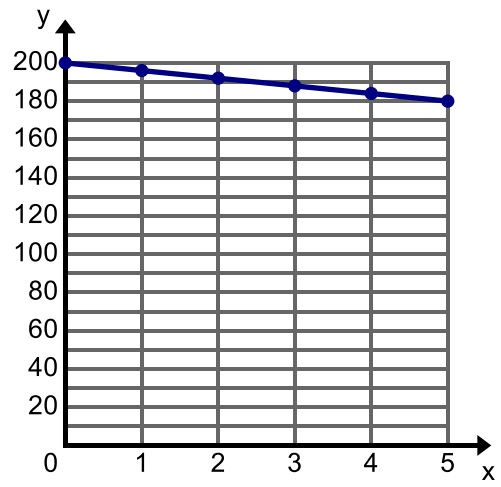
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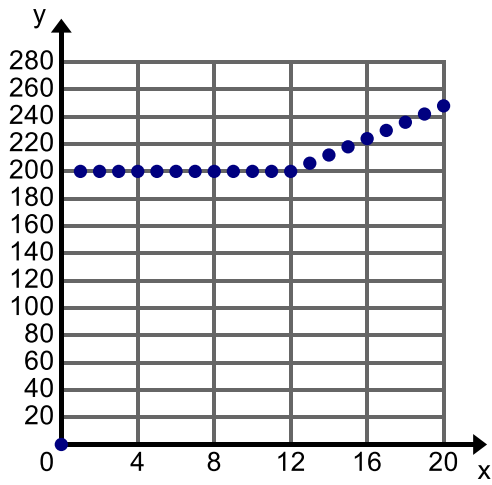
M



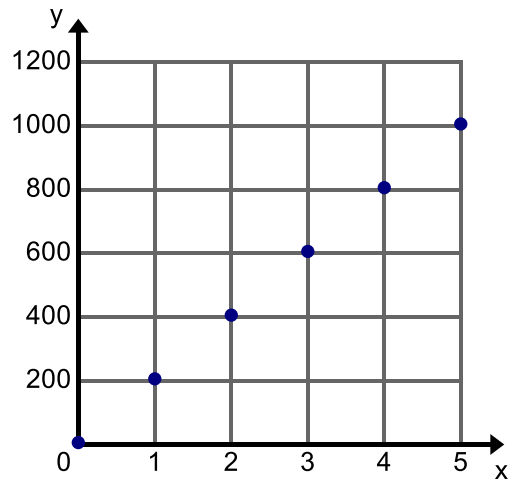
N



O



P



Q

$$y = -4x + 200$$

R

$$y = -16x^2 + 110x + 6$$

S

$$y = 3x + 6$$

T

$$y = 200x + 6$$

U

$$y = 6 \cdot 2^x$$

V

$$y = \begin{cases} 200, & \text{if } 0 < x \leq 12 \\ 200 + 6x, & \text{if } x > 12 \end{cases}$$

W

$$y = \frac{1}{5}x + 6$$

X

$$y = \frac{1}{3}x + 6$$

Y

A certain bacteria reproduces by binary fission every hour. This means that one bacterium grows to twice its size, replicates its DNA, and splits in 2. If 6 of these bacterium are placed in a petri dish, how many will there be after 5 hours?

Dependent Variable:
Independent Variable:

Z

The state is building a road 4.5 km long from point A to point B. Six meters of the road have already been completed when a new crew starts the job. It takes the crew 3 weeks to complete 600 meters of the road. How much of the road will be completed after 5 weeks if the crew works at a constant rate?

Dependent Variable:
Independent Variable:

AA

Talen loves to help his mom clean to earn money for his cash box. He currently has \$6 in his cash box. He earns \$1 for every 3 jobs he does. How much money will Talen have if he does 15 jobs?

Dependent Variable:
Independent Variable:

BB

Josh is draining a swimming pool at a constant rate of 4 gallons per minute. If the swimming pool starts with 200 gallons of water, how many gallons will remain after 5 minutes?

Dependent Variable:
Independent Variable:

CC

Kendall's mom and dad have agreed to sponsor her in a school walk-a-thon to raise money for soccer uniforms. Her mom is donating \$6 to her. Her dad is donating \$3 for each mile she walks. How much money will she collect if she walks 5 miles?

Dependent Variable:
Independent Variable:

DD

The Planetarium charges \$200 for a birthday party for up to 12 guests. Each additional guest is \$6. How much will it cost for a birthday party with 20 guests?

Dependent Variable:
Independent Variable:

EE

Suppose a rocket is fired from a platform 6 ft. off the ground into the air vertically with an initial speed of 110 feet/second. Where will the rocket be after 5 seconds? *Note:* The gravitational force of the earth on the rocket is -16 ft./sec^2 .

Dependent Variable:
Independent Variable:

FF

Suzy is helping her mom fill Easter eggs with jelly beans for a community egg hunt. Before they get started, Suzy eats 6 jelly beans. Her mom tells her that after that she can eat 1 jelly bean for every 5 eggs she fills. How many jelly beans total did Suzy eat if she filled 25 eggs?

Dependent Variable:
Independent Variable:

5.2e Homework: Matching Representations of Functions

Directions: Create each of the following representations.

1. a table of data that represents a linear function	2. a table of data that represents a nonlinear function
3. a graph that represents a linear function	4. a graph that represents a nonlinear function
5. an equation that defines a linear function	6. an equation that defines a nonlinear function
7. a context (story) that can be modeled by a linear function	8. a context (story) that can be modeled by a nonlinear function

5.2f Self-Assessment: Section 5.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Distinguish between linear and nonlinear functions given a context, table, graph, or equation.				

1. On Tamara's first day of math class, her teacher asked the students to shake hands with everyone in the room to introduce themselves. There are 26 students total in the math class. Can the relationship between number of people and number of handshakes exchanged be modeled by a linear function? Why or why not? Can you determine the number of handshakes that took place in Tamara's math class on the first day of class? Justify your answer.



Section 5.3: Model and Analyze a Functional Relationship

Section Overview:

In this section, students will analyze functional relationships between two quantities given different representations. For relationships that are linear, students will construct a function to model the relationship between the quantities. Students will also compare properties of linear functions represented in different ways, examining rates of change and intercepts, and using this information to solve problems. Next, students will learn about some of the key features of graphs of functions and apply this knowledge in order to describe qualitatively the functional relationship between two quantities. Lastly, students will sketch graphs that display key features of a function given a verbal description of the relationship between two quantities.

Concepts and Skills to Master:

By the end of this section, students should be able to:

- Determine whether the relationship between two quantities can be modeled by a linear function and construct a function to model a linear relationship between two quantities.
- Compare properties of linear functions (rates of change and intercepts) and use this information to solve problems.
- Identify and interpret key features of a graph that models a relationship between two quantities.
- Sketch a graph that displays key features of a function that has been described verbally.

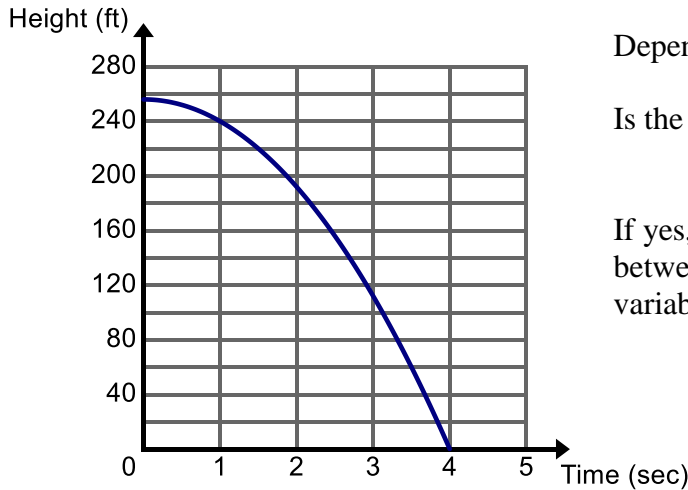
5.3a Class Activity: Constructing Linear Functions



Directions: Identify the dependent and independent variable in the following situations. Determine whether the situations are linear or nonlinear. **For the situations that are linear**, construct a function that models the relationship between the two quantities. Be sure to define your variables.



1. An object is dropped from a bridge into the water below. The graph below shows the height of the object (in feet) with respect to time (in seconds). Consider the relationship between the height of the object and time.



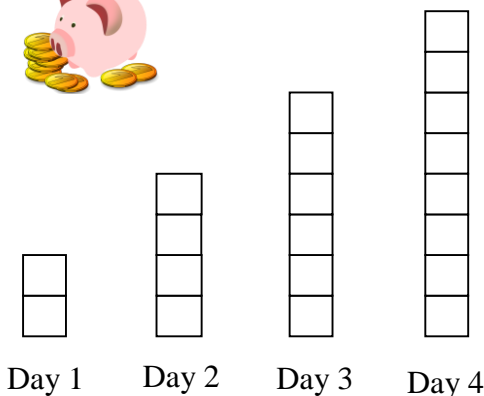
Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two quantities. Be sure to define your variables.

2. Owen is earning pennies each day that he makes his bed in the morning. On the first day, Owen's mom gives him 2 pennies. On the second day, Owen's mom gives him 4 pennies, on the third day 6 pennies, on the fourth day 8 pennies, and so on. Owen makes his bed every day and this pattern continues. The model below shows how many pennies Owen earns each day (each box represents 1 penny). Consider the relationship between **the number of pennies received on a given day** and the **day number**.



Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

3. Refer back to #2 and Owen earning pennies. Consider the relationship between the **total number of pennies** Owen has earned and the **day number**.

Day	# of Pennies Added That Day	Sum of Pennies

Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

4. Carbon-14 has a half-life of 5,730 years. The table below shows the amount of carbon-14 that will remain after a given number of years. Consider the relationship between number of years and amount of carbon-14 remaining.

# of Years	Milligrams of Carbon-14
0	8
5,730	4
11,460	2
17,190	1
22,920	$\frac{1}{2}$

Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

5. The table below shows the amount of time a recipe recommends you should roast a turkey at 325° F dependent on the weight of the turkey in pounds. Consider the relationship between cooking time and weight of the turkey.

Weight of Turkey (lbs.)	12	13	14	15
Cooking Time (hours)	4	$4\frac{1}{3}$	$4\frac{2}{3}$	5

Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

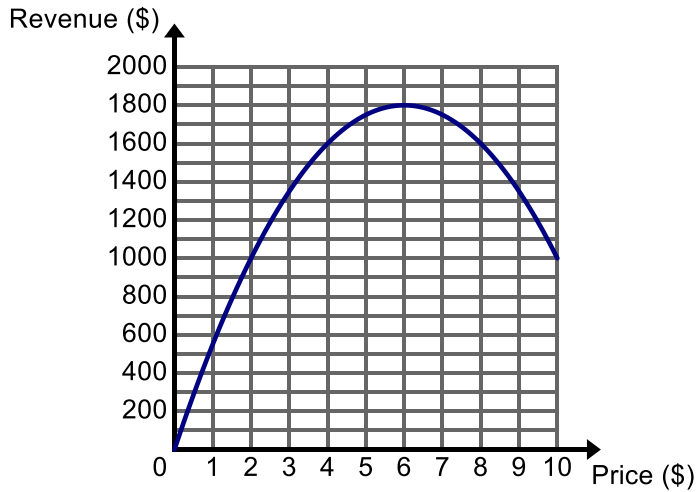
If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

6. Steve is a lifeguard at a local community pool. Each day at noon, he records the temperature and the number of people in the pool. Do you think the relationship between temperature and number of people in the pool is linear? Why or why not?

5.3a Homework: Constructing Linear Functions

Directions: Determine whether the situations represented below are linear or nonlinear. **For the situations that are linear**, construct a function that models the relationship between the two quantities. Be sure to define your variables.

1. The graph below shows the amount of revenue a company will make selling t-shirts dependent on the price of each t-shirt. Consider the relationship between price of each shirt and revenue made.



Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

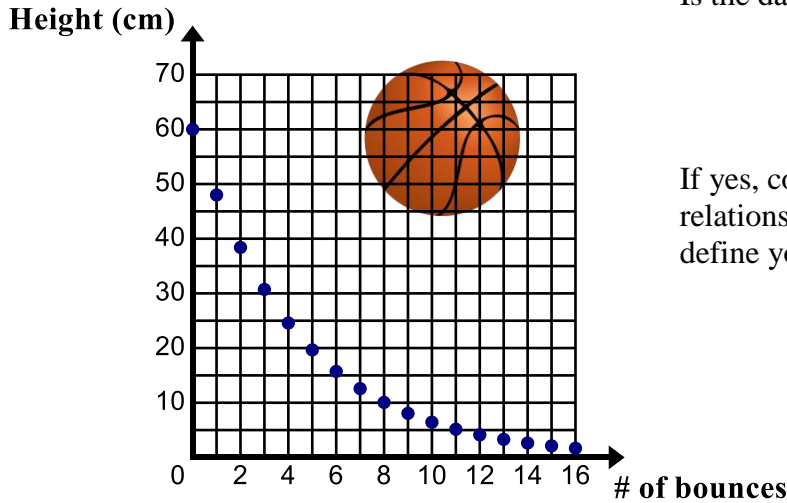
2. When Camilo opened his email this morning he had 140 unread emails. The table below shows the number of remaining unread emails Camilo has in his inbox. Assume that Camilo does not receive any new emails while he is reading his email. Consider the relationship between time and the number of unread emails.

Time (hours)	# of Unread Emails
0	180
0.5	160
1	140
2	100
2.5	80
4	20
4.5	0

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two quantities. Be sure to define your variables.

3. Suppose you drop a basketball from a height of 60 inches. The graph below shows the height of the object after b bounces.



Is the data linear? Why or why not?

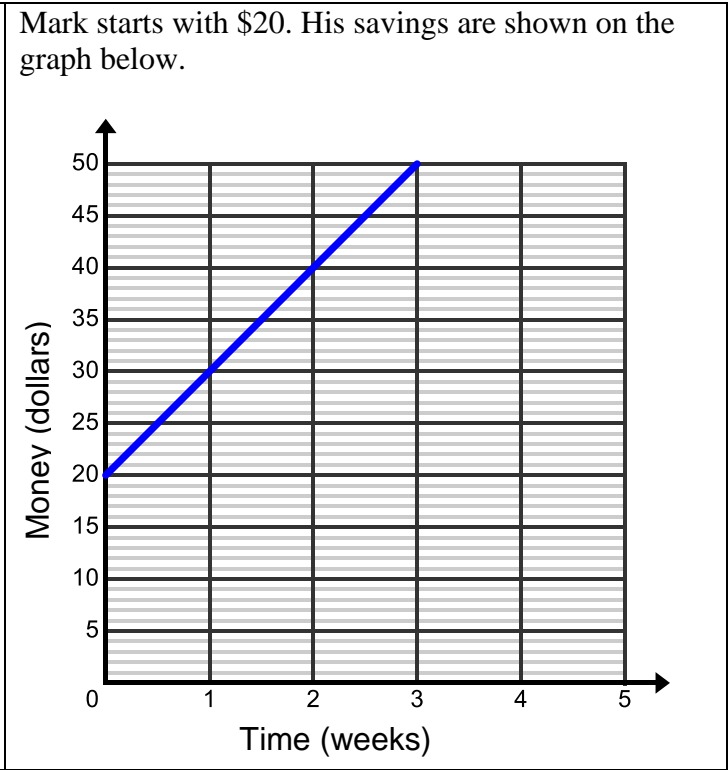
If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

4. Justine and her family are floating down a river. After 1 hour, they have floated 1.25 miles, after 4 hours they have floated 5 miles, and after 6 hours they have floated 7.5 miles. Is the relationship between time (in hours) and distance (in miles) linear? Why or why not? If it is linear, write a function that models the relationship between the two quantities.
5. You and your friends go to a BMX dirt-biking race. For one of the events, the competitors are going off a jump. The winner of the event is the competitor that gets the most air (or jumps the highest). Do you think the relationship between the weight of the bike and the height of the jump can be modeled by a linear relationship? Why or why not?
6. Homes in a certain neighborhood sell for \$117 per square foot. Can the relationship between the number of square feet in the home and the sale price of the home be modeled by a linear function? Why or why not? If it can be modeled by a linear function, write a function that models the relationship between the two quantities.
7. Suppose a certain bank pays 4% interest at the end of each year on the money in an account. When Devon was born, his parents put \$100 in the account and will leave it there until he goes to college. Is the relationship between time (in years) and the amount of money in the account (in dollars) linear or not? Why or why not? If it is linear, write a function that models the relationship between the two quantities.

5.3b Class Activity: Comparing Linear Functions

1. Who will have \$100 first, George or Mark?

George has \$20 and is saving \$15 every week.



2. Put the cyclists in order from slowest to fastest. (Note variables: x = time in seconds, y = meters traveled)

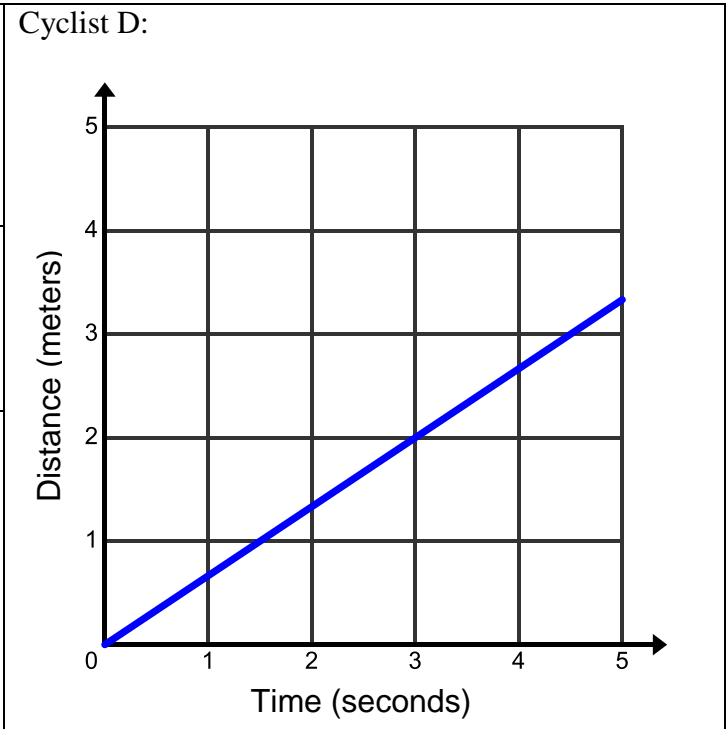
Cyclist A:

Time (x)	Distance (y)
2	1
4	2
6	3

Cyclist B:

Bob has cycled 12 meters in the past 6 seconds.

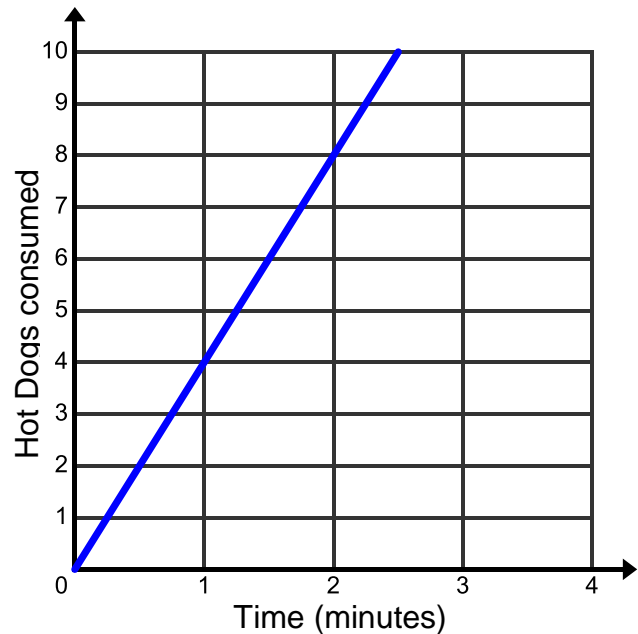
Cyclist C:

$$y = \frac{1}{3}x$$


3. Assume the rates below will remain constant. Who will win the hot dog eating contest? Why?

Helga, who has eaten 18 hot dogs in 5 minutes.

Pablo whose eating record is shown below.



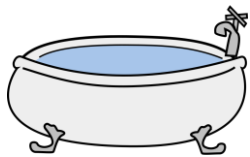
4. Based on the information below, which bathtub will be empty first? Why?

Bathtub A:

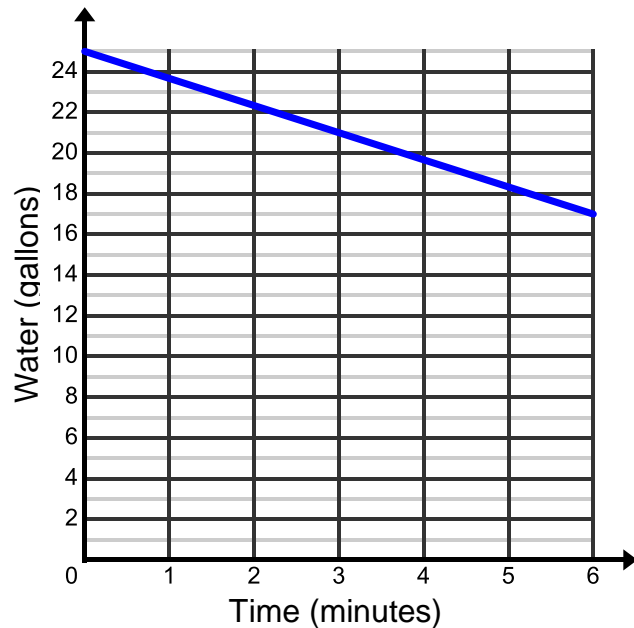
Starts with 25 gallons and is draining 1.5 gallons a minute.

Bathtub B:

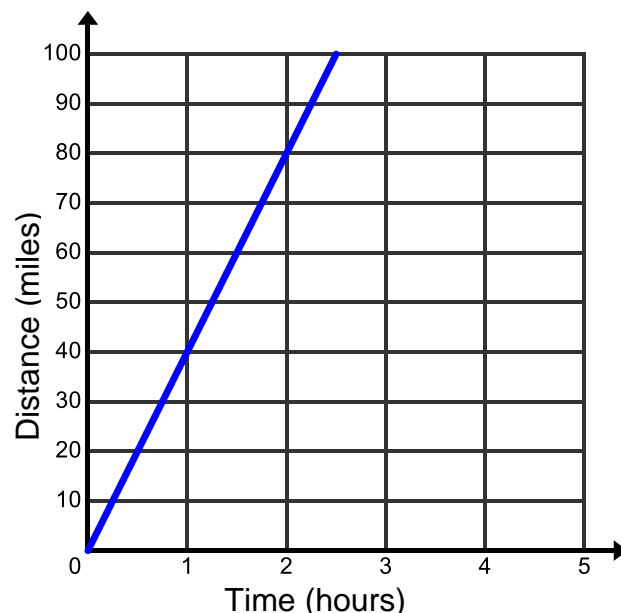
Minutes	Gallons
0	25
3	20
6	15



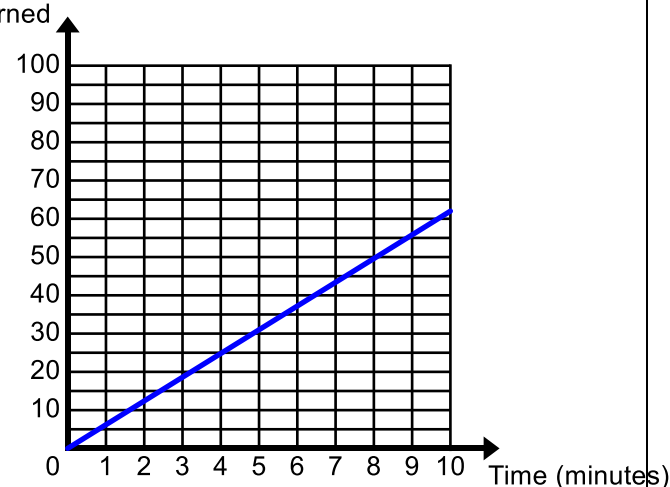
Bathtub C:



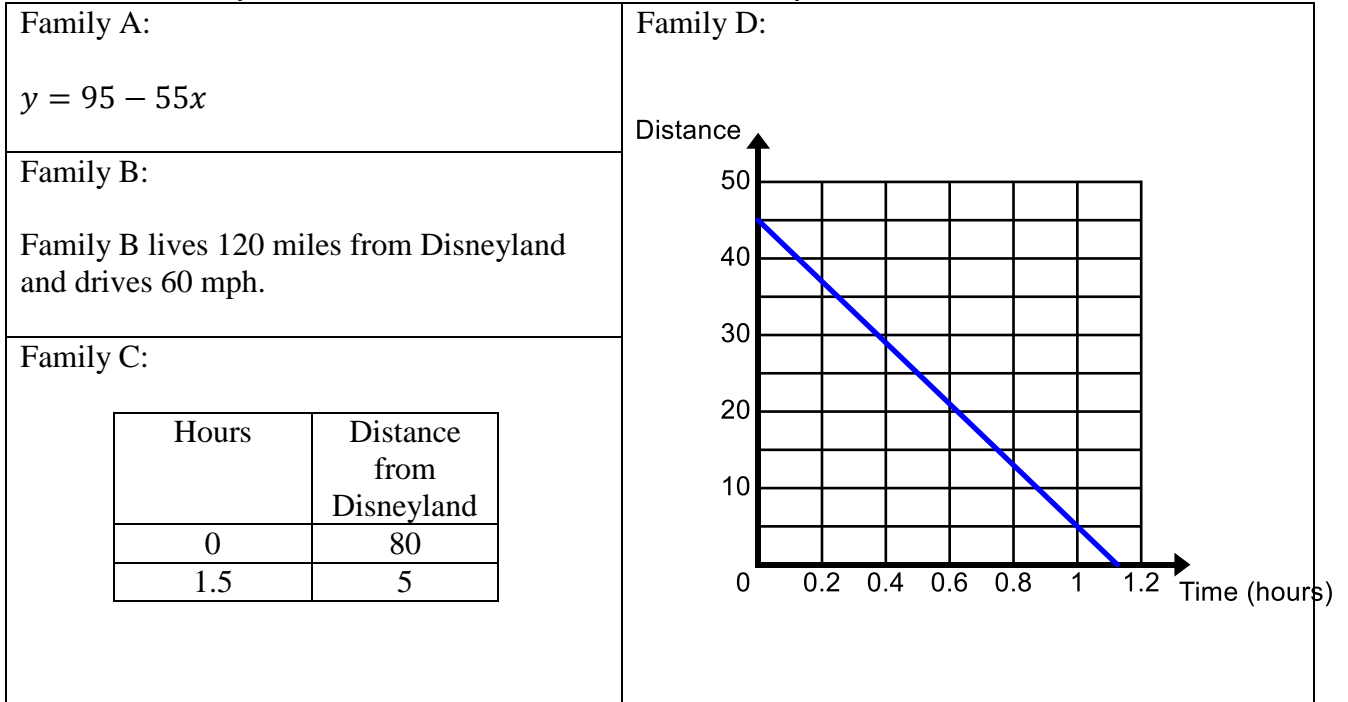
5. Put the cars in order from fastest to slowest. (Note variables: x = time in hours, y = miles traveled). Assume all cars travel at a constant rate.

<p>Car A:</p> $y = 65x$	<p>Car D:</p> 					
<p>Car B:</p> <p>The Bakers completed the 64-mile trip to Salt Lake City in 50 minutes.</p>						
<p>Car C:</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Hours</th> <th style="padding: 5px;">Miles</th> </tr> </thead> <tbody> <tr> <td style="text-align: center; padding: 5px;">2</td> <td style="text-align: center; padding: 5px;">150</td> </tr> <tr> <td style="text-align: center; padding: 5px;">4</td> <td style="text-align: center; padding: 5px;">300</td> </tr> </tbody> </table>		Hours	Miles	2	150	4
Hours	Miles					
2	150					
4	300					

6. Put the exercises below in order from burns the most calories to burns the least calories. (Note variables: x = time in minutes, y = calories burned). Assume the rate at which you burn calories in each of the exercises is constant.

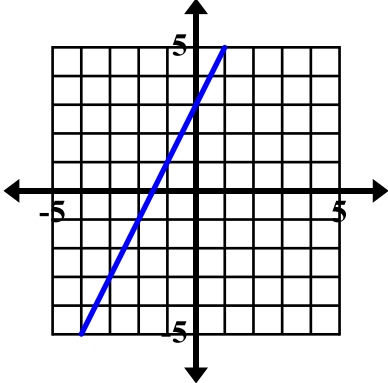
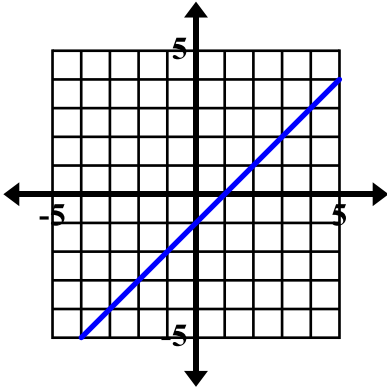
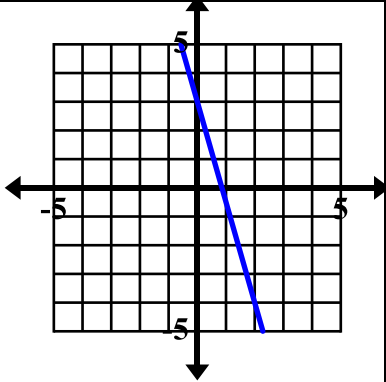
<p>Cycling:</p> $y = 10.8x$	<p>Swimming:</p> 							
<p>Yoga:</p> <p>Meredith burned 429 calories in a 1-hour long yoga class.</p>								
<p>Running:</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Minutes</th> <th style="padding: 5px;">Calories Burned</th> </tr> </thead> <tbody> <tr> <td style="text-align: center; padding: 5px;">10</td> <td style="text-align: center; padding: 5px;">137</td> </tr> <tr> <td style="text-align: center; padding: 5px;">15</td> <td style="text-align: center; padding: 5px;">205.5</td> </tr> <tr> <td style="text-align: center; padding: 5px;">25</td> <td style="text-align: center; padding: 5px;">342.5</td> </tr> </tbody> </table>		Minutes	Calories Burned	10	137	15	205.5	25
Minutes	Calories Burned							
10	137							
15	205.5							
25	342.5							

7. Four families are meeting up in Disneyland. Each family starts driving from home. The representations below show the distance each family is from Disneyland over time. (Note variables: x = time in hours, y = distance from Disneyland.) Assume the families drive to Disneyland at a constant rate.



- a. Which family lives the closest to Disneyland?
- b. Which family lives the farthest from Disneyland?
- c. Which family is traveling at the fastest speed?
- d. Which family is traveling at the slowest speed?
- e. Who will get to Disneyland first?
- f. Who will get to Disneyland last?

Directions: For each problem, circle the representation with the greatest rate of change. Put a star by the representation with the greatest y-intercept. Assume all representations have a constant rate of change.

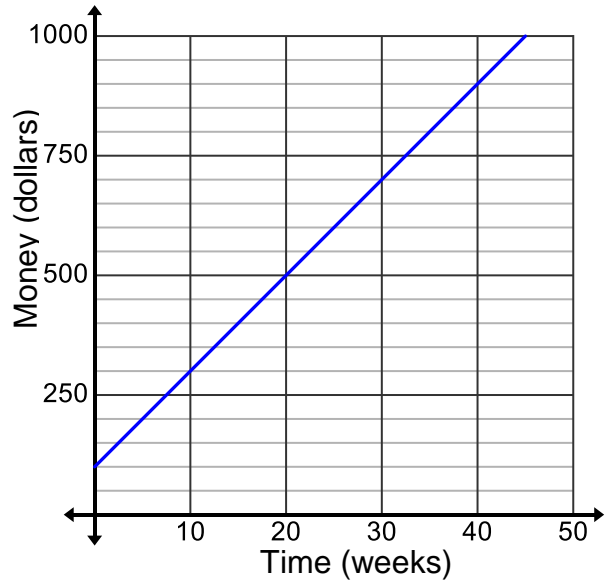
<p>8. $y = 2 + 3.5x$</p>	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>8</td> </tr> <tr> <td>5</td> <td>20</td> </tr> <tr> <td>7</td> <td>26</td> </tr> </tbody> </table>	x	y	1	8	5	20	7	26	
x	y									
1	8									
5	20									
7	26									
<p>9. $y = \frac{3}{2}x$</p>		<p>(0,1)(1, 2.2)</p>								
<p>10.</p> 	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>4</td> </tr> <tr> <td>2</td> <td>-2</td> </tr> <tr> <td>5</td> <td>-11</td> </tr> </tbody> </table>	x	y	0	4	2	-2	5	-11	<p>(0, 3)(2, -5)</p>
x	y									
0	4									
2	-2									
5	-11									

5.3b Homework: Comparing Linear Functions

1. Who will have \$1,000 first, Becky or Olga?

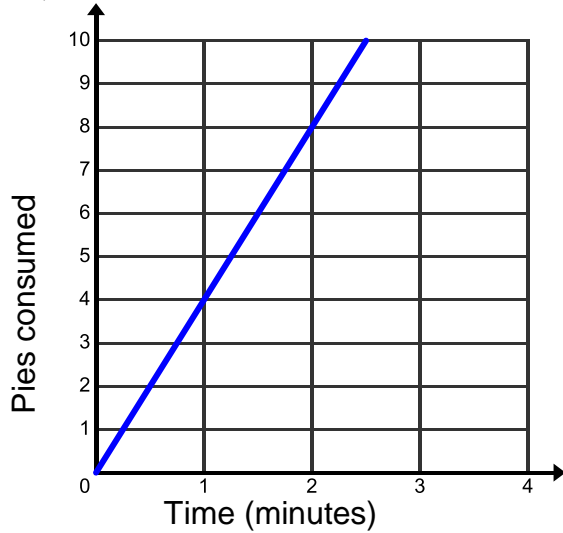
Becky has \$100 and is saving \$10 every week.

Olga's information is shown on the graph below.



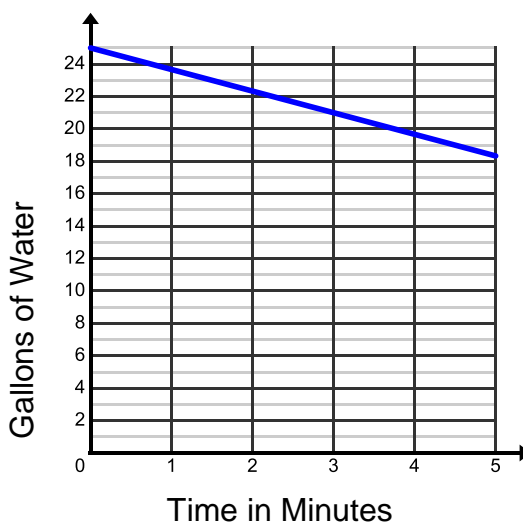
2. Assume the rates below will remain constant. Who will win the pie eating contest? Why?

Joe, whose information is shown below.

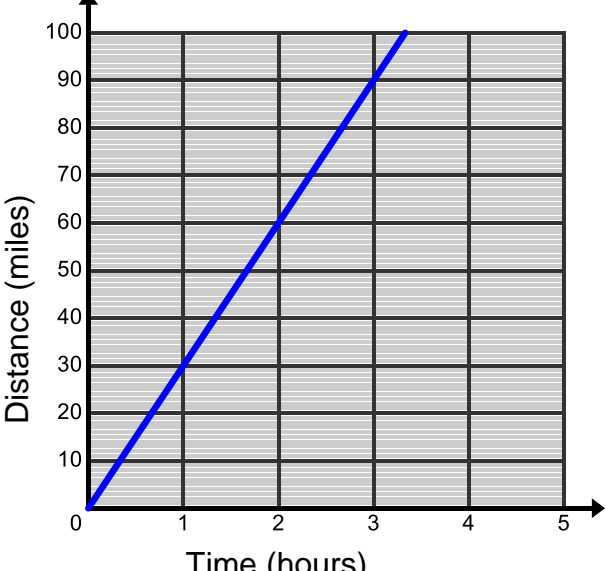


Donna, who has eaten 11 pies in 2.5 minutes.

3. Based on the information below, which hot water heater will use up the available hot water first?

<p>Water Heater A: Starts with 50 gallons and drains 1.5 gallons a minute.</p>	<p>Water Heater C:</p> 								
<p>Water Heater B:</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Time in Minutes</th> <th style="padding: 5px;">Gallons</th> </tr> </thead> <tbody> <tr> <td style="text-align: center; padding: 5px;">0</td> <td style="text-align: center; padding: 5px;">30</td> </tr> <tr> <td style="text-align: center; padding: 5px;">2</td> <td style="text-align: center; padding: 5px;">27.5</td> </tr> <tr> <td style="text-align: center; padding: 5px;">4</td> <td style="text-align: center; padding: 5px;">25</td> </tr> </tbody> </table>	Time in Minutes	Gallons	0	30	2	27.5	4	25	
Time in Minutes	Gallons								
0	30								
2	27.5								
4	25								

4. Put the cars in order from fastest to slowest. (Note variables: x = time in hours, y = miles traveled.)

<p>Car A: $y = 35x$</p>	<p>Car D:</p> 								
<p>Car B: The Andersons completed the 24 mile trip to Salt Lake City in 30 minutes.</p>									
<p>Car C:</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Time (hours)</th> <th style="padding: 5px;">Distance (miles)</th> </tr> </thead> <tbody> <tr> <td style="text-align: center; padding: 5px;">0.5</td> <td style="text-align: center; padding: 5px;">18</td> </tr> <tr> <td style="text-align: center; padding: 5px;">2.5</td> <td style="text-align: center; padding: 5px;">90</td> </tr> <tr> <td style="text-align: center; padding: 5px;">4</td> <td style="text-align: center; padding: 5px;">144</td> </tr> </tbody> </table>	Time (hours)	Distance (miles)	0.5	18	2.5	90	4	144	
Time (hours)	Distance (miles)								
0.5	18								
2.5	90								
4	144								

5. Put the runners in order from slowest to fastest. (Note variables: x = time in minutes, y = miles traveled.)

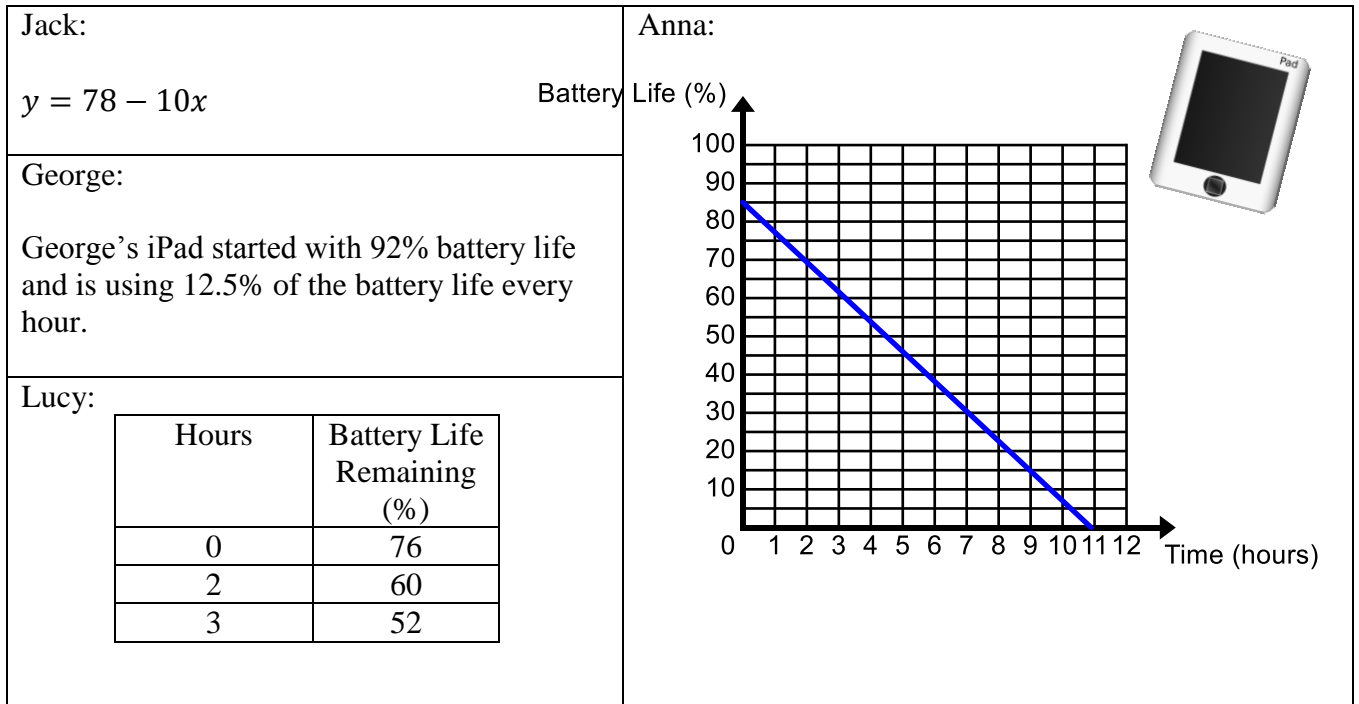
Ellen ran 1 mile in the last 10 minutes	Samantha: $y = \frac{2}{13}x$								
<p>Jason:</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Time (minutes)</th> <th style="padding: 5px;">Distance (miles)</th> </tr> </thead> <tbody> <tr> <td style="text-align: center; padding: 5px;">3</td> <td style="text-align: center; padding: 5px;">0.5</td> </tr> <tr> <td style="text-align: center; padding: 5px;">6</td> <td style="text-align: center; padding: 5px;">1</td> </tr> <tr> <td style="text-align: center; padding: 5px;">9</td> <td style="text-align: center; padding: 5px;">1.5</td> </tr> </tbody> </table>	Time (minutes)	Distance (miles)	3	0.5	6	1	9	1.5	<p>Dale:</p>
Time (minutes)	Distance (miles)								
3	0.5								
6	1								
9	1.5								

6. Use the representations below to answer the questions that follow. (Note variables: x = time in weeks, y = amount of money remaining.)

Imiko starts with \$60 and spends \$2 per week	Henry: $y = 80 - 5x$								
<p>Garek:</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Time (weeks)</th> <th style="padding: 5px;">Amount of Money Remaining</th> </tr> </thead> <tbody> <tr> <td style="text-align: center; padding: 5px;">2</td> <td style="text-align: center; padding: 5px;">59</td> </tr> <tr> <td style="text-align: center; padding: 5px;">5</td> <td style="text-align: center; padding: 5px;">35</td> </tr> <tr> <td style="text-align: center; padding: 5px;">7</td> <td style="text-align: center; padding: 5px;">19</td> </tr> </tbody> </table>	Time (weeks)	Amount of Money Remaining	2	59	5	35	7	19	<p>Roslyn:</p>
Time (weeks)	Amount of Money Remaining								
2	59								
5	35								
7	19								

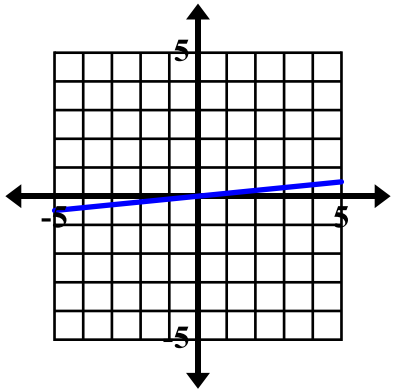
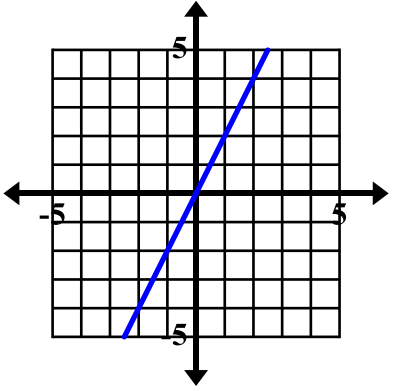
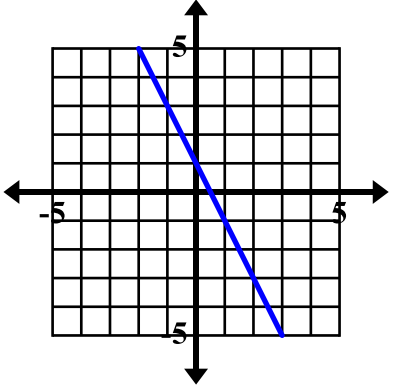
- a. Who starts with the most money?
- b. Who is spending his/her money at the fastest rate?
- c. Who will run out of money first at the current rate of spending?

7. Jack, George, Lucy, and Anna are playing games on their iPads. The representations below show the battery life remaining on each child's iPad over time. (Note variables: x = time in hours, y = battery life remaining as a percent.) Use these representations to answer the questions that follow.



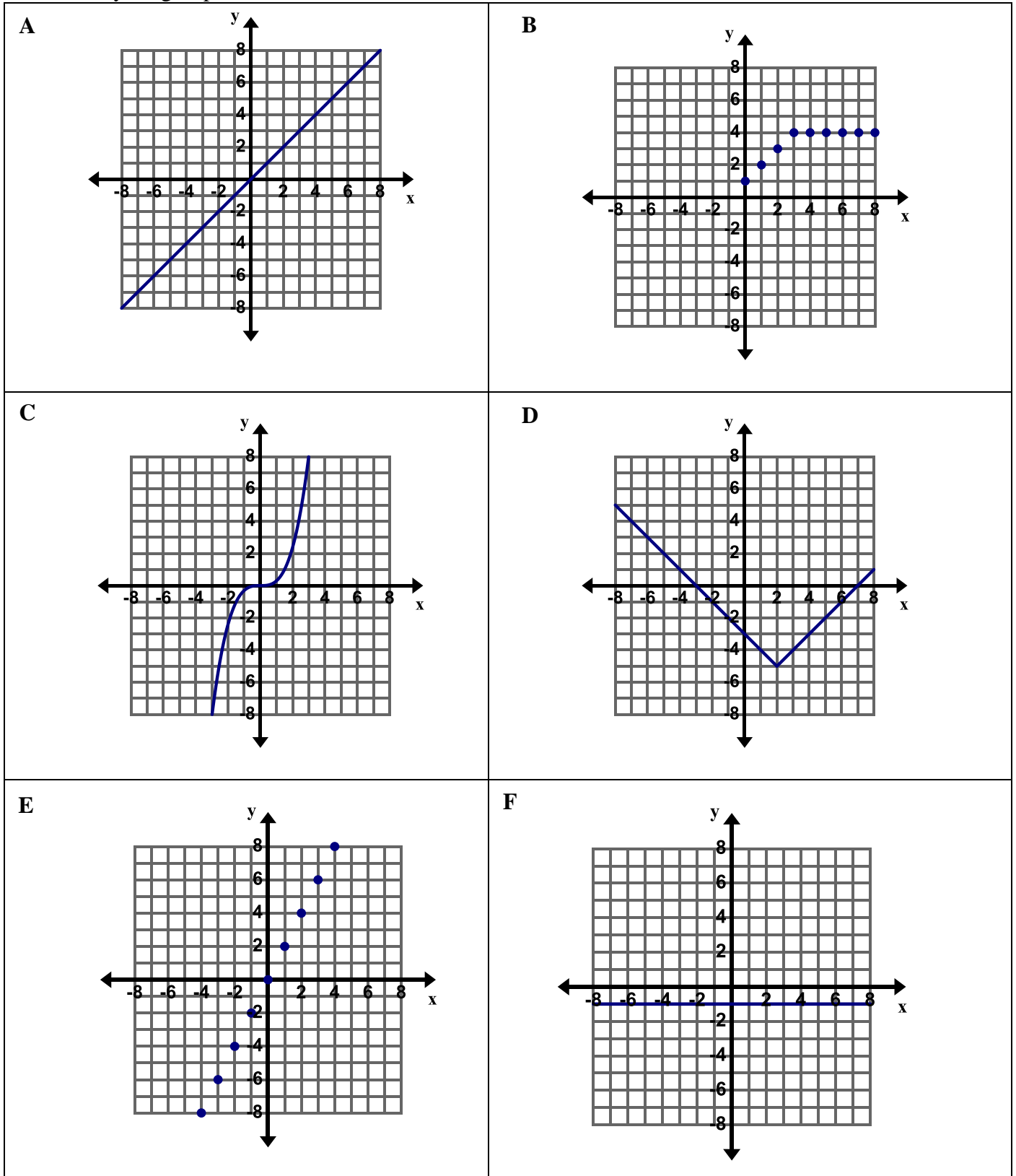
- a. Whose iPad had the most battery life when the kids started playing?
- b. Whose iPad is using the battery at the fastest rate? At the slowest rate?
- c. Who will run out of battery life first?
- d. Whose will be able to play their iPad for the longest amount of time?

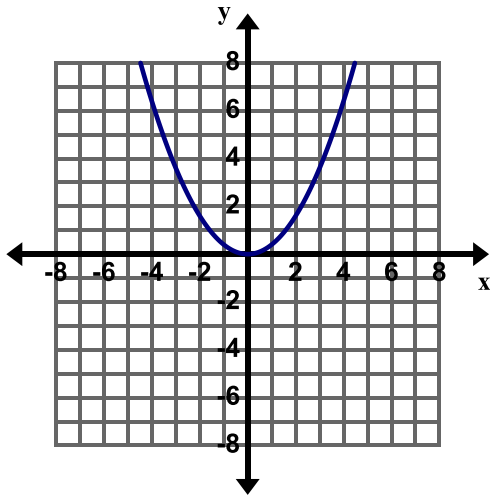
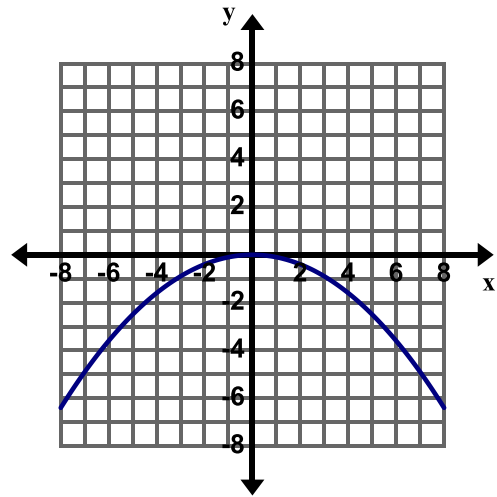
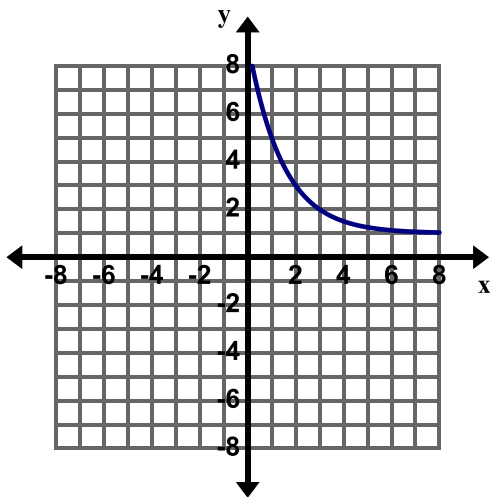
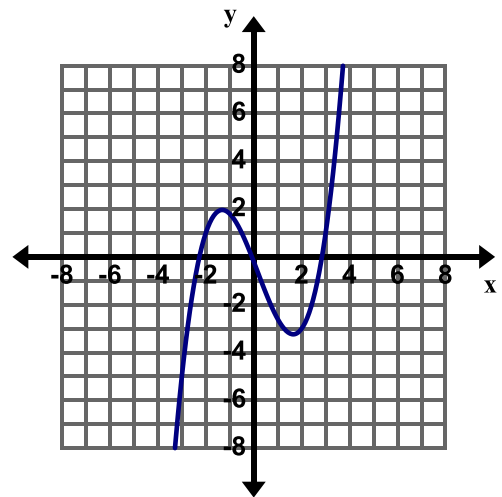
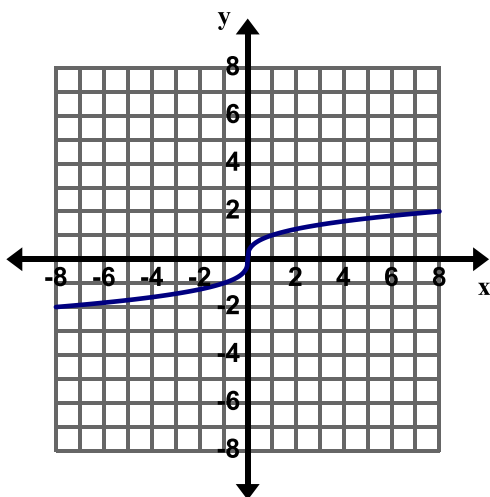
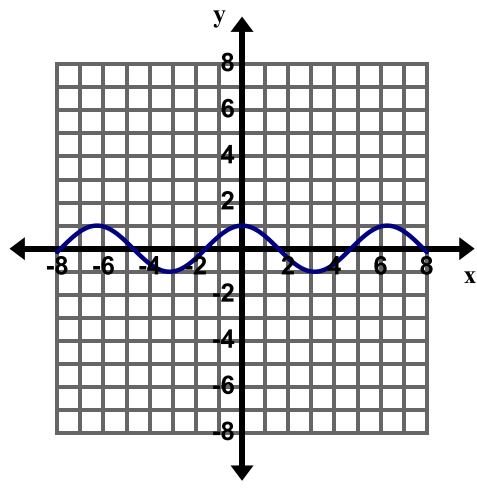
Directions: For each problem, circle the representation with the greatest rate of change. Put a star by the representation with the greatest y-intercept. Assume all representations have a constant rate of change.

<p>8. $y = x$</p>	<table border="1" data-bbox="667 218 954 438"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1.5</td> </tr> <tr> <td>2</td> <td>2</td> </tr> <tr> <td>3</td> <td>2.5</td> </tr> </tbody> </table>	x	y	1	1.5	2	2	3	2.5	
x	y									
1	1.5									
2	2									
3	2.5									
<p>9. $y = \frac{7}{4}x + 2$</p>	<p>(1,1.5)(2, 3)</p>									
<p>10. $y = -0.5x$</p>	<table border="1" data-bbox="667 1064 954 1285"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> </tr> <tr> <td>2</td> <td>-2</td> </tr> <tr> <td>5</td> <td>-5</td> </tr> </tbody> </table>	x	y	0	0	2	-2	5	-5	
x	y									
0	0									
2	-2									
5	-5									

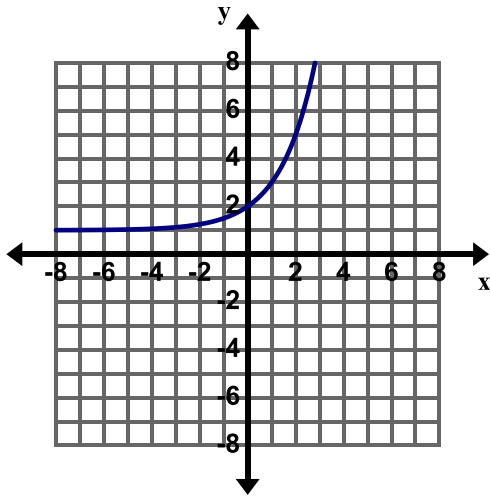
5.3c Class Activity: Features of Graphs

1. Cut out each graph. Sort the graphs into groups and be able to explain why you grouped the graphs the way you did. In the table that follows, name your groups, describe your groups, and list the graphs that are in your group.

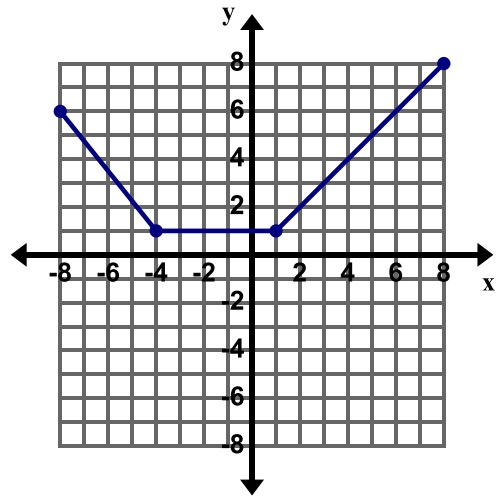


G**H****I****J****K****L**

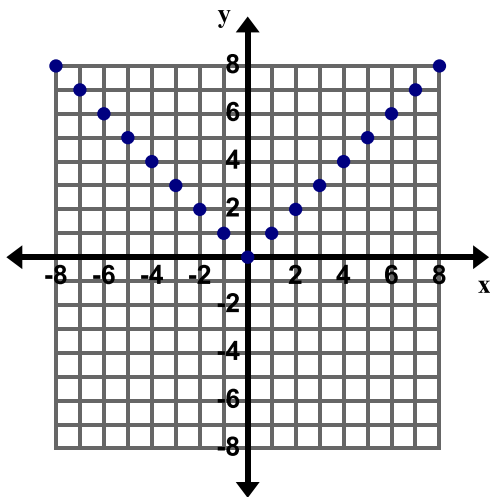
M



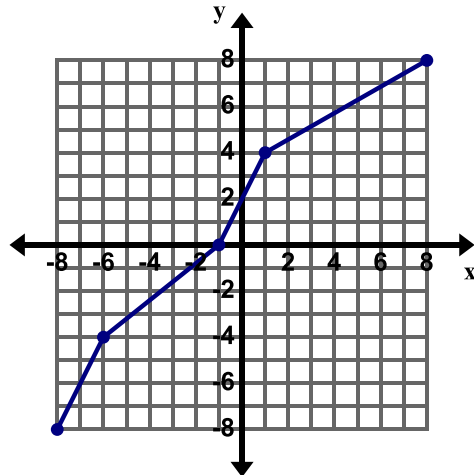
N



O

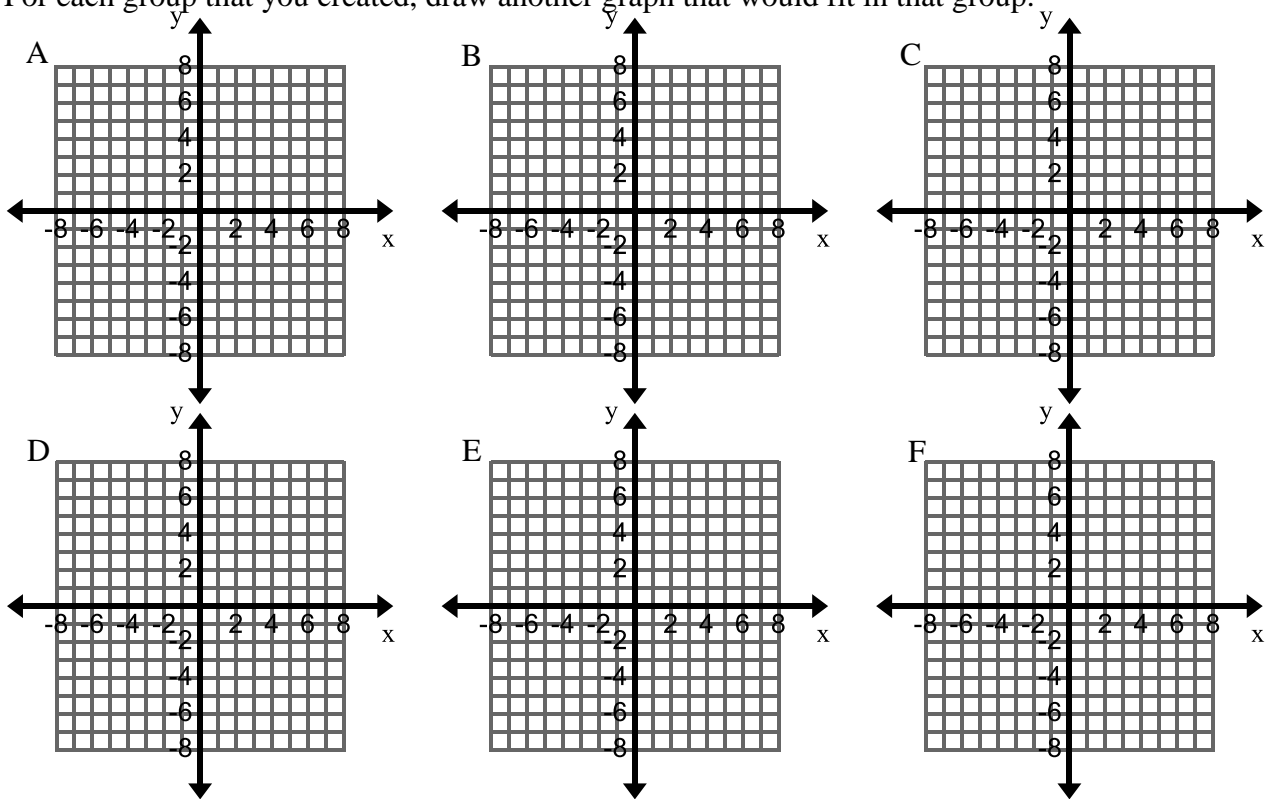


P



Name of the group	Description of the group	Graphs in the group
A.		
B.		
C.		
D.		
E.		
F.		

2. For each group that you created, draw another graph that would fit in that group.

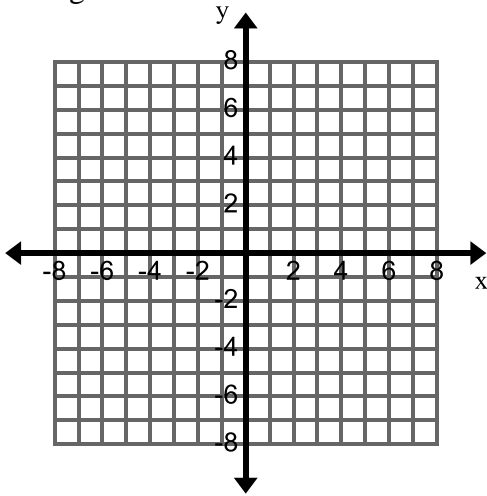


3. Lucy grouped hers as follows:

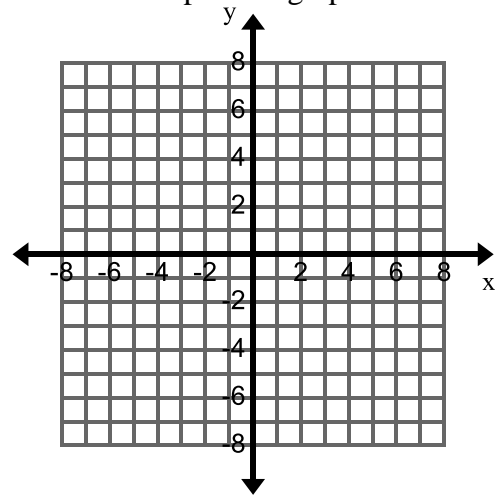
Increasing on the entire graph: A, C, E, K, M, P
Decreasing on the entire graph: I
Constant on the entire graph: F
Increasing on some parts of the graph, decreasing on some parts of the graph: D, G, H, J, L, O
Increasing on some parts of the graph, decreasing on some parts of the graph, constant on some parts of the graph: N
Increasing on some parts of the graph, constant on some parts of the graph: B

4. Define **increasing**, **decreasing**, and **constant** in your own words.

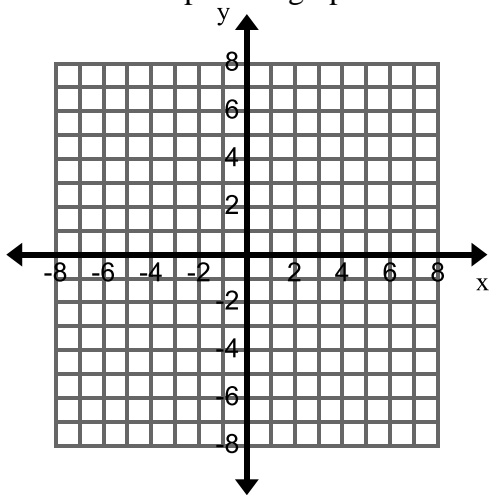
5. Draw an example of a graph that is increasing.



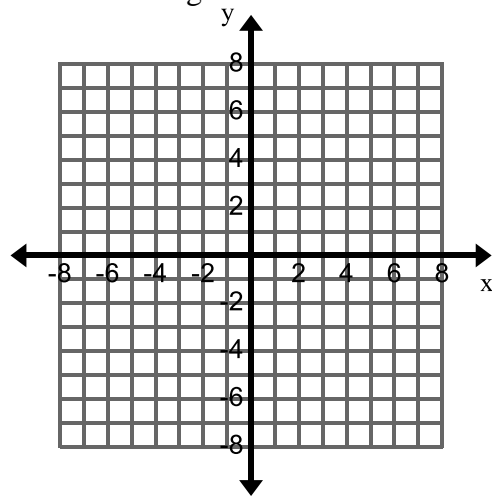
6. Draw an example of a graph that is decreasing.



7. Draw an example of a graph that is constant.



8. Draw an example of a graph that is increasing then decreasing.



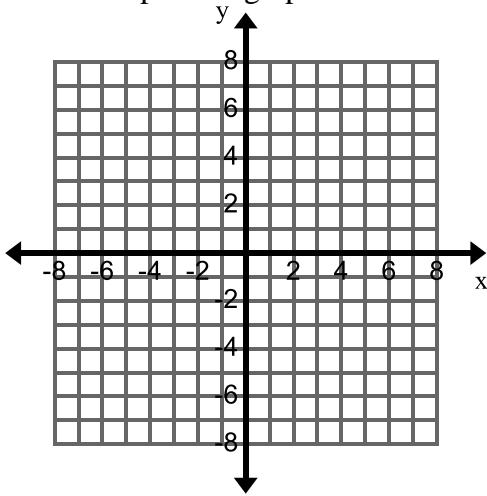
9. Ellis grouped hers as follows:

Discrete: **B, E, O**

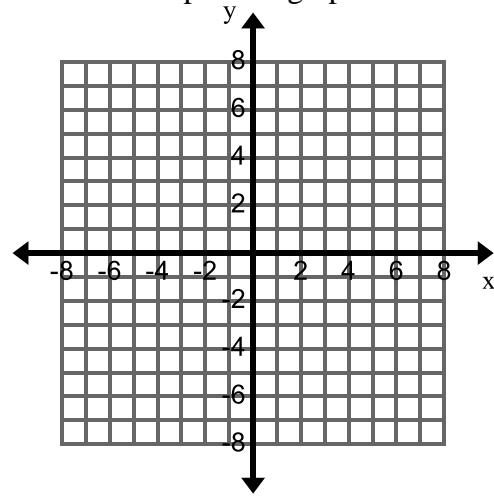
Continuous: **A, C, D, F, G, H, I, J, K, L, M, N, P**

10. Define **discrete** and **continuous** in your own words. Can you think of a real world situation that has a discrete graph? Why doesn't it make sense to connect the points in this situation?

11. Draw an example of a graph that is discrete.



12. Draw an example of a graph that is continuous.



13. Grace grouped hers as follows:

Linear: A, E, F
Nonlinear: C, G, H, I, J, K, L, M
Made up of pieces of different linear functions: B, D, N, O, P

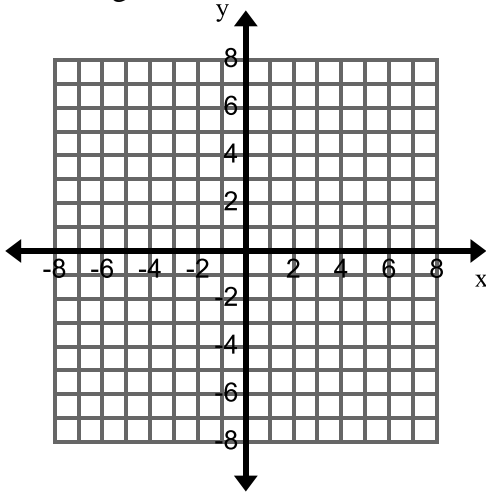
14. Define **linear** in your own words.

15. Define **nonlinear** in your own words.

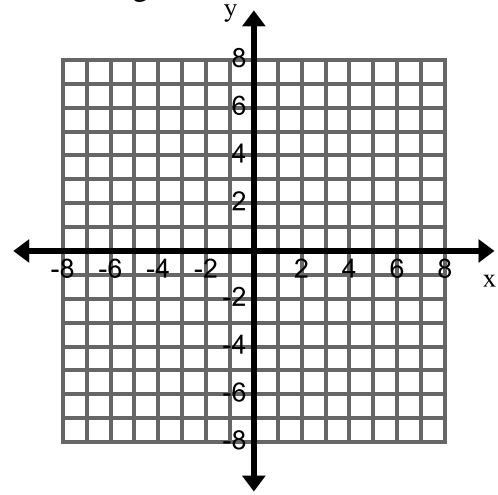
5.3c Homework: Features of Graphs

Directions: Draw a graph with the following features.

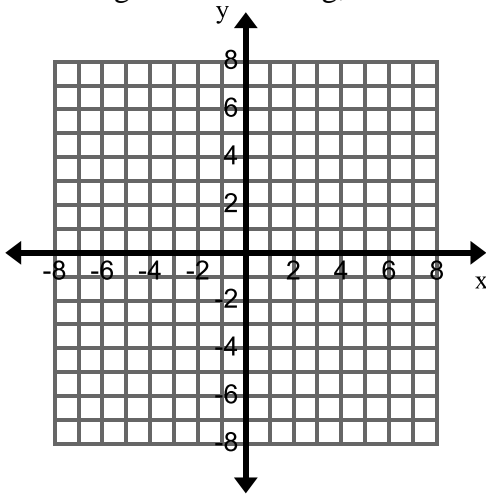
1. increasing, linear, continuous



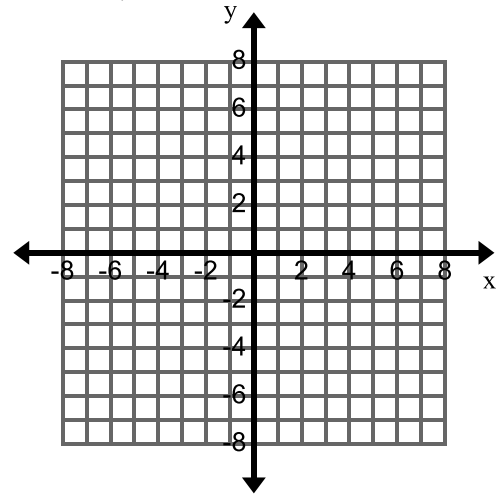
2. Decreasing, linear, and discrete



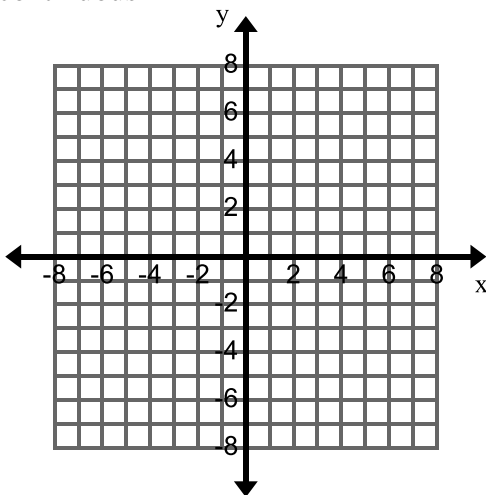
3. Increasing then decreasing, continuous



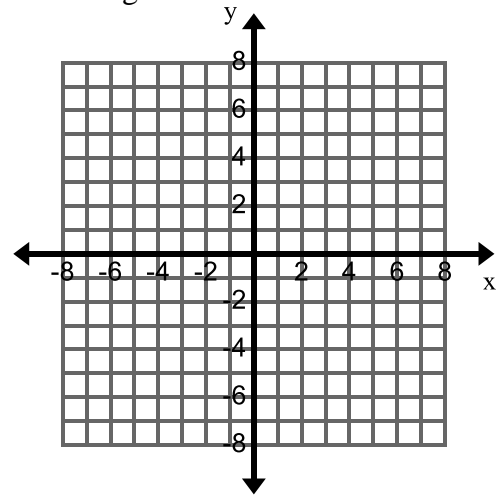
4. Constant, discrete



5. Decreasing, then constant, then increasing, continuous

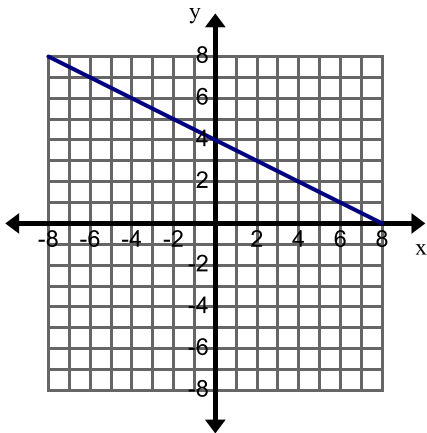


6. Increasing and nonlinear



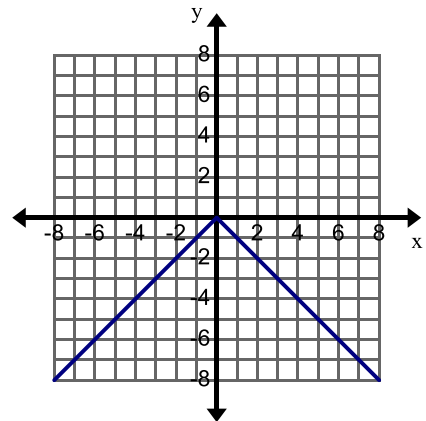
Directions: Describe the features of each of the following graphs (increasing/decreasing/constant; discrete/continuous; linear/nonlinear). Label on the graph where it is increasing, decreasing, or constant. Identify the intercepts of the graph.

7.



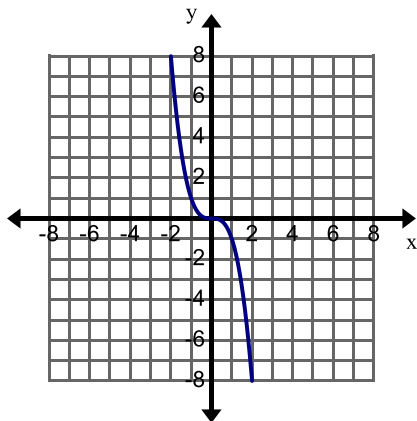
Features:

8.



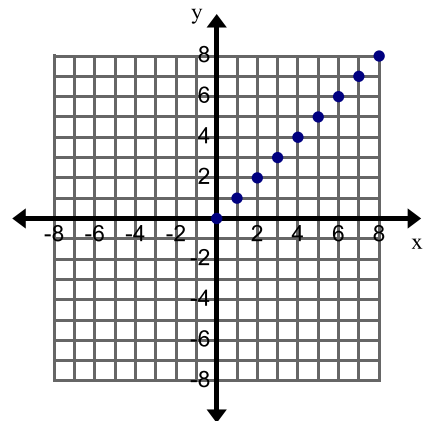
Features:

9.



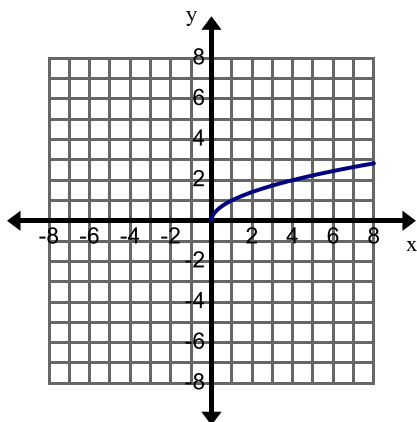
Features:

10.



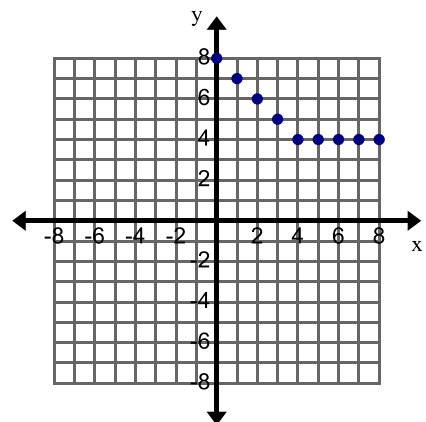
Features:

11.



Features:

12.

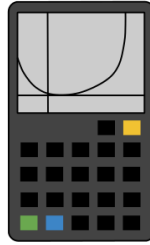


Features:

5.3d Class Activity: CBR Activity

You will be using the *DIST MATCH* application in the *CBR™ Ranger* program on the TI 73 (or other) graphing calculators. Instructions for CBR/calculator use:

- Firmly attach the TI 73 to the CBR Ranger.
- Choose the APPS button on the TI 73.
- Choose 2: CBL/CBR.
- Choose 3: RANGER.
- Choose 3: APPLICATIONS.
- Choose 2: FEET.
- Choose 1: DIST MATCH. Get your first graph onto the calculator screen.

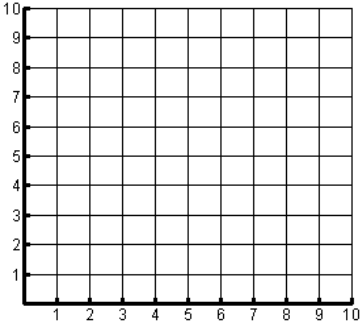
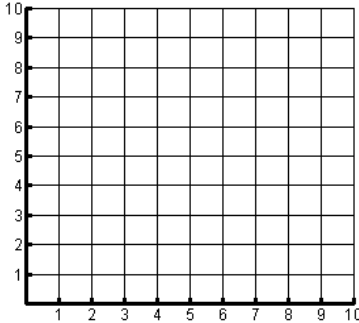
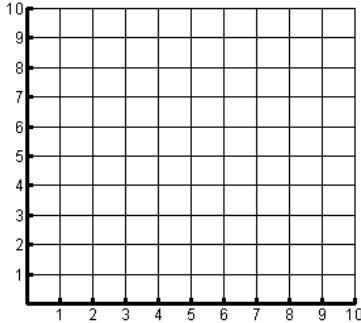


1. Try to match the graph given to you in the program. You will reproduce the graph by walking. Then trace the graph onto the grids below.

Be sure to model a few examples with your class before you begin in teams!

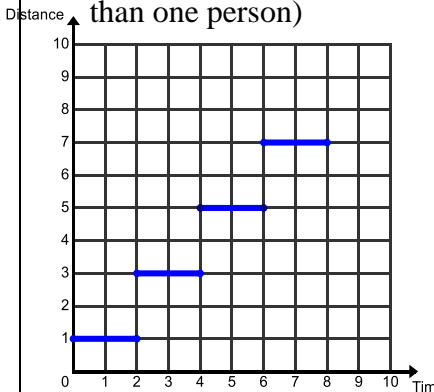
- a. Get a graph to match ready in the calculator.
- b. Decide how far away from the sensor you should stand to begin.
- c. Talk through the walk that will make a graph match. (how far away to begin, walk forward or backward, how fast to move forward or backward, how long to walk forward or backward, when to change directions or speed, etc.)
- d. You may wish to write the story of the graph first (before you walk it)—see below.
- e. Have a member of your group hold the CBR so that the CBR sensor is up and directed toward the person that is walking
- f. Have a group member press start on the calculator. Then walk toward or away from the sensor trying to make your walk match the graph on the calculator screen.
- g. Each member of your group should walk to match at least one graph on the calculator.
- h. Sketch each graph below. Write the story for the graph.

Graph 1:	Graph 2:	Graph 3:
Story:	Story:	Story:

<p>Graph 4:</p> 	<p>Graph 5:</p> 	<p>Graph 6:</p> 
<p>Story:</p>	<p>Story:</p>	<p>Story:</p>

Extra for Experts

If you finish early try to create the following graphs, write a description/story that matches the graph.

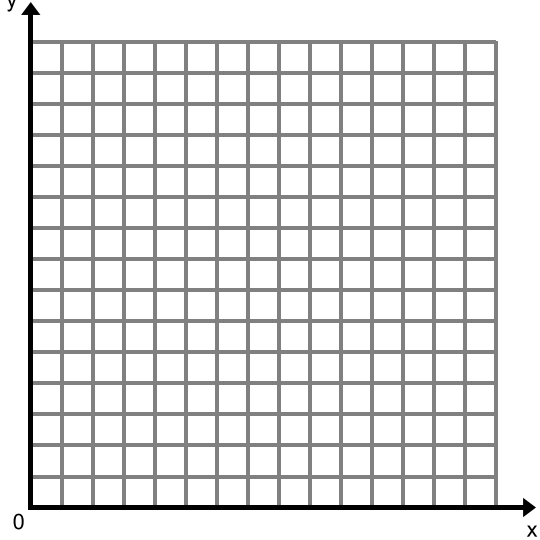
<p>1. A line that rises at a steady rate. Story:</p>	<p>2. A line that falls at a steady rate. Story:</p>	<p>3. A horizontal line Story:</p>
<p>4. A “V” Story:</p>	<p>5. A “U” Story:</p>	<p>6. An “M” Story:</p>
<p>7. Try creating an O. Are you successful? Why or why not?</p>	<p>8. Name a letter you could graph using the CBR. Name a letter you cannot graph using the CBR. Explain your choices.</p>	<p>9. Try creating this graph. (Hint: It will take more than one person)</p> 

5.3d Homework: Stories and Graphs

Directions: Sketch graphs to match the stories.

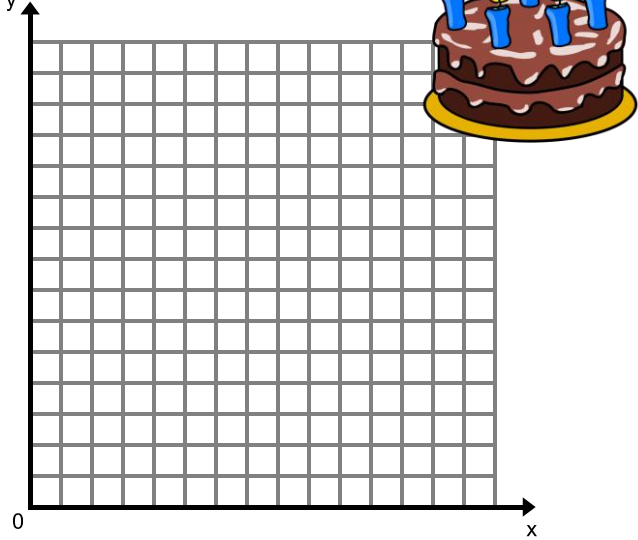
1. Before School

Create a graph to match the story below (distance in feet, time in minutes). (Note: This graph will show distance traveled related to time passing—consider the student to be continually moving forward.)

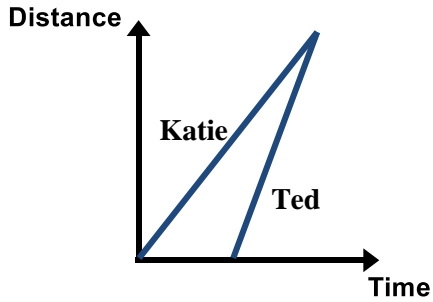
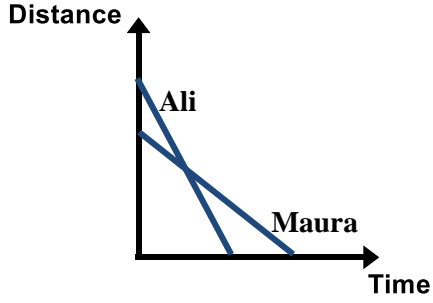
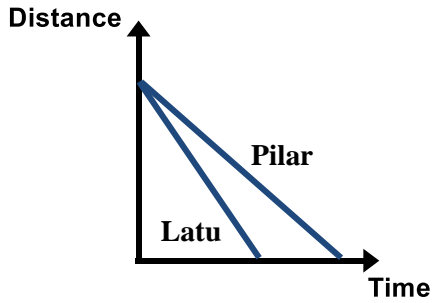
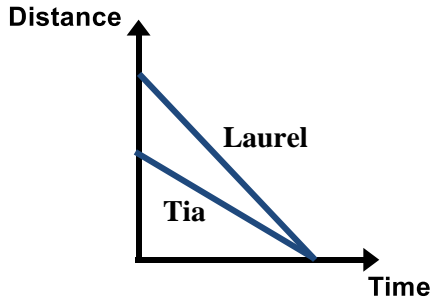
<p>Story: A student walks through the halls before school. He/she begins at the front door, stops to talk to at least three different friends, stops at his/her locker, stops in the office.</p>	<p>Graph</p> 
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2. Birthday Cake

- Write a story about your family eating a birthday cake. You want to talk about amount of cake eaten related to passing time.
- Create the graph to tell the same story. You decide on the labels.

<p>Story:</p>	<p>Graph</p> 
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3. Make up stories to go with the following graphs. In this problem, distance represents the **distance from school**. Include in the stories specific details about starting points and slopes. Answer the additional questions.

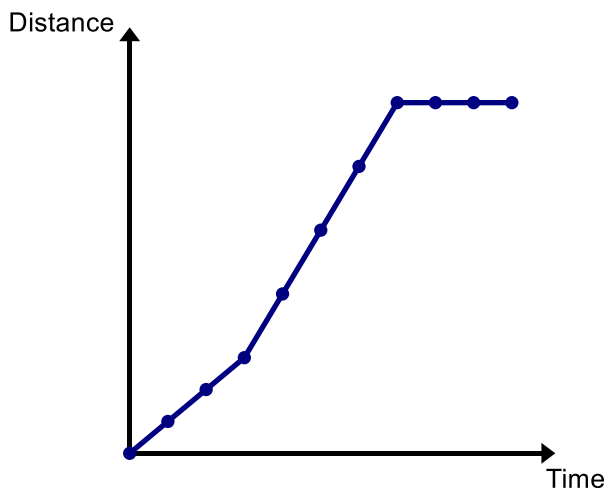
<p>a. Tell the story of this graph.</p> <p>Draw and label a line that represents Izzy who started at the same time as Katie but walked away from school at a faster rate than Katie.</p>	
<p>b. Tell the story of this graph.</p> <p>Draw and label a line that represents Gabi who walks slower than both Ali and Maura.</p>	
<p>c. Tell the story of this graph.</p> <p>Draw and label a line that represents Carmen who starts the same distance from school as Pilar and Latu but gets to school faster than both of them.</p>	
<p>d. Tell the story of this graph.</p> <p>Draw and label a line that represents Colin who lives closer to school, leaves at the same time as Laurel and Tia to walk to school, and arrives at the same time as Laurel and Tia.</p>	

5.3e Class Activity: School's Out

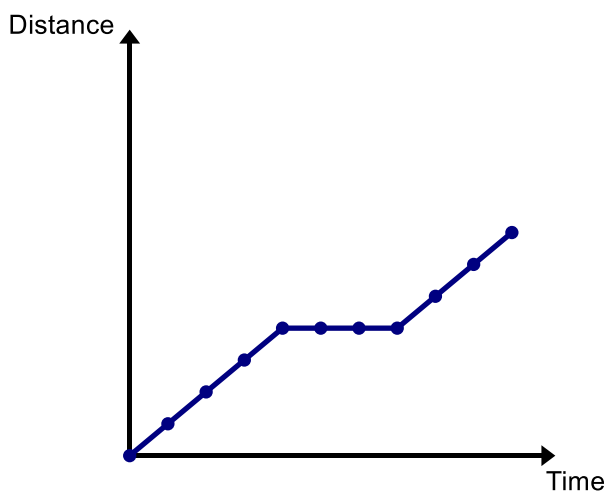
Directions: The following graphs tell the story of five different students leaving school and walking home. Label the key features of the graph. Write a story for each graph describing the movement of each of the

students. 

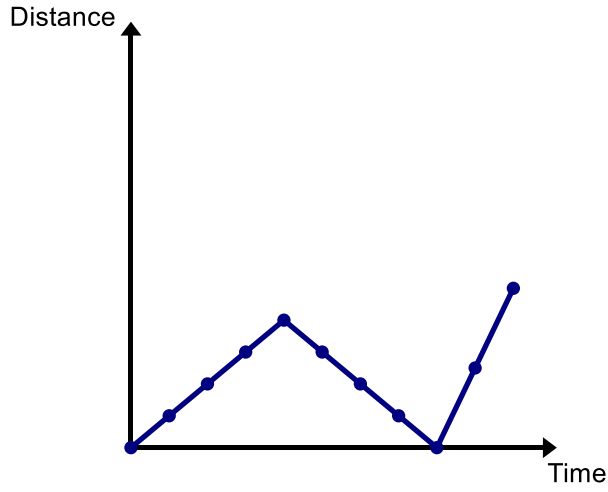
Abby's Journey Home:



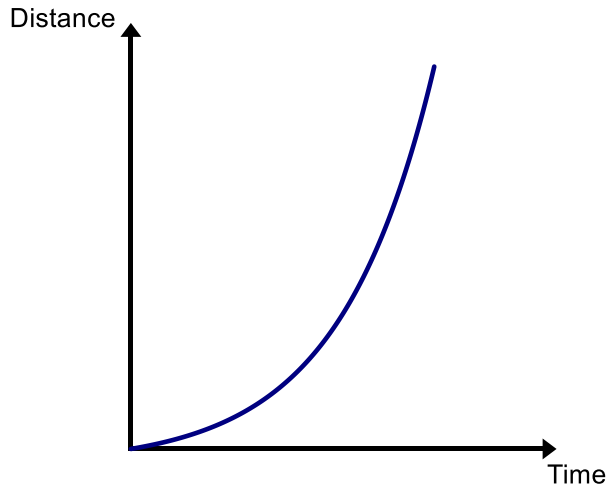
Beth's Journey Home:



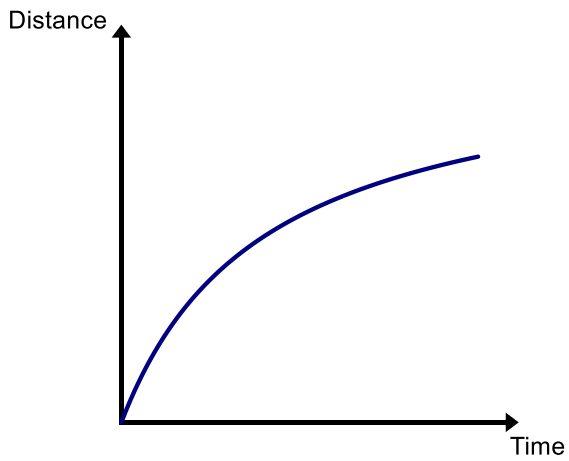
Chad's Journey Home:



Drew's Journey Home:

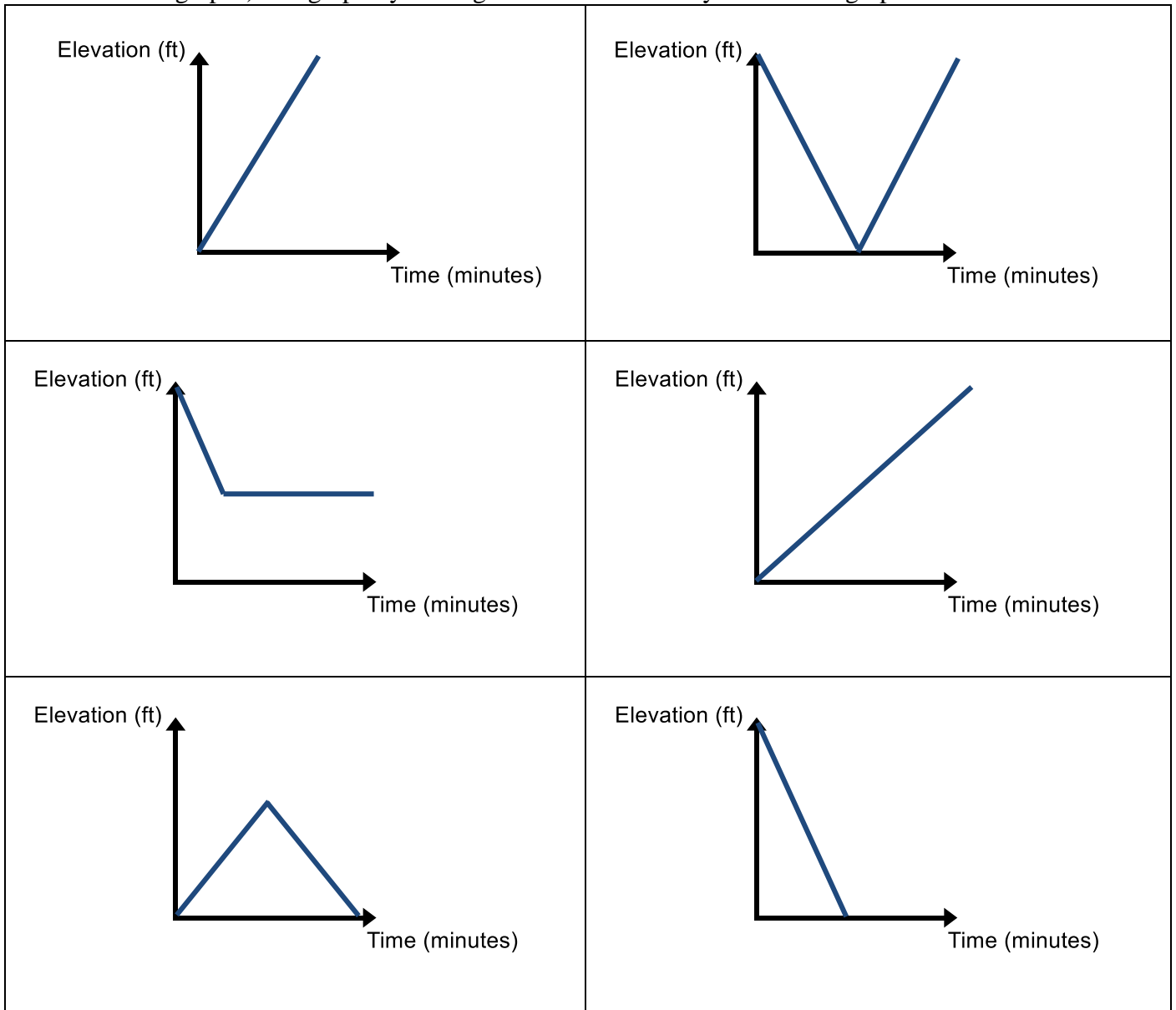


Eduardo's Journey Home:



5.3e Homework: School's Out

1. The graphs below show Estefan's elevation (height above the ground) over time as he is playing around on a flight of stairs. Assume the bottom of the stairs has an elevation of 0 feet. Match each story (shown below the graphs) to a graph by writing the letter of the story under each graph.



Story A: Estefan starts at the bottom of the stairs and walks up the stairs at a constant rate.

Story B: Estefan starts at the bottom of the stairs and sprints up the stairs at a constant rate.

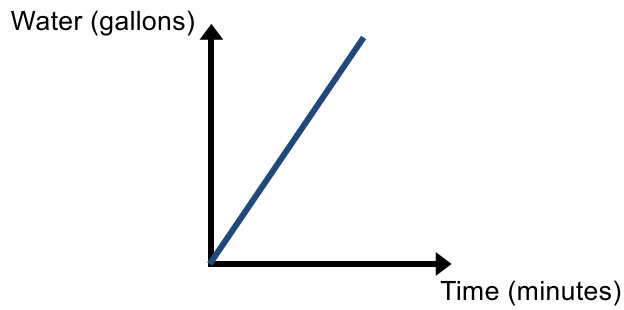
Story C: Estefan starts at the bottom of the stairs, runs half-way up the stairs, turns around and runs back down the stairs.

Story D: Estefan starts at the top of the stairs and sprints down the stairs until he reaches the bottom.

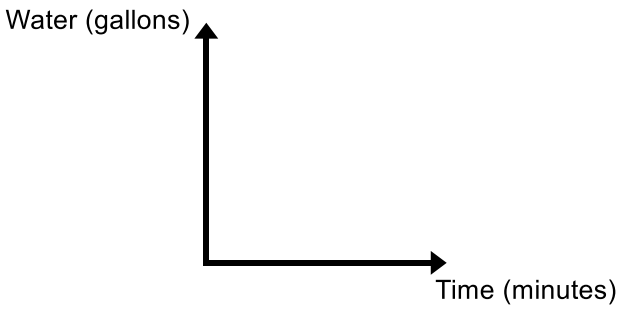
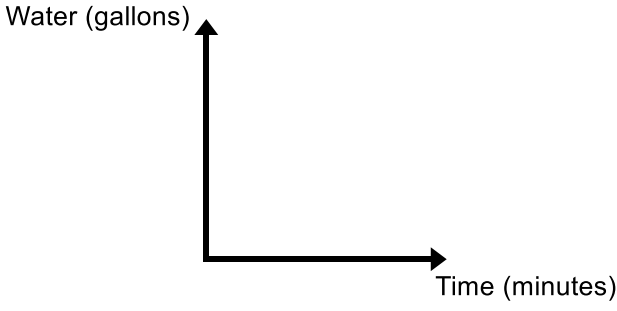
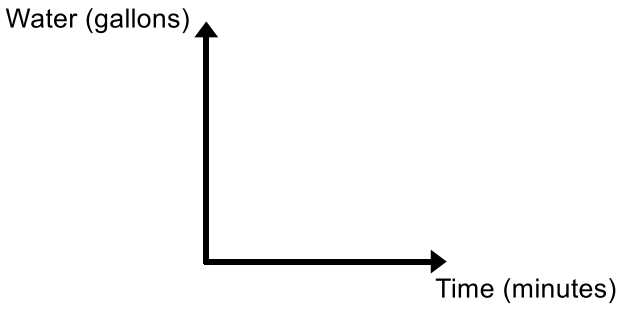
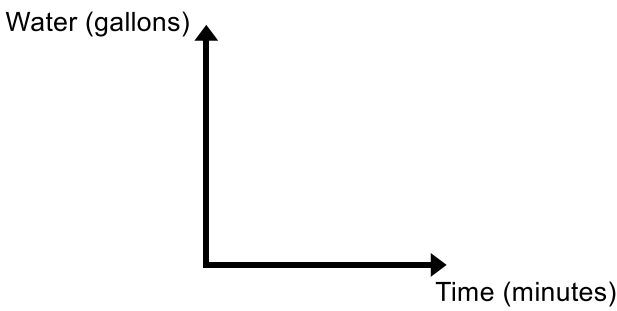
Story E: Estefan starts at the top of the stairs, sprints down the stairs, and stops when he is half-way down the stairs.

Story F: Estefan starts at the top of the stairs, runs down to the bottom, turns around and runs back up to the top of the stairs.

2. The graph below tells the story of Kelii filling up her empty swimming pool with a hose at a constant rate. Create new graphs based on the changes described below.

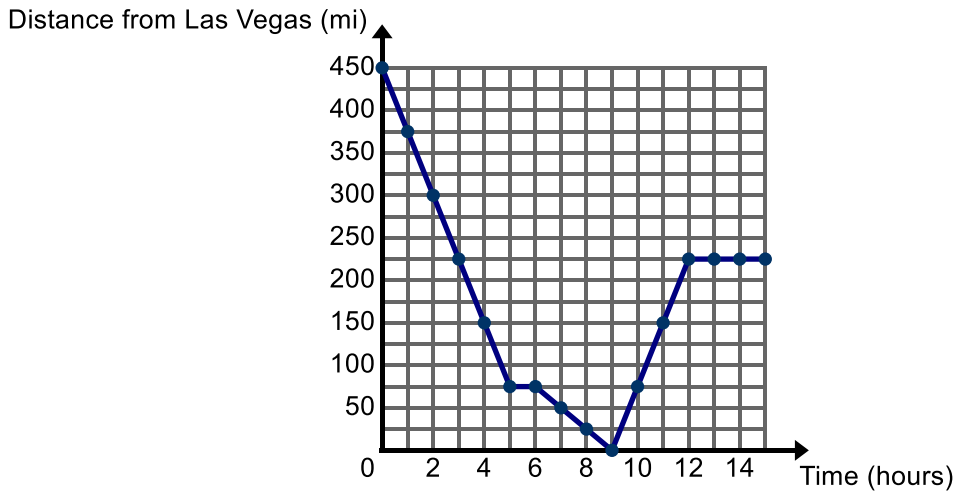


<p>a. After Kelii has been filling the pool for a few minutes, her friend decides to help her and puts a second hose in the pool.</p>	
<p>b. After Kelii has been filling the pool for a few minutes, the hose gets a hole in it so the water is coming out at a slower rate.</p>	
<p>c. The pool started with some water in it.</p>	

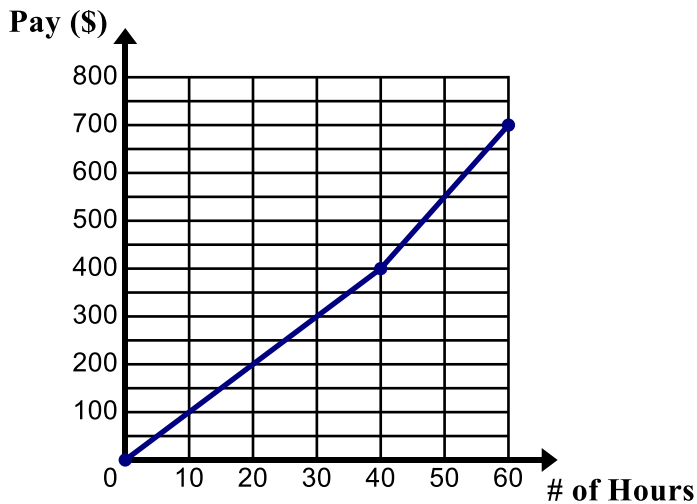
<p>d. Kelii fills the pool up and realizes that she filled it up too much so she drains some of the water out at a constant rate.</p>	
<p>e. Kelii fills up the pool, her brother drops his ice cream into the pool, so she drains the water back out at a constant rate until the pool is empty.</p>	
<p>f. Kelii starts filling the pool, stops to go eat lunch, and then comes back out and starts filling up the pool again.</p>	
<p>g. Kelii fills the pool half-way and decides that is enough.</p>	

5.3f Class Activity: From Graphs to Stories

1. Ben and his family took a road trip to visit their cousins. The graph below shows their journey. Label the key features of the graph.

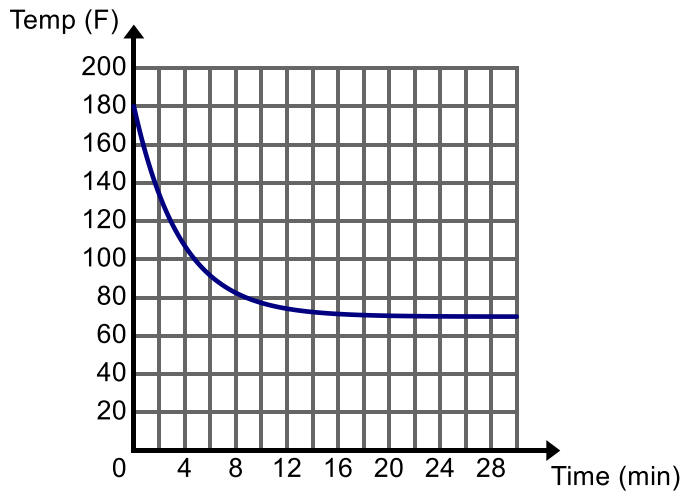


- a. Tell the story of the graph.
2. The graph below shows the amount Sally makes based on how many hours she works in one week. Label the key features of the graph.



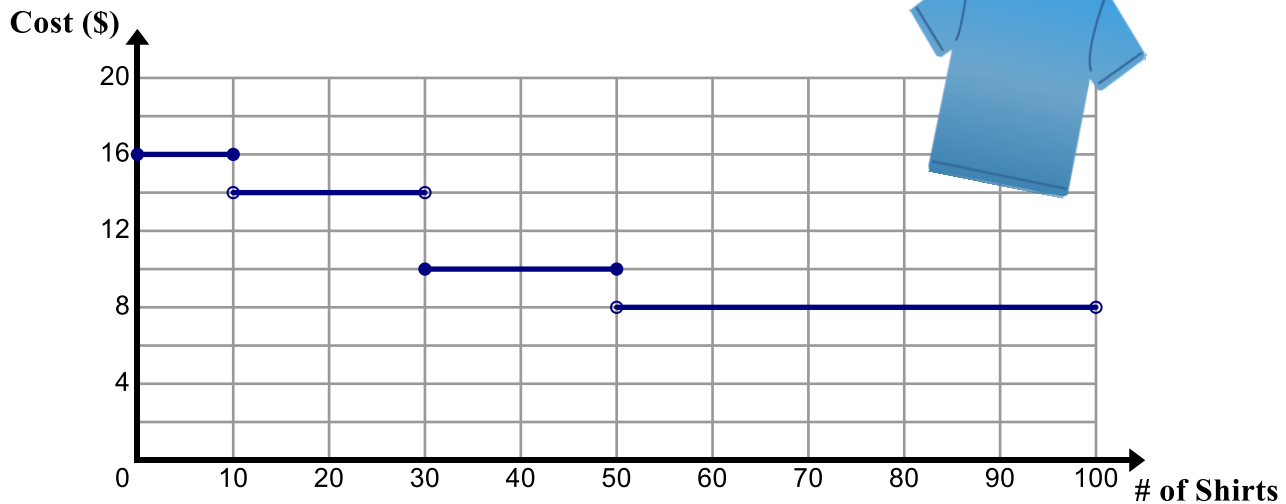
- a. Tell the story of the graph.

3. Cynthia is doing research on how hot coffee is when it is served. The graph below shows the temperature of a coffee (in °F) as a function of time (in minutes) since it was served. Label the key features of the graph.



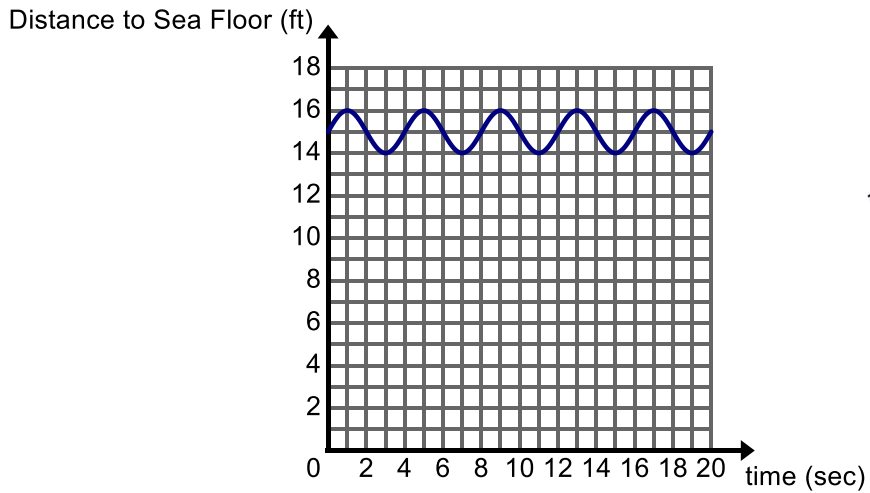
- a. Tell the story of the graph.

4. Jorge is the team captain of his soccer team. He would like to order shirts for the team and is looking into how much it will cost. He called Custom T's to ask about pricing and the manager sent him the following graph.



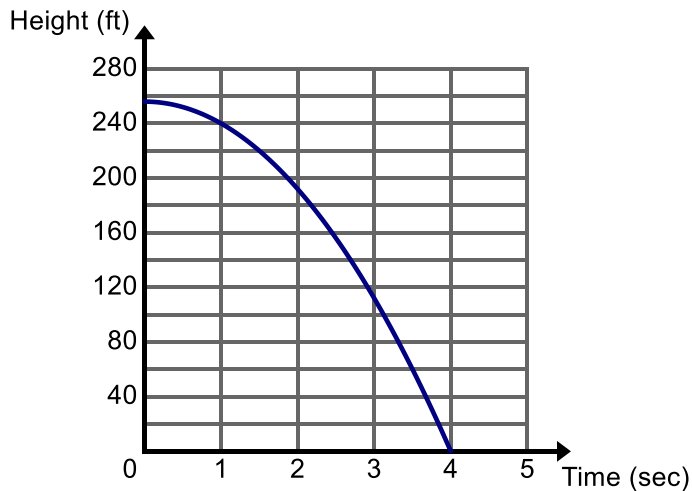
- a. Tell the story of the graph.

5. A boat is anchored near a dock. The graph below shows the distance from the bottom of the boat to the sea floor over a period of time.



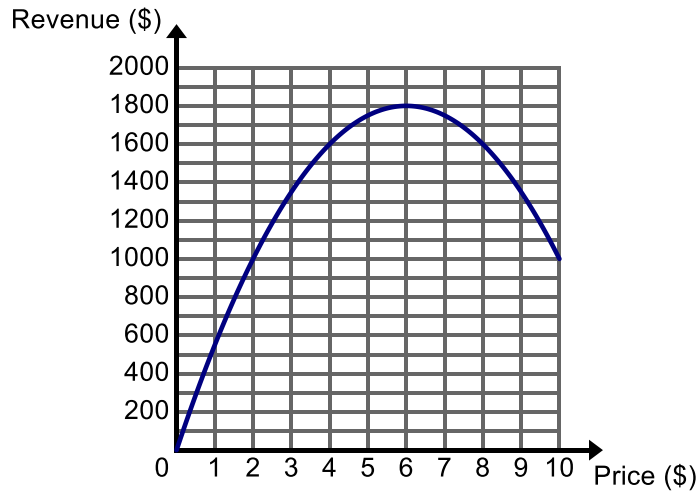
- a. Tell the story of the graph.

6. An object is dropped from a bridge into the water below. The graph below shows the height of the object (in feet) with respect to time (in seconds). Consider the relationship between the height of the object and time.



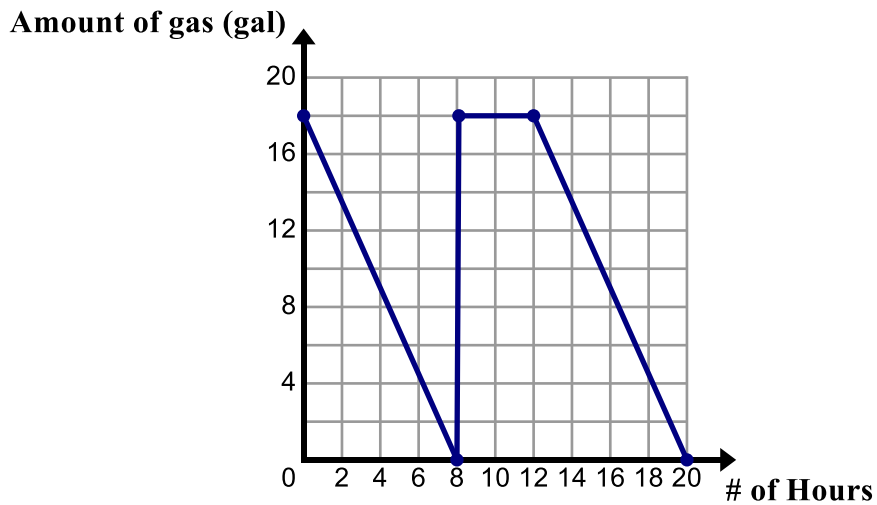
- a. Tell the story of the graph.

7. The graph below shows the amount of revenue a company will make selling t-shirts dependent on the price of each t-shirt.



- a. Tell the story of the graph.

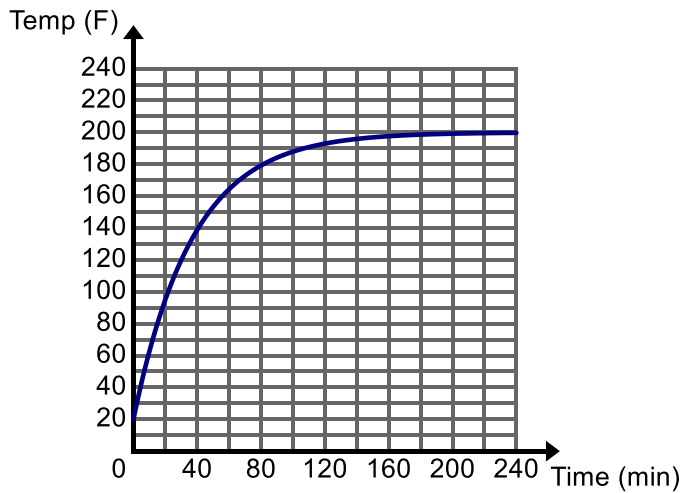
8. The graph below shows the amount of gas remaining in a vehicle over time.



- a. Tell the story of the graph.

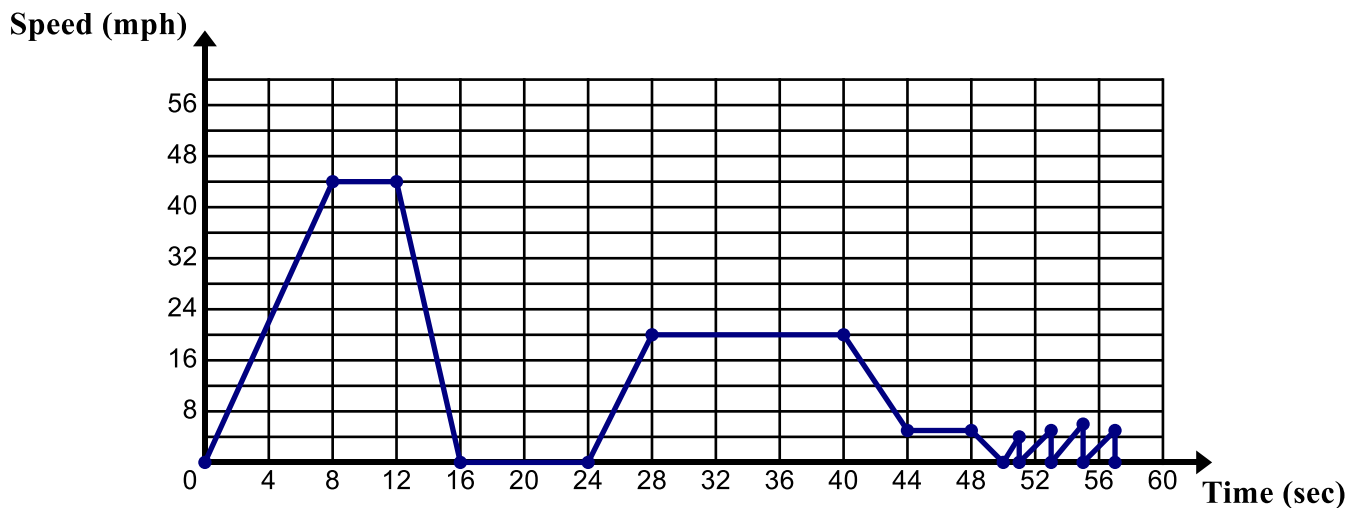
5.3f Homework: From Graphs to Stories

1. Tessa is cooking potatoes for dinner. She puts some potatoes in an oven pre-heated to 200° F. The graph below shows the temperature of the potatoes over time. Label the key features of the graph. The y-intercept of the graph is (0, 20).

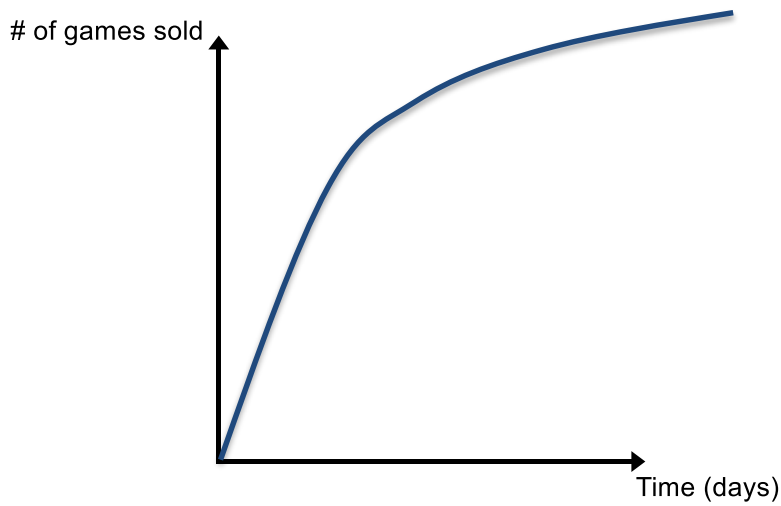


- a. Tell the story of the graph.

2. Steve is driving to work. The graph below shows Steve's speed over time. Label the key features of the graph to tell the story of the speed of Steve's car over time. Use words like accelerating, decelerating, driving at a constant speed, stopped. You can abbreviate these words using the first letter of each word (i.e. A for accelerating, D for decelerating, C for driving at a constant speed, S for stopped). Explain what might be happening at the end of the graph.

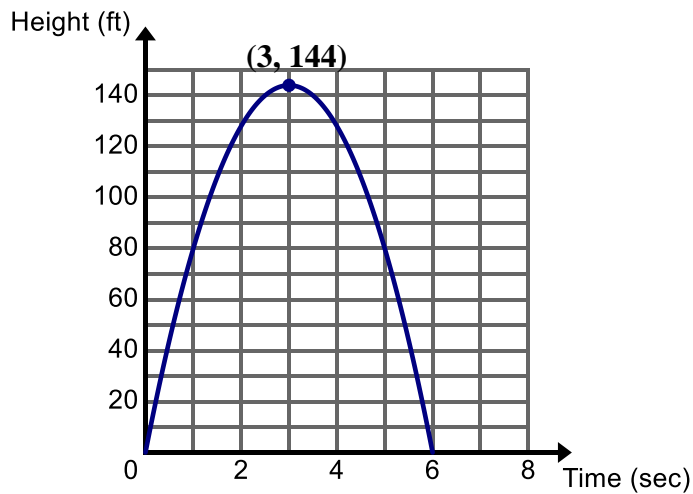


3. Microsoft is releasing the most anticipated new Xbox game of the summer. The graph below shows the total number of games sold as a function of the number of days since the game was released.



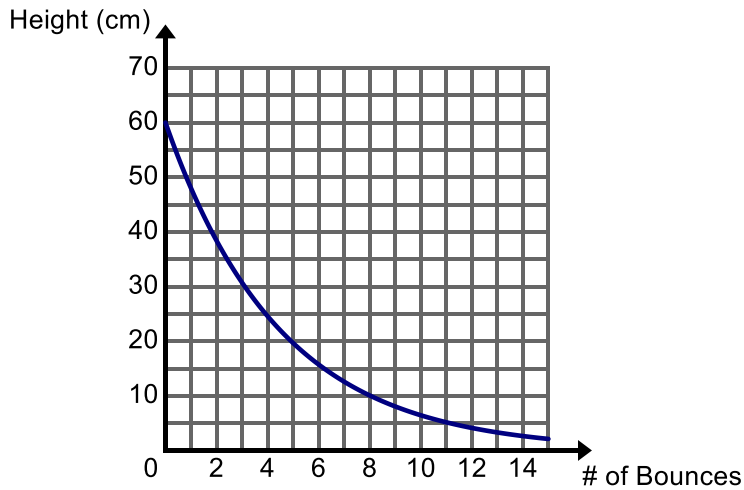
- a. Tell the story of the graph.

4. A toy rocket is launched straight up in the air from the ground. It leaves the launcher with an initial velocity of 96 ft./sec. The graph below shows the height of the rocket in feet with respect to time in seconds. Label the key features of the graph.



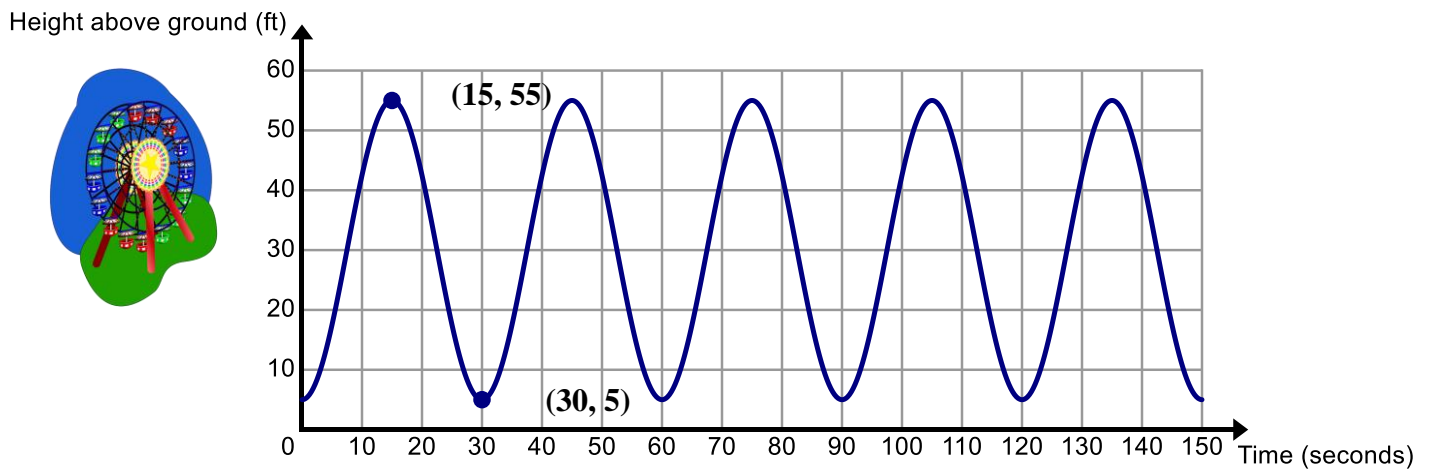
- a. Tell the story of the graph.

5. Suppose you drop a basketball from a height of 60 inches. The graph below shows the height of the object after b bounces.



- a. Tell the story of the graph.

6. You are riding a Ferris wheel. The graph below shows your height (in feet) above the ground as you ride the Ferris wheel.



- a. Tell the story of the graph.

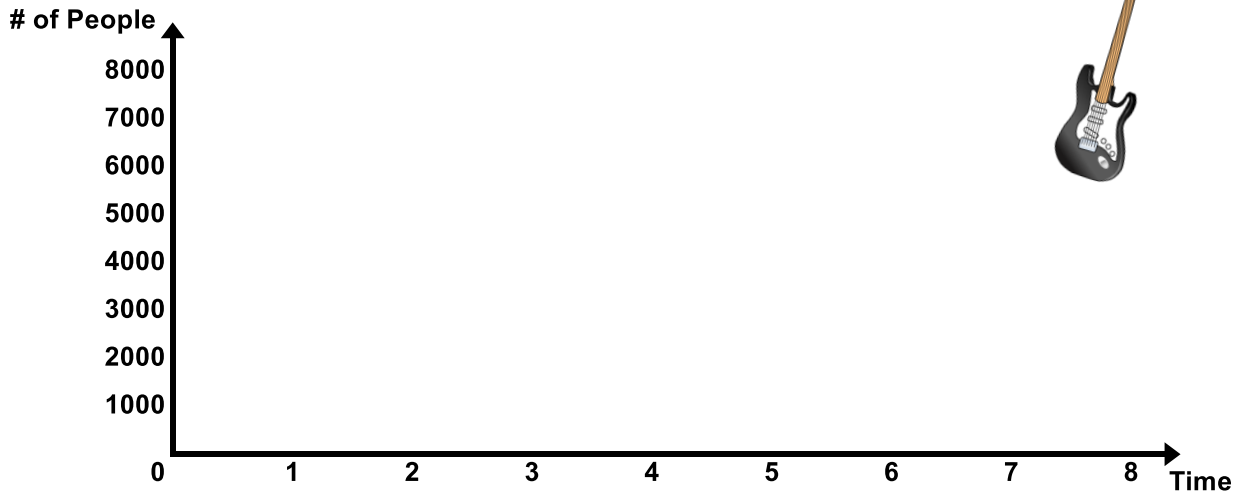
5.3g Class Activity: From Stories to Graphs



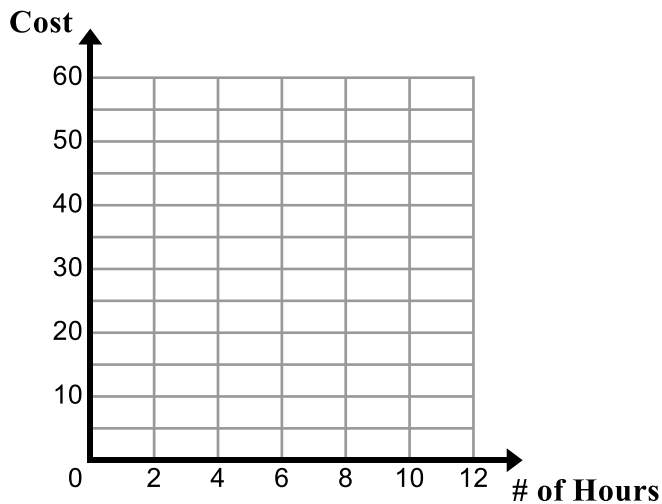
Directions: Sketch a graph to match each of the following stories. Label key features of your graph.

Story	Graph
<p>1. Zach walks home from school each day. Sketch a graph of Zach's distance from school as a function of time since the bell rang if the following happens: When the bell rings, Zach runs to his locker to grab his books and starts walking home. When he is about halfway home, he realizes that he forgot his math book so he turns around and runs back to school. After retrieving his math book, he realizes that he is going to be late so he sprints home.</p>	
<p>2. Solitude is offering a ski clinic for teens. The cost of the class is \$30 per student. A minimum of 5 students must sign up in order for Solitude to hold the class. The maximum number of students that can participate in the class is 12. Sketch a graph that shows the revenue Solitude will bring in dependent on the number of students that take the class.</p>	
<p>3. A biker is riding up a hill at a constant speed. Then he hits a downhill and coasts down the hill, picking up speed as he descends. At the bottom of the hill, he gets a flat tire. Sketch a graph that shows the distance traveled by the biker as a function of time.</p>	

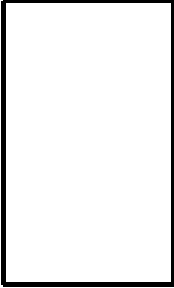

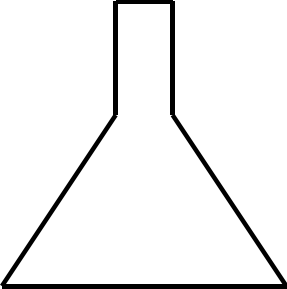

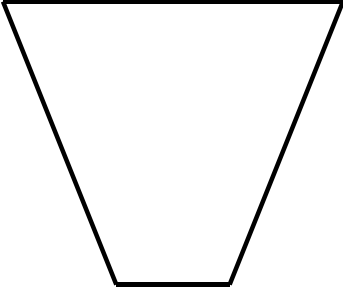

4. A concert for a popular rock group is sold out. The arena holds 8,000 people. The rock group is scheduled to take the stage at 8 pm. A band that is not very well known is opening for the rock band at 6:30 pm. The rock band is scheduled to play for 2 hours and the staff working the concert have been told that the arena must be cleared of people by 11:30 pm. Sketch a graph of the number of people in the arena from 5 pm to midnight. Time 0 on the grid below is 5 pm.

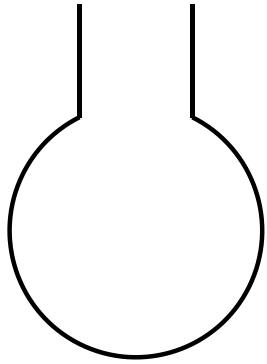

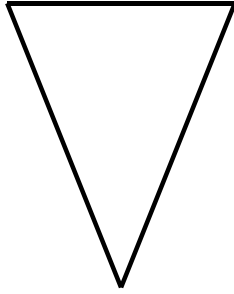



5. A parking garage charges \$5 per hour and has a maximum cost of \$40 for 12 hours. Sketch a graph of the total cost depending on how many hours a car is in the garage.



6. Your science teacher has the beakers shown below. He is going to fill them with water from a faucet that runs at a constant rate. Your job is to sketch a graph of the height of the water in each of the beakers over time.

Beaker	Graph of the height of the water over time
 <p data-bbox="321 594 428 621">Beaker 1</p>	
 <p data-bbox="321 1079 428 1106">Beaker 2</p>	
 <p data-bbox="321 1541 428 1568">Beaker 3</p>	

 <p style="text-align: center;">Beaker 4</p>	
 <p style="text-align: center;">Beaker 5</p>	

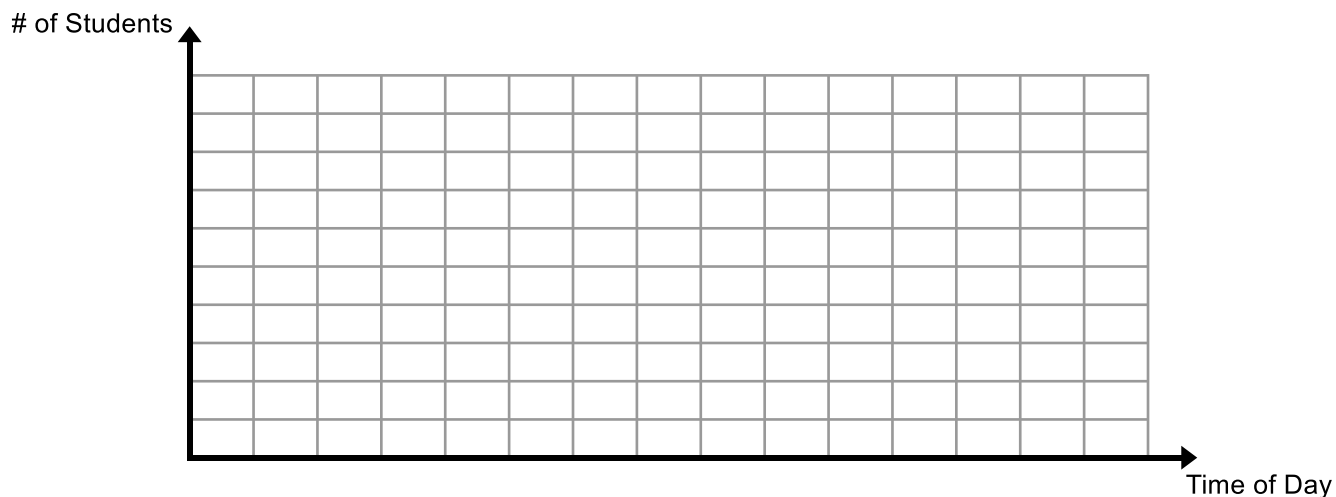
7. Now consider the volume of the water in each of the beakers over time. Sketch a graph of the volume of the water in each of the beakers over time.



5.3g Homework: From Stories to Graphs

Directions: Sketch a graph for each of the stories below.

1. Sketch a graph of the number of students in the cafeteria as a function of time throughout the school day at your school. Tell the story of your graph.



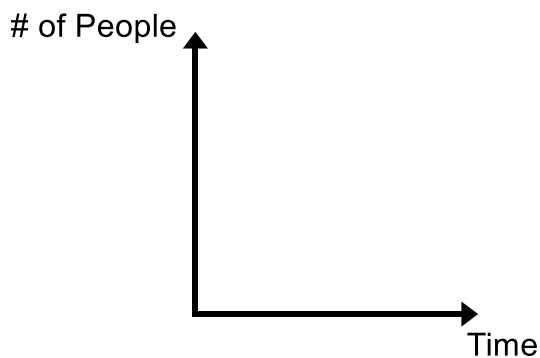
2. Two thousand, five hundred students attend a local high school. School starts at 8 am and ends at 2:30 pm. Many students stay after school for clubs, sports, etc. The school has a one-hour lunch at noon and seniors are allowed to leave campus for lunch. Sketch a graph of the number of cars in the student parking lot from 6 am to 4 pm. Time 0 on the grid below is 6 am.



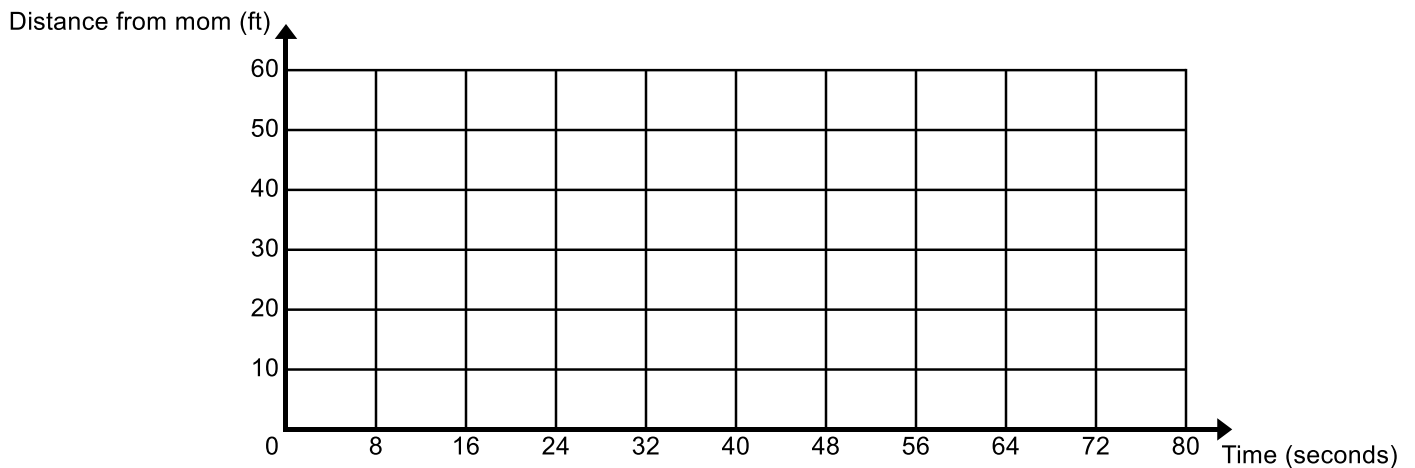
3. A train that takes passengers from downtown back home to the suburbs makes 5 stops. The maximum speed at which the train can travel is 40 mph. Sketch a graph of the speed of the train a function of time since leaving the downtown train station.



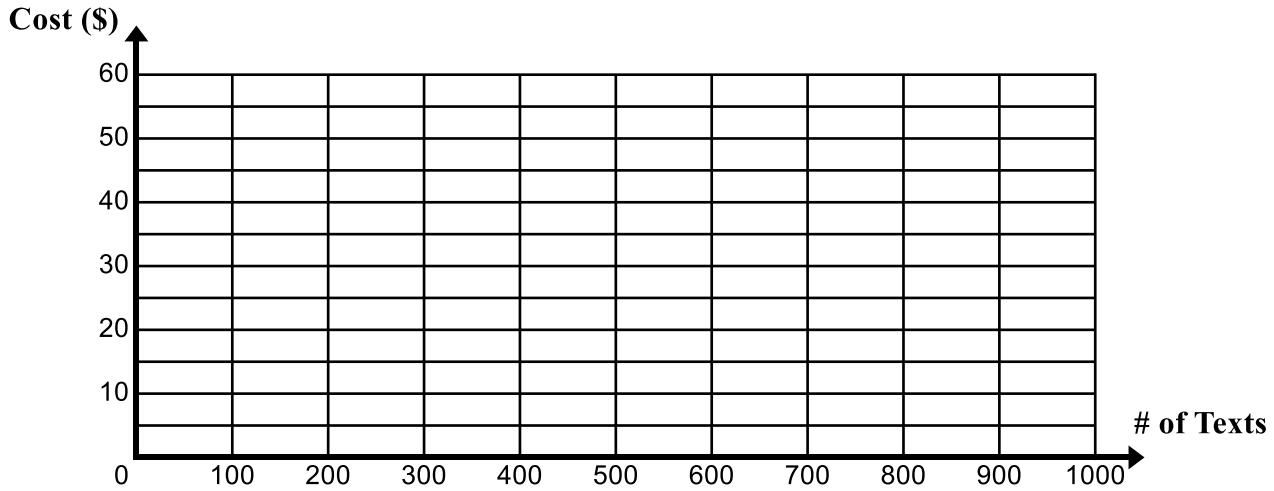
4. Sketch the graph of the total number of people that have seen the hit movie of the summer as a function of the time since opening day of the movie.



5. A little girl is going around on a merry-go-round. Her mom is standing at the entrance to the ride. Sketch a graph of the distance the little girl is from her mom as she goes around if the minimum distance she is from her mom during the ride is 5 feet and the maximum distance she is from her mom is 45 feet. Assume it takes 16 seconds to make one full revolution on the merry-go-round.



6. Yvonne is researching cell phone plans. Company A offers charges \$0.05 for each text message sent. Company B offers unlimited texting for \$25 per month. Company C charges \$10 per month for up to 500 text messages and an additional \$0.10 for each text message over 500. Sketch and label a graph that shows the relationship between number of texts sent and total monthly cost for each of the plans.



5.3h Self-Assessment: Section 5.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Determine whether the relationship between two quantities can be modeled by a linear function. Construct a function to model a linear relationship between two quantities.				
2. Compare properties of linear functions (rates of change and intercepts) and use this information to solve problems.				
3. Identify and interpret key features of a graph that models a relationship between two quantities.				
4. Sketch a graph that displays key features of a function that has been described verbally.				

For use with skill/concept #1

- Which of the following representations/situations can be modeled by the function $y = 2x + 10$? Circle all that apply.
 - A pool has 10 gallons of water in it and water is being added to the pool at a rate of 2 gallons per minute.
 - A pool has 2 gallons of water in it and water is being added to the pool at a rate of 10 gallons per minute.
 - There are 10 bacterium in a petri dish. Each hour, the number of bacteria in the dish doubles.
 - There are currently 10 shoes on the shelf in a store. The owner is adding boxes with pairs of shoes inside to the shelf.
 - Penny has 10 pennies in a jar. Each day, she adds 2 pennies to the jar.
- Create 3 different representations (a table, graph, and context) that can be modeled by the function $y = 4x$.

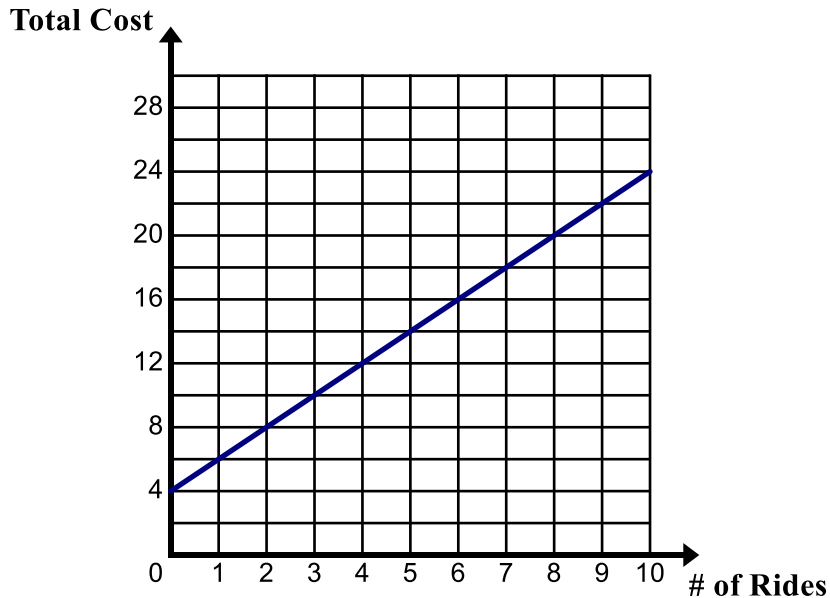
3. Circle the letter of the representations that can be modeled by a linear function. Construct a linear function for those that are linear.

a.

Radius (in)	Area (in ²)
1	3.14
2	12.56
3	28.26
4	50.24
5	78.50

- b. The cost of a frozen yogurt at Callie's Custard Shop is \$4.50. Each additional topping is \$0.25.

- c. The graph below shows the total cost dependent on the number of rides taken.



- d. Nick receives a 3% raise every year.

- e. A plane starts its descent from an elevation of 35,000 feet. The table below shows the elevation of the plane as it is descending.

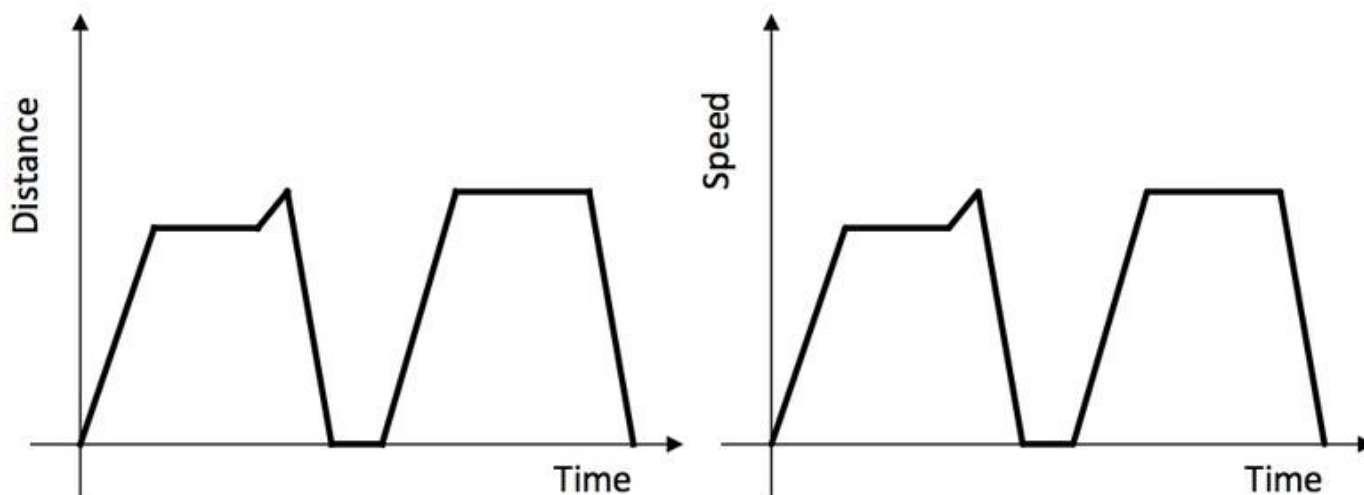
Time (min.)	Elevation (ft.)
0	35,000
3	27,500
5	22,500
6	20,000

For use with skill/concept #2

1. Maya and her brother each brought a seedling plant home from the store. The plants are both growing at a constant rate. Maya's plant was 8 cm. tall 2 weeks after she brought it home and 20 cm. tall 8 weeks after she brought it home. The height h of her brother's plant in centimeters t weeks after he brought it home can be modeled by the equation $h = \frac{3}{2}t + 6$. Which plant is growing at a faster rate? Which plant was taller when they brought the plants home?

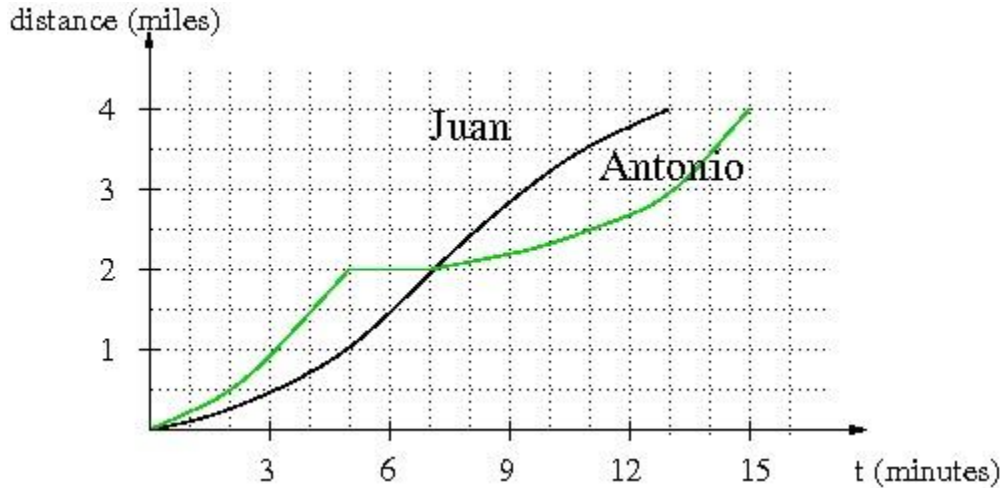
For use with skill/concept #3

1. Below are two graphs that look the same. Note that the first graph shows the distance of a car from home as a function of time and the second graph shows the speed of a different car as a function of time. Describe what someone who observes the car's movement would see in each case.



This is an Illustrative Mathematics Task: <https://www.illustrativemathematics.org/illustrations/632>

2. Antonio and Juan are in a 4-mile bike race. The graph below shows the distance of each racer (in miles) as a function of time (in minutes).

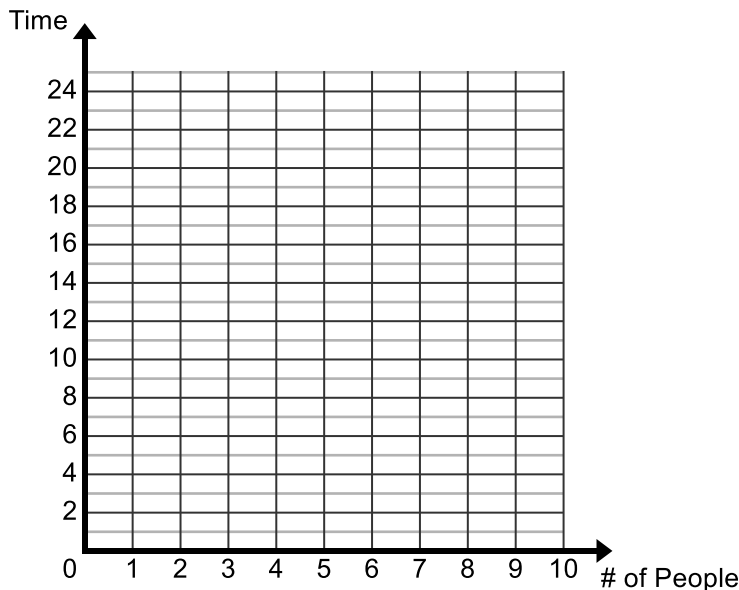


- Who wins the race? How do you know?
- Imagine you were watching the race and had to announce it over the radio. Write a little story describing the race.

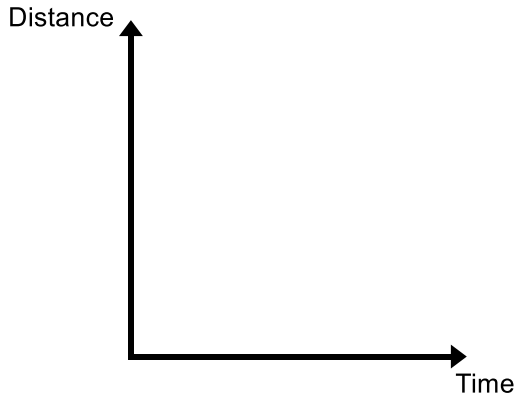
This is an Illustrative Mathematics Task: <https://www.illustrativemathematics.org/illustrations/633>

For use with skill/concept #4

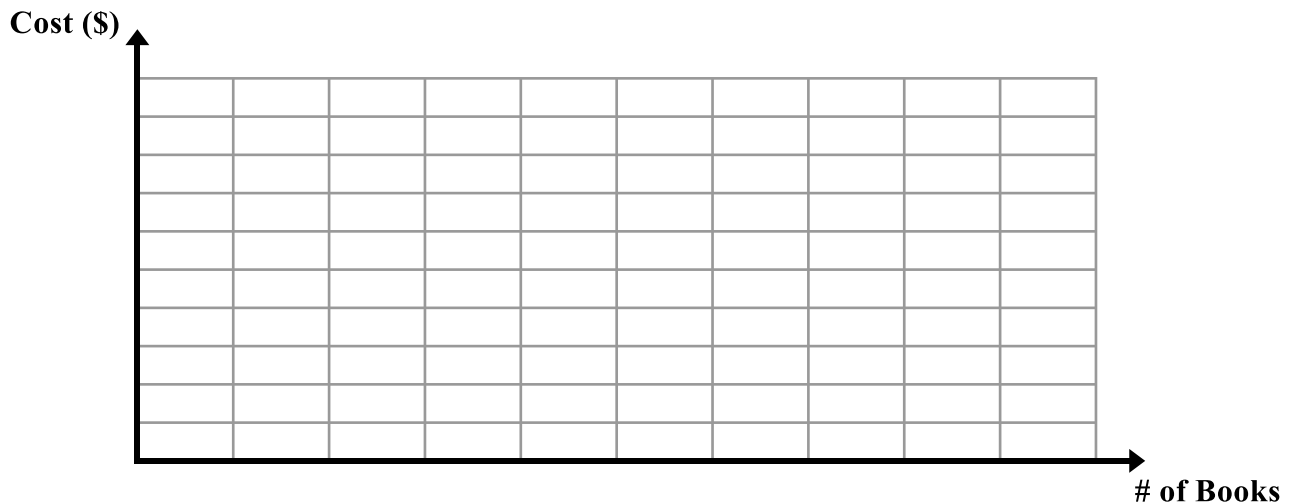
- It will take Rick 24 hours to paint a fence in his backyard. Rick is trying to get some friends to help him paint the fence. Sketch a graph of the amount of time it will take to paint the fence dependent on how many friends Rick gets to help.



- Sketch a graph of Carrie's distance from home. Carrie starts at home, walks to the neighbors to play, stays at the neighbors to play, then runs home.



- Sketch the graph of the total cost of ordering books dependent on the number ordered given the following criteria: it costs \$110 per book if you order 0 – 50 books, \$90 if you order 51 – 100 books, and \$75 if you order more than 100 books.



- Sketch a graph of your energy level during the day from the time you wake up until the time you go to sleep at night. Label key features and events of the day.
- Sketch a graph of the distance the second hand of a clock is from the number 6 as it moves around the clock.



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Chapter 6: Statistics-Investigate Patterns of Association in Bivariate Data (2 weeks)

Utah Core Standard(s):

- Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (8.SP.1)
- Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (8.SP.2)
- Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (8.SP.3)
- Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* (8.SP.4)

Academic Vocabulary: Experiment, outcomes, sample space, random variables, realizations, quantitative (numerical) variables, categorical variables, univariate data, bivariate data, scatter plot, association, positive association, negative association, no apparent association, linear association, non-linear association, weak association, strong association, perfect association, cluster, outlier, line of best fit, linear model, prediction function, two-way frequency table, marginal frequencies, relative frequencies.

Chapter Overview:



Up to this point, students have been studying data that falls on a straight line. Most of the time data given in the real world is not perfect; however, often the data is associated with patterns that can be described mathematically. In this chapter, students will investigate patterns of association in bivariate data by constructing and interpreting scatter plots, fitting a linear function to scatter plots that suggest a linear association, and using the prediction function to solve real world problems and make predictions. In addition they explore categorical bivariate data by constructing and interpreting two-way frequency tables.


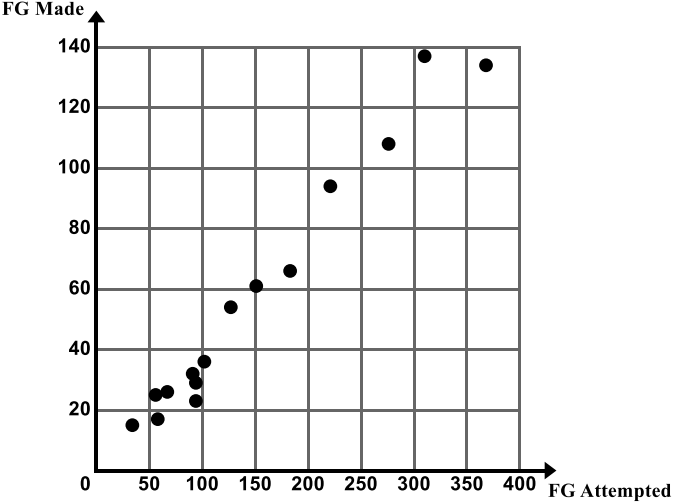


Connections to Content:



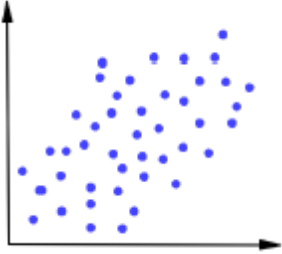
Prior Knowledge: Until 8th grade, the study of statistics has centered on univariate data. Students have created and analyzed univariate data displays, describing features of the data and calculating numerical measures of center and spread. In 8th grade, students have the opportunity to apply what they have learned about the coordinate plane and linear functions in order to analyze and interpret bivariate data and construct linear models for data sets that suggest a linear association.

Future Knowledge: Students will more formally fit a linear, as well as additional types of functions, to bivariate data using technology. They will also calculate correlation coefficients, a numerical measure for determining the strength of a linear association. Students will also use residual plots as a tool for assessing the fit of a linear model. Students will also continue with the study of two-way frequency tables.

MATHEMATICAL PRACTICE STANDARDS:

	<p>Make sense of problems and persevere in solving them.</p>	<p>Emina loves to eat tomatoes from her garden in Salt Lake City. She asked her friend Renzo, “Don’t you just love tomatoes?” Renzo crinkled his nose and replied, “Ew, tomatoes gross me out! When I see them in the grocery store, I just keep on walking.” Renzo’s response prompted Emina to think, “I don’t buy tomatoes at the grocery store either, because I grow them in my garden. The tomatoes from my garden are delicious, whereas grocery store tomatoes look less appealing to me. I wonder if there is an association between enjoying tomatoes and having a garden at home?”</p> <p><i>In the problem above the student must help Emina determine if there is an association between liking tomatoes and having a garden at home. They organize collected data into a two-way frequency table and then analyze it. Students must problem solve as they decide how to organize their data and as they determine what the data is telling them.</i></p>
	<p>Reason abstractly and quantitatively.</p>	<p>The table gives data relating the number of oil changes every two years to the cost of car repairs. <i>Table not shown due to space.</i></p> <p>Plot the data on the graph provided, with the number of oil changes on the horizontal axis. You will need to define your own scale.</p> <p>Write a prediction function in slope-intercept form that you could use to predict the cost of repairs, y, for any number of oil changes, x. Compare your prediction with that of a partner.</p> <p>Use your prediction function to predict how much a person would spend on car repairs if they were to get 8 oil changes. Compare your prediction with that of a partner</p> <p><i>Throughout the chapter, students analyze displays of numeric data sets (in tables and in graphs). If the data sets suggest a linear association, students construct a linear function to model the situation. These functions are an abstract way to represent the associations suggested by the data sets.</i></p>

	<p>Construct viable arguments and critique the reasoning of others.</p>	<p>Using the scatter plot, determine if there is a relationship between field goals attempted and field goals made. Describe any trends or patterns you observe in the data.</p>  <p><i>Throughout the chapter, students are asked to create a scatter plot of a given data set and analyze the scatter plot to determine if there is an association between two variables. They look for trends and patterns, including clusters and outliers. They provide explanations related to the context for the associations, trends, and patterns. Students are making arguments about the data and are asked to support their arguments with data and critical thinking about the context and limitations of the data.</i></p>
	<p>Model with mathematics.</p>	<p>Students will say a selected tongue twister one at a time. In the first trial, only the first student will say the tongue twister; in the second trial, only the first and second students will say the tongue twister, etc. In each trial, one person will be added to the chain of tongue twisters and the total elapsed time will be recorded.</p> <p>Tongue twisters:</p> <ul style="list-style-type: none"> A. Work will win when wishy-washy wishing won't. B. Three witches wished three wishes, but which witch wished which wish. C. Peter Piper picked a peck of pickled peppers. D. Picky people pick Peter Pan peanut butter it is the only peanut butter picky people pick. <p><i>Throughout the chapter students will fit a linear model to several real-life situations that suggest a linear association. Students will construct prediction functions for lines of best fit and use the functions to make predictions and solve real-world problems.</i></p>
	<p>Use appropriate tools strategically.</p>	<p><i>Online software and graphing calculators are important tools that can be used to display and analyze large data sets and construct functions to model data sets. Additionally, many of the skills that students have learned up to this point will become a tool they will rely on in order to construct linear functions for data sets that suggest a linear association.</i></p>

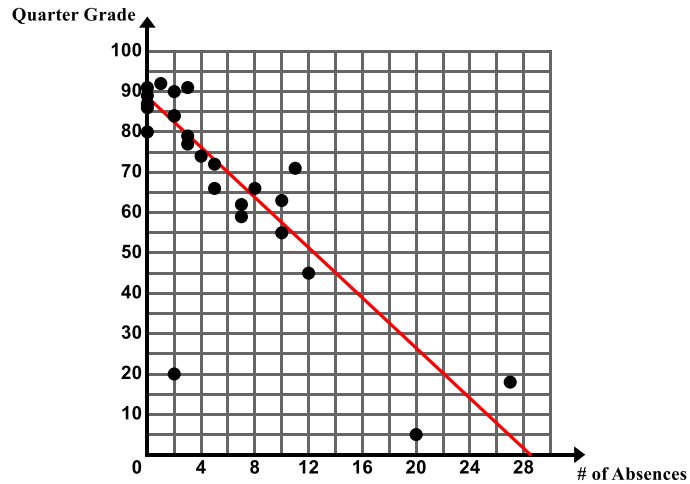
	<p>Attend to precision.</p>	<p>The following table shows the weight of an English Mastiff from birth to age 60 weeks. <i>Table not shown due to space.</i></p> <p>Create a scatter plot of the data on the grid below. Describe any patterns of association you see in this scatter plot. Use the context to give possible explanations as to why these trends, patterns, and associations exist.</p> <p><i>When students create scatter plots in this chapter, they must determine how to scale each axis appropriately and ensure that they are graphing the data points accurately in order to determine whether an association exists between the two variables and in order to write a function that models the data.</i></p>
	<p>Look for and make use of structure.</p>	<p>Describe the association between x and y. Circle any clusters in the data. Put a star by any points that appear to be outliers.</p>  <p><i>In order to describe the association between x and y, students must examine the structure of the data points on the graph. If there is an association, students must determine the following: Is it linear or non-linear? Is the association positive or negative? Is the association weak or strong? Do there appear to be any outliers or clusters?</i></p>



Look for and express regularity in repeated reasoning.

The following scatter plot shows the final grade in Ms. Ganchero's math class for students and the number of times they are absent.

Explain the meaning of the slope and y-intercept in the context.



Throughout the chapter, students must determine whether the relationship between two quantities suggests a linear association. In the case of a linear association, slope is a calculation that is repeated – linear functions grow at a constant rate. For data that resembles a line, students will write a prediction function for a line of best fit drawn through the data and explain the meaning of the slope in the context.

6.0 Anchor Problem: Tongue Twisters



Students will say a selected tongue twister one at a time. In the first trial, only the first student will say the tongue twister; in the second trial, only the first and second students will say the tongue twister, etc. In each trial, one person will be added to the chain of tongue twisters and the total elapsed time will be recorded.

Tongue twisters:

- A. Work will win when wishy-washy wishing won't.
- B. Three witches wished three wishes, but which witch wished which wish.
- C. Peter Piper picked a peck of pickled peppers.
- D. Picky people pick Peter Pan peanut butter it is the only peanut butter picky people pick.

1. In the table below, record the class data for each Tongue Twister.

Number of people	Tongue Twister A (time)	Tongue Twister B (time)	Tongue Twister C (time)	Tongue Twister D (time)
1				
2				
3				
4				
5				
6				
7				
8				
9				

Section 6.1: Construct and Interpret Scatter Plots for Bivariate Data

Section Overview: In this section we continue our study of bivariate data, specifically quantitative or numerical data. In 7th grade students engaged in the study of univariate data. We begin this section with a problem that deals with univariate data and then use the same context to explore a bivariate data set. As in the case of univariate data, analysis of bivariate measurement data graphed on a scatterplot proceeds by describing shape, center, and spread. Later, we are introduced to Izumi and her basketball statistics and use her data throughout the chapter to build upon the concepts of analyzing bivariate data. In this section students learn how to construct, read, and interpret a scatter plot. Throughout the section students investigate and describe trends and patterns of association between two variables and interpret these associations in a variety of real-world situations.

Concepts and Skills to be Mastered:

By the end of this section students should be able to:

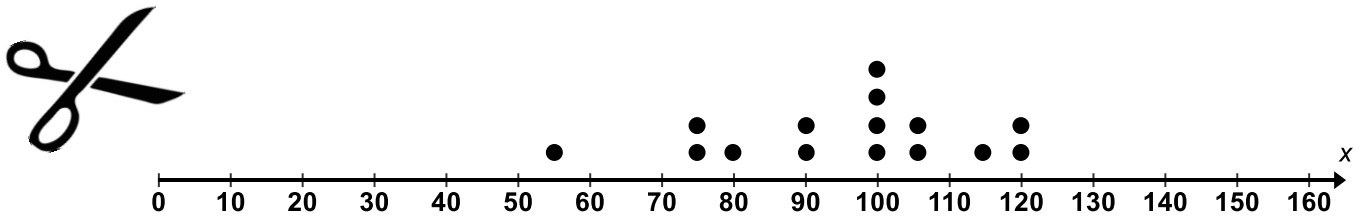
1. Read and interpret a scatter plot.
2. Construct a scatter plot for bivariate data.
3. Describe patterns of association in a scatter plot.

6.1a Class Activity: Read and Interpret a Scatter Plot

1. Jenny is a hair stylist. She decides to record the amount of money she makes in tips over a 15-day period. She records the following data:

Day	Amount of Money Made in Tips (dollars)
1	120
2	75
3	80
4	100
5	115
6	100
7	55
8	90
9	100
10	120
11	90
12	105
13	105
14	75
15	100

To better visualize the data, Jenny makes a dot plot of the data.



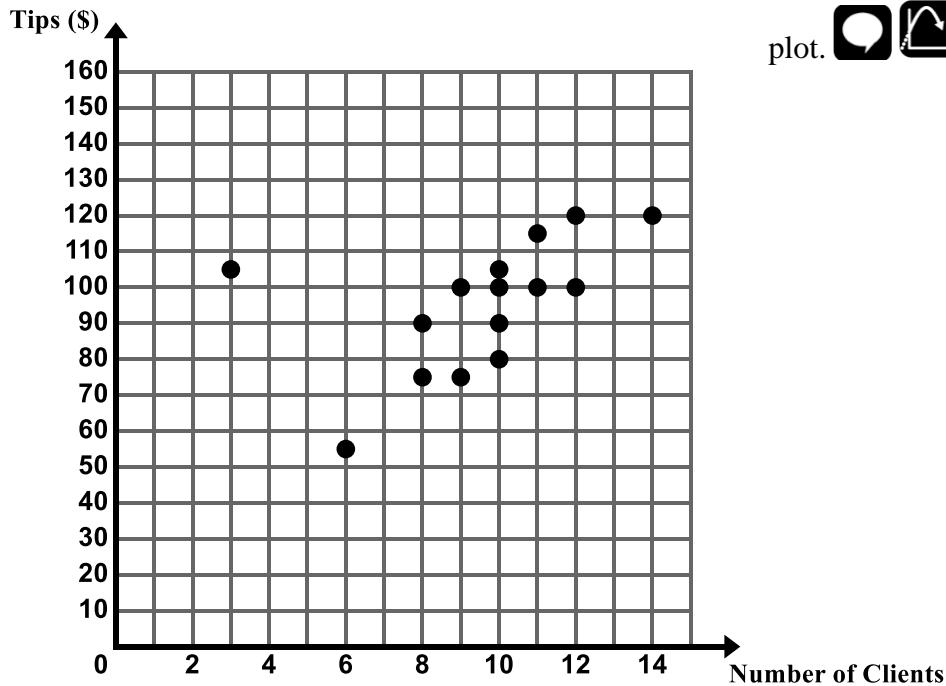
- a. Make some observations about the data shown in the dot plot.



2. Jenny then asks herself the following question: “I wonder if the amount I make in tips is associated to the number of clients I have each day?” She looks back through her appointment book and records the number of clients she had on each of the 15 days. She records the following data.

Day	Number of Clients	Amount of Money Made in Tips (dollars)
1	12	120
2	8	75
3	10	80
4	12	100
5	11	115
6	9	100
7	6	55
8	8	90
9	10	100
10	14	120
11	10	90
12	10	105
13	3	105
14	9	75
15	11	100

To better visualize the data, Jenny makes a **scatter plot** of the data. A **scatter plot** is a graph in the coordinate plane of the set of all (x, y) ordered pairs of bivariate data.



- a. Make some observations about the scatter plot.



Directions: Determine if the following scenarios represent univariate or bivariate data.

3. Lucas conducts an experiment where he records the number of speeding tickets issued in Iron County in a given year along with the average price of gasoline for that same given year. He collects this data from the year 1972 through 2012.
4. Lea conducts an experiment where she records the heights of all the NBA basketball players on the Miami Heat's roster for the 2014 season.
5. Adel conducts an experiment where she records the selling price of several homes in a neighborhood.
6. Adel conducts an experiment where she records the selling price and square footage of homes in a neighborhood.
7. Lisa conducts an experiment on the number of times a person works out a week and the person's weight.

In this chapter, we will focus our study on bivariate data sets and we will explore the relationship between two variables of interest.

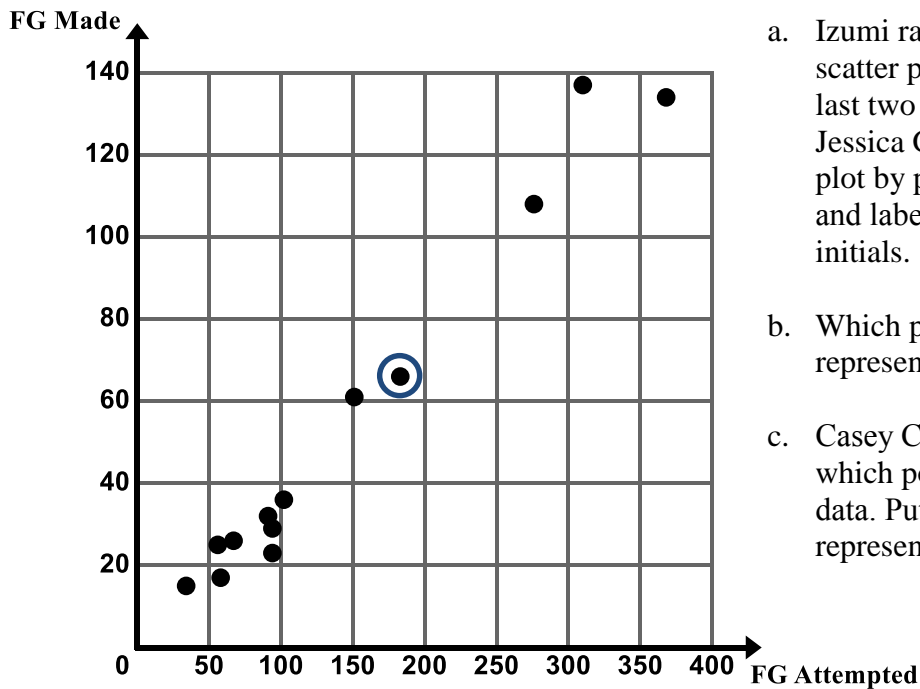
Izumi is the score keeper for her school's basketball team. Izumi's responsibilities as score keeper are to keep a record for several plays during the 2012-2013 season. The basketball plays are listed below.

- Total number of field goals made.
In basketball a field goal is the result of the player successfully shooting the basketball through the hoop, regardless of whether it is a two point shot or a three point shot. This does not include foul shots.
- The total number of field goals attempted.
A field goals attempt results when a player tries to make a field goal, an attempt is made whether or not the ball goes through the hoop.
- The total number of assists.
An assist results when the player passes the ball to a teammate who then scores.
- The total number of rebounds
A rebound results when the player retrieves the ball from an unsuccessful field goal attempt.



The table given below shows the record that Izumi made regarding the number of field goals attempted and the number of field goals made.

Player	Field Goals Attempted	Field Goals Made
Amber Carlson	34	15
Casey Corbin	368	134
Joan O'Connell	94	23
Monique Ortiz	102	36
Maria Ferney	91	32
Amelia Krebs	310	137
Tonya Smith	56	25
Juanita Martinez	58	17
Sara Garcia	151	61
Alicia Mortenson	67	26
Parker Christiansen	94	29
Rachel Reagan	183	66
Paula Lyons	276	108
Thao Ho	221	94
Jessica Geffen	127	54

8. As Izumi examines the data she wonders, “Is there is an association between the number of field goals made and the number of field goals attempted?” To further investigate the relationship between these two random variables, “Field Goals Made” and “Field Goals Attempted” Izumi makes a **scatter plot** of the data as shown below.



- Izumi ran out of time while creating her scatter plot and did not plot the data for the last two players in the table, Thao Ho and Jessica Geffen. Help Izumi finish the scatter plot by plotting the data for these players and labeling the points with these players' initials.
- Which player does the circled data point represent?
- Casey Corbin sees Izumi's graph and asks which point on the scatter plot represents her data. Put Casey's initials by the point that represents his data.

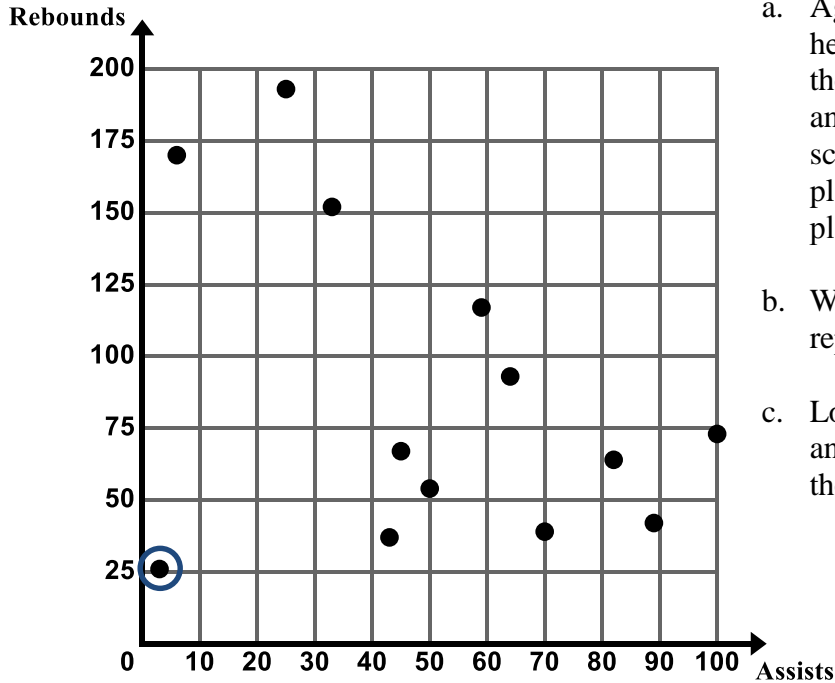
- d. Using the scatter plot, determine if there is a relationship between field goals attempted and field goals made. Describe any trends or patterns you observe in the data.  

- e. Can you think of another variable that when graphed with field goals made would have a positive association?

9. In addition to data about field goals, Izumi is curious about the relationship between the number of assists and the number of rebounds a player makes in a season. In order to study this relationship, Izumi gathers data on the number of assists and rebounds each player makes during the season. Izumi's Assist and Rebound data are given in the following table.

Player	Assists	Rebounds
Amber Carlson	82	64
Casey Corbin	6	170
Joan O'Connell	43	37
Monique Ortiz	50	54
Maria Ferney	89	42
Amelia Krebs	25	193
Tonya Smith	70	39
Juanita Martinez	3	26
Sara Garcia	100	73
Alicia Mortenson	33	152
Parker Christiansen	64	93
Rachel Reagan	45	67
Paula Lyons	59	117
Thao Ho	15	179
Jessica Geffen	30	113

Izumi made the scatter plot of assists and rebounds shown below to help her better visualize the data.



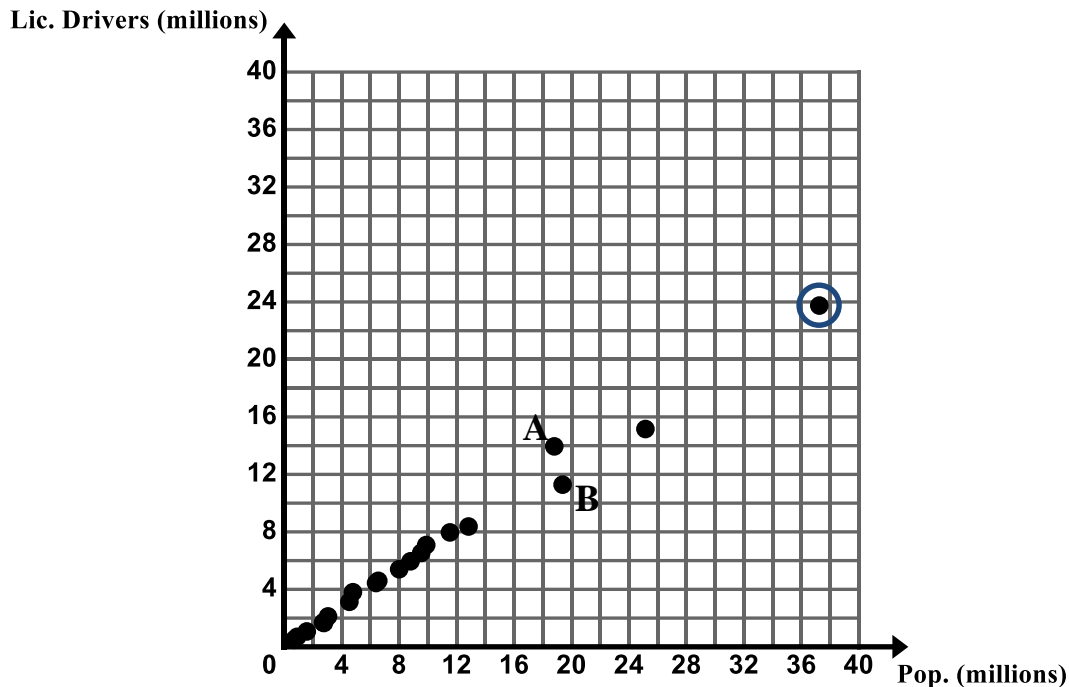
- Again, Izumi ran out of time while creating her scatter plot and did not plot the data for the last two players in the table, Thao Ho and Jessica Geffen. Help Izumi finish the scatter plot by plotting the data for these players and labeling the points with these players' initials.
- Which player does the circled data point represent?
- Locate the data points for 3 different players and put the initials of the players next to their data point.

- Izumi notices the circled data point stands out noticeably from the general behavior of the data set. We call this point an **outlier**. Provide an explanation as to why this player's data does not fit with the rest of the data.
- Using the scatter plot, determine if there is a relationship between number of assists and number of rebounds. Describe any trends or patterns you observe in the data.
- Can you think of another variable that when graphed with field goals made would have a negative association?

10. Which data set appears to have a stronger association: the relationship between number of field goal made and number of field goal attempts or the relationship between number of rebounds and number of assists?

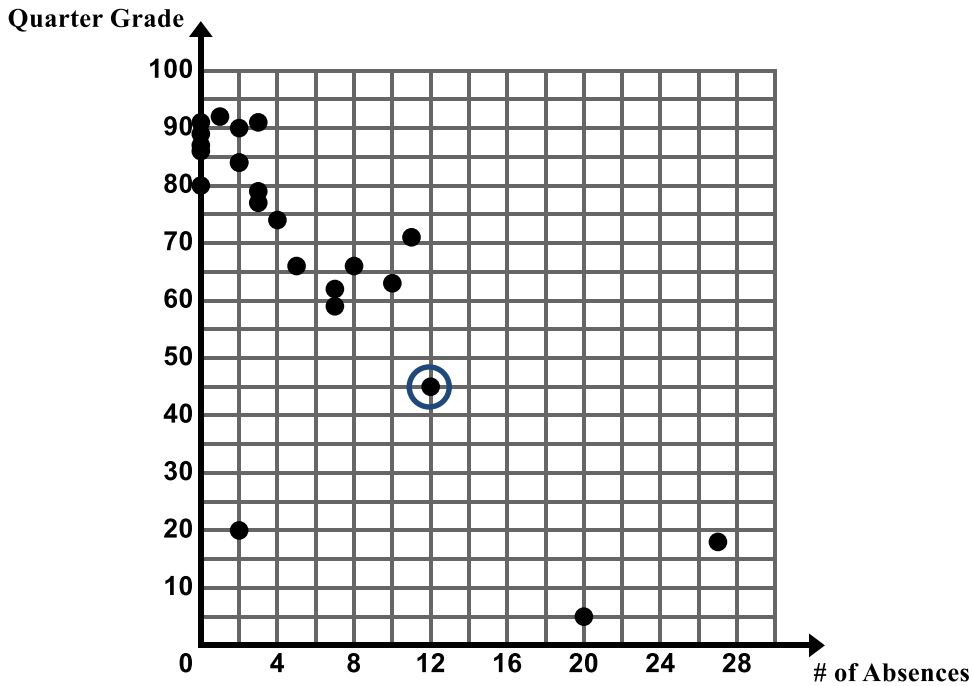
6.1a Homework: Read and Interpret a Scatter Plot


1. The U.S. Census Bureau collects data about the people and economy in the United States. The graph below shows the population (in millions) and the number of licensed drivers (in millions) for 20 different states for the year 2010.



- a. What does the circled data point (37.25, 23.75) represent in the context?
- b. In 2010, Texas had a population of approximately 25.15 million people and had approximately 15.2 million licensed drivers. Put a star by the data point that represents Texas.
- c. What does the graph show about the relationship between a state's population and the number of licensed drivers in the state?
- d. If a state has a population of approximately 32 million people, approximately how many licensed drivers would you expect to find in the state based on the trend in the scatter plot?
- e. If a state has approximately 12 million licensed drivers in a state, what would you expect the population to be in that state based on the trend in the scatter plot?
- f. Compare data points A and B.
- g. Data point A represents the state of Florida and data point B represents the state of New York. Provide an explanation as to why New York has more total people than Florida but fewer licensed drivers.

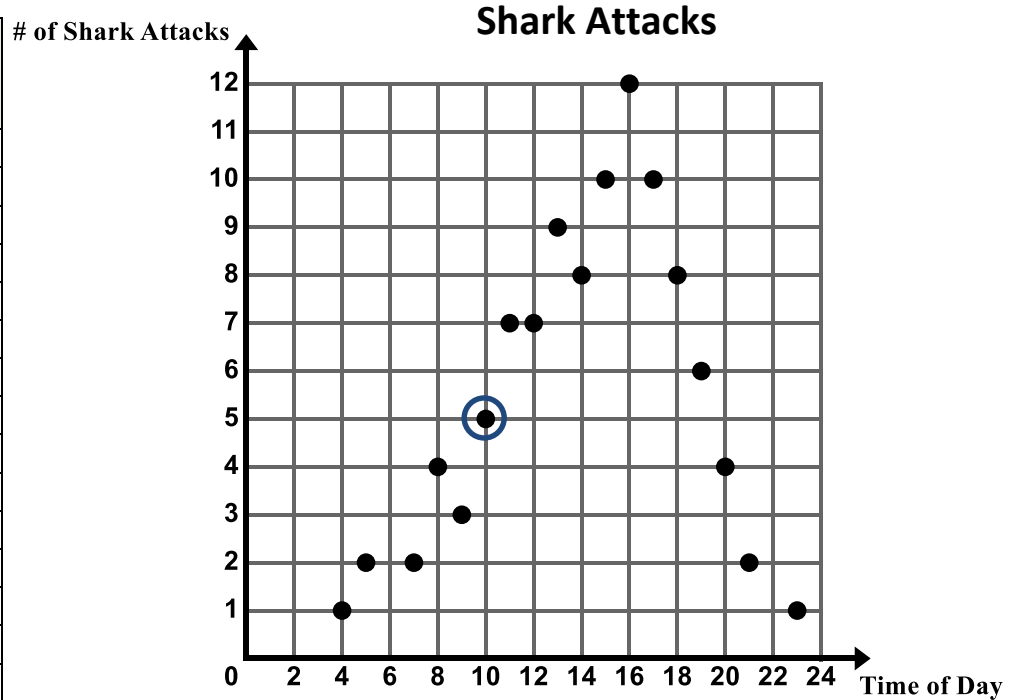
2. Ms. Ganchero is a math teacher. She wonders if there is an association between the number of absences a student has in her class and the grade they earn at the end of the quarter. In order to analyze this relationship, Ms. Ganchero created the scatter plot below which shows the number of absences a student has in a quarter and their final grade at the end of the quarter.



- While reviewing the scatter plot, Ms. Ganchero realized that she did not plot the data for two students. Rachel was absent 5 times and received a final grade of 72 and Lydia was absent 10 times and received a final grade of 55. Plot and label these two data points on the scatter plot above.
- What does the circled data point represent in the context?
- Provide an explanation for the cluster of points in the upper left corner of the graph. 
- Do there appear to be any outliers in the data? If yes, what are they? Provide an explanation for the outlier(s).
- Does the scatter plot suggest a relationship between absences and grade? Describe any trends or patterns you observe in the data.

3. A long stretch of a popular beach is overseen by the local coast guard. Over a period of 60 years the coast guard has kept track of the number of shark attacks occurring along the coast as well as the hour during the day in which the attack occurred. The table and corresponding scatter plot show this data.
- *Note:** The time of day is given by a 24 hour clock, also known as military time.

Hour during the day	Number of Shark Attacks
04:00	1
05:00	2
07:00	2
08:00	4
09:00	3
10:00	5
11:00	7
12:00	7
13:00	9
14:00	8
15:00	10
16:00	12
17:00	10
18:00	8
19:00	6
20:00	4
21:00	2
23:00	1



- What does the circled data point represent in the context?
- Describe the association that exists between the time of day and the number of shark attacks. Give a possible explanation as to why this graph is shaped the way it is.

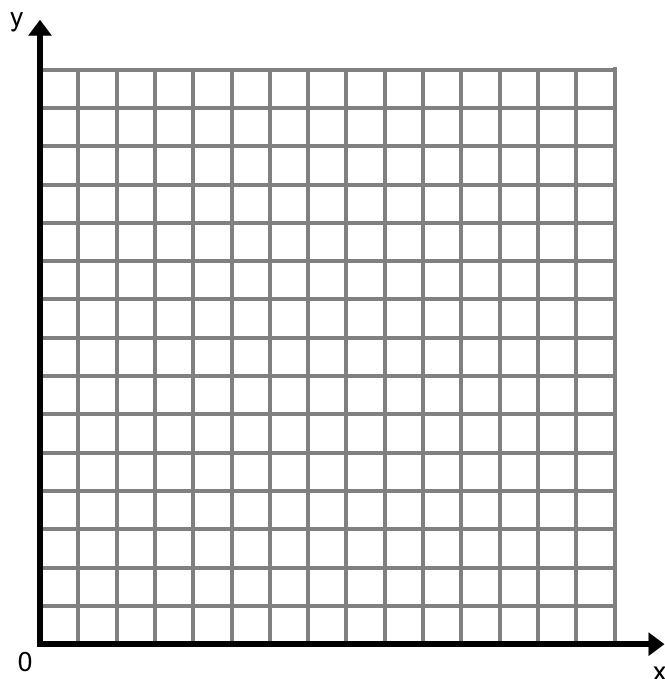
For tomorrow's class, you will need data on the height and shoe size of 5 people. Be sure to gather this data from different aged people – younger siblings, older siblings, parents, grandparents. Record your data here for tomorrow's class.

6.1b Class Activity: Create and Analyze a Scatter Plot

1. Do you anticipate an association between a person's height and their shoe length?
 - a. Make a prediction.
 - b. Collect your class data in the table below.

	Height	Shoe Length
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		
11.		
12.		
13.		
14.		
15.		

- c. Make a scatter plot of the data.




- d. Using the scatter plot, determine if there is an association between a person's shoe length and height. Describe any trends or patterns you observe in the data including clusters and outliers.

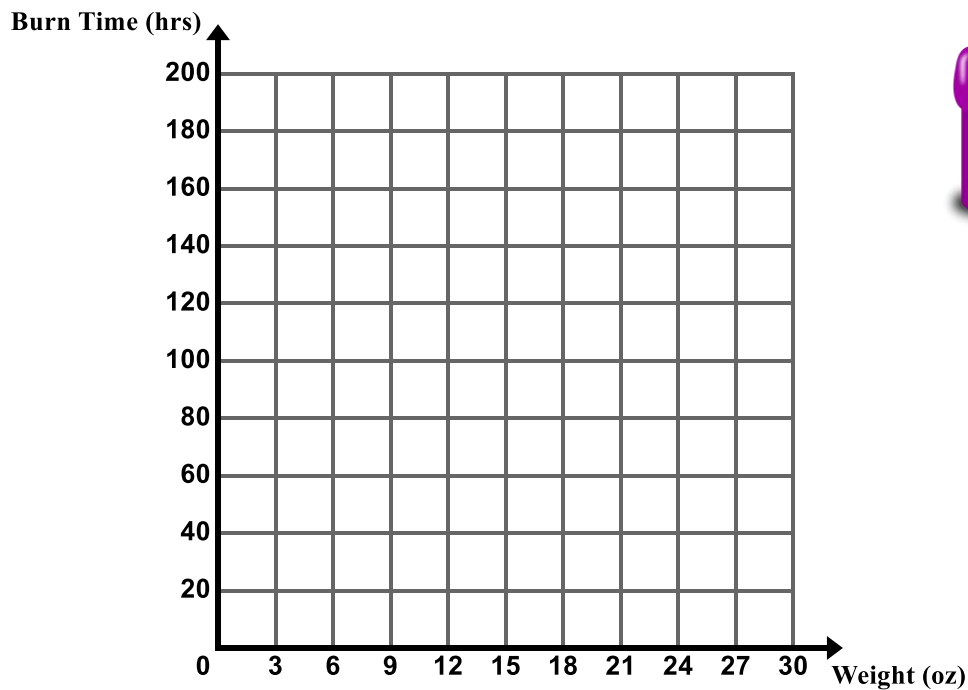
6.1b Homework: Create and Analyze a Scatter Plot

1. Is there an association between the weight of a candle and the amount of time it burns?
 - a. Make a prediction.


A company that manufactures candles tests the amount of time it takes for several candles of several different weights to burn. The results are shown in the table below.

Candle Weight (ounces)	2	2	2	3	3	4	4	5	5	5	10	10	10	16	16	16	22	22	22	26	26
Burn Time (hours)	15	16	18	20	33	34	35	38	40	36	40	80	80	95	100	98	120	125	175	174	180

- b. Make a scatter plot of the data on the graph provided. 



- c. Using the scatter plot, determine if there is an association between the weight of a candle and how long it burns. Describe any trends or patterns you observe in the data including clusters and outliers.

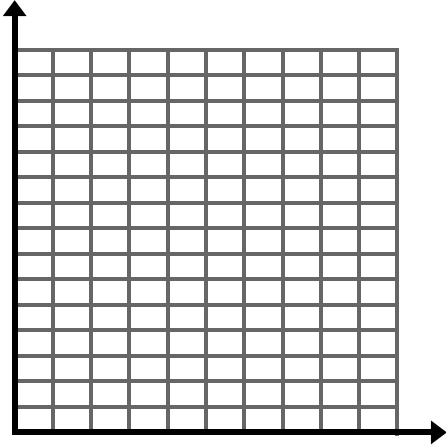
- d. **Bonus:** How much would a candle have to weigh to burn for one year? 

2. Create scatter plots of the following sets of data. Think about how to scale each axis based on the data



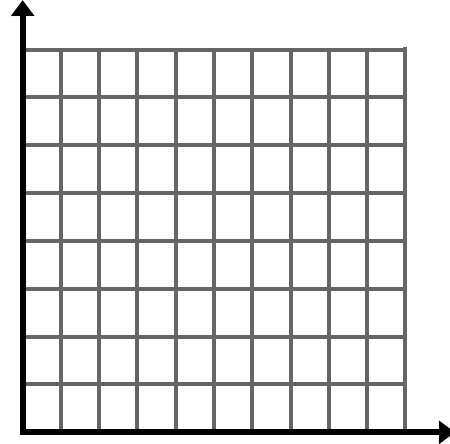
a.

x	1	2	3	4	5
y	2	5	9	12	14



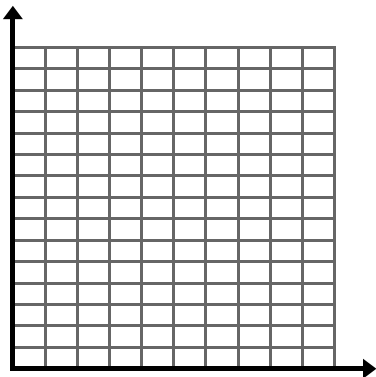
b.

x	0	2	2	4	4	5	6	6	7	8
y	5	6	5	5	7	6	4	6	5	6



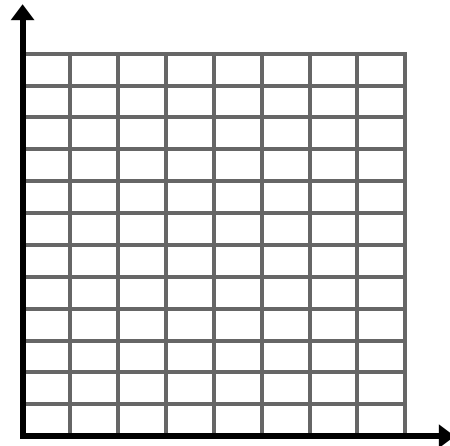
c.

x	0	1	2	3	4	5
y	1.4	1.7	2	2.2	2.4	2.8



d.

x	10	10	20	30	30	40	40	50	60	80
y	9	10	9	8	9	7.5	8	7	6	5



6.1c Classwork: Patterns of Association

So far in our study of bivariate data, we have seen data sets that show different **types of association** between two variables. There are many ways that we can describe the association (if there is one) between two variables. Common ways to talk about the association of two variables are shown in the table below. **Sketch scatter plots that correspond to each of the four associations described.**

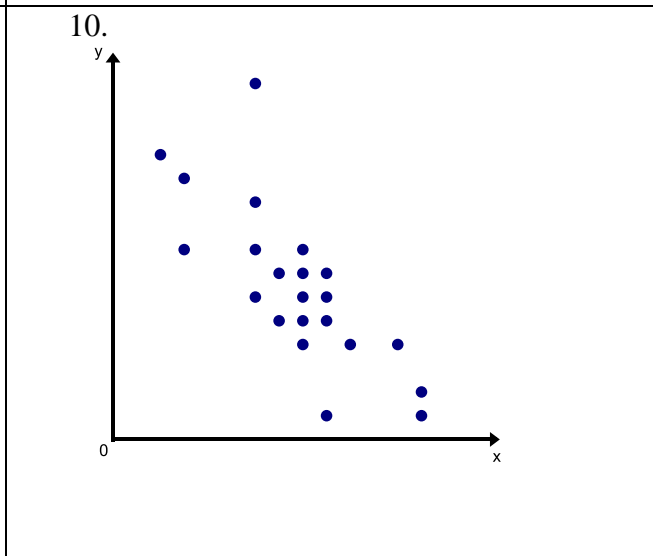
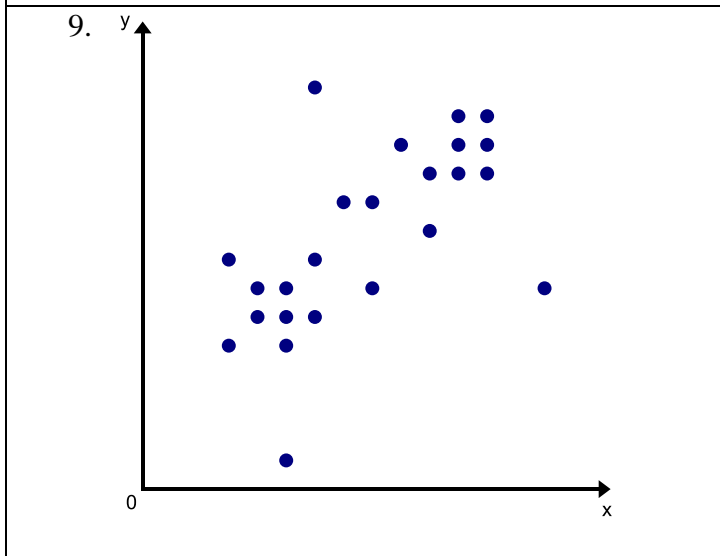
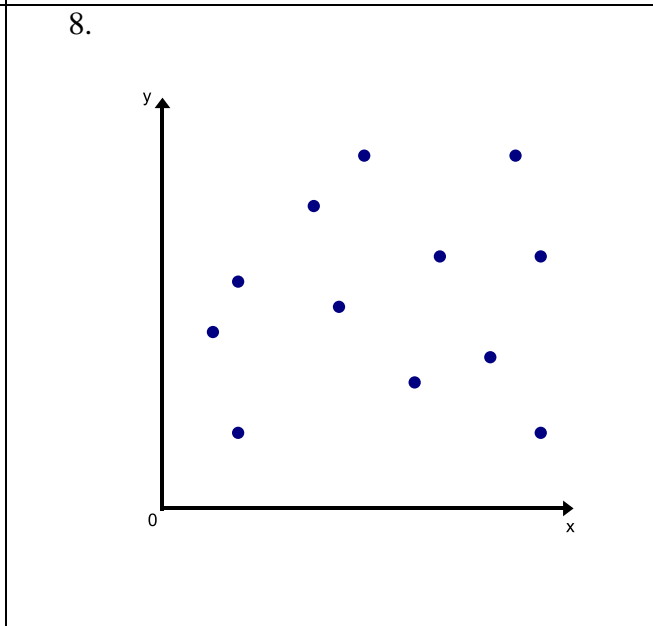
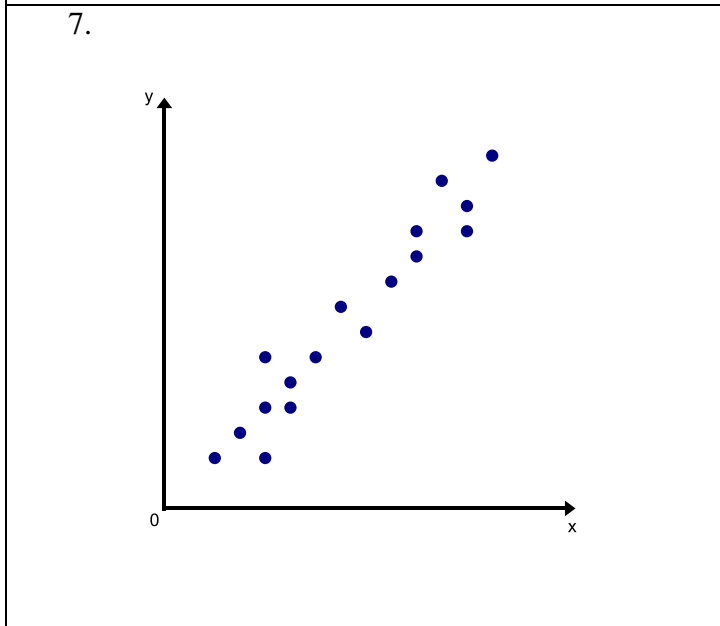
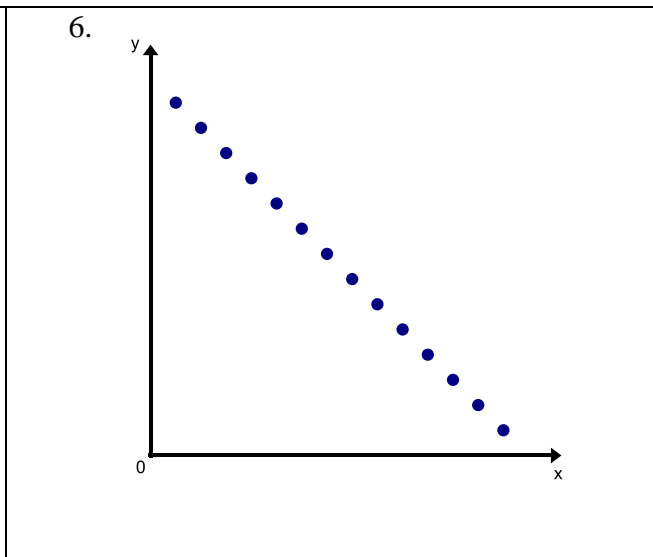
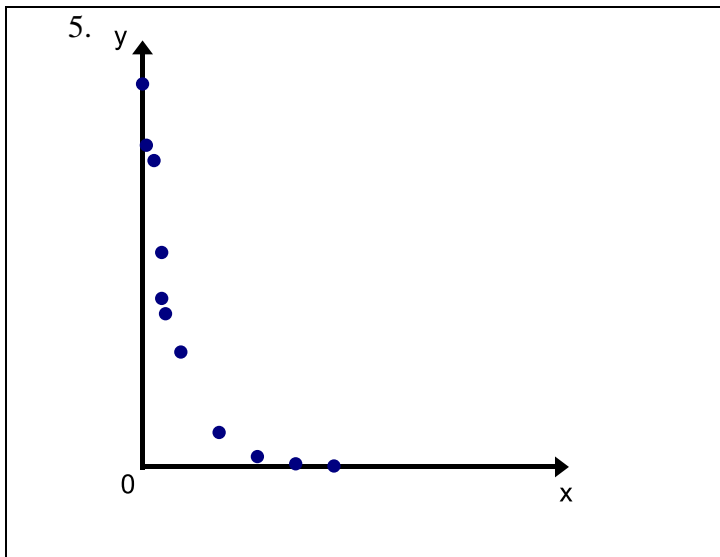
1. Positive Linear Association	2. Negative Linear Association
3. No Apparent Association	4. Nonlinear Association

If the variables show a linear association, we can determine whether that relationship is **strong**, **weak**, or **perfect**. Imagine drawing a line through the center of the points—**EYEBALLING** the line. If the data points are closely packed around your line, the linear relationship is a **strong** one. If the data points are more spread out from the line, the linear relationship is a **weak** one. If your data points fall on a straight line, the linear association is **perfect**.

We may also observe the following patterns in our data:

- **Clusters** - A cluster is a set of points that are in close proximity to each other.
- **Outliers** - An outlier is a data point that noticeably stands out from the general behavior of the data set.

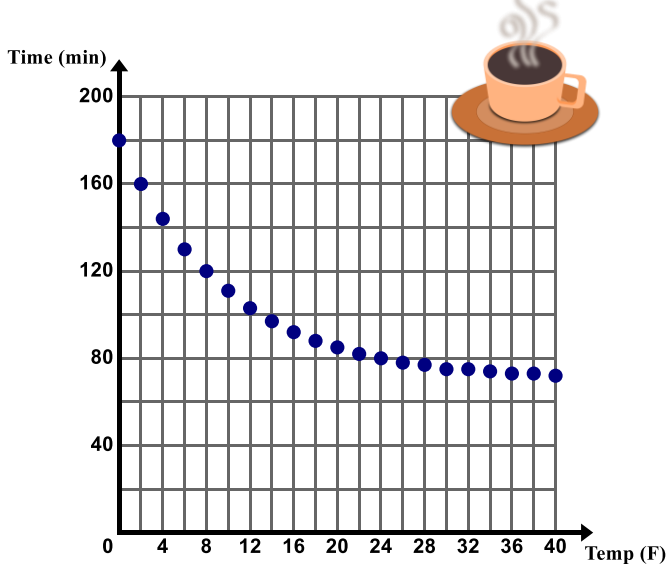
Directions: Describe the association between x and y using the terms from the previous page. Circle any clusters in the data. Put a star by any points that appear to be outliers.



Directions: Examine the following scatter plots. Describe the association between the two variables. Circle any clusters in the data. Put a star by any points that appear to be outliers. Use the context to give possible explanations as to why these trends, patterns, and associations exist.

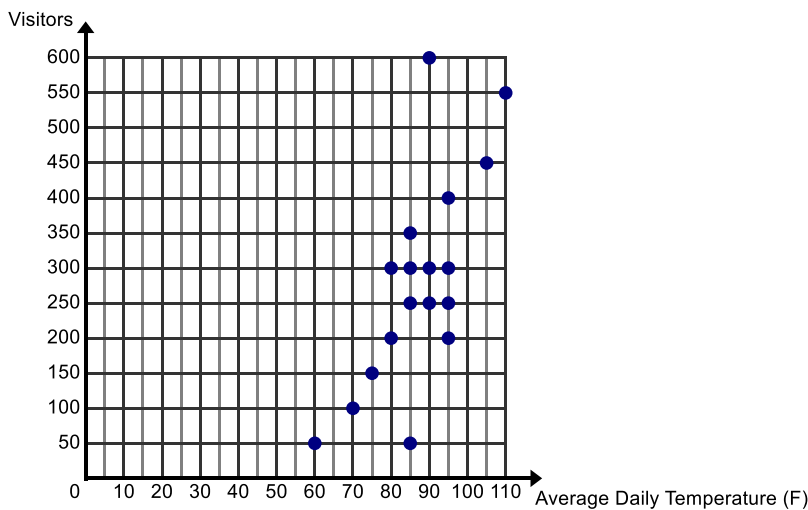


11. The scatter plot given below shows the temperature of a cup of tea sitting on the counter for 30 minutes. The cup of tea is sitting in a room that is 70 degrees.

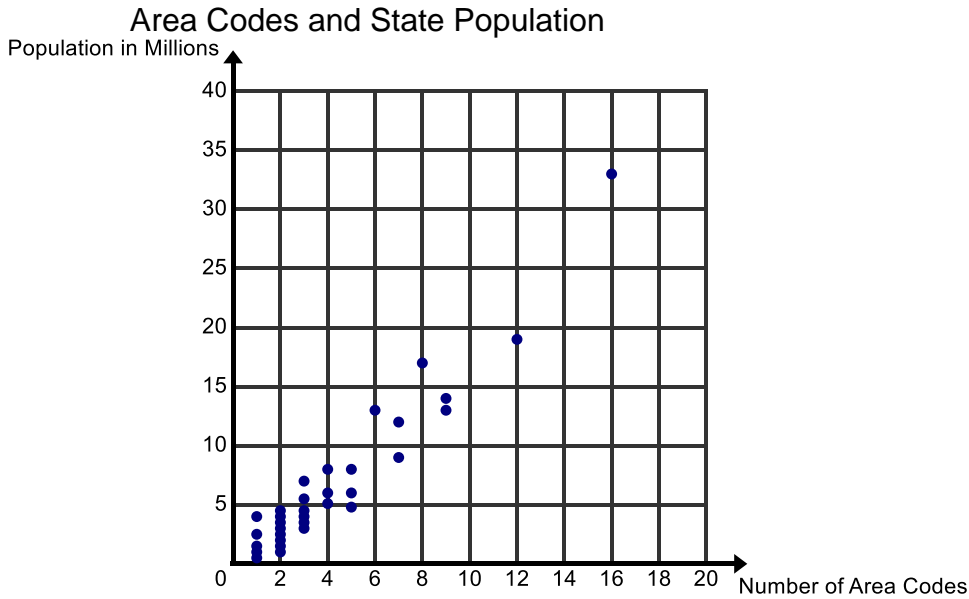


12. The Paradise Pool records the average daily temperature and the number of visitors to their pool for 18 days throughout the month of July. On July 24th, to celebrate Pioneer Day, admission is half off. The average daily temperature on that day is 90 degrees.

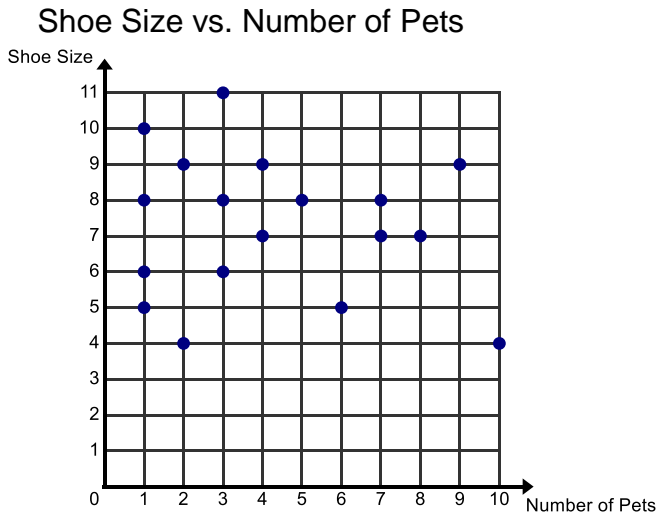
Visitors vs. Temperature at a Swimming Pool



13. The scatter plot below show the population (in millions) and number of area codes for some states in the United States.



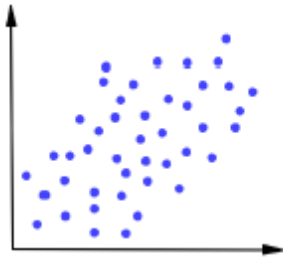
14. Holly’s math teacher asks her to conduct her own survey to study different types of association. She chooses to investigate the number of pets a person has and their shoe size.



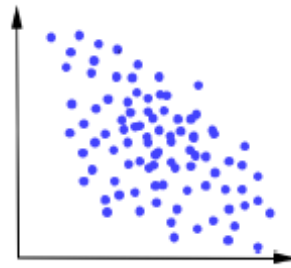
6.1c Homework: Patterns of Association

Directions: Describe the association between x and y . Circle any clusters in the data. Put a star by any points that appear to be outliers.

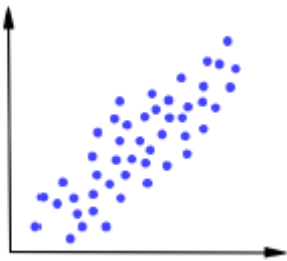
1.



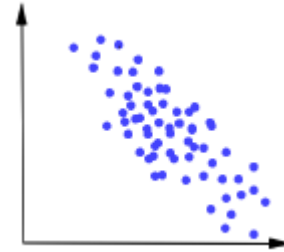
2.



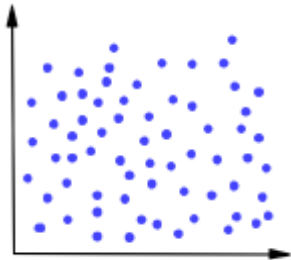
3.



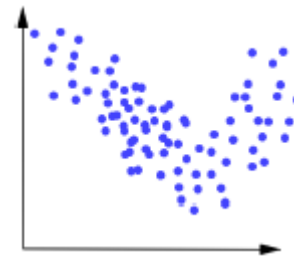
4.



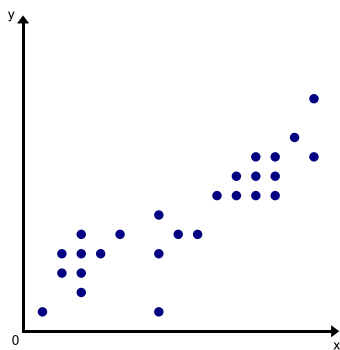
5.



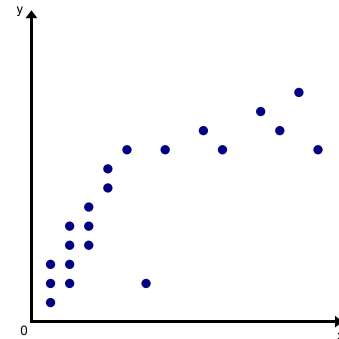
6.



7.



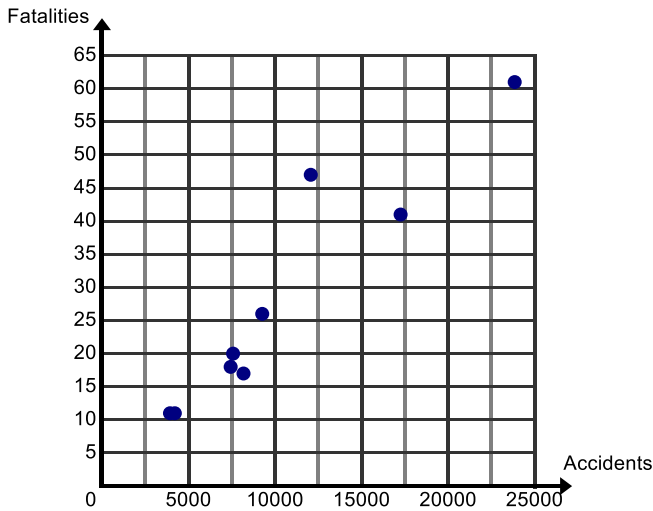
8.



Directions: Examine the following scatter plots. Describe the association between the two variables. Circle any clusters in the data. Put a star by any points that appear to be outliers. Use the context to give possible explanations as to why these trends, patterns, and associations exist.

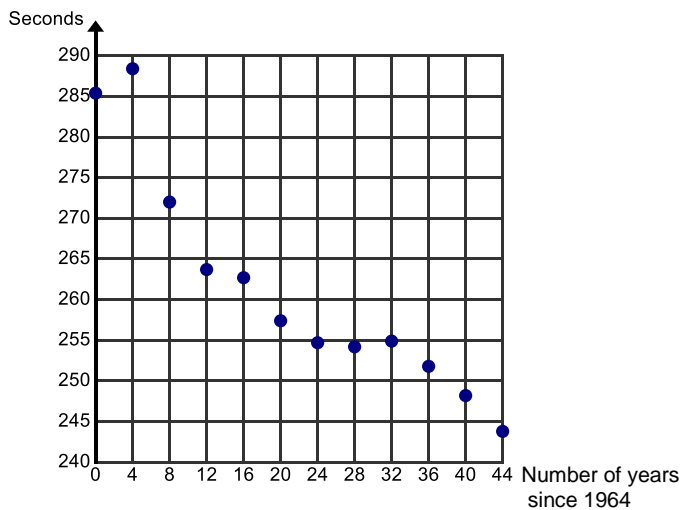
9. For Heidi’s Driver’s Education class, she finds data about the number of car accidents and fatalities (deaths) from car accidents for teens in the Western United States.

Fatalities vs. Accidents for Teen Drivers in 2006 in the Western United States



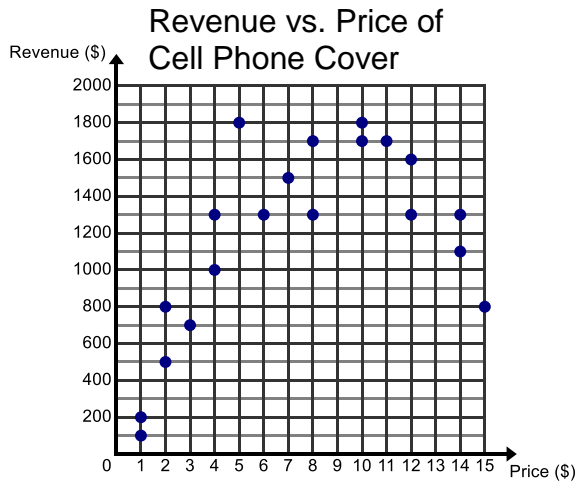
10. Winning times for the Men’s Individual Swimming Medley in the Olympics from 1964-2008 are in the plot below. Michael Phelps’ times are the last two entries.

400-Meter Individual Swimming Medley in Olympics (1964 – 2008)



Bonus: Research, collect, and analyze Olympic data for other events that interest you.

11. Hannah has a kiosk in the mall where she is selling Cell Phone Covers. She records how much money she makes (revenue) based on the price she charges for the covers.

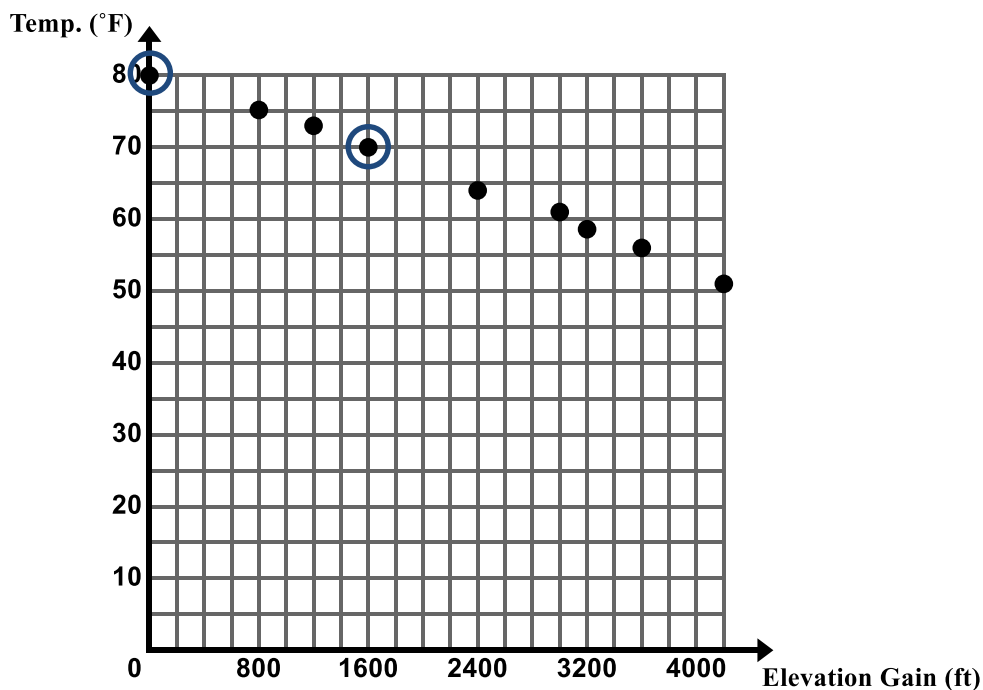


6.1d Self-Assessment: Section 6.1

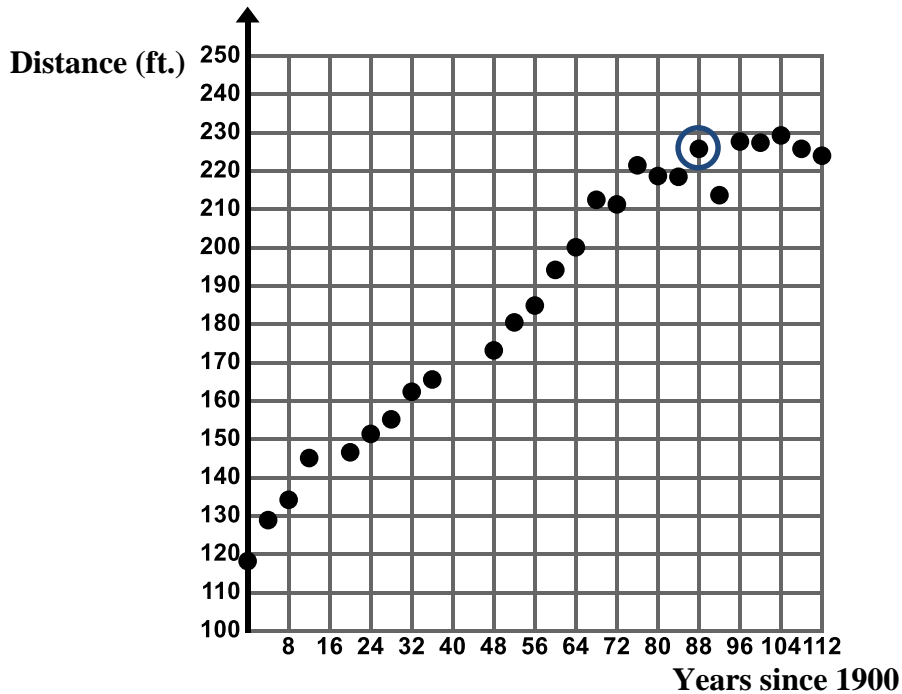
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Read and interpret a scatter plot.				
2. Construct a scatter plot for bivariate data.				
3. Describe patterns of association in a scatter plot.				

1. The following graph shows the temperature at the start of a popular hiking trail and at various points along the hike (for use with Skill/Concepts #1 and #3).
 - a. What do the circled data points represent in the context?
 - b. Describe any patterns of association you see in this scatter plot. Use the context to give possible explanations as to why these trends, patterns, and associations exist.

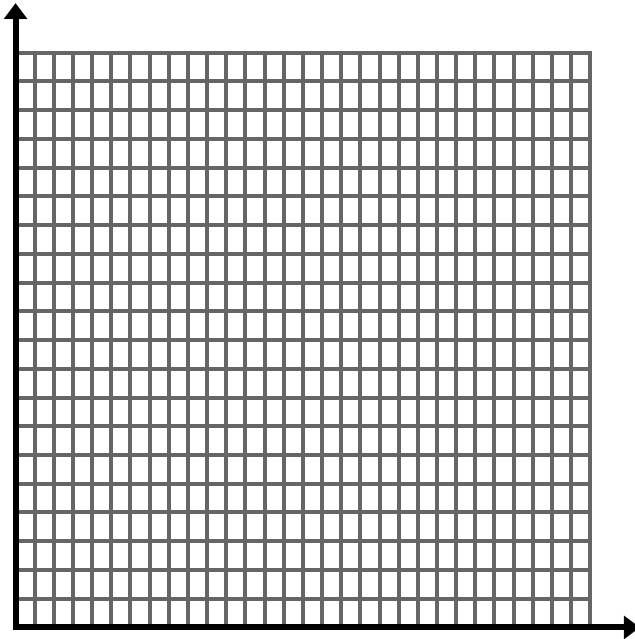


2. The following graph shows the distance, in feet, of the winning Olympic discus throws for men from 1900 to 2012 (for use with Skill/Concepts #1 and #3).
- What does the circled data point (88, 225.8) represent in the context?
 - Virgilijus Alekna of Lithuania holds the Olympic record for discus in the 2004 Summer Olympics in Athens. Circle this data point on the scatter plot.
 - Describe any patterns of association you see in this scatter plot. Use the context to give possible explanations as to why these trends, patterns, and associations exist.

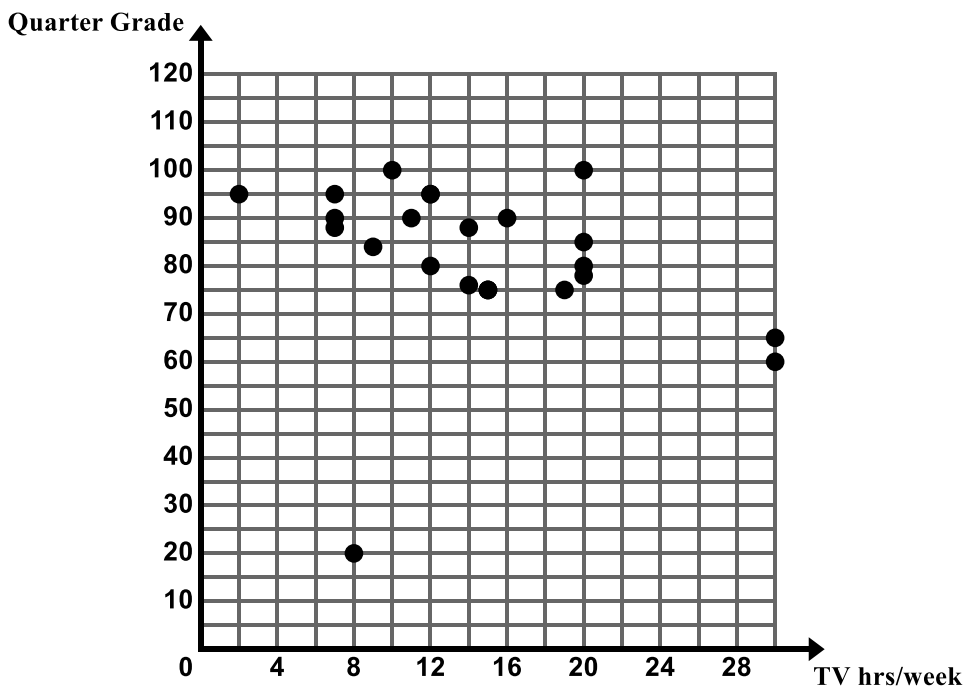


3. The following table shows the weight of an English Mastiff from birth to age 60 weeks (for use with Skill/Concepts #1, 2 and #3).
- Create a scatter plot of the data on the grid below.
 - Describe any patterns of association you see in this scatter plot. Use the context to give possible explanations as to why these trends, patterns, and associations exist.

Age (weeks)	0	4	8	9	10	11	12	16	20	24	28	32	36	40	44	48	52	56	60
Weight (lbs.)	1.4	15	29	33	36	40	45	60	80	100	125	140	155	165	170	175	180	185	188



4. Mr. Clark's math classes gathered data on the average number of hours of television a student watches each week and the student's final grade at the end of the quarter. The scatter plot below shows the data. (for use with Skill/Concepts #1 and #3).
- Describe any patterns of association you see in this scatter plot. Use the context to give possible explanations as to why these trends, patterns, and associations exist.
 - Can you think of a variable that when graphed with quarter grade would have a positive association?
 - Can you think of a different variable that when graphed with quarter grade would have a negative association?
 - Can you think of a variable that when graphed with quarter grade would have no apparent association?



Section 6.2 Construct a Linear Model to Solve Problems

Section Overview:

In this section, students continue to construct and interpret scatter plots. For scatter plots that suggest a linear association, students informally fit a straight line to the data and assess the model fit by judging the closeness of the data points to the line. They also analyze how outliers affect a line of best fit and reason about whether to drop outliers from a data set. Students then construct functions to model the data sets that suggest a linear association and use the functions to make predictions and solve real-world problems, noting that limitations exist for extreme values of x . Students will interpret the slope and y -intercept of the prediction function in context. Throughout the section students must use a critical eye, keeping in mind that most statistical data is subjective and has limitations. Students will also rely on their knowledge of the subject matter as they analyze the data.

Concepts and Skills to be Mastered:

By the end of this section students should be able to:

1. Draw a line of best fit for linear models.
2. Informally assess the model fit by judging the closeness of the data points to the line.
3. Write a prediction function for the line of best fit.
4. Explain the meaning of the slope and y -intercept of the prediction function in context.
5. Use the prediction function of a linear model to solve problems.


These practice standards are central to this entire section and chapter.



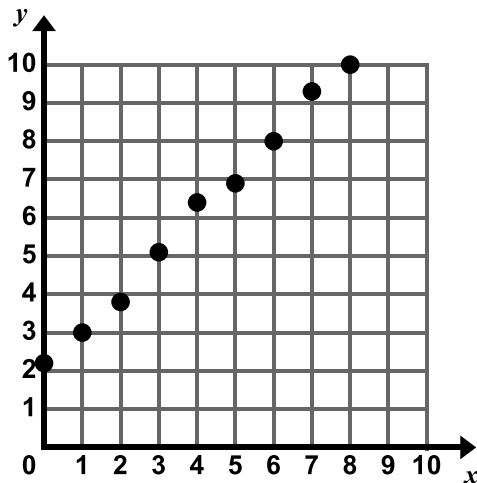
6.2a Classwork: Lines of Best Fit

Most real-world data does not fall perfectly on a line. However, if the data on a scatter plot resembles a line, we can fit a line to the data, write a function for the line, and use this function to solve problems and make predictions.

The line that you use to represent the data is called **the line of best fit**. We will refer to the function you write for the line of best fit as the **prediction function**. The most common way to find the line of best fit is to use the “eyeballing” technique. Simply try to draw a straight line that best fits the data.

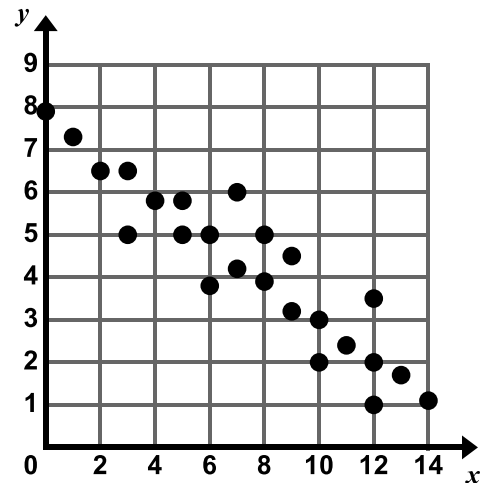
Directions: In #1 and 2, observe the data sets and take note of any associations you see, draw a line of best fit, write a prediction function, and use your function to predict the value of y when $x = 12$ and when $x = 100$. 

1.



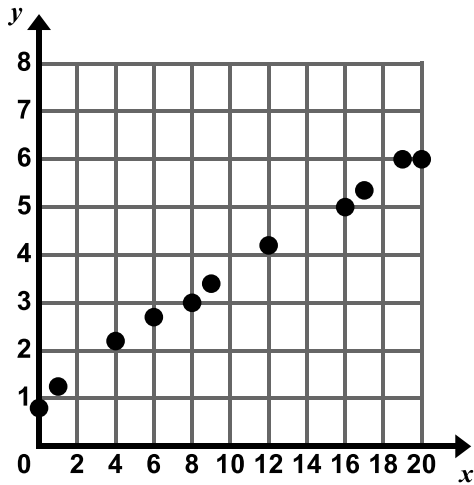
- Observations:
- Using a ruler, draw a line of best fit through the data points that captures the general trend of the data.
- Estimate the slope and y-intercept of your line.
 $m \approx$ _____ $b \approx$ _____
- Write a prediction function for the data set.
- Use your prediction function to find the value of y when $x = 12$ and when $x = 100$.

2.



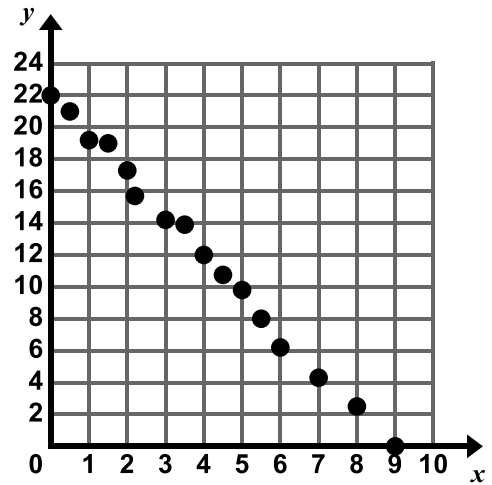
- Observations:
- Using a ruler, draw a line of best fit through the data points that captures the general trend of the data.
- Estimate the slope and y-intercept of your line.
 $m \approx$ _____ $b \approx$ _____
- Write a prediction function for the data set.
- Use your prediction function to find the value of y when $x = 12$ and when $x = 100$.

3.



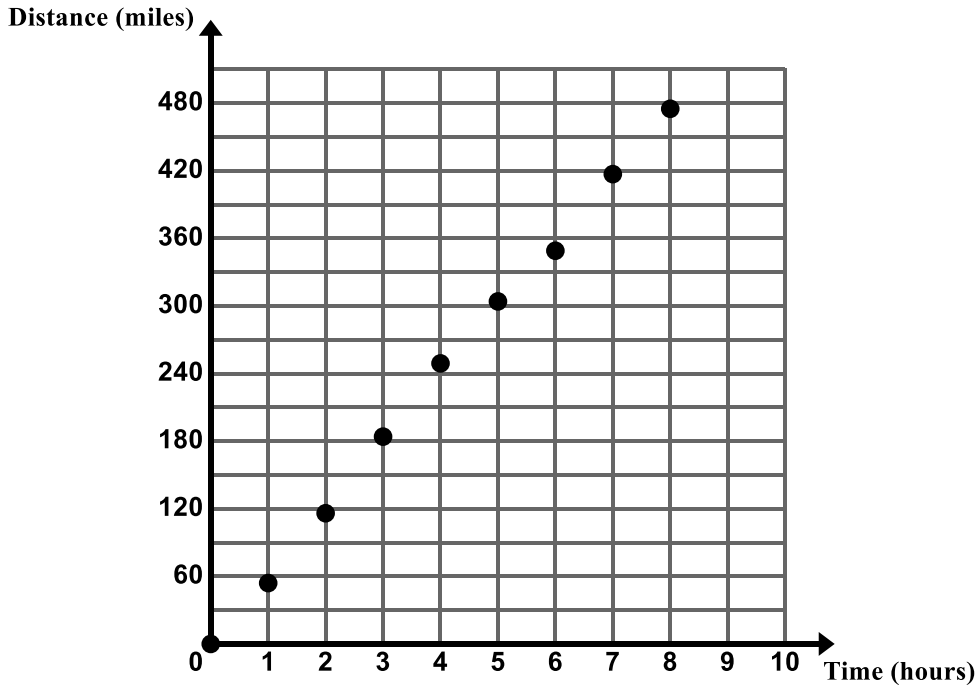
- Observations:
- Using a ruler, draw a line of best fit through the data points that captures the general trend of the data.
- Estimate the slope and y-intercept of your line.
 $m \approx \underline{\hspace{2cm}}$ $b \approx \underline{\hspace{2cm}}$
- Write a prediction function for the data set.
- Use your prediction function to find the value of y when $x = 12$ and when $x = 100$.

4.



- Observations:
- Using a ruler, draw a line of best fit through the data points that captures the general trend of the data.
- Estimate the slope and y-intercept of your line.
 $m \approx \underline{\hspace{2cm}}$ $b \approx \underline{\hspace{2cm}}$
- Write a prediction function for the data set.
- Use your prediction function to find the value of y when $x = 12$ and when $x = 100$.

5. Camilo and his family are taking a road trip. The graph below shows the total distance the family traveled over an eight hour period.



- a. Using a ruler, draw a line of best fit through the data points that captures the general trend of the data.
- b. Estimate the slope and y-intercept of your line.

$$m \approx \underline{\hspace{2cm}} \quad b \approx \underline{\hspace{2cm}}$$

- c. Write a prediction function for the data set.



- d. What does the slope represent in the context?



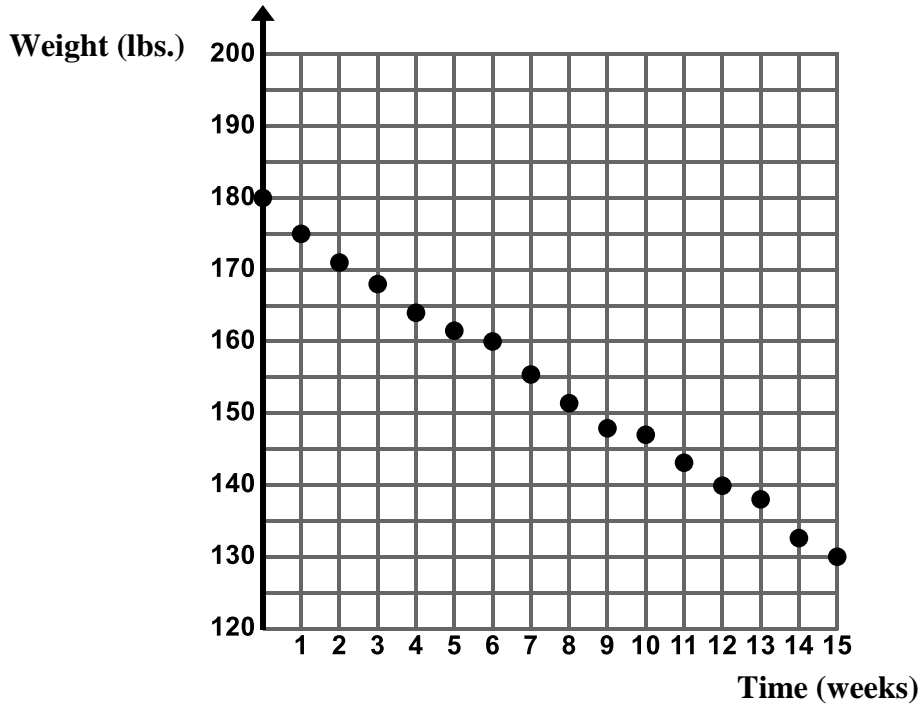
- e. What does the y-intercept represent in the context?



- f. Predict how far Camilo and his family will have driven after 10 hours if this trend continues.



6. The scatter plot below shows the weight, in pounds, of a person who is on a strict diet.

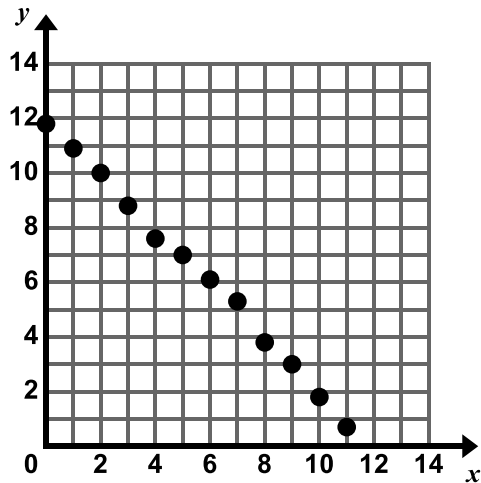


- Using a ruler, draw a line of best fit through the data points that captures the general trend of the data.
- Estimate the slope and y -intercept of your line.
 $m \approx$ _____ $b \approx$ _____
- Write a prediction function for the data set.
- What does the slope represent in the context?
- What does the y -intercept represent in the context?
- Predict this person's weight after 18 weeks if this trend continues.

6.2a Homework: Lines of Best Fit

Directions: In #1 and 2, observe the data sets and take note of any associations you see, draw a line of best fit, write a prediction function, and use your function to predict the value of y when $x = 12$ and when $x = 100$.

1.



a. Observations:

b. Using a ruler, draw a line of best fit through the data points that captures the general trend of the data.

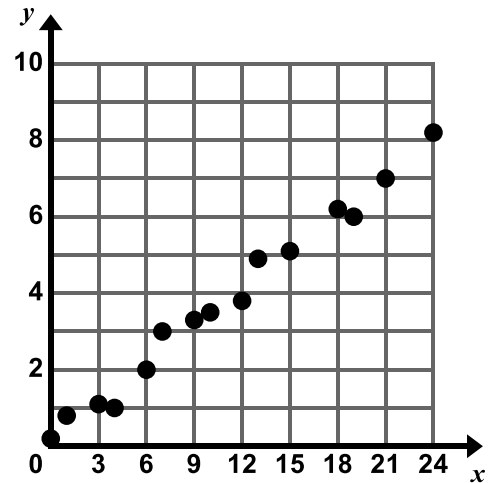
c. Estimate the slope and y-intercept of your line.

$$m \approx \underline{\hspace{2cm}} \quad b \approx \underline{\hspace{2cm}}$$

d. Write a prediction function for the data set.

e. Use your prediction function to find the value of y when $x = 12$ and when $x = 100$.

2.



a. Observations:

b. Using a ruler, draw a line of best fit through the data points that captures the general trend of the data.

c. Estimate the slope and y-intercept of your line.


$$m \approx \underline{\hspace{2cm}} \quad b \approx \underline{\hspace{2cm}}$$

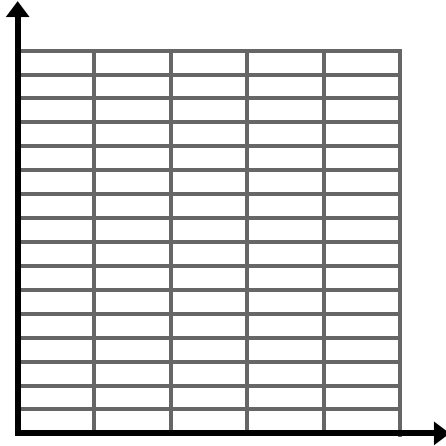
d. Write a prediction function for the data set.

e. Use your prediction function to find the value of y when $x = 12$ and when $x = 100$.

3. Use the table of data shown below to answer the questions that follow.

x	1	1	1	2	2	3	3	3	4	4	4	5
y	5	6	8	8	10	9	10	12	11	12	15	14

- a. Create a scatter plot of the data on the grid below. 

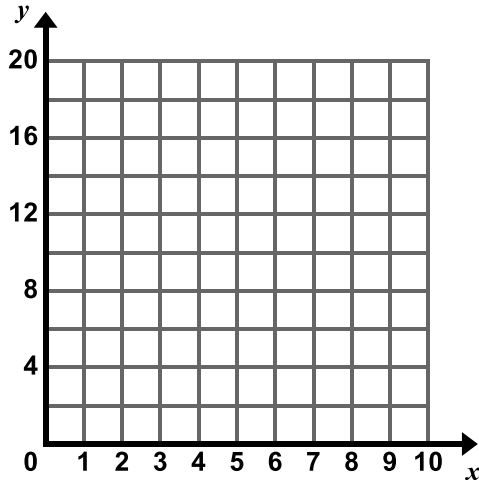


- b. Using a ruler, draw a line of best fit through the data points that captures the general trend of the data.
- c. Estimate the slope and y -intercept of your line.
- $m \approx$ _____ $b \approx$ _____
- d. Write a prediction function for the data set.

4. Use the table of data shown below to answer the questions that follow.

x	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7
y	16	15	12	13	12	11	10	8	7	6	5	4	3	2	0

a. Create a scatter plot of the data on the grid below.



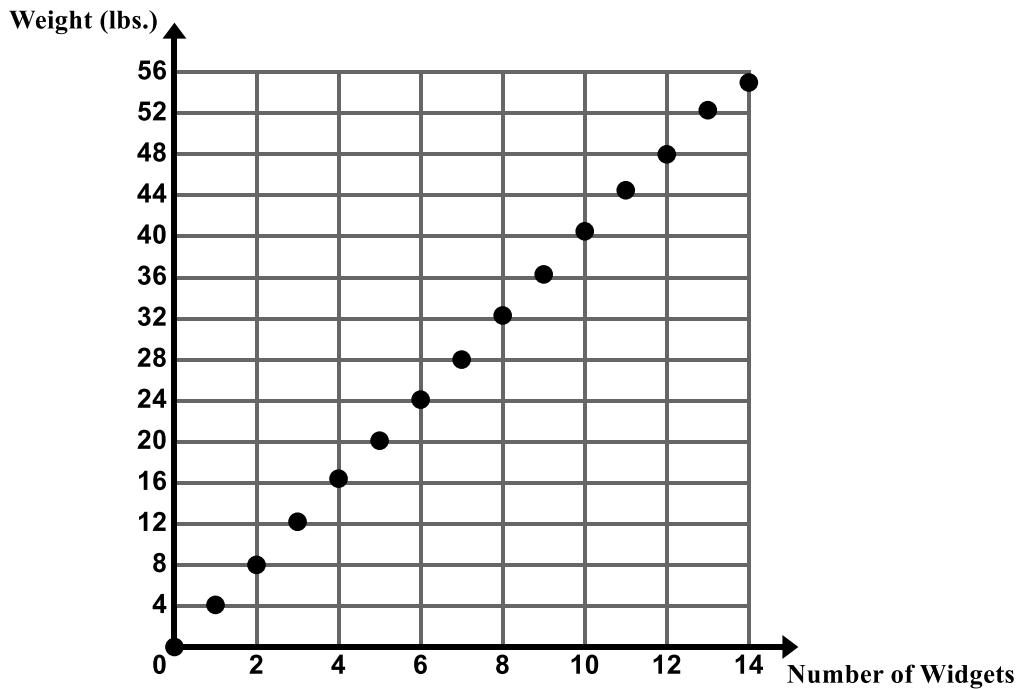
b. Using a ruler, draw a line of best fit through the data points that captures the general trend of the data.

c. Estimate the slope and y -intercept of your line.

$$m \approx \underline{\hspace{2cm}} \quad b \approx \underline{\hspace{2cm}}$$

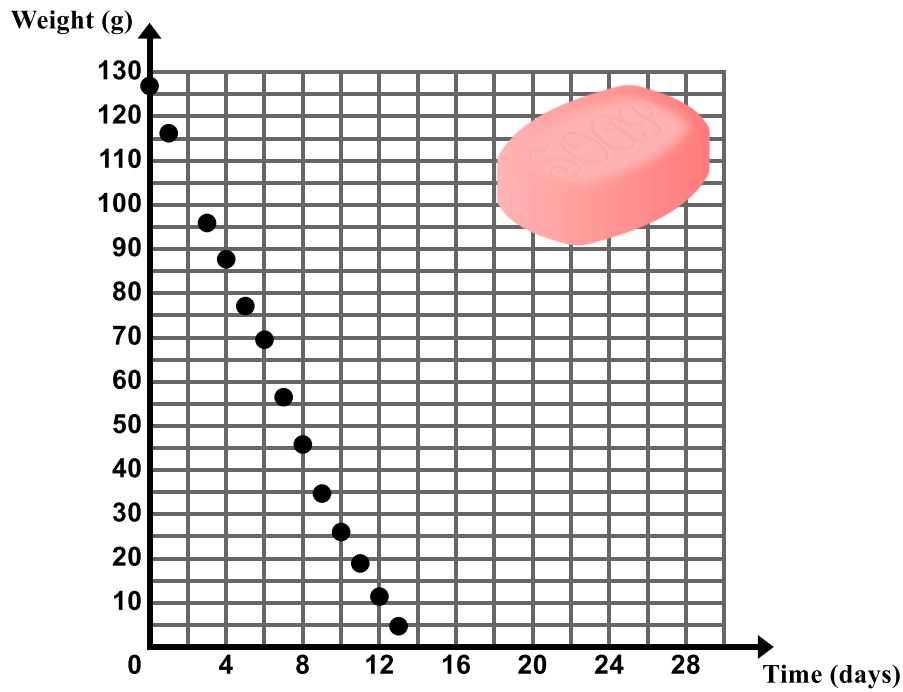
d. Write a prediction function for the data set.

5. Company XYZ makes and sells widgets. The following graph shows the weight of widgets and the number of widgets put on a scale.



- Using a ruler, draw a line of best fit through the data points that captures the general trend of the data.
- Estimate the slope and y-intercept of your line.
 $m \approx$ _____ $b \approx$ _____
- Write a prediction function for the data set.
- What does the slope represent in the context?
- What does the y-intercept represent in the context?
- Predict the weight of 50 widgets.

6. Chad was trying to determine how quickly his family goes through a bar of soap in the shower. He took the weight of the soap in the shower over a period of several days.

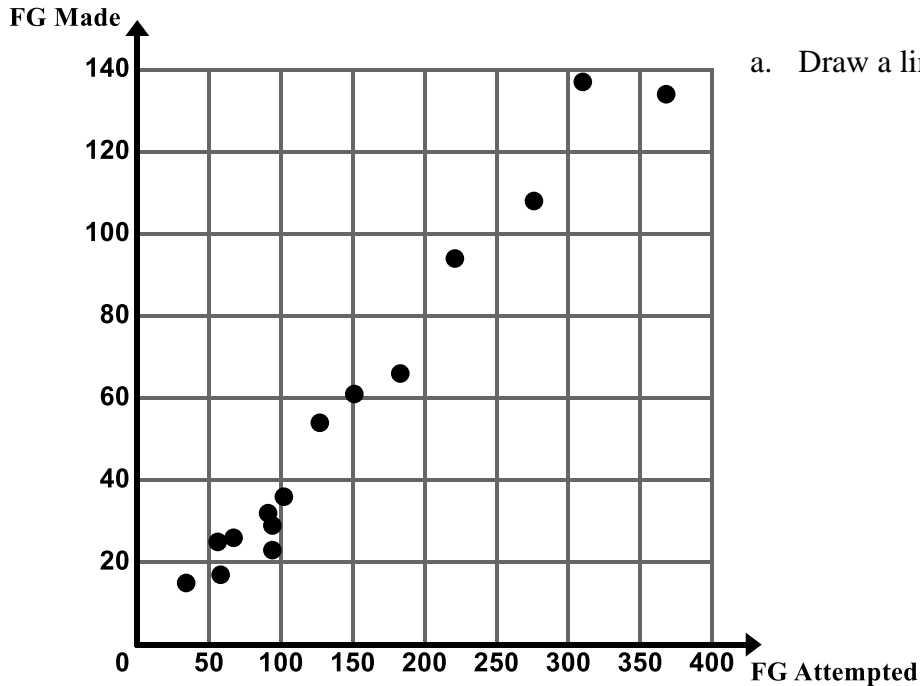


- Using a ruler, draw a line of best fit through the data points that captures the general trend of the data.
- Estimate the slope and y -intercept of your line.
 $m \approx$ _____ $b \approx$ _____
- Write a prediction function for the data set.
- What does the slope represent in the context?
- What does the y -intercept represent in the context?

6.2b Class Activity: Fit a Linear Model to Bivariate Data

Let's revisit some examples from section 1 where the two variables of interest had a linear association and determine a line of best fit for the data.



1. Once again refer back to Izumi's basketball statistics. Look at the scatter plot for Field Goals Made and Field Goals Attempted.



- a. Draw a line of best fit on the scatter plot.

- b. Write a prediction function for the line of best fit you drew.



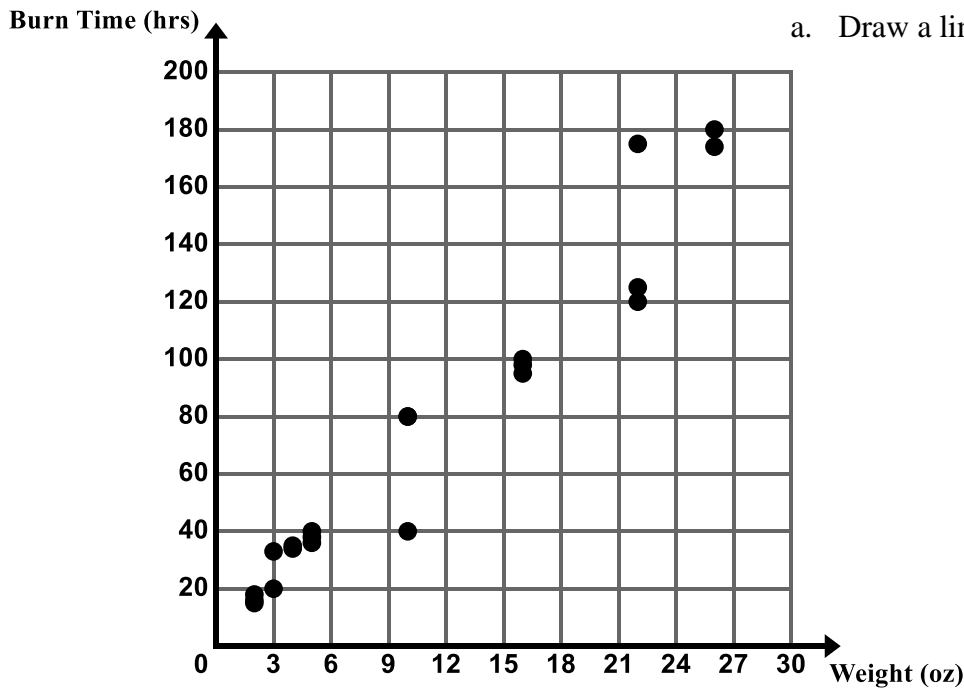
c. Explain the meaning of the slope and y-intercept in the context.  

d. Use your prediction function to predict the number of field goals a person would make if they attempted 500 field goals.

e. Use your prediction function to predict the number of field goals a person would make if they attempted 102 field goals.

f. Is the association between number of field goals attempted and number of field goals made strong or weak? Justify your answer.

2. The following scatter plot shows the burn time for candles of various weights.



a. Draw a line of best fit on the scatter plot.

b. Write a prediction function for the line of best fit you drew.

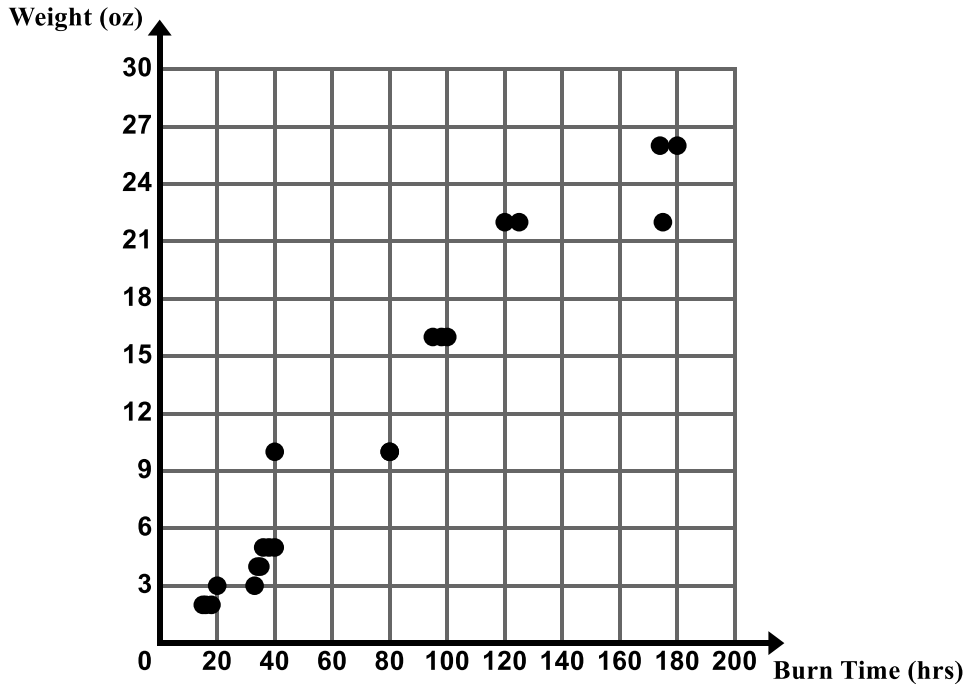
c. Explain the meaning of the slope and y-intercept in the context.

d. Use your prediction function to predict the burn time for a candle that weighs 40 ounces.

e. If candle burns out at 500 hours, predict how much the candle weighs.

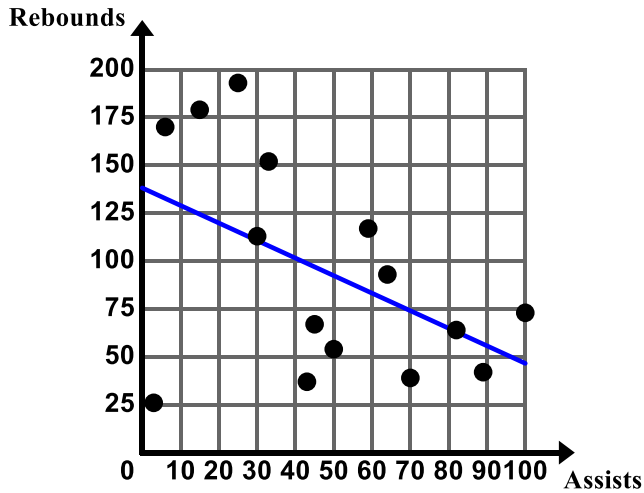
f. What do you think would happen if we changed the graph above so that burn time was on the x -axis and weight was on the y -axis? Would our data still resemble a line? What would happen to the slope and y -intercept of the line of best fit?

3. The following scatter plot shows the burn time for candles of various weights. This time, burn time has been graphed on the x -axis and weight has been graphed on the y -axis.



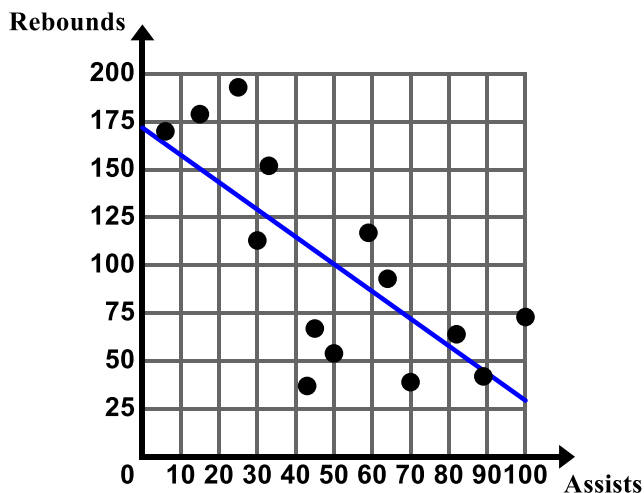
- Was your prediction on the previous page correct?
- Draw a line of best fit on the scatter plot.
- Write a prediction function for the line of best fit you drew.
- How does this new function compare to your equation in #2? What accounts for this change?


4. Software programs and graphing calculators can be used to draw lines of best fit. Izumi used a graphing calculator to generate a line of best fit for her data on assists and rebounds. The graph below shows the line of best fit generated by the calculator.



- a. After creating this line of best fit, Izumi decided that it might be best to drop the outlier (3, 26) from her data set. Is it reasonable for Izumi to drop the outlier from her data set? Why or why not? Assume this player joined the team midway through the season.

After dropping the outlier, Izumi used the calculator to generate a new line of best fit.



- b. Analyze the differences in the two lines. What did the outlier do to the line of best fit generated by the calculator? 
- c. Write a prediction function for the line of best fit generated by the calculator with the data set that does **not** include the outlier.

d. Explain the meaning of the slope and y-intercept in the context.



e. Use your function to predict the number of rebounds a random player would have if they made 110 assists throughout the season? 150 assists? Explain the limitations that the data exhibits.



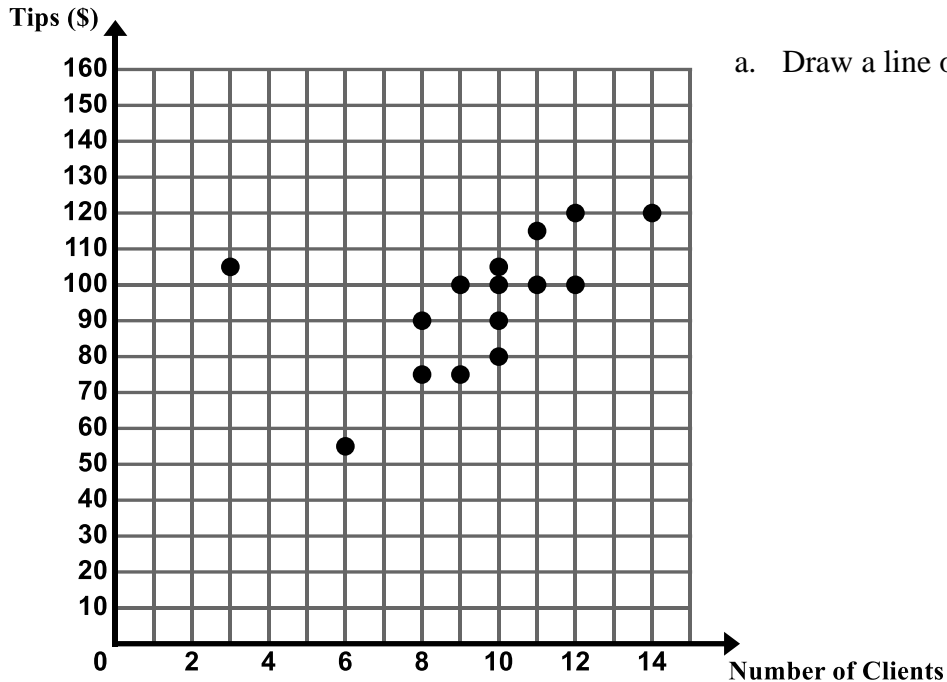
f. Similarly use your function to predict the number of assists a random player would have if they made 150 rebounds throughout the season.

5. Which scatter plot, the Field Goals Made vs. Field Goals Attempts or Rebounds vs. Assists, is more closely aligned with its line of best fit? Justify your answer. What does this tell us about the strength of each of the associations? What does this tell us about the accuracy of using each of the prediction functions to make predictions?

6.2b Homework: Fit a Linear Model to Bivariate Data

Directions: For the following problems, draw a line of best fit, write a prediction function, and use your function to make predictions. **Prior to drawing your line of best fit, determine whether you should remove any outliers from your data set.**

1. The following scatter plot shows the amount of money Jenny makes in tips based on how many clients she has in a day.



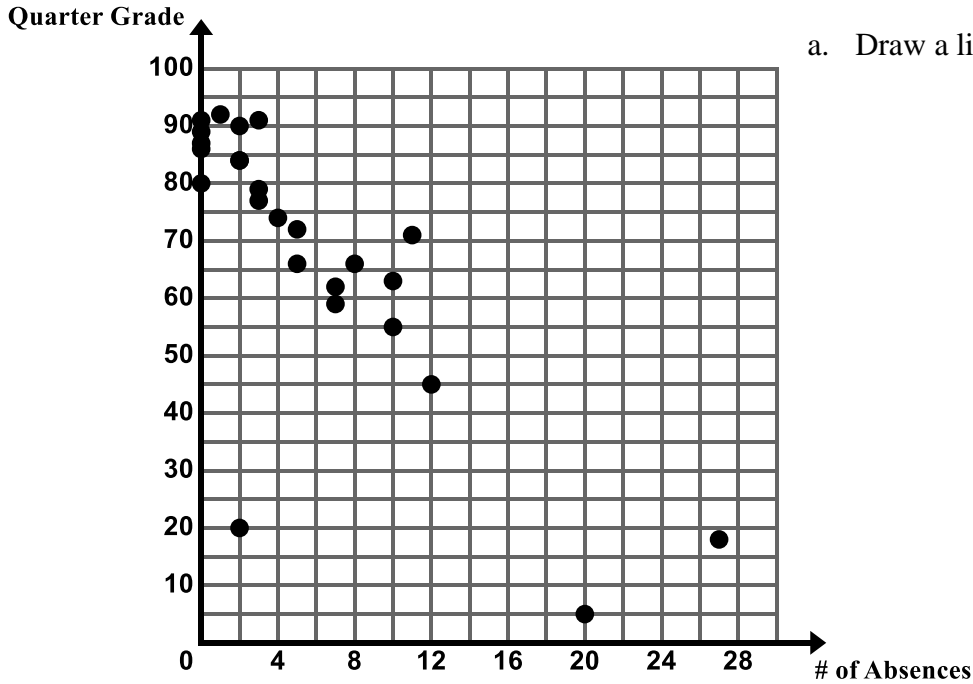
- a. Draw a line of best fit on the scatter plot.

- b. Write a prediction function for the line of best fit you drew.

- c. Explain the meaning of the slope and y-intercept in the context.

- d. Use your prediction function to predict the amount Jenny would make in tips if she had 18 clients in one day.

2. The following scatter plot shows the final quarter grade in Ms. Ganchero's math class for students vs. the number of times they are absent.



a. Draw a line of best fit on the scatter plot.

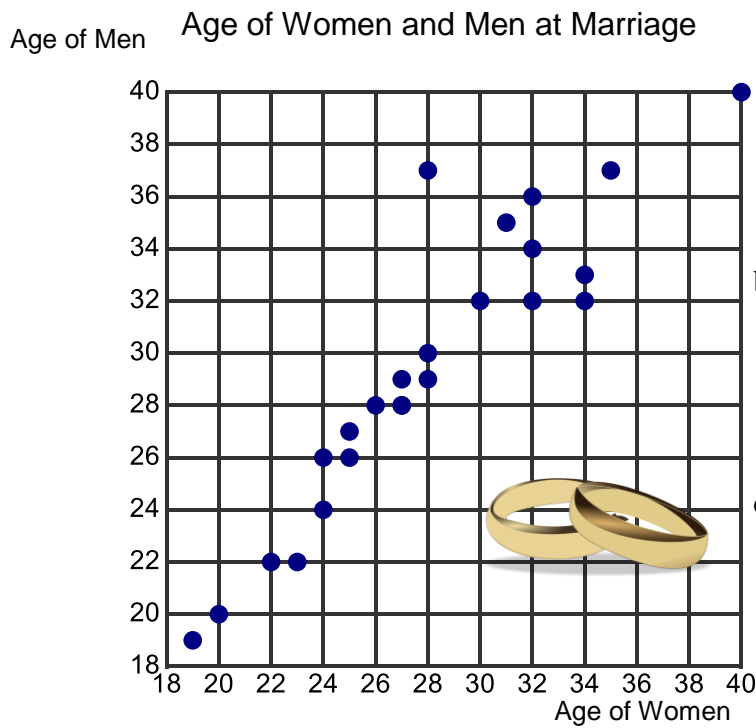
b. Write a prediction function for the line of best fit you drew.

c. Explain the meaning of the slope and y-intercept in the context.

d. Use your prediction function to predict the final grade of a student who is absent 16 times.

e. Use your prediction function to predict how many times a student is absent who receives a final grade of 5 in the class.

3. Bethany is interested in the relationship between the age of when men and women get married. She surveys 24 couples and asks them the age in which they got married for the first time. A scatter plot of her data is below.



- a. Describe the association between the two variables. Circle any clusters in the data. Put a star by any points that appear to be outliers.

- b. Provide an explanation for any clusters of data or outliers.

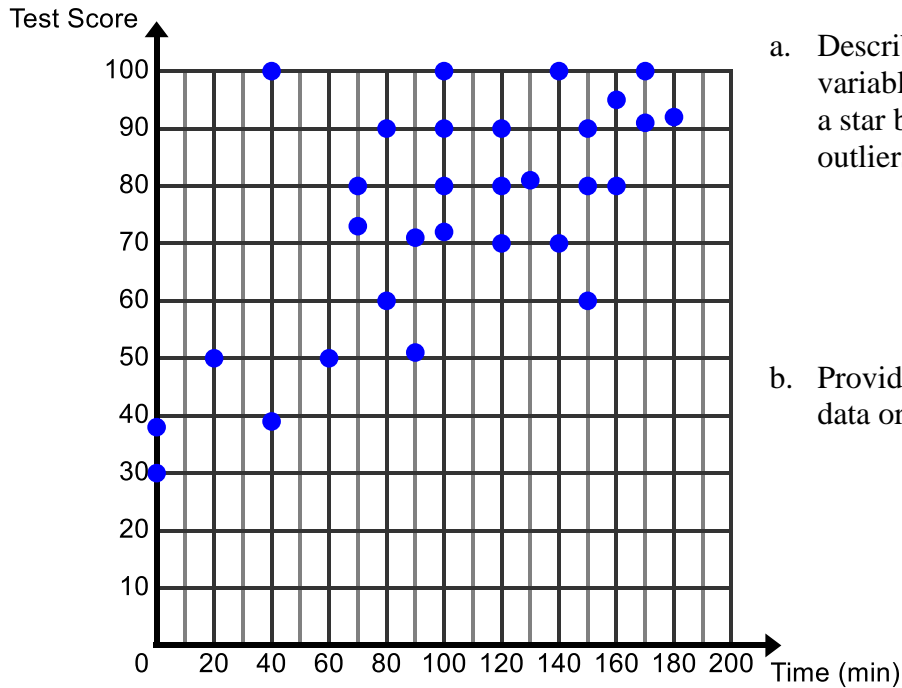
- c. Draw a line of best fit on the scatter plot.

- d. Write a prediction function for the line of best fit you drew.

- e. Use your prediction function to predict the age of a man when he gets married if the woman that he marries is 38.

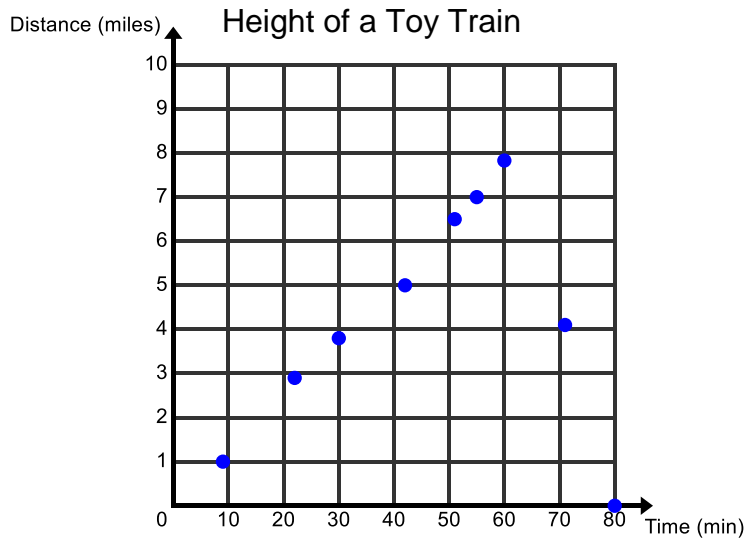
4. Jenna is interested in the association between the time spent studying for a test and the score that is earned. She surveys 30 people about the time they spent studying for a test and the score that they earned on the test. Her data is in the scatter plot below.

Test Score vs. Time Spent Studying



- Describe the association between the two variables. Circle any clusters in the data. Put a star by any points that appear to be outliers.
 - Provide an explanation for any clusters of data or outliers.
 - Draw a line of best fit on the scatter plot.
- Write a prediction function for the line of best fit you drew.
 - Explain the meaning of the slope and y -intercept of your line of best fit in the context.
 - Use your prediction function to predict the score for a person who studies for 160 minutes.
 - Compare and contrast the prediction calculated using the equation with the actual data points of the people who studied for 160 minutes.
 - Does the association between these two variables appear to be weak or strong? Provide an explanation regarding why the strength is this way.

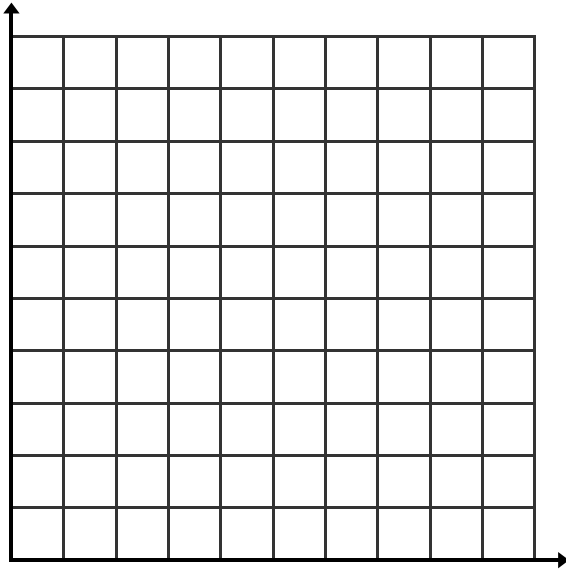
5. A scatter plot given below is about the height of a toy train attached to a weather balloon. A GPS (global positioning system) records the height of the toy train about every ten minutes that it is in the air. When the train reaches the stratosphere the weather balloon pops.



- a. What kind of association exists for this data?
- b. Would it be feasible to draw a line of best fit for this data? Why or why not.

6. The table gives data relating the number of oil changes every two years to the cost of car repairs.
- a. Plot the data on the graph provided, with the number of oil changes on the horizontal axis. You will need to define your own scale.

Oil Changes	3	5	2	3	1	4	6	4	3	2	0	10	7
Repair Costs	\$300	\$300	\$500	\$400	\$700	\$400	\$100	\$250	\$450	\$650	\$600	\$0	\$150



- b. Write a sentence describing the association between the number of oil changes and the cost of car repairs. Is the association weak or strong?
- c. Are there any outliers or clusters that affect the data?
- d. Draw a line of best fit for the data. Assess how well the line fits the data.
- e. What is the slope of the line of best fit and what does it represent?
- f. What is the y-intercept of the line and what does it represent?

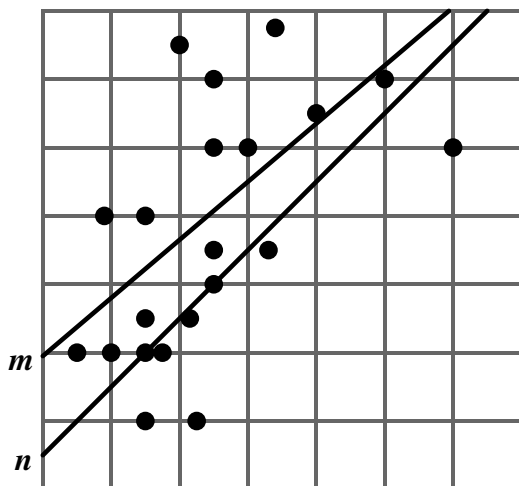
- g. Write a prediction function in slope-intercept form that you could use to predict the cost of repairs, y , for any number of oil changes, x . Compare your prediction with that of a partner.
- h. Use your prediction function to predict how much a person would spend on car repairs if they were to get 8 oil changes. Compare your prediction with that of a partner.
- i. If a person spent \$1,000 dollars on car repairs how many oil changes would you expect them to have?
- j. Based off of this data what would you recommend as the ideal number of oil changes to get every two years.

6.2c Self-Assessment: Section 6.2

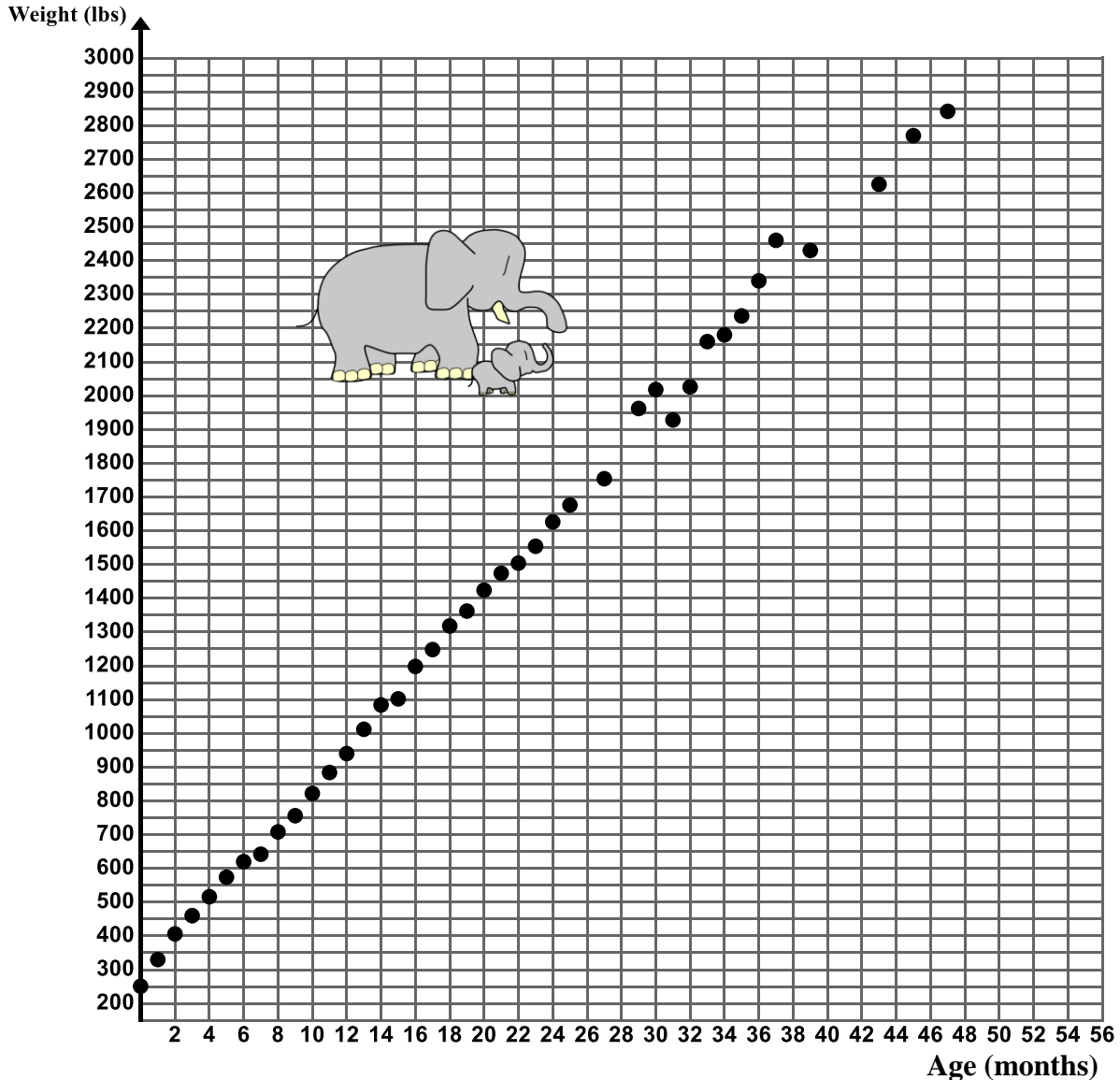
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Draw a line of best fit for linear models.				
2. Informally assess the model fit by judging the closeness of the data points to the line.				
3. Write a prediction function for the line of best fit.				
4. Explain the meaning of the slope and y-intercept of the prediction function in context.				
5. Use the prediction function of a linear model to solve problems.				

1. Which line, m or n , is the best fit for the data? Justify your answer (for use with Skill/Concepts #2).



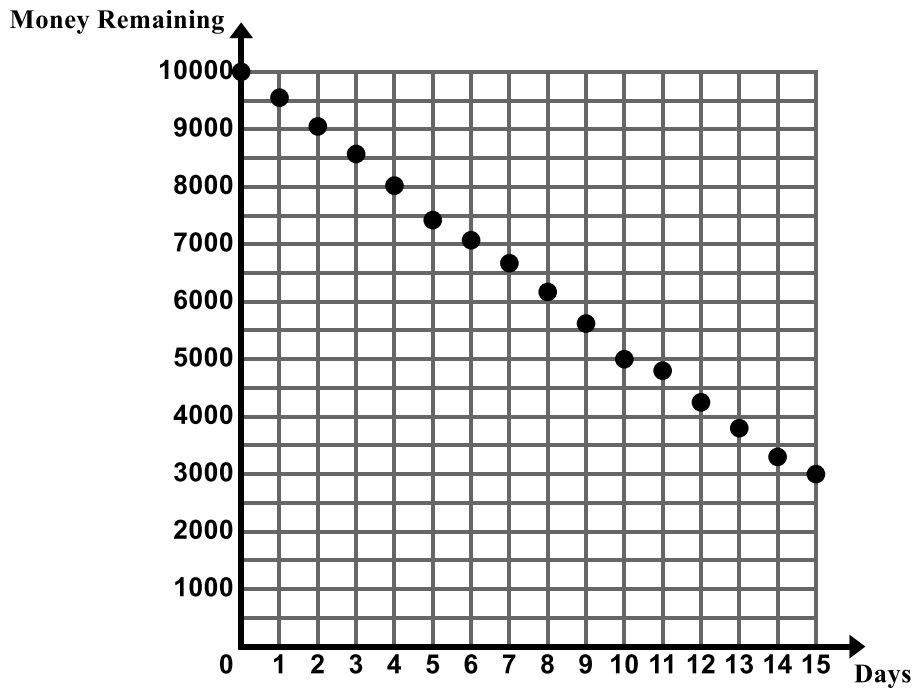
2. The following scatter plot show the weight of Zuri, a female African elephant born at Utah’s Hogle Zoo on August 10, 2009 (for use with Skill/Concepts #1 – 5).



- Describe the association between the two variables.
- Draw a line of best fit on the scatter plot.
- Write a prediction function for the line of best fit you drew.
- Explain the meaning of the slope and y-intercept of your line of best fit in the context.
- Use your prediction function to predict the weight of Zuri at 56 months.
- Adult female African elephants typically weigh between 8,000 and 11,000 pounds. If Zuri’s growth rate continues to follow the pattern shown in the graph above, how long will it take for her to be full grown?

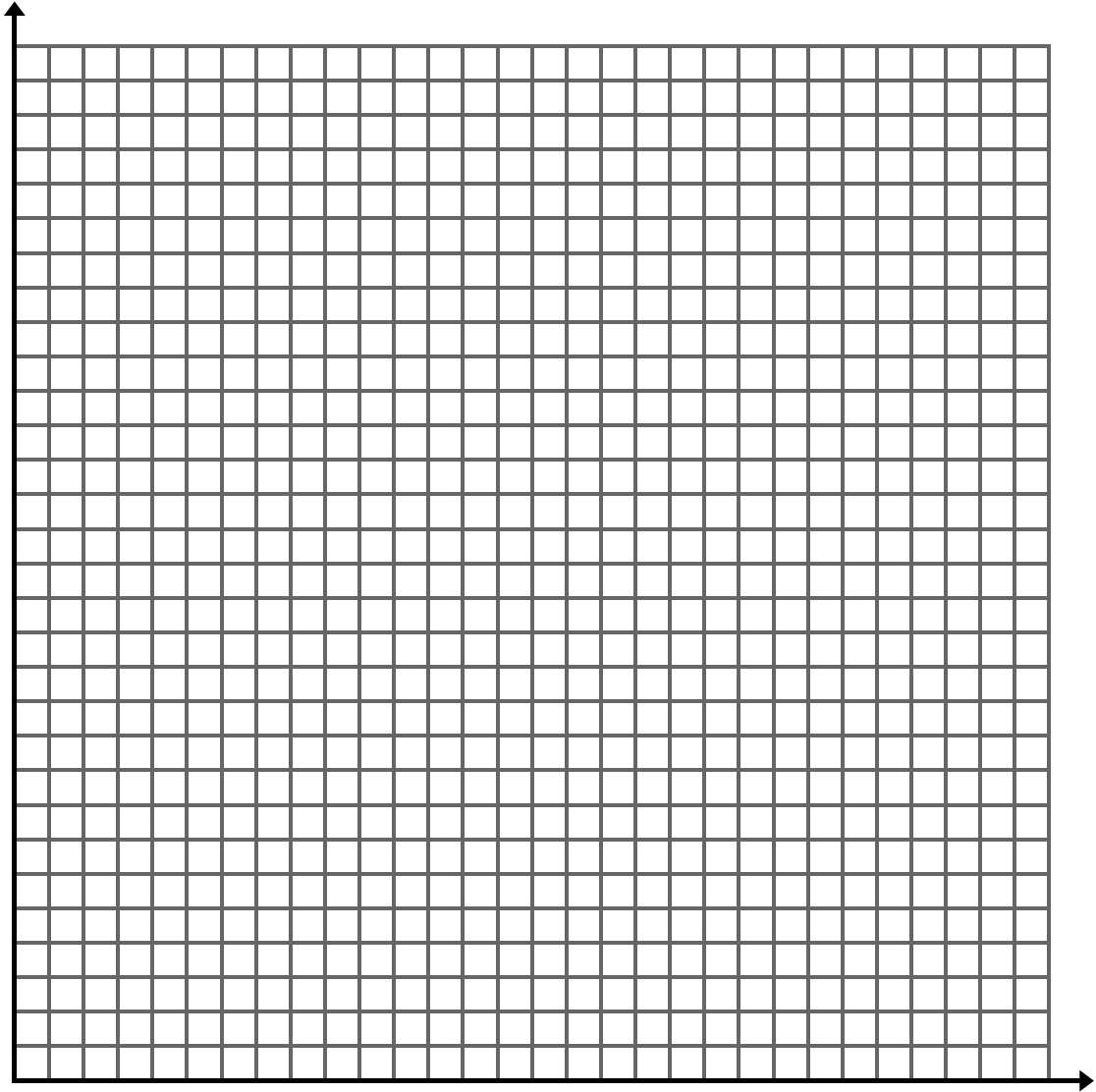
Source: Data provided by Utah’s Hogle Zoo

3. The Burgess family took a 15-day vacation to southern California and visited several popular theme parks during their trip. The graph below shows the amount of money the Burgess family had remaining at the end of each day of their trip (for use with Skill/Concepts #1 – 5).



- Describe the association between the two variables.
 - Draw a line of best fit on the scatter plot.
 - Write a prediction function for the line of best fit you drew.
- d. Explain the meaning of the slope and y-intercept of your line of best fit in the context.
- e. Use your prediction function to predict how much money the Burgess family will have at the end of Day 18 if they extend the length of their trip.

4. Gather data to determine whether there is an association between the height of a person and the length of their arm span. The arm span of a person is the length from one end of an individual's arms (measured at the fingertips) to the other end when the arms are raised parallel to the ground at shoulder height (for use with Skill/Concepts #1 – 5).
- Create a scatter plot of the data on the grid below.
 - Describe any patterns of association you see in this scatter plot. Use the context to give possible explanations as to why these trends, patterns, and associations exist.
 - If the plot suggests a linear association, draw a line of best fit and write a prediction function.
 - If the plot suggests a linear association, explain the meaning of the slope and y-intercept in the context.



Section 6.3 Construct and Interpret Two-Way Frequency Tables to Analyze Categorical Data

Section Overview:


At the beginning of this section students are introduced to a new type of random variable – a categorical random variable. Up to this point in the chapter, students have been studying quantitative random variables. Quantitative random variables have a cardinal numerical value. Categorical random variables are those that represent some quality or name. Categorical data is often represented and summarized in a two-way frequency table. In this section, students learn what a two-way frequency table is and how to read it. They complete two-way frequency tables by filling in missing data. As the section progresses, students begin to formally interpret the frequency tables. They calculate and analyze relative frequencies (for rows, columns, and the entire table) to describe possible associations between the two variables and use these associations to make decisions. Finally, students conduct a survey of their own involving categorical random variables, summarize their data in a two-way frequency table, and analyze the data to determine if an association exists between the two variables of interest.

Concepts and Skills to be Mastered:

By the end of this section students should be able to:

1. Read and understand a two-way frequency table.
2. Construct a two-way frequency table for categorical data.
3. Calculate and analyze relative frequencies (for rows, columns, and the entire table) to describe possible associations between the two variables and to make decisions.

6.3a Class Activity: Construct Two-Way Frequency Tables using Categorical Data

There are two different types of random variables when looking at bivariate data; **quantitative random variables** and **categorical random variables**. So far in this chapter, we have been studying **quantitative random variables**. Quantitative random variables can be counted or measured. For example, we can count the number of assists and rebounds that a player on Izumi's team had during the team. We can count the amount that Jenny made in tips each day. We can measure a person's shoe size and their height. We can measure the amount of time it takes to say a tongue twister. A **categorical random variable** represents a quality or a name. Suppose we were interested in determining if there is an association between a person's gender and whether or not that person has pierced ears. We would interview people and classify them as male or female and as yes (ears pierced) or no (ears not pierced). Suppose we were interested in whether a person's favorite color is associated with their favorite holiday. We would categorize a person according to their favorite color (red, orange, yellow, etc.) and their favorite holiday (Christmas, Thanksgiving, Halloween, Hanukah, etc.) 

Directions: Determine if the following random variables represent data that is Quantitative or Categorical.

1. Gender of babies born in the Riverton Hospital for the month of June
2. Thickness of the plastic for various types of water bottles
3. Favorite ice cream flavor chosen from the following options; chocolate, vanilla, or strawberry
4. The number of pages you can read of your favorite book before you fall asleep

In the previous sections we summarized and displayed quantitative data using a **scatter plot**. In this section, we will summarize and display categorical bivariate data using a **two-way frequency table**. A two-way frequency table is "two-way" because each bivariate data entry is composed of an ordered pair from two categorical random variables.

Suppose we were interested in whether there is an association between a person's gender (male/female) and whether or not they smoke (smoker/non-smoker). The following ordered pairs are possible outcomes for our experiment:

(female, non-smoker) (female, smoker) (male, non-smoker) (male, smoker)

The table is a "frequency" table because the cell entries count the number of data points that fall into each combination of categories.

In this section, we will construct two-way frequency tables and analyze the tables to determine if there is an association between the two variables of interest.

5. Carlos enjoys spending time with his friends. He feels sad when one of his friends cannot hang out with him. Often when one of his friends cannot hang out with him it is because they are either doing their chores or they cannot stay out late at night. Carlos notices that it tends to be the same group of friends that have curfews on school nights who also have chores to do at home. He wonders, “In general, do students at my school who have chores to do at home tend to also have curfews at night?”

Carlos decides to conduct an experiment to help answer his question. He randomly surveys 52 students at his school, asking each student if they have a curfew and if they have to do household chores. He organizes his findings into the frequency table below.

	Has A Curfew	No Curfew	Total
Has Chores	26	9	
No Chores	5	12	
Total			

Directions: Use the table to answer each question below.

- How many students have a curfew and have chores?
- How many students have no curfew and have chores?
- How many students have no curfew and no chores?

It is also possible to calculate the frequencies for “Total” column and “Total” row. These frequencies represent the total count of one variable at a time.

- Find the frequencies for the Total column and Total row by adding up the numbers in each column and row. Write these numbers in the table above.
- How many of the students surveyed have chores?
- How many of the students surveyed have a curfew?

You can also calculate how many total students that were surveyed by adding up the frequencies in the “Total” row and “Total” column.

- Add the entries in the Total row and the Total column and put this number in the cell in the bottom left corner. Does this number match how many students that Carlos said he was going to survey?

6. Emina loves to eat tomatoes from her garden in Salt Lake City. She asked her friend Renzo, “Don’t you just love tomatoes?” Renzo crinkled his nose and replied, “Ew, tomatoes gross me out! When I see them in the grocery store, I just keep on walking.” Renzo’s response prompted Emina to think, “I don’t buy tomatoes at the grocery store either, because I grow them in my garden. The tomatoes from my garden are delicious, whereas grocery store tomatoes look less appealing to me. I wonder if there is an association between enjoying tomatoes and having a garden at home.”

She decides to survey 100 randomly selected Salt Lake City vegetable eating residents and asks each of them two questions: 1. Do you primarily obtain your vegetables at the grocery store (including food pantry), the farmer’s market, or your home garden (assume they grow tomatoes in their home garden)? Do you like tomatoes? Her results are summarized in the table below.

	Grocery Store	Farmer’s Market	Home Garden	Total
Likes Tomatoes	50	4	12	
Dislikes Tomatoes	30	1	3	
Total				

- Fill in the frequencies for the Total column and Total row in the table.
- Check to make sure that you found the above frequencies correctly by finding the total number of people surveyed.
- How many people get their tomatoes at the farmer’s market and dislike tomatoes?
- How many people get their tomatoes from a home garden and like tomatoes?
- How many people get their tomatoes from the grocery store?
- How many people like tomatoes?



Emina is not quite sure if her data suggests an association between enjoying tomatoes and having a garden. We will further investigate this relationship in the next section.

7. Use the given information to complete the two-way frequency table about the eating habits of 595 students at Copper Ridge Middle School.
- 190 male students eat breakfast regularly out of 320 total males surveyed.
 - 295 students do not eat breakfast regularly
 - 165 females do not eat breakfast regularly

a. Fill in the missing information.

	Male	Female	Total
Eat breakfast regularly			
Do not eat breakfast regularly			
Total			

- b. How many females total were surveyed?
- c. How many people surveyed eat breakfast regularly?
- d. How many people total were surveyed?
- e. How many males surveyed do not eat breakfast regularly?
- f. How many females surveyed eat breakfast regularly?
- g. What percentage **of the total number of people surveyed** eat breakfast regularly?
- h. What percentage **of the females surveyed** eat breakfast regularly?
- i. What percentage **of the people who eat breakfast regularly** are male?
- j. What percentage **of the total number of people surveyed** are females who do not eat breakfast regularly?
- k. Make up your own problem similar to the problems in parts g. – j. Have a partner answer your question.
- l. Make up a different problem similar to the problems in parts g. – j. Have a partner answer your question.

8. The data given in the table below is about modes of transportation to and from school at Brookside High School.

a. Fill in the missing information.

	Walk	Car	Bus	Cycle	Total
Male		28			129
Female	46		12	17	92
Total		45	27	69	

b. How many males ride their bikes to school?

c. How many females take the bus to school?



d. How many females were surveyed?

e. How many students were surveyed?

f. What percentage **of the total number of people surveyed** walk to school?

g. What percentage **of the total number of people surveyed** are females that bike to school?

h. What percentage **of the males surveyed** cycle to school?

i. Make up your own problem similar to the problems in parts f. – h. Have a partner answer your question.

j. Make up a different problem similar to the problems in parts f. – h. Have a partner answer your question.

9. Keane collects data about the number of people who own a smart phone and if they also own an MP3 player. He gives you the following information.

- 25 people surveyed owned smart phones
- 20 people that own a smart phone do not own an MP3 player
- 9 people do not own smart phones but they do own an MP3 player
- 24 people do not own an MP3 player

a. Design and complete a two-way frequency table to show the display the data.

b. How many people did Keane survey?

c. How many people own a smart phone and an MP3 player?

d. How many people own an MP3 player?

10. Tamra wondered if there is an association between age and favorite flavor of ice cream (choices: chocolate, strawberry, and vanilla). She surveyed 200 children in different age ranges. The table below shows the results of her survey.

Tamra gives you the following information.

- $\frac{1}{2}$ of the children surveyed chose chocolate as their favorite flavor
- 25% of the children surveyed were in the age range of 8 – 12 years old
- $\frac{2}{5}$ of the children surveyed were in the age range of 13 – 17 years old
- 50% of the children in the age range of 3 – 7 years old chose chocolate as their favorite flavor
- 50 children chose strawberry as their favorite flavor

a. Complete the two-way frequency table to display the data.

	Chocolate	Vanilla	Strawberry	Total
Ages 3 – 7				
Ages 8 – 12	25		12	
Ages 13 – 17			12	
Total				200



6.3a Homework: Construct a Two-Way Frequency Table

1. In Miss Marble's music collection there are...
 - 208 songs in total
 - She has 150 songs in her "Workout Music" playlist
 - 162 of the songs in the total music collection are Pop songs
 - 38 Classical songs are in her "Music for Studying" playlist

a. Complete the table for about the Miss Marble's music collection.

	Workout Music	Music for Studying	Totals
Classical			
Pop			
Totals			

- b. How many total songs are in her "Music for Studying" playlist?
- c. How many classical songs are in her "Workout Music" playlist?
- d. What percentage **of songs in the collection** are pop?
- e. What percentage **of songs in the collection** are for studying?
- f. What percentage **of the classical music** is music for studying?
- g. What percentage **of songs in the collection** are classical music for studying?

2. Laura was driving home from school and texting her mom at the same time. She did not notice that she was speeding and a police officer pulled her over and gave her a traffic citation. She wonders if there is an association between people who regularly text while driving and if they have received a traffic citation in the last 2 years. She conducts a survey among 50 drivers and records some data in the table below.

a. Fill in the missing information in the frequency table below.

	Regularly Texts While Driving	Never Texts While Driving	Totals
No traffic citations			
Has received a traffic citation in the last two years.	18	5	
Totals	25		50

b. How people regularly text while driving?

c. How many people have no traffic citations and regularly text while driving?

3. Paul tosses a dice and spins a coin 150 times as part of an experiment. He records 71 heads and a six 21 times. On 68 occasions, he gets neither a head nor a six. Complete the table.

	Six	Not a Six	Totals
Head			
Tail			
Totals			

a. How many times did he toss a tails and a six?

b. How many times did he toss a heads?



4. The 300 members of a tennis club are classified by gender and whether or not they are over 18. You are given the following information about the members of the club.
- 36 are under 18 and female
 - 159 are over 18 and male
 - 180 are male

a. Design and complete a two-way table to show this information.

- b. How many members of the club are female?
- c. How many member of the club are over 18 and female?
- d. What percentage **of the members** are female?
- e. What percentage **of the members** are age 18 and over?
- f. What percentage **of the members** are males under age 18?
- g. What percentage **of the members age 18 and over** are male?

5. Susan loves social media and is interested in at what age people prefer different social media outlets. She groups people into the following age groups, middle school age, high school age, and college age. She then asks 75 people what their favorite form of social media is, Twitter, Instagram, or Facebook.

a. Fill in the missing information in the frequency table below.

	Facebook	Instagram	Twitter	Totals
Middle School		5	3	
High School	10	10		27
College		7		24
Total	31		22	

- b. How many Middle School aged people were surveyed?
- c. How many people prefer Instagram?
- d. How many college age people prefer Facebook?
- e. How many high school aged people prefer Twitter?

6. Julie wants to know if there is an association between gender and the type of movie a person prefers. She surveys 500 people and discovers the following.

- 35% of the people surveyed prefer comedy movies
- $\frac{3}{10}$ of the people surveyed prefer action movies
- 95 people surveyed prefer romance movies
- Of the females surveyed, $\frac{2}{7}$ prefer romance movies
- 35% of the males surveyed prefer comedy movies




a. Complete the two-way frequency table to display the data.

	Romance	Comedy	Action	Drama	Totals
Male					
Female				52	280
Total					

6.3b Class Activity: Interpret Two-Way Frequency Tables



Now that we are comfortable making a two-way frequency table we are going to see what conclusions we can draw from them. 

1. The table below displays the data Julie gathered on gender and the type of movie a person prefers. Use **numerical evidence** from the table to answer the questions below.

	Romance	Comedy	Action	Drama	Totals
Male	15	77	100	28	220
Female	80	98	50	52	280
Total	95	175	150	80	500

- a. Julie is showing a movie at a party at which males and females will be present. Which type or types of movies should Julie show?
- b. Julie is showing a movie at a party at which only males will be present. Which type or types of movies should Julie show?
- c. Julie is showing a movie at a party at which only females will be present. Which type or types of movies should Julie show?
- d. Determine whether the following statement is true or false based on the data in the table. Put a “T” on the line if it is true and an “F” on the line if it is false. **Use numerical evidence to support your answer.**

_____ Males and females have an equal likelihood of choosing comedy movies.

2. The table below show the results of the data Tamra collected on age and favorite flavor of ice cream (choices: chocolate, strawberry, and vanilla). Use **numerical evidence** from the table to answer the questions below.

	Chocolate	Vanilla	Strawberry	Total
Ages 3 – 7	35	9	26	70
Ages 8 – 12	25	13	12	50
Ages 13 – 17	40	28	12	80
Total	100	50	50	200

- a. Tamra is in charge of buying ice cream for a pre-school carnival. Which type or types of ice cream should she purchase?
- b. Tamra is in charge of buying ice cream for a neighborhood picnic at which all ages of children will attend. What type or types of ice cream should she buy?
- c. Determine whether the following statements are true or false based on the data in the table. Put a “T” on the line if the statements are true and an “F” on the line if the statements are false. Use **numerical evidence to support your answer.**

_____ Children in all of the age ranges have an equal likelihood of choosing chocolate.

_____ Children in the age ranges 8 – 12 and 13 – 17 have an equal likelihood of choosing strawberry.

_____ As students get older they tend to like vanilla more.

3. Refer back to Carlos' data regarding chores and curfew.

	Has A Curfew	No Curfew	Totals
Has Chores	26	9	35
No Chores	5	12	17
Totals	31	21	52

- a. Analyze the two-way table. What arguments can you make about the data? Use numerical evidence to support your answer.
- b. Is there an association between kids having chores and having a curfew? Use numerical evidence to support your answer.

4. Let's revisit Emina and her tomatoes.

	Grocery Store	Farmer's Market	Home Garden	Totals
Likes Tomatoes	50	4	12	66
Dislikes Tomatoes	30	1	3	34
Totals	80	5	15	100

a. Analyze the two-way table. What arguments can you make about the data? Use numerical evidence to support your answer.

b. Is there an association between growing your own tomatoes (having a home garden) and whether or not you like tomatoes?

5. In the previous section you made a frequency table about gender and eating breakfast.

	Male	Female	Totals
Eat breakfast regularly	190	110	300
Do not eat breakfast regularly	130	165	295
Totals	320	275	595

a. Is there an association between gender and whether or not a person eats breakfast regularly.

6. Eddy wanted to determine whether there is an association between gender and whether or not a person has their ears pierced. He collected data from a random sample of young adults ages 13 – 18.

	Has Pierced Ears	Does not have Pierced Ears	Totals
Male	19	71	90
Female	84	4	88
Totals	103	75	178

a. Is there an association between gender and whether or not a person has their ears pierced?

6.3b Homework: Interpret Two-Way Frequency Tables

1. **Modes of Transportation:** Recall the data gathered from Brookside High School about modes of transportation and gender. Use **numerical evidence** from the table to answer the questions below.

	Walk	Car	Bus	Cycle	Total
Male	34	28	15	52	129
Female	46	17	12	17	92
Total	80	45	27	69	221

Directions: Answer the following questions about the data collected:

- What percentage **of students surveyed** take the bus to school?
- What percentage **of students surveyed** are males who walk to school?
- Based off of the table above what is the most popular mode of transportation for the sample population.
- What is the preferred method of transportation for females? Use numerical evidence to support your answer.
- What is the preferred method of transportation for males? Use numerical evidence to support your answer.
- Is taking the bus more common with males or females?

2. **Cell Phones and MP3 Players:** Recall the two-way table you made in the previous section about Keane’s data on Cell Phones and MP3 Players below. Use **numerical evidence** from the table to answer the questions below.

	Owens a smart phone	Does not own a smart phone	Total
Owens an MP3 player	5	9	14
Does not own an MP3 Player	20	4	24
Total	25	13	38

- What percentage **of the people surveyed** own a smart phone?
- What percentage **of the people surveyed** do not own a smart phone but own an MP3 player?
- What percentage **of the people surveyed** own a smart phone and an MP3 player?
- Is there an association between owning a smart phone and owning an MP3 player? Use numerical evidence to support your answer.

3. **Music:** Use the two-way frequency table given below about Miss Marbles’ music playlists to answer the following questions.

	Workout Music	Music for Studying	Totals
Classical	8	38	46
Pop	142	20	162
Totals	150	58	208

- Is there an association between what Miss Marble is doing (exercising or studying) and what she is listening to? Use numerical evidence to support your answer.

4. **Texting While Driving:** Use the two-way given below about texting while driving to answer the questions that follow.

	Regularly Texts While Driving	Never Texts While Driving	Totals
No traffic citations	7	20	27
Has received a traffic citation in the last two years.	18	5	23
Totals	25	25	50

- a. What percentage of people regularly text while driving?
- b. What percentage of people have not received a traffic citation in the last two years?
- c. What percentage of people regularly text and have received a traffic citation in that last two years?
- d. What percentage of people who never text have no traffic citations?
- e. What percentage of people who regularly text while driving have received a traffic citation in the last two years?
- f. Out of all the people who have received a traffic citation in the last two years, what percentage of them text regularly?
- g. What type of association exists between texting while driving and receiving traffic citations? Use numerical evidence to support your answer.

5. **Social Media:** Use the two-way frequency table given below to answer the questions that follow.

	Facebook	Instagram	Twitter	Totals
Middle School	16	5	3	24
High School	10	10	7	27
College	5	7	12	24
Total	31	22	22	75

a. Analyze the two-way table. What arguments can you make about the data? Use numerical evidence to support your answer.

6.3c Class Activity: Conduct a Survey

Is there an association between whether a student plays a sport and whether he or she plays a musical instrument? **This problem was adapted from an Illustrative Mathematics task.*

To investigate these questions, ask 20 students in your class to answer the following two questions:

1. Do you play a sport? (yes or no)
2. Do you play a musical instrument? (yes or no)
3. Record the answers in the table below.

Student Name	Sport?	Musical Instrument?
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

4. Summarize the data into a clearly labeled frequency table.

Use the tables that you made above to answer the following questions.

5. What percentage of students play a sport and a musical instrument?
6. What percentage of students that play a sport also play a musical instrument?
7. What percentage of students that do not play a sport play a musical instrument?
8. What percentage of musical instrument players do not play a sport?
9. Based on the class data, do you think there is an association between playing a sport and playing an instrument? Use numerical evidence to support your answer.

6.3d Self-Assessment: Section 6.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Read and understand a two-way frequency table.				
2. Construct a two-way frequency table for categorical data.				
3. Calculate and analyze relative frequencies (for rows, columns, and the entire table) to describe possible associations between the two variables and to make decisions.				

1. Lisa is the owner of a local gym and is trying to determine if there is an association between gender and a person's favorite workout class. She gathers data and organizes it into the two-way frequency table shown below.

	Zumba	Spinning	Weight Lifting	Step	Total
Male	2		25	5	
Female		16			
Total	45	30	35		150

- Complete the table.
- How many females chose Zumba as their favorite workout class?
- How many males chose spinning as their favorite workout class?
- How many females were surveyed?
- How many people were surveyed?
- What percentage of the people surveyed chose step as their favorite class?
- What percentage of the people who chose spinning as their favorite class are male?
- What percentage of the males surveyed chose weight lifting as their favorite class?
- Based on the data, do you think there is an association between gender and a person's favorite workout class? Use numerical evidence to support your claim.
- Are there any other conclusions you can draw from the table? Use numerical evidence to support your claims.

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Chapter 7: Rational and Irrational Numbers (3 weeks)

Utah Core Standard(s):

- Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. (8.EE.2)
- Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers, show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. (8.NS.1)
- Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. (8.NS.2)

Academic Vocabulary: square, perfect square, square root, $\sqrt{\quad}$, cube, perfect cube, cube root, $\sqrt[3]{\quad}$, quadratic equation, cubic equation, inverse operation, decimal expansion, repeating decimal, terminating decimal, rational number, irrational number, truncate, decimal approximation, real number, real number line

Chapter Overview:

In 8th grade, students begin to think more carefully about the real line by asking the question, “Is there a number associated with every point on the line?” Up to this point, students have worked only with rational numbers, numbers they generated from iterations of a whole unit or portions of whole units. Part of their work included identifying a point on the real line associated with each rational number. Students explore the question posed above through an activity that has them constructing the lengths of non-perfect squares, thus introducing students to irrational numbers.

The chapter starts by having students examine the relationship between the area of a square and its side length. This activity introduces students to the idea of what it means to take the square root of a number. Additionally, students begin to surface ideas about the limitations of rational numbers. In the activity, students construct physical lengths of irrational numbers and begin to realize that we cannot find an exact numerical value for these numbers. Students then use the knowledge gained from this activity to simplify square roots and solve simple quadratic and cubic equations (i.e. $x^2 = 40$ and $x^3 = 64$).




In the last section, students deepen their understanding of what an irrational number is and in the process solidify their understanding of rational numbers. They realize that even though they cannot give an exact numerical value for the lengths of non-perfect squares, they can transfer these lengths to a number line to show the exact location of these numbers. Once the numbers are placed on the real line, students can approximate the value of these irrational numbers and compare their value to rational numbers. At the end of the chapter, students learn additional methods for approximating the value of irrational numbers to desired degrees of accuracy, estimate the value of expressions containing irrational numbers, and compare and order rational and irrational numbers.

Connections to Content:

Prior Knowledge: Students have worked a great deal with rational numbers up to this point. They have defined and worked with the subsets of rational numbers. They have represented rational numbers on a number line, expressed rational numbers in different but equivalent forms, and operated with rational numbers. Students have also worked a great deal with slope, have an understanding of area, and know how to find the area of polygons and irregular shapes which will help them to access the tilted square material.

Future Knowledge: Later in this book, students will study exponent rules and deepen their understanding of the connection between taking the square root of a number and squaring a number. In subsequent courses, students will continue to extend their knowledge of the number system even further. For example, students will learn about complex numbers as a way to solve quadratic equations that have a negative discriminant. They will also continue to work with irrational numbers, learning how to operate on irrational numbers.

MATHEMATICAL PRACTICE STANDARDS:

	<p>Make sense of problems and persevere in solving them.</p>	<p>A hospital has asked a medical supply company to manufacture intravenous tubing (IV tubing) that has a minimum opening of 7 square millimeters and a maximum opening of 7.1 square millimeters for the rapid infusion of fluids. The medical design team concludes that the radius of the tube opening should be 1.5 mm. Two supervisors review the design team’s plans, each using a different estimation for π.</p> <p>Supervisor 1: Uses 3 as an estimation for π</p> <p>Supervisor 2: Uses 3.1 as an estimation for π</p> <p>The supervisors tell the design team that their designs will not work. The design team stands by their plans and tells the supervisors they are wrong. Who is correct and why? Recall that the formula for the area of a circle is $A = \pi r^2$.</p> <p><i>In this problem, students realize the effects of approximating the value of irrational numbers. They must decide which estimation of π is appropriate for the given situation, appreciating that the precision of the estimation may have profound impact on decisions people make in the real world.</i></p>
	<p>Reason abstractly and quantitatively.</p>	<p>The decimal $0.\overline{3}$ is a repeating decimal that can be thought of as 0.33333... where the “...” indicates that the 3s repeat forever. If they repeat forever, how can we write this number as a fraction? Here’s a trick that will eliminate our repeating 3s.</p> <p><i>To solve this problem, students create and solve a system of linear equations. The skills and knowledge they learned about systems of equations become an abstract tool that allows students to write repeating decimals as fractions, proving that they do in fact fit the definition of a rational number.</i></p>
	<p>Construct viable arguments and critique the reasoning of others.</p>	<p>Directions: The table below contains statements about rational and irrational numbers. If the statement is true, put a check in the box. If the statement is not true, write a correct statement.</p> <ul style="list-style-type: none"> • You can always use a calculator to determine whether a number is rational or irrational by looking at its decimal expansion. • The number 0.256425642564 ... is rational. • You can build a perfect cube with 36 unit cubes. • If you divide an irrational number by 2, you will still have an irrational number. <p><i>Students must have a clear understanding of rational and irrational numbers to assess whether the statements are true or false. If the statement is flawed, students must identify the flaw, and construct a statement that is true. Due to the fact that there are several possible ways to change the statements to make them true, students must communicate their statements to classmates, justify the statements, and question and respond to the statements made by others.</i></p>



Model with mathematics.

People often wonder how far they can see when they're at the top of the tallest buildings such as the Empire State Building, The Sears Tower in Chicago, etc. The farthest distance you can see across flat land is a function of your height above the ground. If h is the height in meters of your viewing place, then d , the distance in kilometers you can see, can be given by this formula: $d = 3.532\sqrt{h}$

The CN Tower in Toronto, Canada is 555 meters tall. It is near the shore of Lake Ontario, about 50 kilometers across the lake from Niagara Falls. Your friend states that on a clear day, one can see as far as the falls from the top of the Tower. Are they correct? Explain your answer.

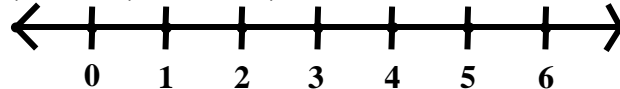
The formula shown above is a model for the relationship between the height of a building and the distance one can see. Students use this model along with their knowledge of square roots to solve problems arising in everyday life.



Use appropriate tools strategically.

Directions: Show the length of the following numbers on the number line below. Use the grid on the following page to construct lengths where needed and transfer those lengths onto the number line. Then answer the questions that follow. **Note:** On the grid, a horizontal or vertical segment joining two dots has a length of 1. On the number line, the unit length is the same as the unit length on the dot grid.

A: $\sqrt{25}$ B: $\sqrt{2}$ C: $\sqrt{8}$ D: $2\sqrt{2}$ E: $\sqrt{5}$ F: $2\sqrt{5}$



1. Use the number line to write a decimal approximation for $\sqrt{2}$.
2. Would 1.41 be located to the right or to the left of $\sqrt{2}$ on the number line?
3. Describe and show how you can put $-\sqrt{2}$ on the number line. Estimate the value of this expression.
4. Describe and show how you can put $(2 + \sqrt{2})$ on the number line. Estimate the value of this expression.
5. Describe and show how you can put $(2 - \sqrt{2})$ on the number line. Estimate the value of this expression.
6. Describe and show how you can put $2\sqrt{2}$ on the number line. Estimate the value of this expression.

To solve this problem, students use dot paper to construct physical lengths of irrational numbers. They can then transfer these segments to the number line using patty (or tracing) paper. Once on the number line, students can use these tools (number line, dot paper, patty paper, constructed segments) to approximate the value of given expressions (i.e. $(2 + \sqrt{2})$).



Attend to precision.

Use the following approximations and calculations to answer the questions below. Do not use a calculator.

Approximation: π is between 3.14 and 3.15

Calculations:

$$3.1^2 = 9.61$$

$$3.2^2 = 10.24$$

$$3.16^2 = 9.9856$$

$$3.17^2 = 10.0489$$

Put the following numbers in order from **least to greatest**.

$\sqrt{10}$, $3\frac{1}{10}$, $3.\bar{1}$, π , side length of a square with an area of 9

Find a number between $3\frac{1}{10}$ and $3.\bar{1}$.

Find a number between 3.1 and $\sqrt{10}$.

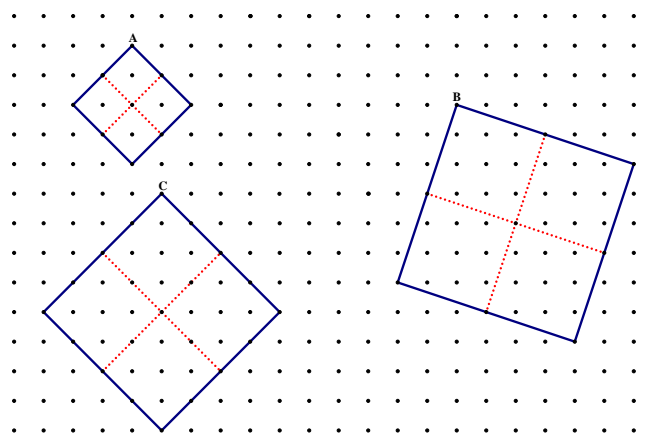
This task demands mastery of the topics learned in the chapter. Students must have a very clear understanding of square roots, repeating decimals, and irrational numbers. They must closely analyze the decimal expansions (approximations) of the numbers as well as the calculations given to be able to compare and order the numbers.




Look for and make use of structure.

Square A shown below has an area of 8 square units. Determine the following measures:

- The area of one of the smaller squares that makes up Square A
- The side length of one of the smaller squares that makes up Square A
- The side length of the large square A (written 2 different ways)



This problem allows students to use structure to understand why $\sqrt{8}$ is the same as $2\sqrt{2}$. They can see the equivalence in the concrete model. A square with an area of 8 (see Square A) has a side length of $\sqrt{8}$ units. This side length is comprised of 2 smaller, congruent segments that

		<p><i>each measure $\sqrt{2}$ units as they are each the side length of a square with an area of 2. This concrete representation builds a conceptual understanding for students as we then move to the algorithm for simplifying square roots.</i></p>
	<p>Look for and express regularity in repeated reasoning.</p>	<p>Change the following rational numbers into decimals without the use of a calculator.</p> $\frac{1}{7}$ <p><i>This problem allows students to understand why the decimal expansion of a rational number either always terminates or repeats a pattern. Working through this problem, and others, students begin to understand that eventually the pattern must repeat because there are only so many ways that the algorithm can go. Once a remainder repeats itself in the division process, the decimal expansion will start to take on a repeating pattern. Students should see this when they begin repeating the same calculations over and over again and conclude they have a repeating decimal.</i></p>

7.0 Anchor Problem: Zooming in on the Number Line

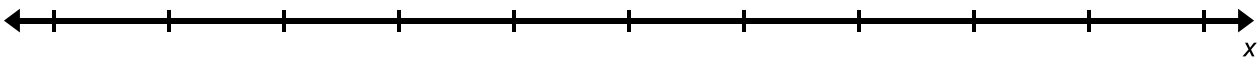


Directions: Place the following sets of numbers on the number lines provided and label each point. You will need to decide where to place 0 and the measure of the intervals for each problem.

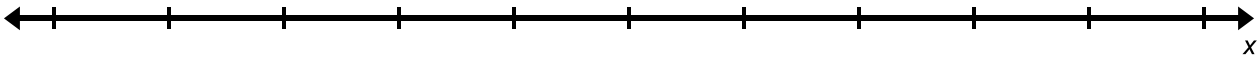
A: 3 B: 4 C: 3.5 D: -4 E: -5 F: -4.5



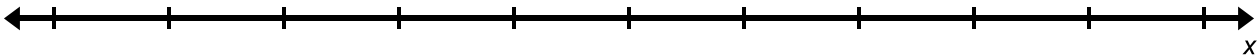
L: $-\frac{1}{4}$ M: $\frac{3}{4}$ N: $-1\frac{1}{2}$ O: 1.75 P: -2



V: $\frac{1}{10}$ W: $\frac{3}{10}$ X: $\frac{1}{2}$ Y: $\frac{9}{10}$ Z: $\frac{10}{10}$



H: 0.1 I: 0.2 J: 0.15 K: 0.11 L: 0.101



A: 3.1

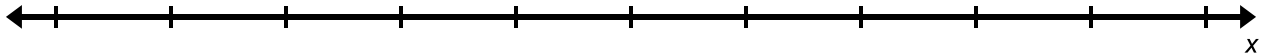
B: 3.2

C: 3.15

D: 3.11

E: 3.105

F: 3.111



Directions: Refer to the number line above to answer the questions that follow.

1. Are there other numbers you can place between 3.1 and 3.11? If yes, find a number.
2. Are there other numbers you can place between 3.11 and 3.111? If yes, find a number.
3. How are you coming up with the numbers? Are there others? How do you know?
4. Where would you put $3.\overline{1}$ on the number line and why?
5. What can you conclude about the real number line based on this activity?

Section 7.1: Represent Numbers Geometrically

Section Overview:

In this section, students are exposed to a new set of numbers, irrational numbers. This chapter starts with a review of background knowledge – finding the area of polygons and irregular shapes, using ideas of slope to create segments of equal length, and reviewing the definition of a square. Then students build squares with different areas and express the measure of the side length of these squares, gaining an understanding of what it means to take the square root of a number. Additionally, students start to surface ideas about irrational numbers. Students create squares that are not perfect and realize they cannot find an exact numerical value for the side length of these squares (e.g. a number that when squared results in the area of the square created). Students also simplify square roots, connecting the simplified answer to a physical model. At the end of the section, students use cubes and volume to gain an understanding of what is meant by the cube root of a number.

Concepts and Skills to Master:

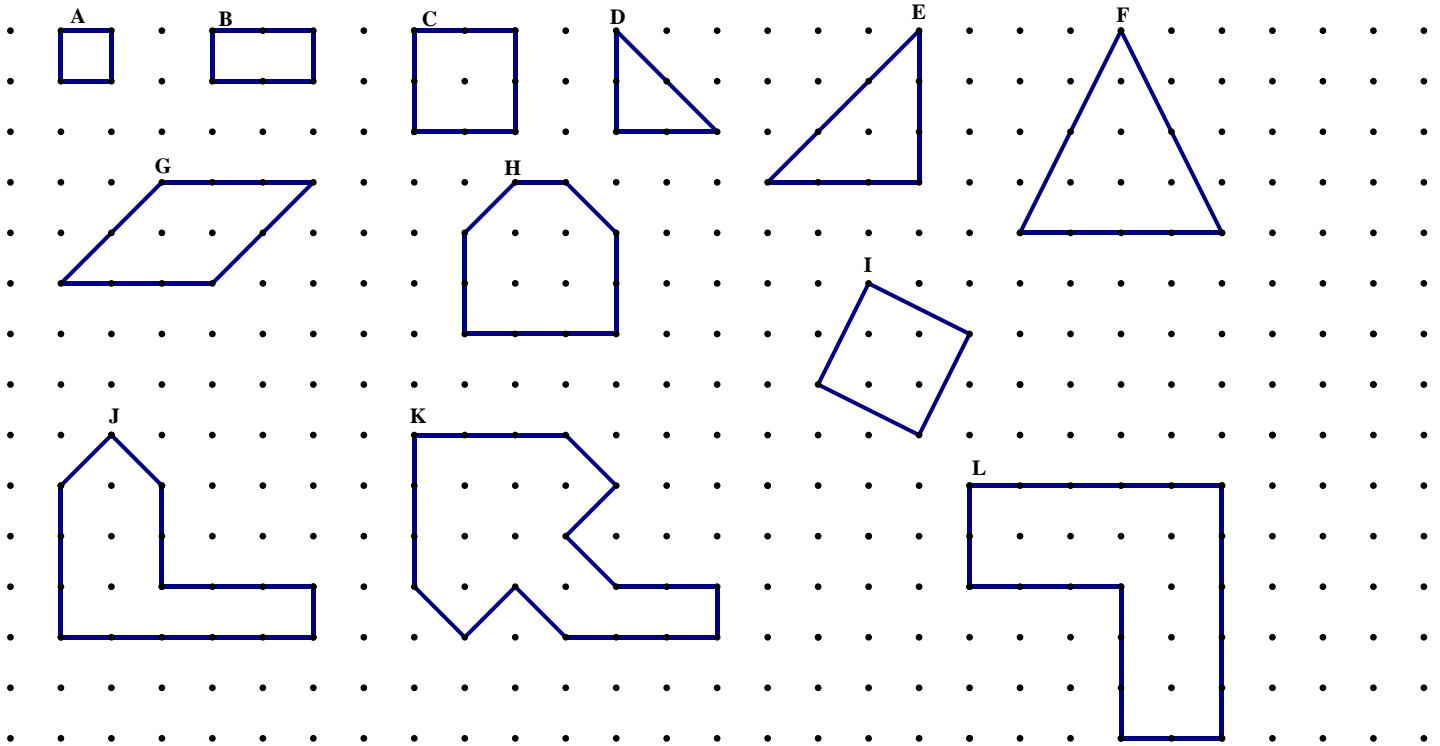
By the end of this section, students should be able to:

1. Understand the relationship between the side length of a square and its area.
2. Understand the relationship between the side length of a cube and its volume.
3. Evaluate the square roots of small perfect squares and the cube roots of small perfect cubes.
4. Simplify square and cube roots.

7.1a Class Activity: Background Knowledge

Activity 1: Finding Area of Irregular Shapes

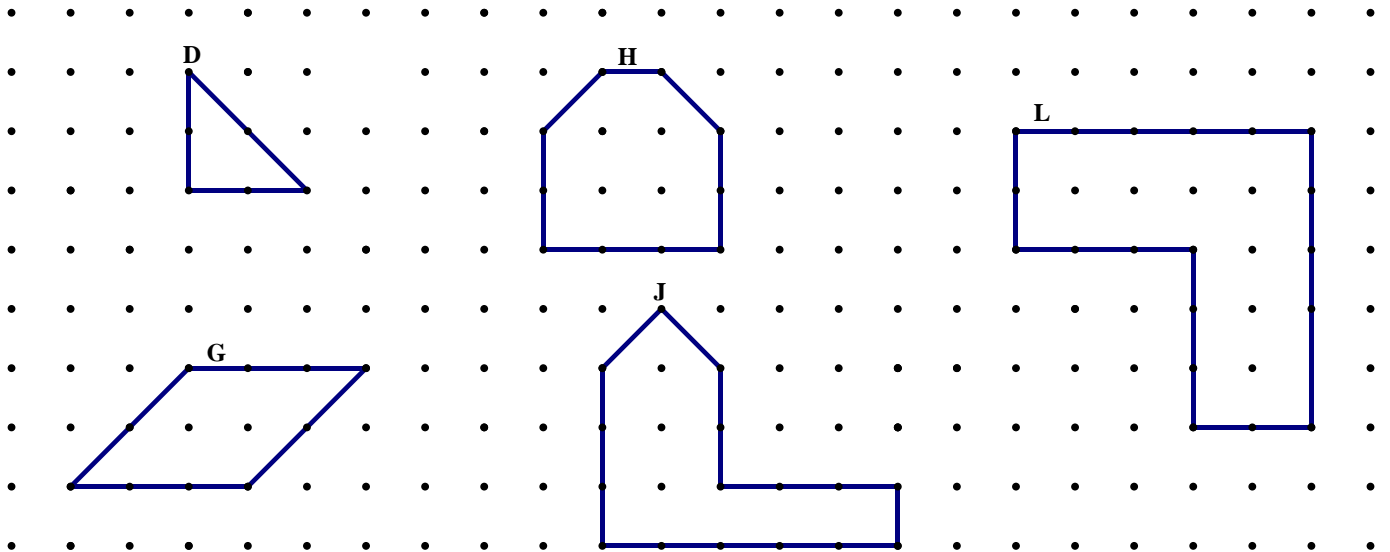
Directions: Find the area of the following shapes. On the grid, a horizontal or vertical segment joining two dots has a length of 1. Put your answers on the lines provided below the grid.



A: _____ B: _____ C: _____ D: _____ E: _____ F: _____

G: _____ H: _____ I: _____ J: _____ K: _____ L: _____

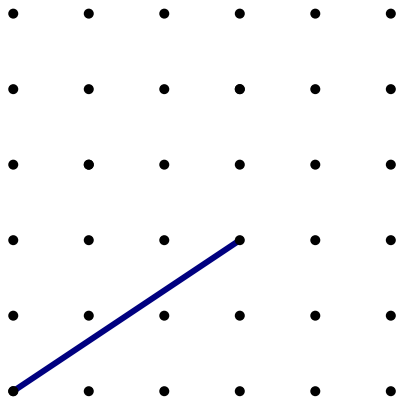
Directions: Use a different method than used above to find the areas of the shapes below.



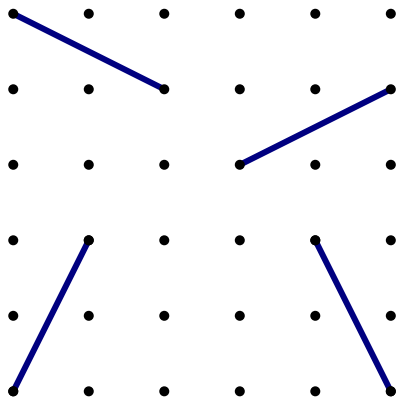
D: _____ G: _____ H: _____ J: _____ L: _____

Activity 2: Slopes and Lengths of Segments

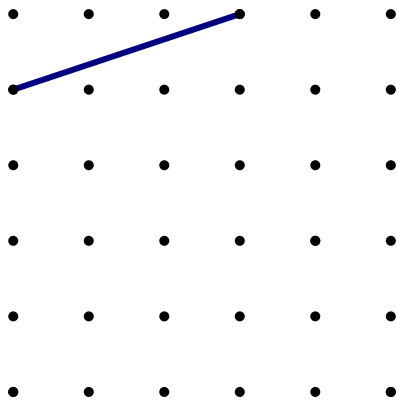
- Using tracing paper, construct 3 additional segments that are the same length as the segment shown below. Your segments cannot be parallel to the segment given and must start and end on a dot on the grid.



- Using the ideas from the previous problem and the one below, write down observations you have about the line segments shown on the grid.

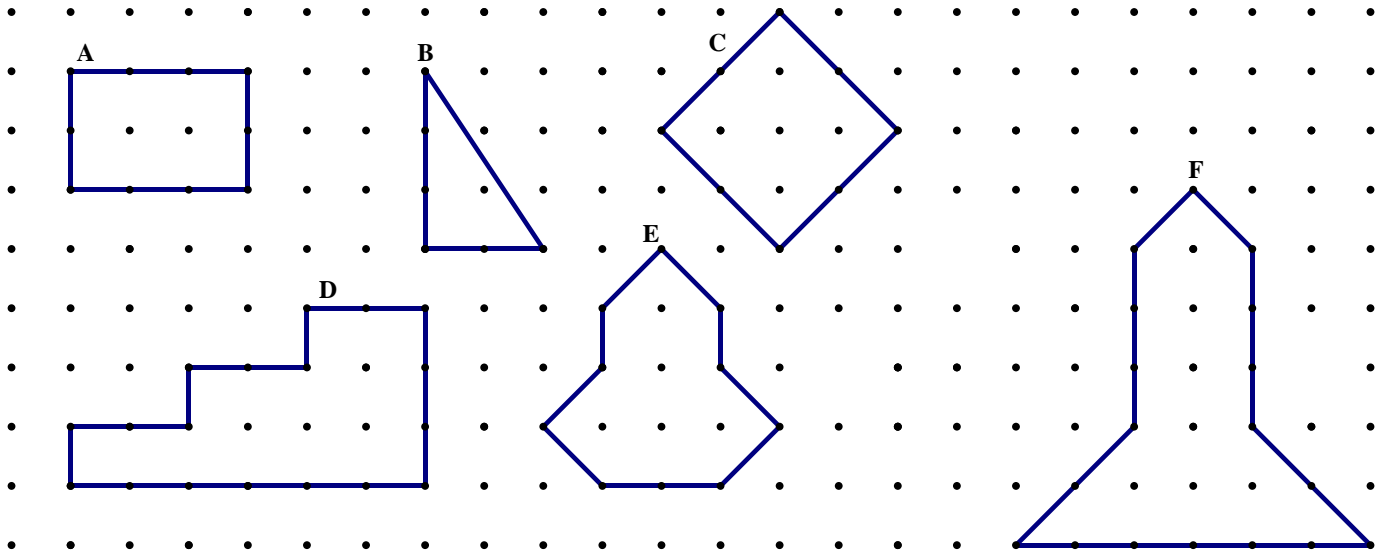


- Create a square on the grid below, using the given segment as one of the sides of the square.



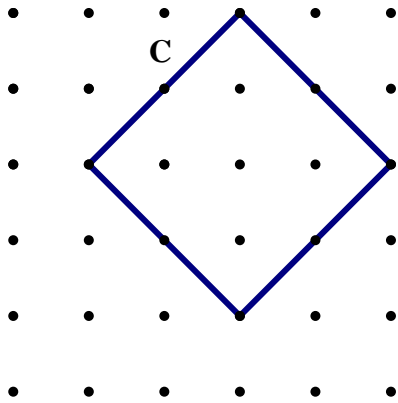
7.1a Homework: Background Knowledge

1. Find the areas of the following shapes. On the grid, a horizontal or vertical segment joining two dots has a length of 1. Put your answers on the lines provided below the grid.

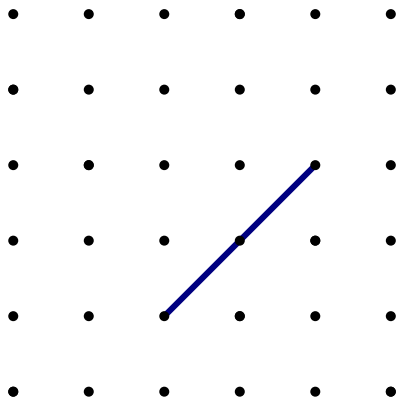


A: _____ B: _____ C: _____ D: _____ E: _____ F: _____

2. Show a second method for finding the area of shape C.

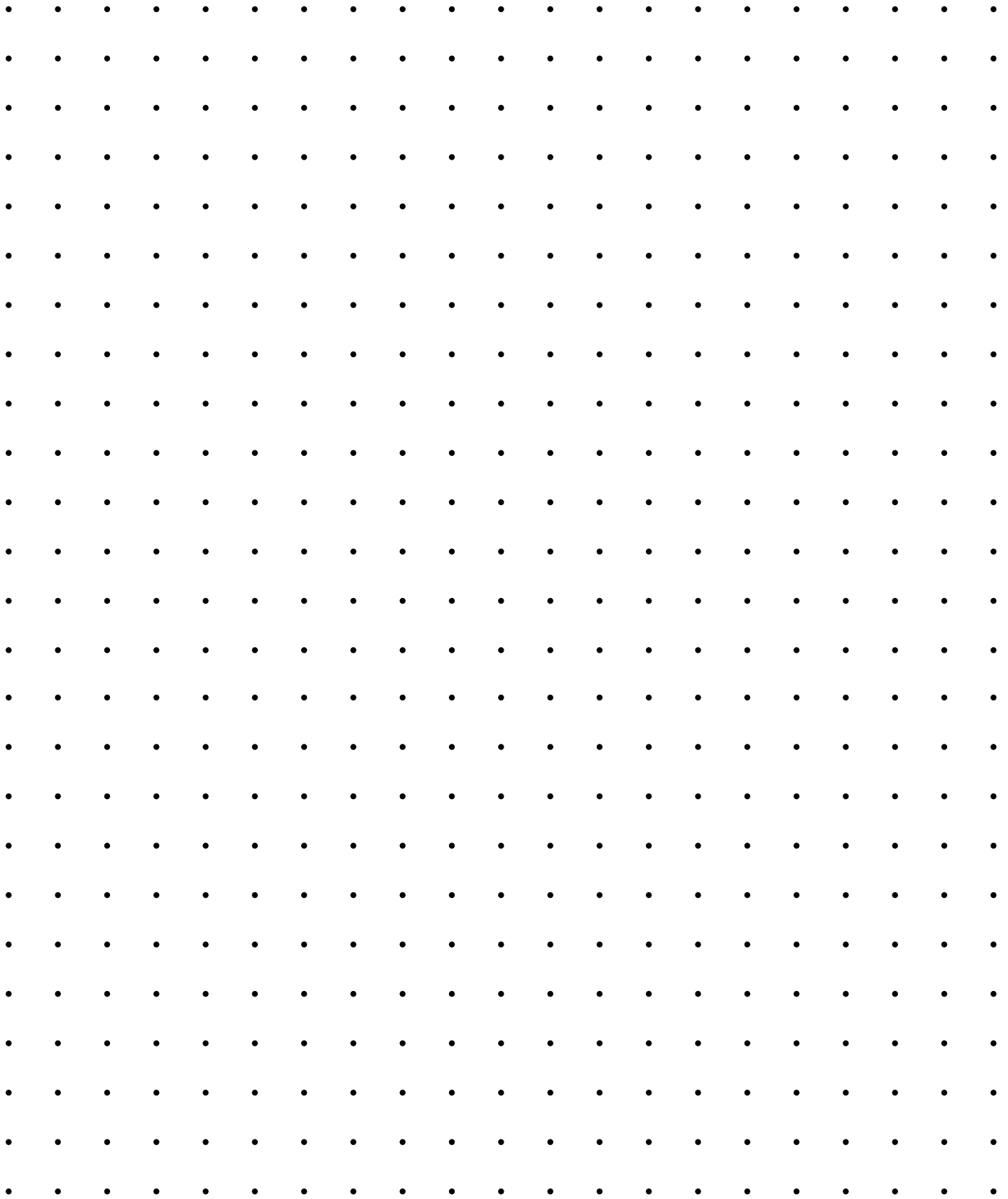


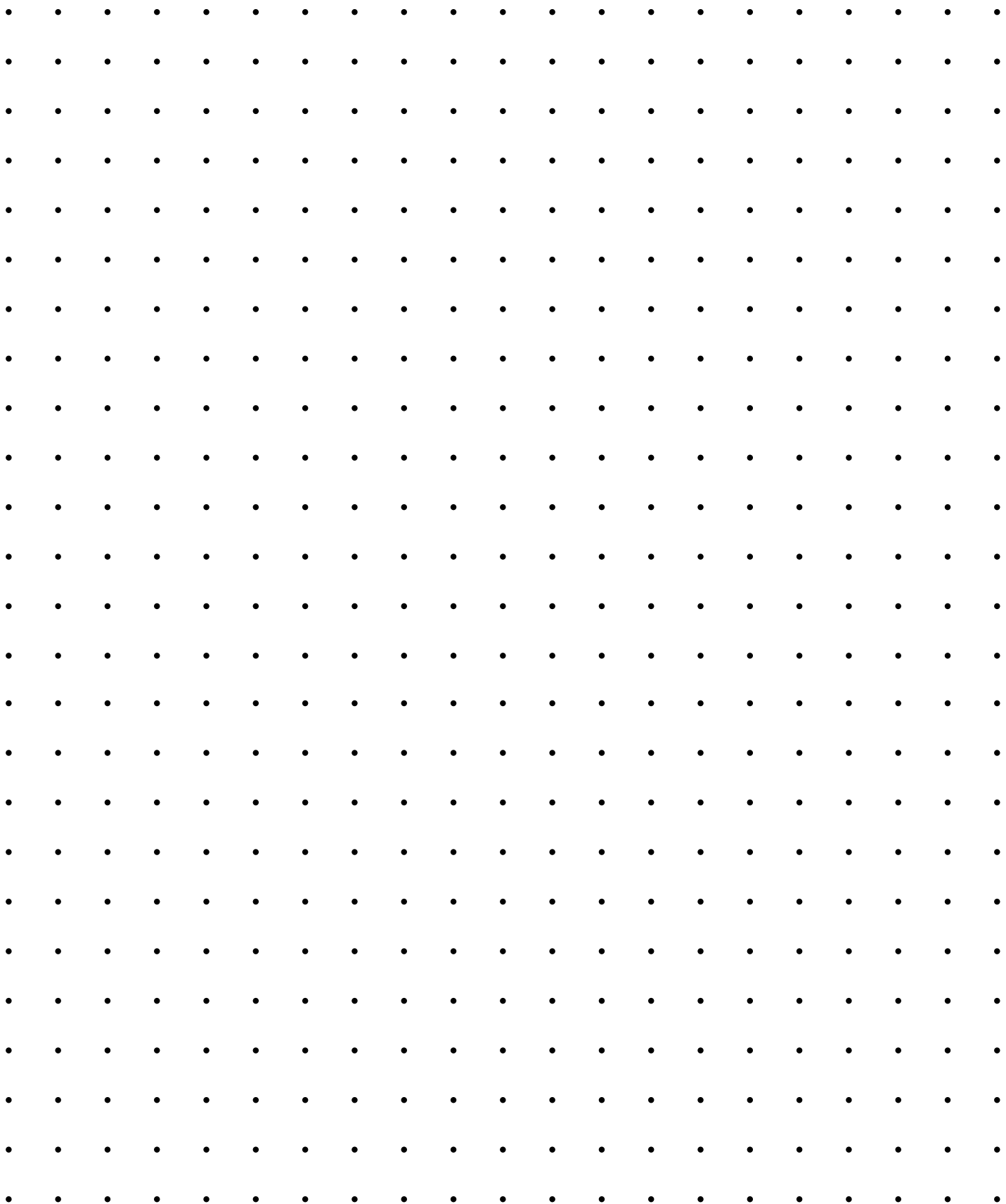
3. Create a square on the grid below, using the given segment as one of the sides of the square.



7.1b Class Activity: Squares, Squares, and More Squares



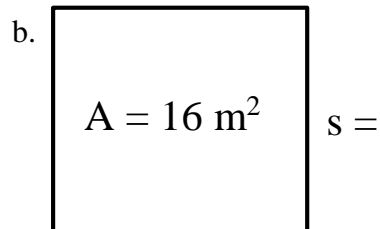
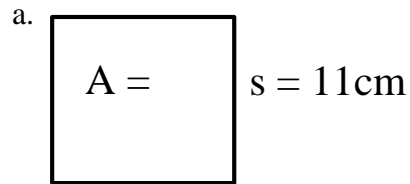




1. Complete the following table...

Area (square units)	Length of Side (units)
1	
9	
	5
2	
5	
	$\sqrt{13}$
	$\sqrt{5}$
100	
	20

2. Find the missing measure.



Directions: Complete the following sentences. **Provide examples to support your statements.**

3. A perfect square is created when...

4. To find the area of a square given the side length of the square...

5. To find the side length of a square given the area of the square...

6. Simplify the following.

a. $\sqrt{36}$

d. $\sqrt{1}$

g. $\sqrt{625}$

b. $\sqrt{121}$

e. $\sqrt{100}$

h. $\sqrt{2500}$

c. $\sqrt{16}$

f. $\sqrt{49}$

i. $\sqrt{225}$

7.1b Homework: Squares, Squares, and More Squares

- List the first 12 perfect square numbers.
- What is the side length of a square with an area of 9 units²?
- What is the area of a square with a side length of 2 units?

4. Complete the following table.

Area (square units)	Length of Side (units)
4	
49	
144	
	10
2	
8	
	$\sqrt{15}$
	$\sqrt{41}$
	1
	9
	s
A	

6. Simplify the following:

a. $\sqrt{9}$

b. $\sqrt{100}$

c. $\sqrt{64}$

d. $\sqrt{4}$

e. $\sqrt{144}$

f. $\sqrt{81}$

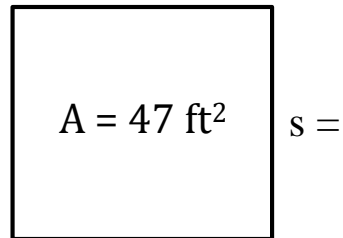
g. $\sqrt{400}$

h. $\sqrt{1600}$

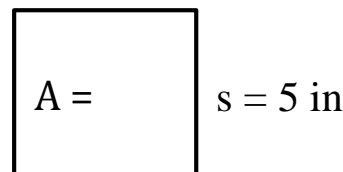
i. $\sqrt{2500}$

5. Find the missing measures of the squares:

a.



b.

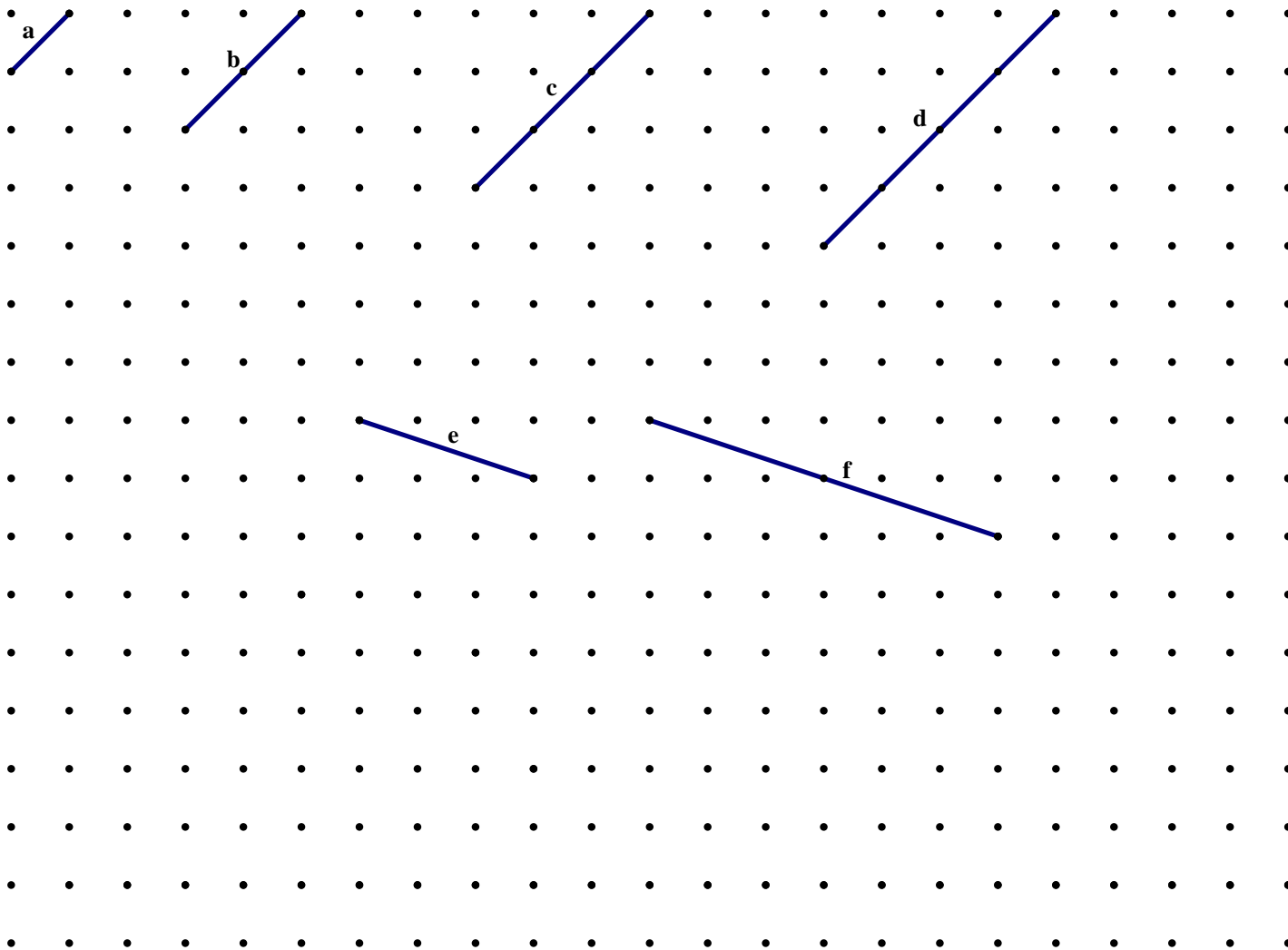


7.1c Class Activity: Squares, Squares, and More Squares Cont.



In the previous sections, we have learned how to simplify square roots of perfect squares. For example, we know that $\sqrt{36} = 6$. What about the square roots of non-perfect squares? How do we know that they are in simplest form? For example, is $\sqrt{5}$ in simplest form? How about $\sqrt{8}$? $\sqrt{147}$? Let's take a look.

- Determine the lengths of line segments a through f without the use of a ruler. Write your answers in the space provided below each grid.



a. _____

b. _____

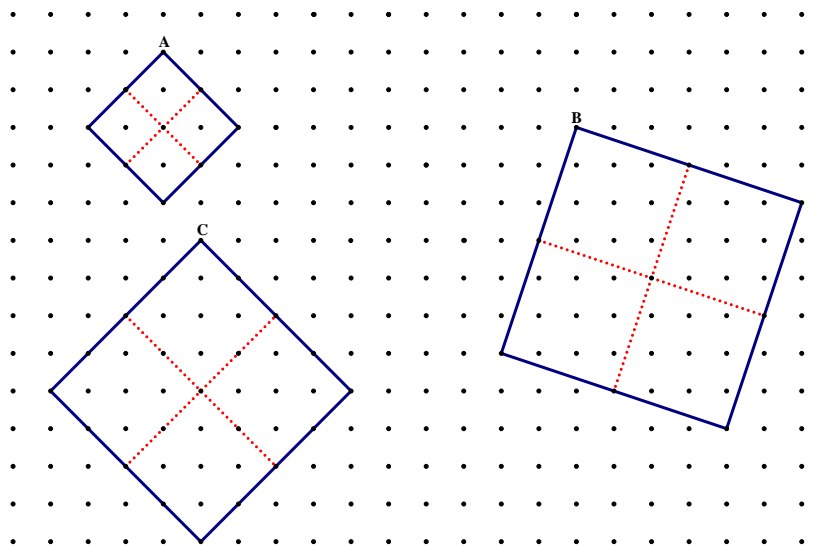
c. _____

d. _____

e. _____

f. _____

Directions: Use the squares on the grid below to answer the questions that follow. Each of the large squares A, B, and C has been cut into four smaller squares of equal size.



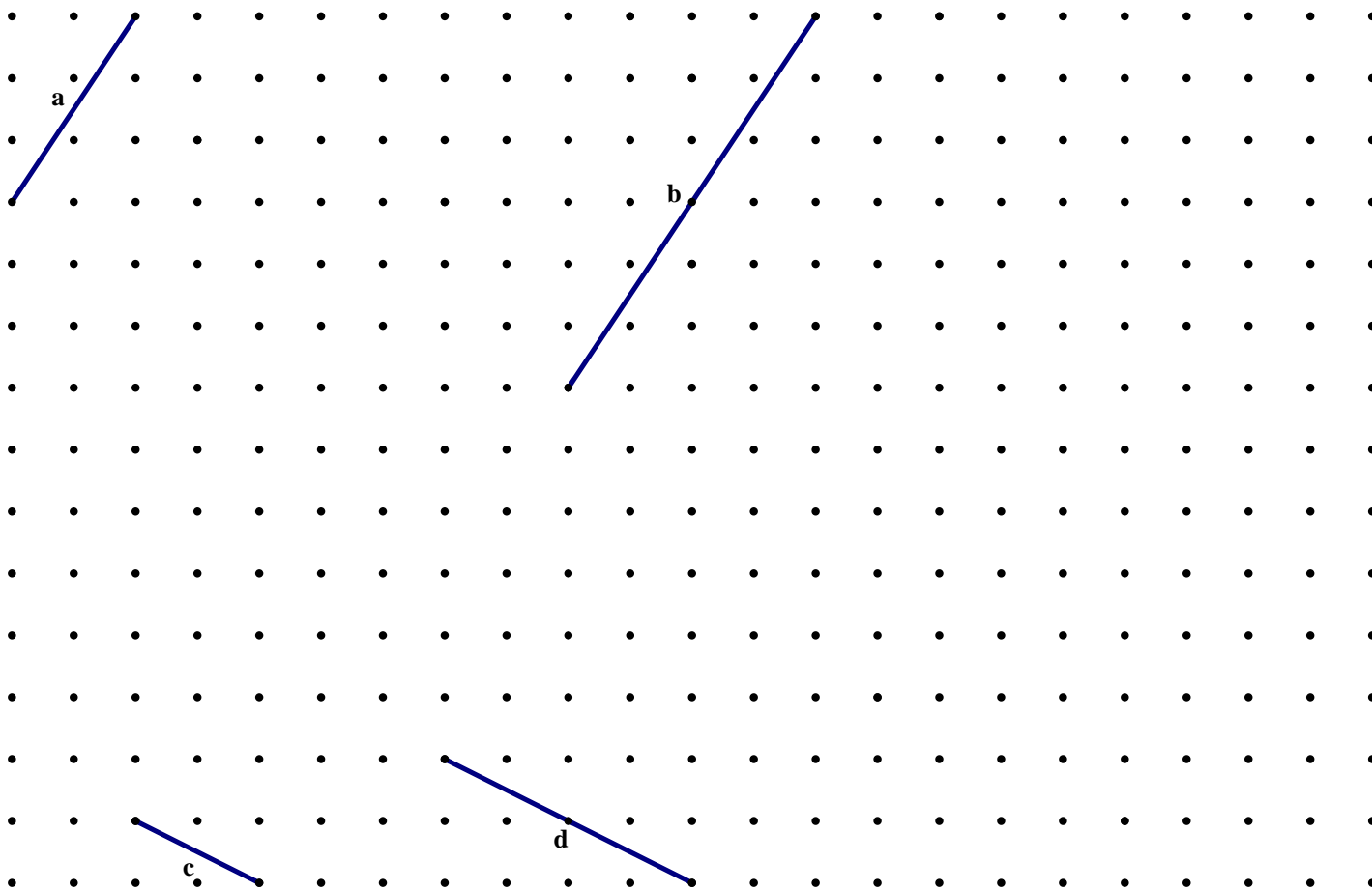
2. Square A has an area of 8 square units. Answer the following questions.
 - a. What is the area of one of the smaller squares that makes up Square A? _____
 - b. What is the side length of one of the smaller squares that makes up Square A? _____
 - c. What is the side length of the large square A (written 2 different ways)? _____

3. Square B has an area of 40 square units. Answer the following questions.
 - a. What is the area of one of the smaller squares that makes up Square B? _____
 - b. What is the side length of one of the smaller squares that makes up Square B? _____
 - c. What is the side length of the large square B (written two different ways)? _____

4. Square C has an area of 32 square units. Answer the following questions.
 - a. What is the area of one of the smaller squares that makes up Square C? _____
 - b. What is the side length of one of the smaller squares that makes up Square C? _____
 - c. What is the side length of the large square (written *three* different ways)? _____

7.1c Homework: Squares, Squares, and More Squares Cont.

1. Determine the lengths of line segments a through f without the use of a ruler. Write your answers in the space provided below each grid.



a. _____

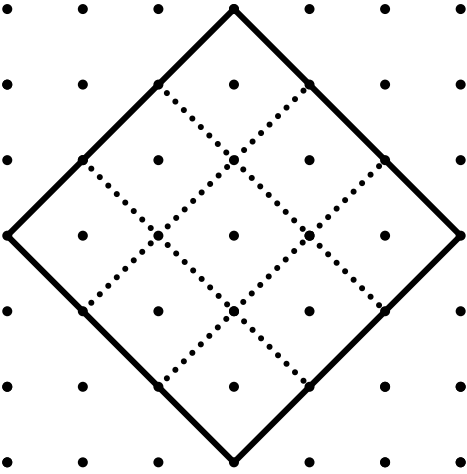
c. _____

b. _____

d. _____

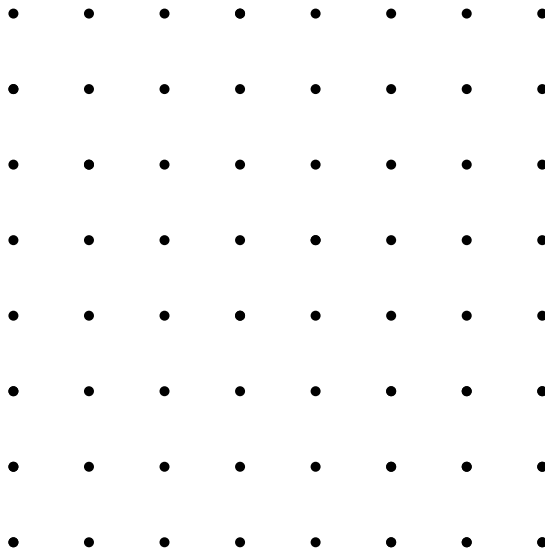
2. On the grid above, construct a segment that has a length of $\sqrt{45} = 3\sqrt{5}$.

3. Use the square on the grid below to answer the questions that follow.



- a. What is the area of the larger square? _____
- b. What is the area of one of the smaller squares? _____
- c. What is the side length of one of the smaller squares? _____
- d. What is the side length of the larger square (written in two different ways)? _____

4. On the grid below, construct a segment with a length of $\sqrt{13}$ units. Explain how you know your segment measures $\sqrt{13}$ units.



7.1d Class Activity: Simplifying Square Roots



In this section we will learn two strategies for simplifying square roots of numbers that are not perfect squares. Both strategies are really doing the same thing, but the methods for each are a little different.

Simplifying Square Roots

Think back to the previous lesson. What does it mean to simplify a square root of a non-perfect square? What was the difference between the simplified version of these square roots as opposed to how they looked before they were simplified?

Let's look at some example from the previous lesson:

$$\begin{aligned}\sqrt{8} &= 2\sqrt{2} \\ \sqrt{18} &= 3\sqrt{2} \\ \sqrt{32} &= 4\sqrt{2} \\ \sqrt{40} &= 2\sqrt{10}\end{aligned}$$

What observations can you make about the simplified versions of these square roots non-perfect squares? List them here:

Two Strategies for Simplifying Square Roots

Strategy 1:

1. Find the greatest perfect square that is a factor of the number inside the square root symbol.
2. Rewrite the number inside the square root symbol as the product of the greatest perfect square and the other factor.
3. Take the square root of the perfect square. Remember: When you take the square root of the perfect square, it is no longer inside the square root symbol.
4. Continue this process until you can no longer find a perfect square other than 1 that is a factor of the number inside the square root symbol.

Examples:

$$\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$$

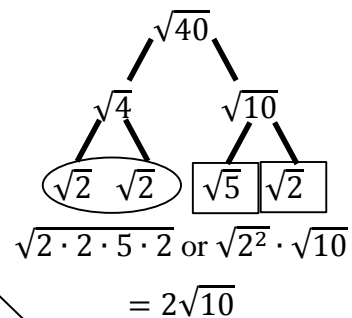
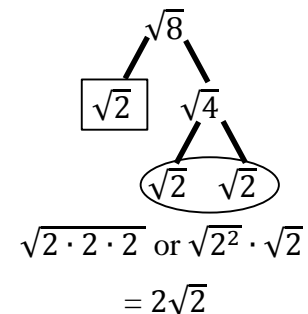
$$\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$$

$$\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$$

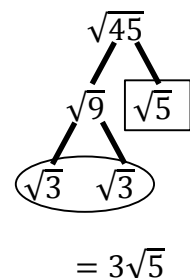
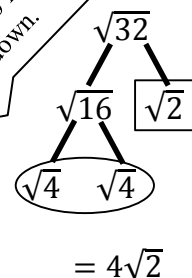
$$\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$$

Strategy 2:

1. Using the factor tree method, factor the number inside the square root symbol.
2. Look for and circle any pairs of numbers among the factors.
3. Put a square around any numbers that are not part of a pair. Re-write the numbers as factors to see that the pairs can be removed, while anything left over must stay under the square root symbol.
4. Remove the pairs and leave any leftover numbers inside the square root symbol. Remember that because we are factoring, all of these numbers are being multiplied, so if you end up with multiple numbers outside or inside the square root symbol, multiply them together.



Hint: You can stop when you find a pair rather than continue to factor it down.



Now you try...

$$\sqrt{50}$$

$$\sqrt{\frac{1}{4}}$$

$$\sqrt{200}$$

$$\sqrt{\frac{4}{25}}$$

$$\sqrt{72}$$

$$\sqrt{\frac{49}{36}}$$

$$\sqrt{147}$$

$$-\sqrt{36}$$

$$\sqrt{128}$$

$$-\sqrt{8}$$

$$10\sqrt{96}$$

$$-5\sqrt{45}$$

What happens when we apply this same method with a perfect square?

$$\sqrt{100} = \sqrt{25 \cdot 4} = \sqrt{25} \cdot \sqrt{4} = 5 \cdot 2 = 10$$

7.1d Homework: Simplifying Square Roots

Directions: Simplify the following square roots.

1. $\sqrt{4} = \underline{\hspace{2cm}}$

2. $\sqrt{36} = \underline{\hspace{2cm}}$

3. $\sqrt{125} = \underline{\hspace{2cm}}$

4. $\sqrt{216} = \underline{\hspace{2cm}}$

5. $\sqrt{80} = \underline{\hspace{2cm}}$

6. $\sqrt{256} = \underline{\hspace{2cm}}$

7. $\sqrt{28} = \underline{\hspace{2cm}}$

8. $\sqrt{99} = \underline{\hspace{2cm}}$

9. $2\sqrt{24} = \underline{\hspace{2cm}}$

10. $3\sqrt{12} = \underline{\hspace{2cm}}$

11. $\sqrt{\frac{1}{64}} = \underline{\hspace{2cm}}$

12. $\sqrt{\frac{25}{49}} = \underline{\hspace{2cm}}$

13. $-\sqrt{72} = \underline{\hspace{2cm}}$

14. $-\sqrt{100} = \underline{\hspace{2cm}}$

15. $-\sqrt{\frac{121}{144}} = \underline{\hspace{2cm}}$

16. $\sqrt{0.16} = \underline{\hspace{2cm}}$

17. $\sqrt{0.0025} = \underline{\hspace{2cm}}$

7.1e Class Activity: Creating Cubes



In the previous lessons, we learned how to find the area of a square given the side length and how to find the side length of a square given the area. In this section, we will study how to find the volume of a cube given its side length and how to find the side length of a cube given its volume.



1. Find the volume of the cube to the left. Describe the method(s) you are using.

2. The cube above is called a perfect cube. A cube is considered a perfect cube if you can arrange smaller unit cubes to build a larger cube. In the example above 27 unit cubes were arranged to build the larger cube shown. Can you build additional perfect cubes to fill in the table below? The first one has been done for you for the cube shown above.

Dimensions	Volume of Cube Exponential Notation (units ³)	Volume of Cube (units ³)	Side Length (units)
$3 \times 3 \times 3$	3^3	27 units ³	3 units

In the previous sections, we learned the following:

- If we are given the side length of a square, s , then its area is s^2 .
- If we are given the area of a square, A , then its side length is \sqrt{A} .

$$A = s^2$$

$$s = \sqrt{A}$$

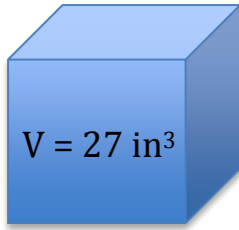
In this section, we see that:

- If we are given the side length of a cube, s , then its volume is s^3 .
- If we are given the volume of a cube, V , then its side length is $\sqrt[3]{V}$.
- Explain in your own words what $\sqrt[3]{V}$ means:

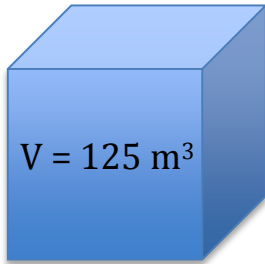
$$V = s^3$$

$$s = \sqrt[3]{V}$$

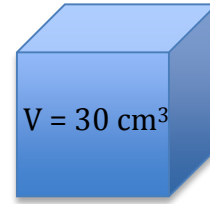
3. Find the side length of the cube: _____



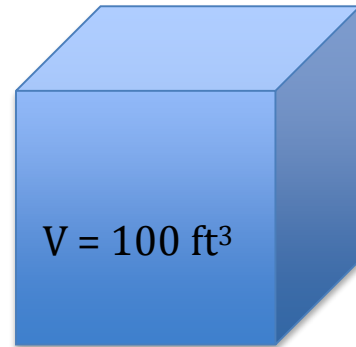
4. Find the side length of the cube: _____



5. Find the side length of the cube: _____



6. Find the side length of the cube: _____



Directions: Fill in the following blanks.

7. $\sqrt[3]{27} = \underline{\hspace{1cm}}$ because $(\underline{\hspace{1cm}})^3 = 27$

8. $\sqrt[3]{64} = \underline{\hspace{1cm}}$ because $(\underline{\hspace{1cm}})^3 = 64$

9. $\sqrt[3]{1} = \underline{\hspace{1cm}}$ because $(\underline{\hspace{1cm}})^3 = 1$

10. $\sqrt[3]{125} = \underline{\hspace{1cm}}$

11. $\sqrt[3]{343} = \underline{\hspace{1cm}}$

12. $\sqrt[3]{\frac{1}{216}} = \underline{\hspace{1cm}}$

13. $\sqrt[3]{\frac{1}{1000}} = \underline{\hspace{1cm}}$

14. $\sqrt[3]{\frac{8}{125}} = \underline{\hspace{1cm}}$

15. $\sqrt[3]{0.001} = \underline{\hspace{1cm}}$

16. $\sqrt[3]{0.027} = \underline{\hspace{1cm}}$

17. $\sqrt[3]{32} = \underline{\hspace{1cm}}$

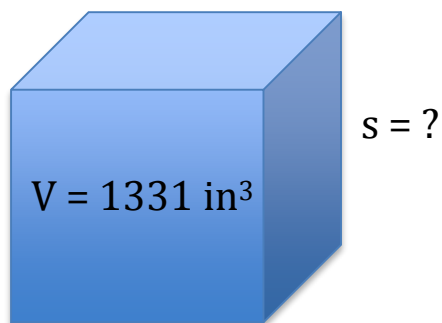
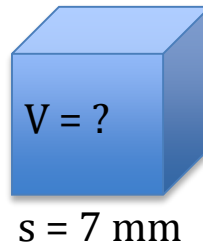
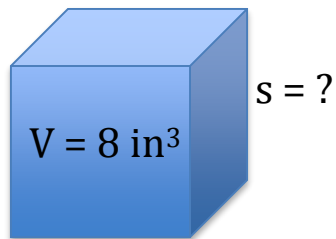
18. $\sqrt[3]{135} = \underline{\hspace{1cm}}$

7.1e Homework: Creating Cubes

1. Fill in the blanks in the table:

Side Length	Volume
1	
	27
4	
	125
6	
	40
	96
$\sqrt[3]{18}$	
$\sqrt[3]{243}$	
	1000
0.2	
	$\frac{1}{64}$
	$\frac{1}{125}$
s	
	V

2. Find the missing measurements:



3. Simplify.

$$\sqrt[3]{512}$$

$$\sqrt[3]{27}$$

$$\sqrt[3]{729}$$

$$\sqrt[3]{\frac{27}{64}}$$

$$\sqrt[3]{24}$$

$$\sqrt[3]{250}$$

$$\sqrt[3]{128}$$

$$\sqrt[3]{40}$$

$$\sqrt[3]{192}$$

7.1g Self-Assessment: Section 7.1

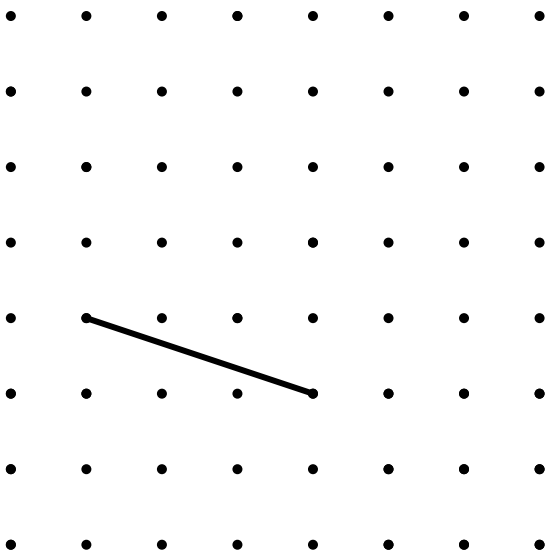
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Understand the relationship between the side length of a square and its area.				
2. Understand the relationship between the side length of a cube and its volume.				
3. Evaluate the square roots of small perfect squares and the cube roots of small perfect cubes.				
4. Simplify square and cube roots.				

1. Find the following:

- The side length of a square with an area of 36 square units. _____
- The side length of a square with an area of 8 square units. _____
- The area of a square with a side length of 5 units. _____
- The area of a square with a side length of $\sqrt{13}$ units. _____

e. Find the length of the segment shown below.



2. Find the following:

a. The side length of a cube with a volume of 125 units³. _____

b. The volume of a cube with a side length of 4 units. _____

3. Evaluate:

a. $\sqrt{4}$

b. $\sqrt{16}$

c. $\sqrt{81}$

d. $\sqrt{121}$

e. $\sqrt[3]{64}$

f. $\sqrt[3]{125}$

g. $\sqrt[3]{1000}$

4. Simplify:

a. $\sqrt{60}$

b. $-\sqrt{90}$

c. $2\sqrt{12}$

d. $\sqrt[3]{81}$

e. $\sqrt[3]{48}$

f. $-\sqrt[3]{108}$

Section 7.2: Solutions to Equations Using Square and Cube Roots

Section Overview:

In this section, students will apply their knowledge from the previous section in order to solve simple square and cubic equations. Building on student understanding of how to solve simple linear equations using inverse operations, students will understand that taking the square root of a number is the inverse of squaring a number and taking the cube root is the inverse of cubing a number. Students will express their answers in simplest radical form.

Concepts and Skills to Master:

By the end of this section students should be able to:

1. Solve simple quadratic and cubic equations.

7.2a Class Activity: Solve Equations using Square and Cube Roots

In the problems below, we review how to solve some basic equations.



1. Write the inverse operation used to solve each of following equations, then show the steps used to solve the equation.

a. $x + 3 = 7$

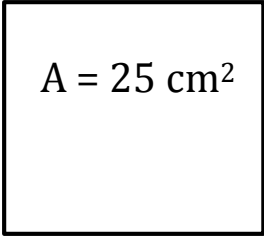
b. $-3x = 18$

c. $x - 6 = -14$

d. $\frac{x}{7} = 3$

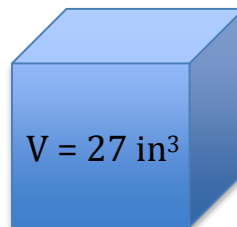
2. What does an inverse operation do?

3. Write and solve an equation to find the side length of a square with an area of 25 cm^2 .


$$A = 25 \text{ cm}^2$$

4. Now consider the equation $x^2 = 25$ out of context. Is 5 the only solution? In other words, is 5 the only number that makes this equation true when substituted in for x ?

5. Write and solve an equation to find the side length of a cube with a volume of 27 in^3 .



6. Now consider the equation $x^3 = 27$ out of context. Is 3 the only solution? In other words, is 3 the only number that makes this equation true when substituted in for x ?

7. State the inverse operation you would use to solve these equations. Solve each equation.

a. $x^2 = 100$

b. $x^2 = 36$

c. $x^3 = 27$

8. Solve the equations below. Express your answer in simplest radical form.

a. $x^2 = 64$

b. $x^2 = -64$

c. $x^3 = 8$

d. $x^3 = -8$

e. $x^3 = 1$

f. $x^2 = 9$

g. $x^2 = 5$

h. $x^2 = 10$

i. $x^3 = 15$

j. $x^2 = -100$

k. $x^3 = -512$

l. $x^2 = 8$

m. $x^2 = 45$

n. $x^3 = 250$

o. $x^3 = 128$

p. $a^2 = \frac{1}{36}$

q. $z^3 = \frac{1}{27}$

r. $y^2 = 0.16$

s. $x^2 + 16 = 25$

t. $x^2 - 5 = 59$

u. $10x^2 = 1440$

v. $2x^2 = 16$

w. $\frac{y^3}{2} = 32$

x. $x^2 = p$ where p is a positive rational number

9. Estimate the solution. Use a calculator to check your estimate.

a. $x^2 = 53$

b. $a^2 = 15$

c. $z^3 = 29$

7.2a Homework: Solve Equations using Square and Cube Roots

1. Solve the equations below. Express your answer in simplest radical form.

a. $x^2 = 121$

b. $x^2 = 81$

c. $y^3 = 125$

d. $x^3 = 216$

e. $x^3 = -1$

f. $x^2 = 18$

g. $x^2 = -36$

h. $x^2 = 2$

i. $y^3 = 81$

j. $x^2 + 12 = 48$

k. $25 + x^2 = 169$

l. $\frac{y^3}{5} = 25$

m. $a^2 = \frac{1}{144}$

n. $z^3 = \frac{1}{8}$

o. $y^2 = 0.25$

p. $a^2 = -\frac{1}{36}$

q. $z^3 = -0.027$

r. $y^3 = \frac{1}{125}$

s. $a^3 = 100$

t. $a^2 + 576 = 625$

u. $64 + b^2 = 289$

v. $x^3 = p$ where p is a positive rational number

w. Solve for r where A is the area of a circle and r is the radius: $A = \pi r^2$

x. Solve for r where V is the volume of a cylinder, r is the radius, and h is the height: $A = \pi r^2 h$

2. Estimate the solution. Use a calculator to check your estimate.

a. $x^2 = 17$

b. $a^2 = 67$

c. $z^3 = 10$

You are designing a bathroom with the following items in it. Your very odd client has asked that each of these items be a perfect square or cube. Use your knowledge of squares and cubes to write an equation that models the area **or** volume of each item. Then solve the equation to find the side length of each item. The first one has been done for you.

3. Rug 1764 in^2

4. Ottoman $3,375 \text{ in}^3$

Let s equal the length of one side of the rug.

$$s^2 = 1764$$

$$\sqrt{s^2} = \sqrt{1764}$$

$$s = 42$$

The side length of the rug is 42 inches.

5. Mirror 1024 cm^2

6. Bar of Soap 27 cm^3

7. Is it probable to have a negative answer for the objects above? Why or why not?
8. Your client tells you that they would like to double the dimensions of the rug. What will happen to the area of the rug if you double the dimensions? Find this new area. What will happen to the area of rug if you triple the dimensions?
9. Your client also tells you that they would like to double the dimensions of the bar of soap. What will happen to the volume of the soap if you double its dimensions? Find this new volume. What will to the volume of the bar of some if you triple the dimensions?
10. Write and solve an equation of your own that has a power of 2 in it.
11. Write and solve an equation of your own that has a power of 3 in it.

7.2b Class Activity: Tower Views

1. Use inverse operations to solve the following problems.

a. $\sqrt{x} = 4$

b. $\sqrt{a} = 9$

c. $2\sqrt{y} = 4$

d. $\sqrt[3]{z} + 5 = 13$



People often wonder how far they can see when they're at the top of really tall buildings such as the Empire State Building, The Sears Tower in Chicago, etc.

The furthest distance you can see across flat land is a function of your height above the ground.

If h is the height in meters of your viewing place, then d , the distance in kilometers you can see, can be given by this formula: $d = 3.532\sqrt{h}$

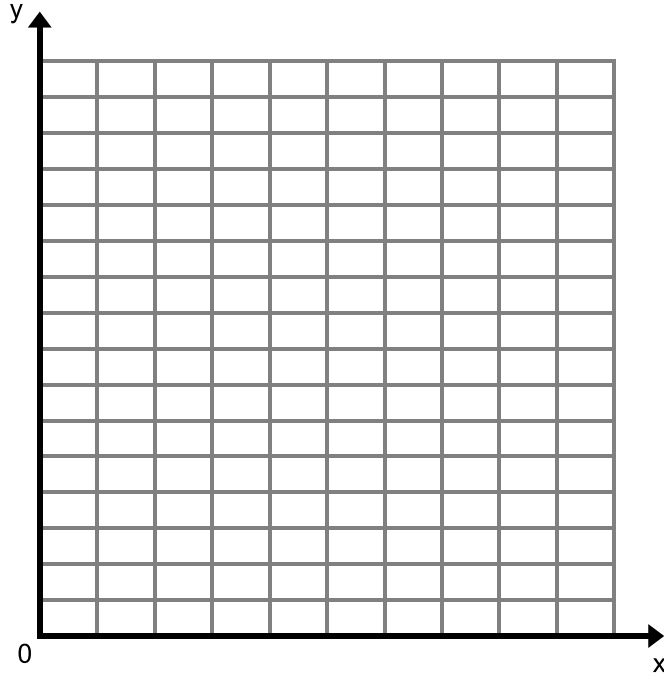
- The equation above can be used to find the distance when you know the height. Rewrite the equation to find height when you know the distance.
- If you were lying down on top of a building that is 100 meters tall, how far could you see? Write an equation to solve this problem. Solve the problem, showing all steps.
- The CN Tower in Toronto, Canada is 555 meters tall. It is near the shore of Lake Ontario, about 50 kilometers across the lake from Niagara Falls. Your friend states that on a clear day, one can see as far as the falls from the top of the Tower. Are they correct? Explain your answer.
- The Washington Monument in Washington D.C. is 170 meters tall. How far can one see from its top? Write the equation you need. Show all steps
- How high must a tower be in order to see at least 60 kilometers? Write the equation you need. Show all steps.
- Advertising for Queen's Dominion Amusement Park claims you can see 40 kilometers from the top of its observation tower. How high is the tower? Write the equation you need. Show all steps.

8. To enhance understanding of the relation between height and viewing distance, first complete the table below. Express each output value to the nearest whole number; then plot the data points on an appropriately labeled graph. Do not connect the points.

a.

Height (m)	0	50	100	150	200	250	300	350	400	450	500
Distance (km)											

b.



- c. What kind of association is shown between height and viewing distance?

7.2b Homework: Driving, Running, and Basketballs

1. Use inverse operations to solve the following problems.

a. $\sqrt{x} = 5$

b. $3 = \sqrt{a}$

c. $3\sqrt{y} = 18$

d. $\sqrt{z} - 3 = 78$

Deven is a civil engineer. He needs to make sure that the design of a curved road ensures the safety of a car driving at the speed limit. The equation $V = \sqrt{2.5r}$ represents the maximum velocity that a car can travel safely on an unbanked curve. V represents the maximum velocity in miles per hour and r represents the radius of the turn in feet.

2. If a curve in the road has a radius of 1690 ft. what is the maximum velocity that a car can safely travel on the curve?
3. The equation above can be used to find the velocity when you know the radius. Rewrite the equation to find radius if you know the velocity.
4. If a road is designed for a speed limit of 55 miles per hour, what is the radius of the curve?
5. If a road is designed for a speed limit of 35 miles per hour, what is the radius of the curve?
6. What type of association exists between the radius of the curve and the maximum velocity that a car can travel safely?

Annie is on the track team her coach tells her that the function $S = \pi \sqrt{\frac{9.8l}{7}}$ can be used to approximate the maximum speed that a person can run based off of the length of their leg. S represents the runner's speed in meters per second and l represents the length of the runner's leg in meters.

7. What is the maximum speed that Annie can run if her leg length is 1.12 meters?
8. The equation given above can be used to find the speed of the runner given their leg length. Rewrite the equation to find the leg length of the runner given their speed.
9. What is the leg length of a runner if their maximum running speed is 2.6 meters per second? Round your answer to the nearest hundredth.
10. What kind of association exists between the length of a person's leg and their maximum running speed?
11. Is leg length the only thing that affects a runner's maximum speed? Explain your answer.

The surface area of a sphere is found by the equation $A = 4\pi r^2$ where A represents the surface area of the sphere and r represents the radius.

12. A basketball has a radius of 4.7 in, what is its surface area?
13. The equation given above can be used to find the surface area given the radius. Rewrite the equation so that you can find the radius if you are given the surface area.
14. The surface area of a dodge ball is 153.9. in². What is the radius of the dodge ball?

7.2c Self-Assessment: Section 7.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Solve simple quadratic and cubic equations.				

1. Solve.

a. $x^2 = 100$

b. $x^3 = 64$

c. $x^2 + 30 = 91$

d. $x^3 - 9 = 134$

e. Solve for r . $A = \frac{1}{2}r^2y$

Section 7.3: Rational and Irrational Numbers

Section Overview:

This section begins with a review of the different sets of rational numbers and why a need arose to distinguish them. Students then explore different ways of representing rational numbers, starting with a review of how to change fractions into decimals. During this process, students are reminded that the decimal expansion of all rational numbers either terminates or repeats eventually. From here, students review how to express terminating decimals as fractions and learn how to express repeating decimals as fractions by setting up and solving a system of equations. This skill allows them to show that all decimals that either terminate or repeat can be written as a fraction and therefore fit the definition of a rational number. After this work with rational numbers, students investigate numbers whose decimal expansion does not terminate or repeat: irrational numbers. With this knowledge, students classify numbers as rational and irrational. Students learn different methods for approximating the value of irrational numbers, zooming in to get better and better approximations of the number. They then use these approximations to estimate the value of expressions containing irrational numbers. Lastly, students compare and order rational and irrational numbers.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Know that real numbers that are not rational are irrational.
2. Show that rational numbers have decimal expansions that either terminate or repeat eventually.
3. Convert a repeating decimal into a fraction.
4. Know that the square root of a non-perfect square is an irrational number.
5. Understand that the decimal expansions of irrational numbers are approximations.
6. Show the location (or approximate location) of real numbers on the real number line.
7. Approximate the value of irrational numbers, zooming in to get better and better approximations.
8. Estimate the value of expressions containing irrational numbers.
9. Compare and order rational and irrational numbers.

7.3 Anchor: Revisiting the Number Line



Directions: Show the length of the following numbers on the number line below. Use the grid on the following page to construct lengths, using tick marks on an index card or tracing paper, and transfer those lengths onto the number line. Then answer the questions that follow. **Note:** On the grid, a horizontal or vertical segment joining two dots has a length of 1. On the number line, the unit length is the same as the unit length on the dot grid.

$A: \sqrt{25}$

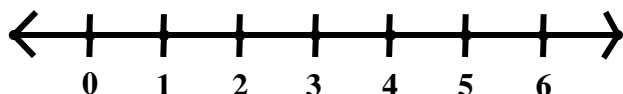
$B: \sqrt{2}$

$C: \sqrt{8}$

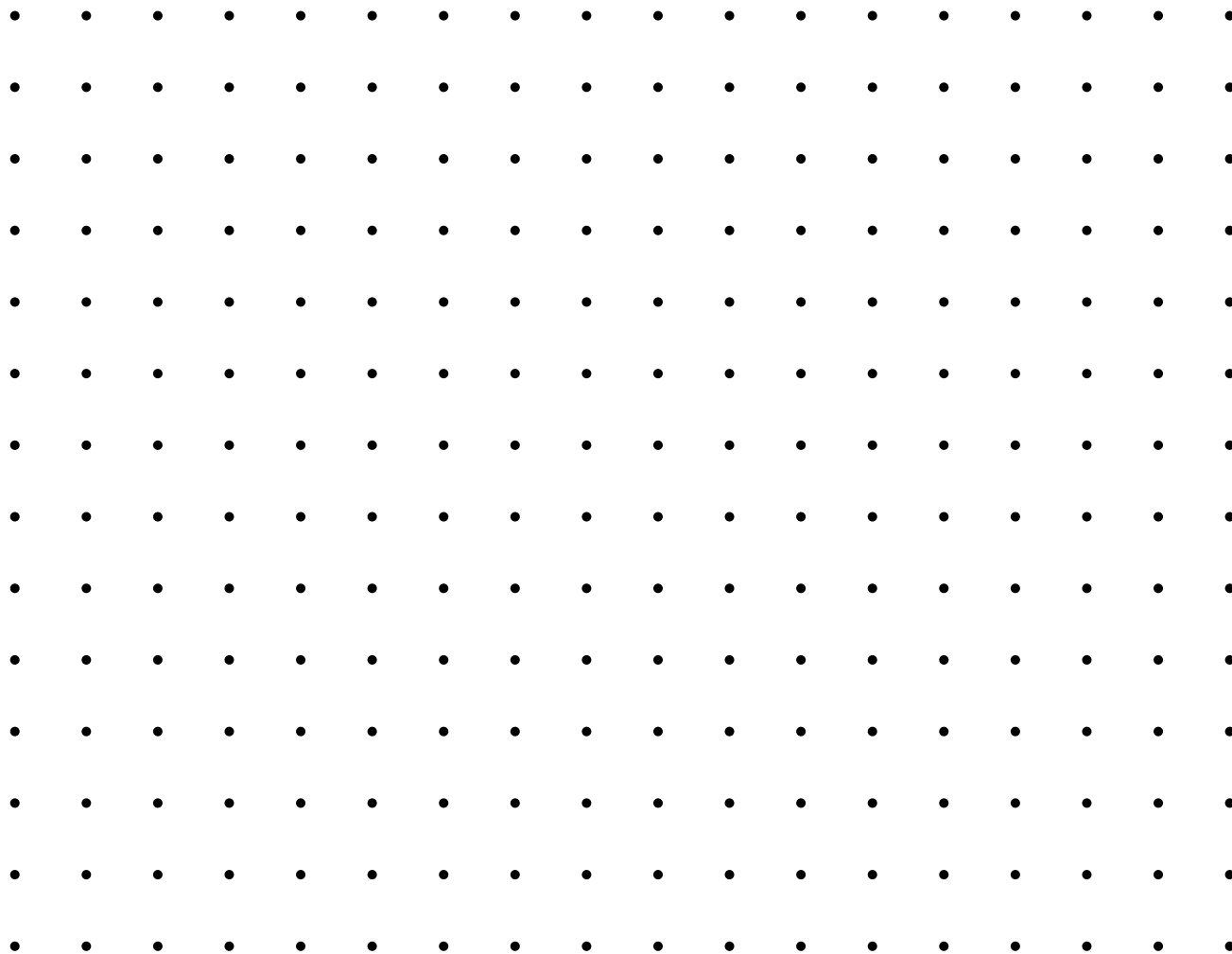
$D: 2\sqrt{2}$

$E: \sqrt{5}$

$F: 2\sqrt{5}$



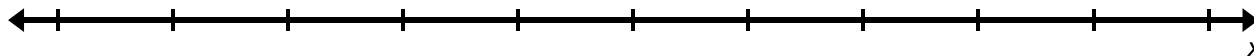
1. Use the number line to write a decimal approximation for $\sqrt{2}$. Verify your estimate with a calculator.
2. Would 1.41 be located to the right or to the left of $\sqrt{2}$ on the number line?
3. Describe and show how you can put $-\sqrt{2}$ on the number line. Write the decimal approximation for $-\sqrt{2}$.
4. Describe and show how you can put $(2 + \sqrt{2})$ on the number line. Estimate the value of this expression.
5. Describe and show how you can put $(2 - \sqrt{2})$ on the number line. Estimate the value of this expression.
6. Describe and show how you can put $2\sqrt{2}$ on the number line. Estimate the value of this expression.
7. Use the number line to write a decimal approximation for $\sqrt{5}$.
8. Would 2.24 be located to the right or to the left of $\sqrt{5}$ on the number line?
9. Describe and show how you can put $1 + \sqrt{5}$ on the number line. Estimate the value of this expression.



7.3a Class Activity: The Rational Number System

Our number system has evolved over time. On the following pages, you will review the subsets of numbers that are included in the set of **rational numbers**.

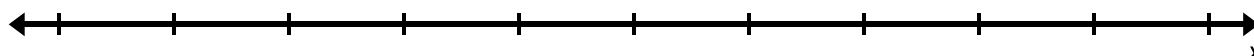
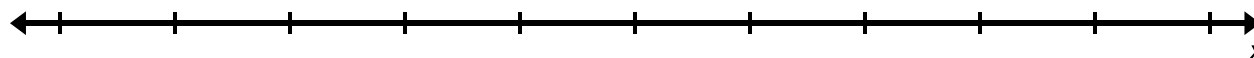
Whole Numbers:



Integers:



Rational Numbers:



Over the years, you have expanded your knowledge of the number system, gradually incorporating the sets of numbers mentioned above. These sets of numbers are all part of the **rational number system**.

A **rational number** is any number that can be expressed as a quotient $\frac{p}{q}$ of two integers where q does not equal 0.

1. Begin to fill out the table below with different subsets, including equivalent forms, of **rational numbers** you know about so far and give a few examples of each. You will continue to add to this list throughout this section.

Subsets of the Rational Numbers	Examples

2. Change the following rational numbers into decimals **without** the use of a calculator.

a. $\frac{1}{2}$	b. $\frac{9}{5}$
------------------	------------------

c. $\frac{3}{8}$	d. $\frac{1}{3}$
e. $\frac{4}{15}$	f. $\frac{1}{7}$

3. What do you notice about the decimal expansion of any rational number? Why is this true?



7.3a Homework: The Rational Number System

1. Change the following rational numbers into decimals **without** the use of a calculator.

a. $\frac{1}{5}$

b. $\frac{7}{4}$

c. $\frac{5}{8}$

d. $\frac{2}{3}$

e. $\frac{2}{9}$

f. $\frac{3}{11}$

7.3b Class Activity: Expressing Decimals as Fractions

As we discovered in the previous section, when we converted fractions into decimals, the result was either a **terminating or repeating decimal**.

If we are given a terminating or repeating decimal, we need a method for changing them into a fraction in order to prove that they fit the definition of a rational number.

In 7th grade, you learned how to convert terminating decimals into fractions. Here are a few examples:

$$0.3 = \frac{3}{10}$$

$$0.25 = \frac{25}{100} = \frac{1}{4}$$

$$0.375 = \frac{375}{1000} = \frac{3}{8}$$

$$-2.06 = -2\frac{6}{100} = -2\frac{3}{50} = -\frac{103}{50}$$

Now you try a few...

$$0.4 =$$

$$0.05 =$$

$$0.275 =$$

$$1.003 =$$

So, how do we express a repeating decimal as a fraction? For example, how would you convert the repeating decimal $0.\overline{45}$ into a fraction? Try in the space below.

We can use a **system of two linear equations** to convert a repeating decimal into a fraction. Let's look at an example:

Example 1:

The decimal $0.\overline{3}$ is a repeating decimal that can be thought of as $0.33333\dots$ where the “...” indicates that the 3s repeat forever. If they repeat forever, how can we write this number as a fraction? Here's a trick that will eliminate our repeating 3s.



Let a represent our number $a = 0.\overline{3}$.

Multiply both sides of the equation by 10 which would give us a second equation $10a = 3.\overline{3}$.

Now we have the following two equations:

$$10a = 3.\overline{3}$$

$$a = 0.\overline{3}$$

Let's expand these out:

$$10a = 3.333333333333 \dots \dots$$

$$a = 0.333333333333 \dots \dots$$

What will happen if we subtract the second equation from the first? Let's try it (remembering to line up the decimals):

$$\begin{array}{r} 10a = 3.333333333333 \dots \dots \\ - a = 0.333333333333 \dots \dots \\ \hline \end{array}$$

$$9a = 3$$

$$a = \frac{3}{9} \quad (\text{Divide both sides by 9})$$

$$a = \frac{1}{3} \quad (\text{Simplify the fraction})$$

Example 2:

The decimal $0.\overline{54}$ is a repeating decimal that can be thought of as $0.54545454\dots$ where the “...” indicates that the 54 repeats forever. Let’s see how to express this as a fraction.

Let a represent our number $a = 0.\overline{54}$.

Why do you think we multiplied the second example by 100 instead of 10 as we did in the first example? What would have happened if we had multiplied by 10 in example 2? Try it below and see.

Example 3: Change the decimal $2.\overline{4}$ into a fraction

The decimal $2.\overline{4}$ is a repeating decimal that can be thought of as $2.4444444\dots$ where the “...” indicates that the 4s repeat forever.

Let a represent our number $a = 2.\overline{4}$.

Example 4: Change the decimal $3.\overline{12}$ into a fraction.

Example 5: Change the decimal $0.\overline{123}$ into a fraction.

Example 6: Change the decimal $4.\overline{1}$ into a fraction.

Example 7: Change the decimal $2.0\overline{15}$ into a fraction.

7.3b Homework: Expressing Decimals as Fractions

Directions: Circle whether the decimal is terminating or repeating then change the decimals into fractions.

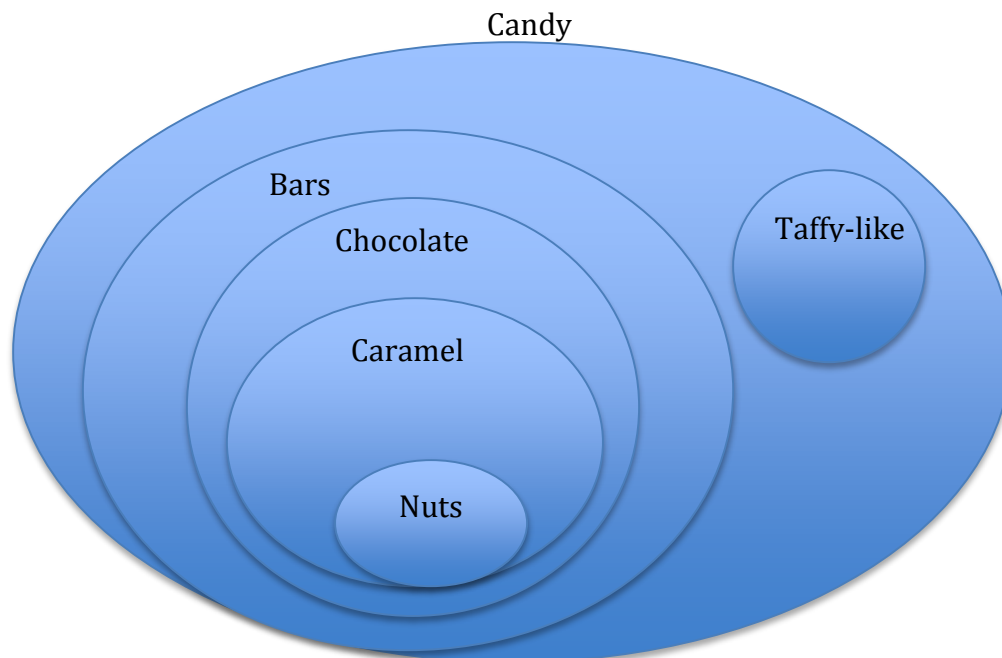
1. 0.6 Terminating Repeating	2. 0.02 Terminating Repeating
3. 1.25 Terminating Repeating	4. $-0.0\overline{64}$ Terminating Repeating
5. $0.\overline{5}$ Terminating Repeating	6. $0.\overline{45}$ Terminating Repeating
7. $0.08\overline{3}$ Terminating Repeating	8. $5.2\overline{6}$ Terminating Repeating
9. -0.24 Terminating Repeating	10. $3.\overline{65}$ Terminating Repeating

11. -0.58 Terminating Repeating	12. $1.1\bar{6}$ Terminating Repeating
13. $2.\bar{6}$ Terminating Repeating	14. 0.2 Terminating Repeating
15. $0.\overline{27}$ Terminating Repeating	16. 0.45 Terminating Repeating

7.3c Class Activity: Expanding Our Number System

Organize the following candy into the Venn diagram.

Snickers, Hershey's Chocolate Bar, Mars Bar, Laffy-Taffy, Starburst



List the sets of numbers we have learned about so far, including equivalent forms.

So are all numbers rational numbers? Are there numbers that cannot be written as a quotient of two integers?

What about $\sqrt{2}$? Can you write $\sqrt{2}$ as a fraction? Why or why not?

Numbers like $\sqrt{2}$, which do not have a terminating or repeating decimal expansion are **irrational numbers**. Irrational numbers **cannot** be expressed as a quotient.

Rational and Irrational numbers together form the set of **real numbers**. Real numbers can be thought of as points on an infinitely long line called the number line. Just like we organized the candy bars in the Venn diagram above we can organize the real number system.

Real Number System



Directions: Classify the following numbers and provide a justification.

Number	Whole number	Integer	Rational number	Irrational number	Real	Justification
1. $\frac{2}{3}$						
2. 0.25						
3. -2						
4. $\sqrt{5}$						
5. 10						
6. 0						
7. $\sqrt{10}$						
8. $\sqrt{36}$						
9. $-\sqrt{121}$						
10. $2\frac{1}{2}$						
11. $0.08\bar{3}$						

Number	Whole number	Integer	Rational number	Irrational number	Real	Justification
12. $\frac{10}{13}$						
13. π						
14. -3π						
15. $0.26\overline{54}$						
16. $\sqrt[3]{27}$						
17. 1.2122122212222...						
18. $\sqrt[3]{30}$						
19. $\frac{\sqrt{2}}{2}$						
20. The side length of a square with an area of 2						
21. The side length of a square with an area of 9						
22. The number half-way between 3 and 4						
23. The number that represents a loss of 5 yards						



7.3c Homework: Expanding Our Number System

Directions: Classify the following numbers as rational or irrational and provide a justification.

Number	Whole number	Integer	Rational number	Irrational number	Real	Justification
1. $\sqrt{2}$						
2. $\sqrt{1}$						
3. $\frac{1}{3}$						
4. -157						
5. $4\frac{1}{9}$						
6. -0.375						
7. $-\sqrt{5}$						
8. $0.\bar{2}$						
9. $\sqrt[3]{125}$						
10. $-\sqrt{81}$						
11. $-2.\bar{24}$						
12. 2π						
13. The side length of a square with an area of 49						
14. The side length of a square with an area of 1						
15. The side length of the side of a square with an area of 5						
16. The side length of a square with an area of 24						
17. The number half-way between 0 and -1						
18. The number that represents 7 degrees below 0.						

19. Give your own example of a rational number.

20. Give your own example of an irrational number.

Directions: The table below contains statements about rational and irrational numbers. If the statement is true, put a check in the box. If the statement is not true, write a correct statement.  

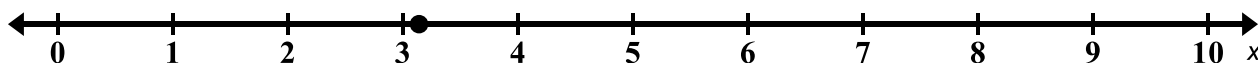
Statement	Check if True or Correct Statement
21. You can show the exact decimal expansion of the side length of a square with an area of 5 square units.	
22. You can construct and show the length $\sqrt{5}$ on a number line.	
23. Square roots of numbers that are perfect squares are rational.	
24. The number 0.256425642564 ... is rational.	
25. You can always use a calculator to determine whether a number is rational or irrational by looking at its decimal expansion.	
26. The number $0.\overline{6}$ is irrational because its decimal expansion goes on forever.	
27. The number half-way between 3 and 4 is rational.	
28. You can build a perfect cube with 36 unit cubes.	
29. If you divide an irrational number by 2, you will still have an irrational number.	
30. The side length of a cube made of 64 unit blocks is irrational.	

Make up **two** of your own statements that are **true** about rational or irrational numbers.

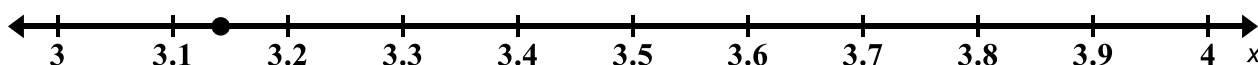
7.3d Class Activity: Approximating the Value of Irrational Numbers

So far, we have seen that we can show the location of an irrational number on the number line. We also know that we cannot show the entire decimal expansion of an irrational number because it is infinitely long and there is no pattern (as far as we know). However, we can come up with good approximations for the numerical value of an irrational number.

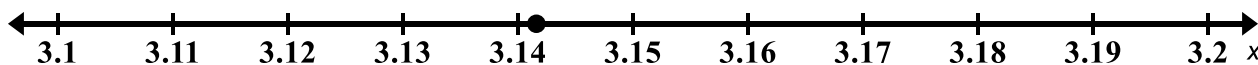
The decimal expansion for π to eight decimal places is 3.14159265... On the number line, we know that π lies somewhere between 3 and 4:



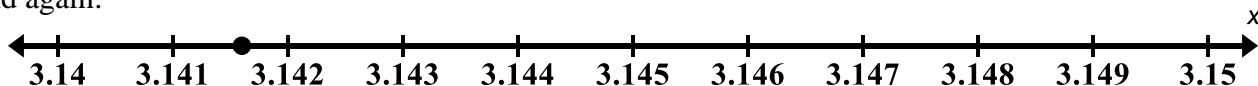
We can zoom in on the interval between 3 and 4 and narrow in on where π lies:



And if we zoom in again on the interval from 3.1 to 3.2:



And again:



We can imagine continuing this process of zooming in on the location of π on the number line, each time narrowing its possible location by a factor of 10.

Once we have an approximation for an irrational number, we can approximate the value of expressions that contain that number.

For example, suppose we were interested in the approximate value of 2π ? We can use our approximations of π from above to approximate the value of 2π to different degrees of accuracy:

Because π is between 3 and 4, 2π is between _____ and _____.

Because π is between 3.1 and 3.2, 2π is between _____ and _____.

Because π is between 3.14 and 3.15, 2π is between _____ and _____.

Because π is between 3.141 and 3.142, 2π is between _____ and _____.

Check the value of 2π on your calculator. How are we doing with our approximations of 2π ?

We can use a method of **guess and check** to give us an estimate of the numerical value of an irrational number that is correct up to as many decimal points as we need.

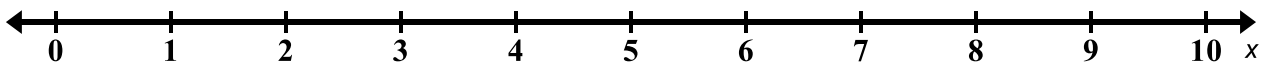


Directions: Approximate the value of the following irrational numbers to the indicated degrees of accuracy. You can use your calculator for the following questions but do not use the square root key.

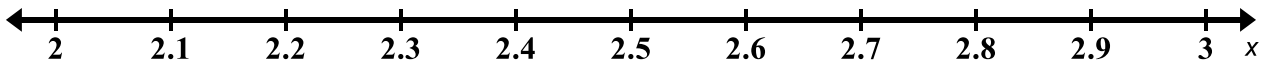
1. Between which two integers does $\sqrt{5}$ lie?

a. Which integer is it closest to?

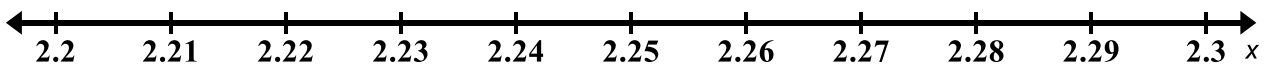
b. Show its approximate location on the number line below.



c. Now find $\sqrt{5}$ accurate to one decimal place. Show its approximate location on the number line below.



d. Now find $\sqrt{5}$ accurate to two decimal places. Show its approximate location on the number line below.

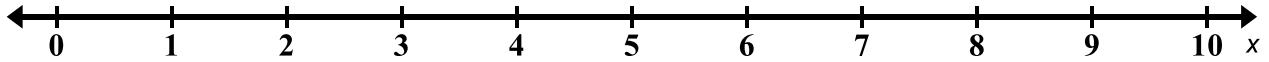


e. Use your work from above to approximate the value of the expression $2 + \sqrt{5}$ to the nearest whole number. The nearest tenth. The nearest hundredth.

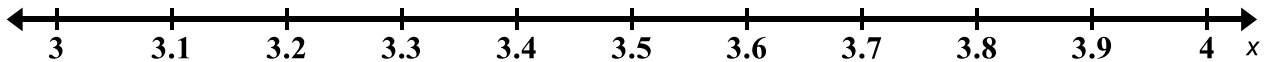
2. Between which two integers does $\sqrt{15}$ lie?

a. Which integer is it closest to?

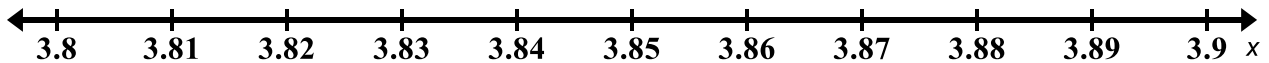
b. Show its approximate location on the number line below.



c. Now find $\sqrt{15}$ accurate to one decimal place. Show its approximate location on the number line below.



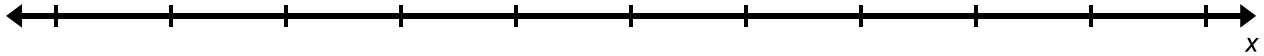
d. Now find $\sqrt{15}$ accurate to two decimal places. Show its approximate location on the number line below.



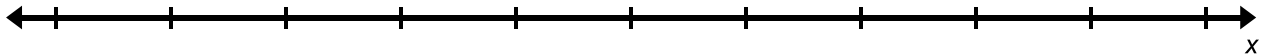
e. Use your work from above to approximate the value of the expression $4\sqrt{15}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

3. Repeat the process above to find $\sqrt{52}$ accurate to two decimal places. Place your numbers on the number lines provided each time you increase the degree of accuracy of your estimate.

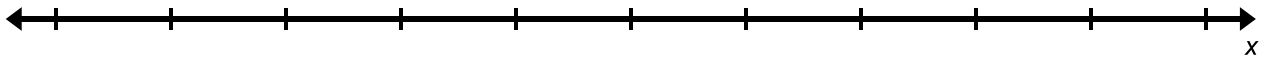
a. To the nearest **whole number**:



b. To the nearest **tenth**:



c. To the nearest **hundredth**:



d. Use your work from above to approximate the value of $3 + \sqrt{52}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

e. Use your work from above to approximate the value of $2\sqrt{52}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

4. Pick a positive integer between 1 and 100, call it A_0 . Find the average of your number (A_0) and $\frac{100}{\text{your number } (A_0)}$ and call this number A_1 . Take the average of A_1 and $\frac{100}{A_1}$ and call this number A_2 . Take the average of A_2 and $\frac{100}{A_2}$ and call this number A_3 . Repeat this process two more times.
5. Pick a different positive integer between 1 and 100 and repeat the process above. What do you notice?
6. Pick a positive integer between 1 and 100, call it A_0 . Find the average of your number (A_0) and $\frac{25}{\text{your number } (A_0)}$ and call this number A_1 . Take the average of A_1 and $\frac{25}{A_1}$ and call this number A_2 . Take the average of A_2 and $\frac{25}{A_2}$ and call this number A_3 . Repeat this process two more times.
7. Compare the number you picked for #6 with that of a neighbor. Compare your end results. What do you notice?
8. Pick a positive integer between 1 and 100, call it A_0 . Find the average of your number (A_0) and $\frac{5}{\text{your number } (A_0)}$ and call this number A_1 . Take the average of A_1 and $\frac{5}{A_1}$ and call this number A_2 . Take the average of A_2 and $\frac{5}{A_2}$ and call this number A_3 . Repeat this process two more times. What do you notice?

Directions: Solve the following problems. Again, do not use the square root key on your calculator.



9. A hospital has asked a medical supply company to manufacture intravenous tubing (IV tubing) that has a minimum opening of 7 square millimeters and a maximum opening of 7.1 square millimeters for the rapid infusion of fluids. The medical design team concludes that the radius of the tube opening should be 1.5 mm. Two supervisors review the design team's plans, each using a different estimation for π .

Supervisor 1: Uses 3 as an estimation for π

Supervisor 2: Uses 3.1 as an estimation for π

The supervisors tell the design team that their designs will not work. The design team stands by their plans and tells the supervisors they are wrong. Who is correct and why? Recall that the formula for the area of a circle is $A = \pi r^2$.

10. A square field with an area of 2,000 square ft. is to be enclosed by a fence. Three contractors are working on the project and have decided to purchase slabs of pre-built fencing. The slabs come in pieces that are 5-ft. long.
- Keith knows that $\sqrt{2000}$ is between 40 and 50. Trying to save as much money as possible, he estimates on the low side and concludes that they will need 160 feet of fencing. Therefore, he concludes they should purchase 32 slabs of the material.
 - Jose also knows that $\sqrt{2000}$ is between 40 and 50 but he is afraid that using Keith's calculations, they will not have enough fencing. He suggests that they should estimate on the high side and buy 200 feet of fencing to be safe. Therefore, he concludes they should purchase 40 slabs of material.

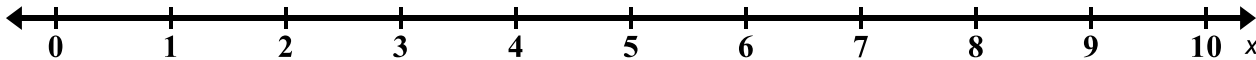
Keith and Jose begin to argue. Sam jumps in and says, "I have a way to make you both happy – we will purchase enough material to enclose the entire field **and** we will minimize the amount of waste." What do you think Sam's suggestion is and how many slabs will be purchased using Sam's rationale?

7.3d Homework: Approximating the Value of Irrational Numbers

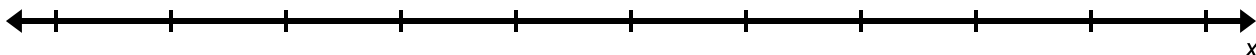
1. Between which two integers does $\sqrt{2}$ lie?

a. Which integer is it closest to?

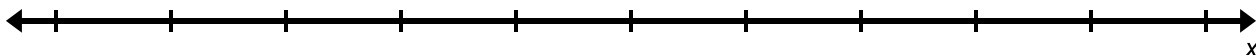
b. Show its approximate location on the number line below.



c. Now find $\sqrt{2}$ accurate to one decimal place. Show its approximate location on the number line below.



d. Now find $\sqrt{2}$ accurate to two decimal places. Show its approximate location on the number line below.

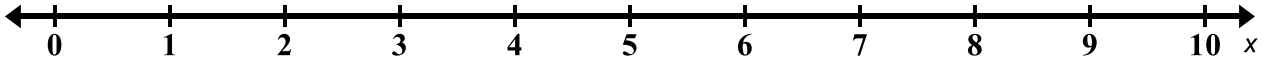


e. Estimate the value of the expression $2 + \sqrt{2}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

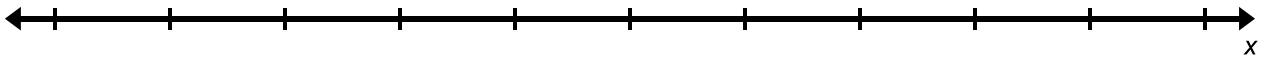
f. Estimate the value of the expression $2\sqrt{2}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

2. Between which two integers does $\sqrt{40}$ lie?

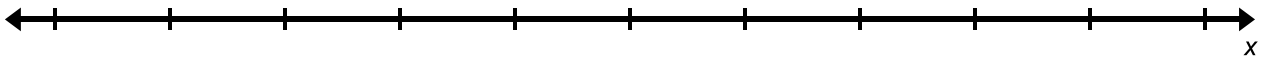
- a. Which integer is it closest to?
- b. Show its approximate location on the number line below.



- c. Now find $\sqrt{40}$ accurate to one decimal place. Show its approximate location on the number line below.



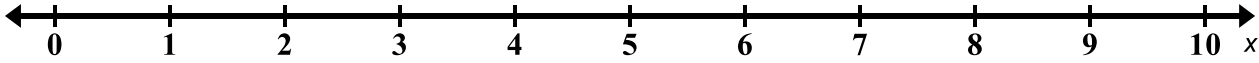
- d. Now find $\sqrt{40}$ accurate to two decimal places. Show its approximate location on the number line below.



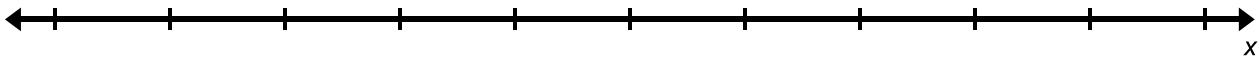
- e. Estimate the value of the expression $2\sqrt{40}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

3. Repeat the process above to find $\sqrt{60}$ accurate to two decimal places. Place your numbers on the number lines provided each time you increase the degree of accuracy of your estimate.

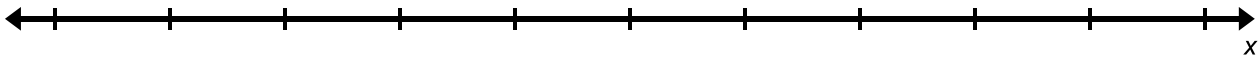
a. To the nearest **whole number**:



b. To the nearest **tenth**:



c. To the nearest **hundredth**:



d. Use your work from above to approximate the value of $\sqrt{60} - 5$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

e. Use your work from above to approximate the value of $1 + \sqrt{60}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

4. Use the approximations of π on page 60 to estimate the value of the following expressions to increasing levels of accuracy. You can use your calculator but don't use the square key or the π key.

a. π^2

b. 10π

c. $3 + \pi$

7.3e Class Activity: Comparing and Ordering Real Numbers

Directions: Do **not** use a calculator for the following problems. Any calculations you may need are given in the problem.



1. Order the following numbers from **least to greatest**. Note that $8.5^2 = 72.25$.
 $\sqrt{80}$, 8, 9, 8.5, $\sqrt{62}$

2. Order the following numbers from **least to greatest**. Note that $3.5^2 = 12.25$.
 $-\sqrt{13}$, -3, -4, -3.5

3. Use the following calculations to answer the questions below.

$$2.2^2 = 4.84$$

$$2.3^2 = 5.29$$

$$2.23^2 = 4.9729$$

$$2.24^2 = 5.0176$$

- a. Put the following numbers in order from **least to greatest**.

$\sqrt{5}$, $\frac{5}{2}$, 2.2, the side length of a square with an area of 4

- b. Find a number between 2.2 and $\sqrt{5}$.

- c. Find an irrational number that is smaller than all of the numbers in part a.

4. Use the following calculations to answer the questions below.

$$6.48^2 = 41.9904$$

$$6.5^2 = 42.25$$

- a. Order the following numbers from **least to greatest**.

$\sqrt{50}$, 6, 7, 6.5, $\sqrt{42}$

- b. Find a rational number that is smaller than all of the numbers in part a.

- c. Find an irrational number that is smaller than all of the numbers in part a.

- d. Find a number between $\sqrt{42}$ and 6.5.

5. Use the following calculations to answer the questions below.

$$2.44^2 = 5.9536$$

$$2.45^2 = 6.0025$$

$$2.449^2 = 5.997601$$

a. Order the following numbers from **least to greatest**.

$\sqrt{6}$, 2.44, $2.\bar{4}$, 2.5, the side length of a square with an area of 9

b. Find an irrational number that is between 0 and the smallest number from part a.

c. Find a number that is between 2.44 and $\sqrt{6}$.

6. Use the approximations of π on page 60 and the calculations given below to answer the questions below.

π is between 3 and 4

π is between 3.1 and 3.2

π is between 3.14 and 3.15

π is between 3.141 and 3.142

$$3.15^2 = 9.9225$$

a. Find a number that is between 3 and π .

b. Find a number that is between 3.14 and π .

c. Which is larger and why? $(\pi + 5)$ or 8

d. Which is larger and why? $(10 - \pi)$ or 7

e. Which is larger and why? 2π or 6.2

f. Which is larger and why? π^2 or 10

7.3e Homework: Comparing and Ordering Real Numbers

Directions: Do **not** use a calculator for the following problems. Any calculations you may need are given in the problem.

1. Give an example of a rational number between $\sqrt{9}$ and $\sqrt{16}$.
2. Give an example of an irrational number between 8 and 9.
3. Use the following calculations to answer the questions below.

$$1.41^2 = 1.9881$$

$$1.42^2 = 2.0164$$

- a. Order the following numbers from **least to greatest**.

$$\sqrt{2}, 1.41, 1.4, 1\frac{1}{2}, 1.4\bar{2}$$

- b. Find a number between 1.4 and $1\frac{1}{2}$.

4. Use the following approximations and calculations to answer the questions below.

π is between 3.14 and 3.15

$$3.1^2 = 9.61$$

$$3.2^2 = 10.24$$

$$3.16^2 = 9.9856$$

$$3.17^2 = 10.0489$$

- a. Order the following numbers from **least to greatest**.

$$\sqrt{10}, 3\frac{1}{10}, 3.\bar{1}, \pi, \text{side length of a square with an area of 9}$$

- b. Find a number between $3\frac{1}{10}$ and $3.\bar{1}$.

- c. Find a number between 3.1 and $\sqrt{10}$.

5. The number e is an important irrational number. In future math classes as well as science and social science, you will see and use this number quite a bit. Use the approximations of e and the calculations given below to answer the questions that follow.

e is between 2 and 3

e is between 2.7 and 2.8

e is between 2.71 and 2.72

e is between 2.718 and 2.719

- a. Find a number that is between 2 and e .
- b. Find a number that is between e and 2.8.
- c. Which is larger and why? $(e + 10)$ or 13
- d. Which is larger and why? $(6 - e)$ or 4
- e. Which is larger and why? $2e$ or 5.4
- f. Which is larger and why? e^2 or 9

6. Order the following numbers from **least to greatest**. Note that $6.2^2 = 38.44$ and $6.4^2 = 40.96$

$-\sqrt{40}$, -7 , -6 , -6.2 , -6.4 , $-6\frac{1}{2}$

7.3f Self-Assessment: Section 7.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Know that real numbers that are not rational are irrational.				
2. Show that rational numbers have decimal expansions that either terminate or repeat.				
3. Convert a repeating decimal into a fraction.				
4. Know that the square root of a non-perfect square is an irrational number.				
5. Understand that the decimal expansions of irrational numbers are approximations.				
6. Show the location (or approximate location) of real numbers on the real number line.				
7. Approximate the value of irrational numbers, zooming in to get better and better approximations.				
8. Estimate the value of expressions containing irrational numbers.				
9. Compare and order rational and irrational numbers.				

1. Circle the numbers that are rational.

a. -4

b. -0.34

c. $\sqrt{7}$

d. 0

e. $\frac{1}{2}$

f. $-\sqrt{11}$

g. $\sqrt{81}$

h. $-\sqrt[3]{27}$

2. Change each fraction to a decimal.

a. $\frac{3}{4}$

b. $\frac{5}{6}$

c. $\frac{8}{3}$

3. Change each decimal to a fraction.

a. $0.\bar{2}$

b. $1.\bar{34}$

c. $2.0\bar{1}$

4. Classify the following numbers as rational or irrational and provide a justification.

a. $\sqrt{10}$

b. $\sqrt[3]{30}$

c. $\sqrt{144}$

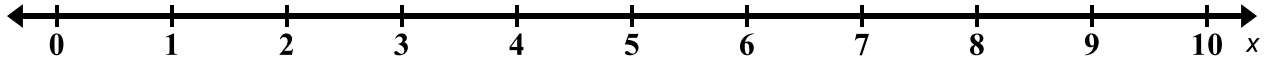
5. Find the decimal approximation of the following numbers to two decimal places without using the square root key on your calculator.

a. $\sqrt{22}$

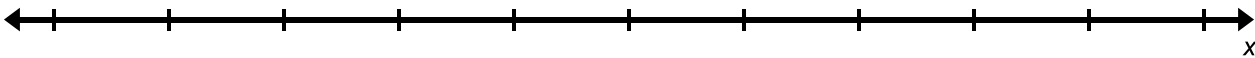
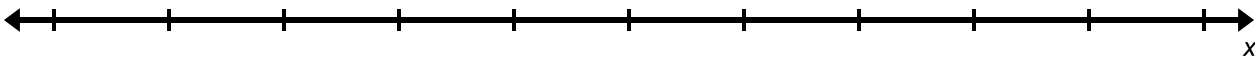
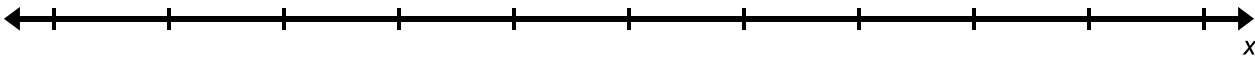
b. $\sqrt{45}$

c. $\sqrt{60}$

6. Describe how you would plot the following points on the number line shown below.
2.0, 2.2, 2.24.

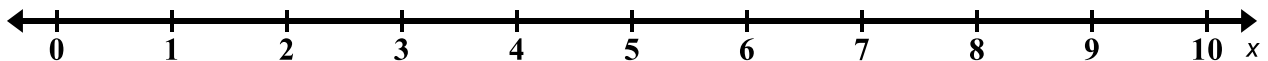


Plot the numbers from above on the three number lines shown below, changing the scale of each number line in order to show the location of the points more precisely.



7. Show the approximate location of the following numbers on the number line below.

A: $\sqrt{3}$, B: $\sqrt{10}$, C: $2\sqrt{5}$, D: $3\frac{1}{10}$, E: 1.5



8. Approximate $\sqrt{31}$ to the...
- Nearest whole number
 - Nearest tenth
 - Nearest hundredth
9. Approximate the value of the following expressions.
- $2\sqrt{2}$ if $\sqrt{2} \approx 1.41$
 - 3π if $\pi \approx 3.14$
 - $4 + \sqrt{2}$ if $\sqrt{2} \approx 1.41$
10. Order the following numbers from least to greatest.
- $1.2, -2\pi, -3\frac{1}{2}, \sqrt{6}, \frac{4}{3}, -6.28, -\sqrt{2}$

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Chapter 8: Integer Exponents, Scientific Notation and Volume (4 weeks)

Utah Core Standard(s):

- Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.* (8.EE.1)
- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.* (8.EE.3)
- Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurement of very large and very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. (8.EE.4)
- Know the formulas for volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. (8.G.9)

Vocabulary: base, cone, cylinder, diameter, estimate, exponent, exponential form, hemisphere, scientific notation, pi, power, powers of ten, radius, scientific notation, slant height, sphere, standard form, volume

Chapter Overview:

Students begin this chapter with the study of integer exponents. They represent repeated multiplication in exponential form and begin to explore the properties of integer exponents as another method for transforming expressions. Students explore problems and patterns that lead them to properties related to negative exponents and an exponent of 0. Through the investigation of these properties they learn to generate equivalent expressions in a quick and efficient way. Their study is then turned to using their knowledge of exponents, place value, and powers of ten to express a number in scientific notation. This notation is used to denote very small and very large numbers. Students learn to change numbers from standard form to scientific notation and vice versa. They also learn to perform operations with numbers in scientific notation. This enables them to work with and analyze real world situations where large and small quantities exist. Finally, students study volume and how exponents play a role in the formulas for the volume of a cylinder, cone, and sphere. They use these formulas to solve a variety of problems related to the volume of these three-dimensional objects.

Connections to Content:




Prior Knowledge:




Prior to 8th grade students have explained patterns in the number of zeros of the product when multiplying a number by a power of ten. They have also analyzed the patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. In addition they have used whole number exponents to denote powers of 10. They extend this knowledge to base numbers other than 10 in 8th grade as they generate the exponent rules. In addition to rewriting numbers as powers of ten to better understand multiplication and place value students have previously rewritten numbers in a variety of ways to perform an indicated skill, i.e., 24 as $2 \times 2 \times 2 \times 3$ and 18 as $2 \times 3 \times 3$, to reduce $\frac{18}{24}$ to $\frac{3}{4}$, or to find the LCM of 72 or GCF of 6. Similarly, in 8th grade, students express numbers as decimals or in scientific notation in order to compare and estimate very large and small quantities. Finally, students must gather their knowledge of area and volume from 6th and 7th grade as they begin their work with volumes of cylinders, cones, and spheres.

Future Knowledge:

A solid foundation with exponent laws and rules will help students significantly as they begin to transform more complicated expressions in high school mathematics courses. For example, in Secondary II they will extend the laws of exponents to rational exponents. In high school students will explore exponential functions and the work they do in 8th grade begins to familiarize them with how exponents work algebraically and how exponential behavior is exhibited. Scientific notation will be used in a variety of contexts in high school mathematics and science courses. By studying the volume formulas for cylinders, cones, and spheres in 8th grade, students will be prepared to investigate informal arguments and proofs, specifically Cavalieri's Principle for the derivation of these formulas in Secondary II.

MATHEMATICAL PRACTICE STANDARDS:

	<p>Reason abstractly and quantitatively.</p>	<ol style="list-style-type: none"> 1. If you fill the hemisphere with water or other filling material, predict what fraction of the cylinder is filled by the volume of one hemisphere. 2. Now try it, what fraction of the cylinder is filled by the volume of one hemisphere? 3. Write down the equation for the volume of the cylinder below the cylinder, be sure to write your height in terms of the radius or r. 4. Manipulate the equation for the volume of the cylinder to show the volume of the hemisphere. 5. Now double your formula to find the formula for the volume of a sphere. <p><i>Students derive the formula for the volume of a sphere by physically comparing the volume of a sphere to that of a cylinder. Based on the physical differences discovered, students manipulate the formula for the volume of a cylinder in order to derive the formula for the volume of a sphere. They use a similar process to derive the formula for the volume of a cone. They must algebraically interpret these changes as they reason abstractly about the dimensions dealt with. They can then make the appropriate changes as they manipulate the formulas.</i></p>
	<p>Look for and make use of structure.</p>	<p>Discuss the multiplication problem $(5 \times 3)(2 \times 8)$ with your class. Write your thoughts below.</p> <p>Of course it most natural to just multiply 15 times 16. But could you rewrite the problem as $(5 \times 2)(3 \times 8)$ or $(5 \times 8)(2 \times 3)$? Is the answer the same? Why can you do this?</p> <p>Rewrite this problem $(5.1 \times 10^5)(6.8 \times 10^3)$ like the problem above (group the powers of 10 together). Then solve the problem (use exponent properties) and write the solution.</p> <p><i>Looking for structure is a big part of this chapter. The example above is showing how the structure of a number written in scientific notation can aid in completing basic operations of very large and small numbers in a fast and efficient way. The students will also make use of structure when they look at how exponents are used to represent repeated multiplication. This in turn points toward the discovery and understanding of the exponent properties and rules.</i></p>
	<p>Attend to precision.</p>	<p>Analyze the pairs of expressions given below; discuss the similarities and differences between them.</p> <p>$(-4)^2$ and -4^2</p> <p>$(-4)^3$ and -4^3</p> <p><i>Students might bring up the fact that the only thing that is different about the first set of expressions are the parentheses. The parentheses are important because they indicate that there are two copies of negative 4. If you expand this expression you get $(-4)(-4) = 16$. The expression -4^2 indicates that there is a coefficient of negative 1. Upon expansion, you get $-4^2 = (-1)(4)(4) = (-1)(16) = -16$. Students will get different answers even though the expressions are similar. They must attend to precision. In the second set of expressions they both equal -64. This is because of the odd exponent. Throughout the chapter students must attend to precision constantly as they grapple with the notation used with exponents and as they decipher what these special notations are communicating to them.</i></p>

	<p>Look for and express regularity in repeated reasoning.</p>	<p>Complete columns 2 and 3 in the table below by writing each expression in expanded form and in exponential form. As you are simplifying try to discover a shortcut or rule that you could use to simplify more quickly. Test your shortcut in the last column and compare your answers.</p> <p>Product of Powers</p> <table border="1" data-bbox="532 289 1515 533"> <thead> <tr> <th>Column 1</th> <th>Column 1 rewritten in expanded form or as repeated multiplication.</th> <th>Column 2 rewritten in exponential form.</th> <th>Test your shortcut and compare.</th> </tr> </thead> <tbody> <tr> <td>$5^3 \cdot 5^2$</td> <td></td> <td></td> <td></td> </tr> <tr> <td>$b^6 \cdot b^2$</td> <td></td> <td></td> <td></td> </tr> <tr> <td>$y^2 y^{10} y^4$</td> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p>Explain how to multiply exponents with the same base without using expanded form.</p> <p>Algebraic Rule for the Product of Powers(explain your rule using symbols):</p> <p><i>Throughout section 8.1 students will expand expressions repeatedly in order to discover the rules and properties of integer exponents.</i></p>	Column 1	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.	$5^3 \cdot 5^2$				$b^6 \cdot b^2$				$y^2 y^{10} y^4$			
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$5^3 \cdot 5^2$																		
$b^6 \cdot b^2$																		
$y^2 y^{10} y^4$																		
	<p>Use appropriate tools strategically.</p>	<p>You are asked to enter the following expression into your scientific calculator.</p> $\left(\frac{2}{3}\right)^4$ <p>Which of the following is not a correct way to enter the expression into a scientific calculator?</p> <table border="1" data-bbox="532 1045 1490 1129"> <tbody> <tr> <td>a. 2 <input type="text" value="÷"/> 3 <input type="text" value="y^x"/> 4</td> <td>b. (2 <input type="text" value="÷"/> 3) <input type="text" value="y^x"/> 4</td> <td>c. 2 <input type="text" value="y^x"/> 4 <input type="text" value="÷"/> 3 <input type="text" value="y^x"/> 4</td> </tr> </tbody> </table> <p><i>Using a calculator appropriately is an important component of this chapter. Students often mistake how to enter exponents into the calculator correctly. They must also learn how to enter and interpret a number expressed in scientific notation on a calculator.</i></p>	a. 2 <input type="text" value="÷"/> 3 <input type="text" value="y^x"/> 4	b. (2 <input type="text" value="÷"/> 3) <input type="text" value="y^x"/> 4	c. 2 <input type="text" value="y^x"/> 4 <input type="text" value="÷"/> 3 <input type="text" value="y^x"/> 4													
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	<p>Make sense of problems and persevere in solving them.</p>	<p><i>Gas 'N' Go Convenience Stores</i> claim that 10% of Utahans fuel up at their stores each week. Decide whether their claim is true using the following information.</p> <ul style="list-style-type: none"> • There are about 2.85×10^6 people in Utah. • There are 2.18×10^2 <i>Gas 'N'Go</i> stores in Utah. • Each stations serves gasoline to about 1.2×10^3 people each week. <p><i>There are several questions in this chapter where students are given the opportunity to problem solve. In the example above students must find an entry point into the problem by determining that they need to figure out what 10% of the Utah population is. They must analyze what is given to them and how they can use it to reach their intended goal. In addition to reaching an outcome they can analyze their solution to see if it makes sense given the context.</i></p>																



Construct viable arguments and critique the reasoning of others.

Edward, Pattie, and Mitch were each simplifying the same exponential expression. Their work is shown below. Determine who has simplified the expression correctly. If they did not simplify the expression correctly, identify the mistake, explain it, and fix it.

Edward

$$\left(\frac{3x^4}{6x}\right)^2 = \left(\frac{2x^4}{x}\right)^2 = (2x^3)^2 = 2x^6$$

Pattie

$$\left(\frac{3x^4}{6x}\right)^2 = \frac{3^2 \cdot x^8}{6^2 \cdot x^2} = \frac{9x^8}{36x^2} = \frac{x^6}{4}$$

Mitch

$$\left(\frac{3x^4}{6x}\right)^2 = \left(\frac{x^4}{2x}\right)^2 = \left(\frac{x^3}{2}\right)^2 = \frac{x^6}{4}$$

In the problem above students must critique the work of others. As they do so they solidify their understanding of different exponent properties. They also analyze common misconceptions and mistakes that are made when simplifying expressions with exponents. As they examine two problems that are correct but simplified differently they see that it is possible to arrive at the same answer in a variety of ways. This enhances their understanding of the structure of these expressions and how they are composed.



Model with mathematics.

A silo is a storage bin that is a cylinder with a hemisphere on top. A farmer has a silo with a base radius of 30 feet and a storage height of 100 feet. The “storage height” is the part which can be filled with grain - it is just the cylinder. A cubic foot of grain weighs 62 lbs.

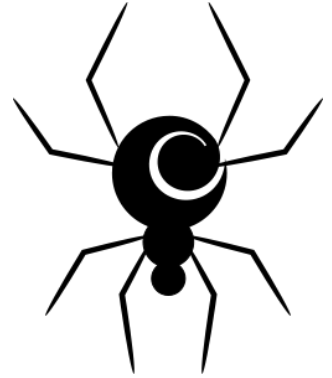
- Draw and label a picture of the silo.
- How many pounds of grain can the farmer store in the silo?
- How high (including the hemispherical top) is the silo?
- 1000 square feet of wheat produces 250 pounds of grain. The farmer’s wheat field is 3,500 ft by 20,000 ft. Is the silo large enough to hold the grain? By how much? Explain your answer.
- If the farmer decides to fill the silo all the way to the top of the hemisphere how many cubic feet of grain can he store?

While working with volume students use geometry to model a variety of situations. In the problem above, a silo is modeled with a hemisphere and cylinder. Student’s use this geometric model to answer questions about the silo. They use a mathematical formula to model the volume of the silo to determine how much grain it will hold and the height of the silo.

8.0 Anchor Problem: Spiders



1. The genetically altered spider that turned Peter Parker into Spider Man with a single bite was about 0.035 ounces. If Spider Man weighs roughly 185 pounds how many spiders does it take to have the same mass as Spider Man?



2. Spider Man fights evil villains in New York City. The size of New York City is roughly 1,214,450,000 square meters. It is estimated that on average there are approximately 1,308 spiders per square meter of land. Use this information to determine how many spiders are in New York City?

Bonus: Determine how many spiders there are in your city or even a bedroom in your basement.

Section 8.1: Integer Exponents

Section Overview:

This section begins with an overview of the structure of exponents and how an exponential expression represents repeated multiplication as opposed to expressions that represent repeated addition. Using the structure of an exponential expression special properties or rules are discovered in this section. These exponent properties and rules aid in simplifying exponential expressions. Students will informally prove why an exponent of zero equals one and also look at the definition of negative exponents, that is, $x^{-1} = \frac{1}{x}$. Once students become familiar with these properties and rules they use them to simplify more complex exponential expressions.

Concepts and Skills to Master:

By the end of this section students should be able to:

1. Apply the properties of integer exponents to simplify algebraic and numerical expressions.

8.1a Class Activity: Get Rich Quick

Mario and Tony both want you to come and drive Go-Karts for their team. They will pay you in gold coins. Each one makes an offer:

Mario: I will give you 3 gold coins on the first day. Then, every day after that, I will pay you 3 times as much as I paid you the day before.

Tony: I will give you 3 gold coins on the first day. Then, every day after that, I will pay you 3 more coins than I paid you the day before.

1. Who would you rather work for? Use the table below to help you decide.

Mario's Deal	Daily Wage	Tony's Deal	Daily Wage
Monday		Monday	
Tuesday		Tuesday	
Wednesday		Wednesday	
Thursday		Thursday	
Friday		Friday	
Total Earnings		Total Earnings	

2. For whom would you rather work and why?

3. Rewrite your earnings for each day as repeated multiplication or repeated addition.



4. You have used exponents previously to represent whole numbers in expanded form as powers of ten. Complete the following table to remind yourself how exponents are used. The first couple of rows have been done for you.

10^1	means	10	which is equal to	10
10^2	means	$10 \cdot 10$	which is equal to	100
10^3	means		which is equal to	
10^5	means		which is equal to	
10^8	means		which is equal to	
10^{10}	means		which is equal to	

5. Now rewrite your earnings for each day using exponents or multiplication. Be ready to discuss the effect that an exponent has on a number; think about the difference between repeated multiplication and repeated addition.

In general, for any number x , and any whole number n ,

$$x^n = \underbrace{(x \cdot x \cdot x \cdots x)}_{n \text{ times}}$$

The expression x^n is read x raised to the n th power. In this expression n is the exponent and x is the base number.

6. Write each expression given below in exponential form.

a. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 =$	b. $\frac{9}{7} \cdot \frac{9}{7} \cdot \frac{9}{7} =$	c. $\underbrace{(4 \cdot 4 \cdot 4 \cdots 4)}_{15 \text{ times}} =$
d. $(-2)(-2)(-2)(-2) =$	e. $\left(\frac{-2}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{-2}{3}\right) =$	f. $x \cdot x \cdot x \cdot x \cdot x \cdot x =$
g. $\frac{1}{x} \cdot \frac{1}{x} \cdot \frac{1}{x} \cdot \frac{1}{x} \cdot \frac{1}{x} \cdot \frac{1}{x} \cdot \frac{1}{x} =$	h. $2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 =$	i. $11 \cdot 11 \cdot 11 \cdot x \cdot x =$
j. $x \cdot x \cdot x \cdot y =$	k. $a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot c \cdot c =$	l. $\underbrace{(r \cdot r \cdot r \cdots r)}_{20 \text{ times}} \underbrace{(q \cdot q \cdot q \cdots q)}_{9 \text{ times}} =$

7. Examine problem *j.* above. What exponent does the variable y have? Do you have to write the exponent?

8. Notice the use of parentheses in problems *d.* and *e.* above. Why do you think they are they used?

9. Evaluate each exponential expression by first re-writing it using multiplication.

$(-3)^2$	means	$(-3)(-3)$	which is equal to	9
$(-3)^3$	means		which is equal to	
$(-3)^4$	means		which is equal to	
$(-3)^5$	means		which is equal to	
$(-3)^6$	means		which is equal to	
$(-3)^7$	means		which is equal to	

10. Describe one pattern that you notice in the table above.

11. Examine the pairs of expressions given below; discuss the similarities and differences between them.

$$(-4)^2 \text{ and } -4^2$$

$$(-4)^3 \text{ and } -4^3$$

March Madness, the NCAA basketball tournament, has the form of a single-elimination tournament. In such a tournament, we start with a certain number of teams, and we pair them off into games; each team plays a game. This is called the first round. All the losers in the first round are eliminated; in the second round all the winning teams are paired off into games, and all the second round losers are eliminated. This process continues until only two teams remain; this is the final round and the winner is the champion of the tournament.

12. March Madness starts with 64 teams. How many rounds are there?



13. How many teams are in the second round? In any round?

14. How many games total are played?

8.1a Homework: Get Rich Quick

1. Write each expression in exponential form

a. $\underbrace{(5 \cdot 5 \cdot 5 \cdots 5)}_{17 \text{ times}} =$

b. $(-4)(-4)(-4)(-4) =$

c. $(3.7)(3.7)(3.7) =$

d. $\frac{5}{8} \cdot \frac{5}{8} \cdot \frac{5}{8} \cdot \frac{5}{8} \cdot \frac{5}{8} \cdot \frac{5}{8} =$

e. $x \cdot x \cdot x \cdot x \cdot y \cdot y =$

f. $3 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot b =$

2. Upon taking a VERY good job, Manuel is given one of the following two options for his retirement plan.

Option A: \$10 the first year, then every year after that you will get 10 times as much as the year before.

Option B: \$100,000 the first year and then every year after that you will get \$100,000 more than the year before.

a. What option should he choose? Justify your answer? Use the table to decide.

Option A	Yearly Retirement	Option B	Yearly Retirement
Year 1		Year 1	
Year 2		Year 2	
Year 3		Year 3	
Year 4		Year 4	
Year 5		Year 5	
Year 6		Year 6	

b. Write each yearly retirement amount as repeated multiplication or repeated addition.

c. Write each yearly retirement amount in exponential form for Option A.

d. Extension: Write an equation or function that could be used to find the amount of retirement Manuel would receive for any given year for Option A or Option B

3. Write an exponential expression with (-1) as its base that will result in a positive product.

4. Write an exponential expression with (-1) as its base that will result in a negative product.

5. Rewrite each number in exponential form, with the number two as its base.

a. $8 =$

b. $32 =$

c. $128 =$

6. Pablo wrote $(-2)^5 = -32$. Is he correct; why or why not?

7. Chantal wrote $-6^2 = 36$. Is she correct; why or why not?

A candy maker is making taffy. He starts with one long piece of taffy and cuts it into 3 pieces. He then takes each resulting piece and cuts it into three pieces. He then takes each of these resulting pieces and cuts it into three pieces. He continues this process.

8. Use exponents to represent the number of pieces of taffy the candy maker has after the first 4 rounds of cuts.

9. How many pieces of taffy will the candy maker have after 8 rounds of cuts?

10. The candy maker gets a special order for 243 pieces of peppermint flavored taffy. How many rounds of cuts will he have to make to get this many pieces?

8.1b Class Activity: Find, Fix, and Justify and Exponent Properties

Part 1: Find, Fix, and Justify



The following statements are **incorrect**. For each of the statements do the following:

- **Find** the mistake(s) in each statement.
- **Fix** the mistake.
- **Justify** your reasoning. You may use pictures if needed.

1. In the expression below; 4 is called the base number and 5 is called the exponent.

$$5^4$$

2. $2^6 = 12$

3. $2^5 = 5^2$

4. $3x = x^3$

5. $(-2)^3 = 8$

6. $-7^2 = 49$

As each problem is discussed with the class, make changes or add notes to your work above if needed. Use the space below to write down important notes about exponents.

Part 2: Exponent Properties

You are going to further investigate expressions with exponents by combining them through multiplication and division. There are special properties that help to transform exponential expressions with a shortcut; they are called Exponent Properties or Rules. The problems given below show the special properties that hold true for all exponential expressions.



Complete columns 2 and 3 in the table below by writing each expression in expanded form and in exponential form. The first one has been done for you. As you are simplifying try to discover a shortcut or rule that you could use to simplify more quickly. Test your shortcut in the last column and compare your answers.

Product of Powers

Column 1	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.
$5^3 \cdot 5^2$	$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$	$5^5 = 3125$	
$b^6 \cdot b^2$			
$y^2 y^{10} y^4$			
$3x^2 y \cdot 2x^2 y^2$			

Explain how to multiply exponents with the same base without using expanding.

Algebraic Rule for the Product of Powers (explain your rule using symbols):

$$x^a \cdot x^b$$

Simplify each expression:

a. $x^5 x^3 =$

b. $x^{40} x^3 =$

c. $a^3 b^2 \cdot a^6 b^3 =$

d. Make up your own problem that requires the Product of Powers to simplify.

e. How would you simplify $a \cdot a^3$.

Try these problems:

f. $ab^5 \cdot 8a^2 b =$

g. $(2xy)(4x^2 y^3 z) =$

Quotient of Powers

	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.
$\frac{3^5}{3^2}$			
$\frac{a^4}{a^3}$			
$\frac{b^3}{b^5}$			
$\frac{4^5 \cdot x^7}{4^2 \cdot x^4}$			

Explain how to divide exponents with the same base without expanding.

Algebraic Rule for the Quotient of Powers:

$$\frac{a^m}{a^p}$$

Simplify each expression:

a. $\frac{6c^5}{c^2}$

b. $\frac{a^7b^{10}}{ab^3c^2}$

c. $\frac{x^{20}y^{15}}{x^{15}y^{14}}$

d. $\frac{6x^2}{2x^4}$

e. $\frac{8s^{12}t^4u^3}{2s^{10}t^4u}$

f. $\frac{2m^2n^5}{8m^5n^2}$

8.1b Homework: Product of Powers and Quotient of Powers Properties

1. For parts a. through c. write each expression in expanded form. Then write the simplified expression in exponential form.

Product of Powers

#	Expression	Expanded Form	Exponential Form
a.	x^4x^6		
b.	$a^3b^2 \cdot a^3b^5$		
c.	$(3abc)(2a^2b)$		

Simplify each expression.

d. x^6x^{12}	e. $y^{50}y^{200}$	f. $a^{13}b^5 \cdot a^3b^{20}$	g. $5a^6 \cdot -4ab^7$	h. $3y^2 \cdot x^4$
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2. For parts a. through c. write each expression in expanded form. Then write the simplified expression in exponential form.

Quotient of Powers

#	Expression	Expanded Form	Exponential Form
a.	$\frac{4^3}{4^2}$		
b.	$\frac{x^2}{x^6}$		
c.	$\frac{a^4b^7}{a^3b^6}$		

Simplify each expression.

d. $\frac{b^{20}}{b^5}$	e. $\frac{c^{200}}{c^{146}}$	f. $\frac{p^{10}r^{20}}{p^3r^{10}}$	g. $\frac{s^2t^3}{t^7}$	h. $\frac{5^4a^4b^2}{5^3ab^2}$
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3. Simplify each expression.

Mixed Practice

a. $(2xy^2)(4x^2y)$	b. $(-4x^2t^3)(-6r^5x^2t)$	c. $\frac{yz^2 \cdot y^3z}{x^2yz}$	d. $3x^2\left(\frac{1}{2}y^3\right)$
e. $\frac{15x^3}{5x^3}$	f. $\frac{x^4 \cdot x^3}{x^9}$	g. $\frac{-ab^5a^4}{a^3bc^5}$	h. $\frac{4^2x^2y^4x}{x^2} \cdot \frac{1}{2y}$

8.1c Class Activity: Power of a Power, Power of a Product, and Power of a Quotient



Complete columns 2 and 3 in the table below by writing each expression in expanded form and in exponential form. As you are simplifying try to discover a shortcut or rule that you could use to simplify more quickly. Test your shortcut in the last column and compare your answers.

Power of a Power

	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.
$(3^3)^2$			
$(a^2)^4$			
$[(b^3)^2]^2$			
$[(x^4)^{10}]^5$	<i>This is too long to expand, find a short cut.</i>		

Explain how to find the power of a power without expanding.

Algebraic Rule for the Power of a Power:

$$(a^m)^p$$

Simplify each expression:

a. $(a^3)^5$

b. $(b^7)^{11}$

c. $[(c^5)^6]^7$

d. $[(-d^2)^{10}]^3$

e. Explain the difference between $(x^2)^3$ and $(x^2)(x^3)$.

Power of a Product

	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.
$(ab)^4$			
$(abc)^3$			
$(a^2b)^3$			
$(4xy^2)^4$			
$(3^2a^{10}b^{11})^4$	<i>This is too long to expand, find a short cut.</i>		

Explain how to find the power of a product without expanding.

Algebraic Rule for the Power of a Product:

$$(ab)^m$$

Simplify each expression:

a. $(ab)^5$

b. $(ab^3)^6$

c. $(2y^2z)^3$

d. $(3^3x^2yz^2)^4$

e. Create two different expressions that simplify to 4^6 .

f. Explain the difference between finding $(xy)^2$ and $(x + y)^2$. Use the values $x = 2$ and $y = 3$ in your explanation.

g. Which value is greater, $(x^1)(x^4)$ or x^5 ? Explain.

Power of a Quotient

	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.
$\left(\frac{a}{b}\right)^3$			
$\left(\frac{x}{5}\right)^2$			
$\left(\frac{xy}{z}\right)^4$			
$\left(\frac{a^3}{b^5}\right)^{10}$	<i>This is too long to expand, find a short cut.</i>		

Explain how to find the power of a quotient without expanding.

Algebraic Rule for the Power of a Quotient:

$$\left(\frac{a}{b}\right)^m$$

Simplify each expression:

a. $\left(\frac{x}{y}\right)^6$

b. $\left(\frac{5}{x^4}\right)^2$

c. $\left(\frac{4x^2}{y}\right)^6$

d. $\left(\frac{2^2x^3y^2}{x}\right)^2$

e. $\left(\frac{3a^5 \cdot 2a^4}{4a^3}\right)^5$

8.1c Homework: Power of a Power, Power of a Product, and Power of a Quotient

1. For parts a. through c. write each expression in expanded form. Then write the simplified expression in exponential form.

Power of a Power

#	Expression	Expanded Form	Exponential Form
a.	$(x^3)^4$		
b.	$(2^3)^5$		
c.	$[(a^2)^3]^2$		

Simplify each expression.

d. $(r^5)^8$	e. $(y^{50})^4$	f. $(-4^5)^3$	g. $[(a^3)^{10}]^5$	h. $(k^9)^5(k^3)^2$
--------------	-----------------	---------------	---------------------	---------------------

2. For parts a. through c. write each expression in expanded form. Then write the simplified expression in exponential form.

Power of a Product

#	Expression	Expanded Form	Exponential Form
a.	$(xy^3)^4$		
b.	$(x^3y^4)^2$		
c.	$(2a^3c^2)^3$		

Simplify each expression.

d. $(a^6b)^7$	e. $(a^2b^5)^5$	f. $(2ab^4)^6$	g. $(6^2ab^3a^3)^4$	h. $(-3r^4)^4 \cdot (r^5)^2$
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3. For parts a. through d. write each expression in expanded form. Then write the simplified expression in exponential form.

Power of a Quotient

#	Expression	Expanded Form	Exponential Form
a.	$\left(\frac{3}{5}\right)^2$		
b.	$\left(\frac{c}{d}\right)^3$		
c.	$\left(\frac{x^2}{y}\right)^3$		
d.	$\left(\frac{2x}{3y}\right)^4$		

Simplify each expression.

e. $\left(\frac{2}{a}\right)^{12}$	f. $\left(\frac{2ab}{a}\right)^4$	g. $\left(\frac{s^2}{t^3}\right)^5$	h. $\left(\frac{4xy^2}{2y}\right)^3$
------------------------------------	-----------------------------------	-------------------------------------	--------------------------------------

4. Simplify each expression.

Mixed practice

a. $4z^5 \cdot 2y^7 \cdot 3z^5y$	b. $(3ab)^4$	c. $\frac{48z^2 \cdot y^3 x^4 r^5}{12z^2 y x r^3}$
d. $(4a^3 \cdot 3b^7)^2$	i. $\left(\frac{4x^6}{2b^3}\right)^3$	e. $\left(\frac{2a^4 \cdot 3a}{3a^2}\right)^2$
f. $(-3xyz)^6$	j. $\frac{27a^8 b^4 a^6}{18a^{10} b}$	g. $\left(\frac{3a^4}{b}\right) \left(\frac{6ab^3}{a^2}\right)^2$

8.1d Class Activity: Find, Fix, and Justify

The following statements are **incorrect**. For each of the statements do the following:

- **Find** the mistake(s) in each statement.
- **Fix** the mistake.
- **Justify** your reasoning. You may use pictures if needed.

1. $x^2 \cdot x^4 = x^8$	2. $a^3b^2 \cdot a^4b^5 = a^5b^9$	3. $a^2b^5 \cdot ab^3 = a^2b^8$
4. $\frac{x^7}{x^4} = x^{11}$	5. $\frac{a^6}{a^2} = a^3$	6. $\frac{b^3}{b^9} = b^6$
7. $\frac{12x^6y^3}{2x^2y^2} = 10x^4y$	8. $(y^3)^4 = y^7$	9. $(tw)^3 = tw^3$

Some of the following statements are **correct** and some are **incorrect**. If the statement is correct justify why it is correct by expanding the expression. If the statement is incorrect:

- Find the mistake(s) in each statement.
- Fix the mistake.
- Justify your reasoning.

10. $[(y^2)^3]^2 = y^{12}$	11. $(a^2b^3)^2 = a^4b^6$	12. $(5y^3)^3 = 5y^9$
13. $\left(\frac{3}{5}\right)^2 = \frac{9}{5}$	14. $\left(\frac{c}{d}\right)^3 = \frac{c^3}{d^3}$	15. $\left(\frac{2x}{yz}\right)^3 = \frac{8x^3}{y^3z^3}$
16. $\left(\frac{a^2}{b^3}\right)^4 = \frac{a^8}{b^3}$	17. $\left(\frac{2x^2}{x^2y^3}\right)^2 = \frac{4}{y^6}$	18. $3^2 + 3^4 = 3^6$

19. Write three expressions equivalent to $3^2 \cdot 9^2$

20. Write three exponential expressions equivalent to 400.

21. $\frac{3^a}{3^b} = 3^2$. Find numbers a and b that satisfy the equation. Can you find different numbers for a and b ?

22. Consider the equation, $x^y = y^x$, where x and y are two different whole numbers. Find the value for x and y .

23. You are asked to enter the following expression into your scientific calculator.

$$(4^2)^3$$

Which of the following is a correct way to enter the expression into a scientific calculator?

a. 4 <input type="text" value="y^x"/> 5	b. 4 <input type="text" value="y^x"/> 6	c. 4 <input type="text" value="y^x"/> 2 <input type="text" value="x"/> 3	d. 4 <input type="text" value="x"/> 2 <input type="text" value="x"/> 3
---	---	--	--

8.1d Homework: Find, Fix , and Justify.

1. Edward, Pattie, and Mitch were each simplifying the same exponential expression. Their work is shown below. Determine who has simplified the expression correctly. If they did not simplify the expression correctly, identify the mistake, explain it, and fix it.

Edward

$$\left(\frac{3x^4}{6x}\right)^2 = \left(\frac{2x^4}{x}\right)^2 = (2x^3)^2 = 2x^6$$

Patti

$$\left(\frac{3x^4}{6x}\right)^2 = \frac{3^2 \cdot x^8}{6^2 \cdot x^2} = \frac{9x^8}{36x^2} = \frac{x^6}{4}$$

Mitch

$$\left(\frac{3x^4}{6x}\right)^2 = \left(\frac{x^4}{2x}\right)^2 = \left(\frac{x^3}{2}\right)^2 = \frac{x^6}{4}$$

- Find the value of 1^8 , 1^9 , 1^{10} , and 1^0 . What can you say about the value of any power of 1?
- What is the area of a square with a side length of $3a^5$?
- What is the area of a rectangle with a length of $12x^3$ units and a width of $6x^2$ units?

- You are asked to enter the following expression into your scientific calculator.

$$(5^2)^5$$

Which of the following is **not a correct** way to enter the expression into a scientific calculator?

a. 5 <input type="text" value="y^x"/> 2 <input type="text" value="y^x"/> 5	b. 5 <input type="text" value="y^x"/> 10	c. 25 <input type="text" value="y^x"/> 5	d. 5 <input type="text" value="y^x"/> 7
--	--	--	---

- You are asked to enter the following expression into your scientific calculator.

$$\left(\frac{2}{3}\right)^4$$

Which of the following **not a correct** way to enter the expression into a scientific calculator?

a. 2 <input type="text" value="÷"/> 3 <input type="text" value="y^x"/> 4	b. (<input type="text" value="2"/> <input type="text" value="÷"/> 3 <input type="text" value=")"/> <input type="text" value="y^x"/> 4	c. 2 <input type="text" value="y^x"/> 4 <input type="text" value="÷"/> 3 <input type="text" value="y^x"/> 4
--	--	---

- Given the statement, $3^a \cdot 3^b = 3^{10}$. Find two numbers for a and b that satisfy the equation. Can you find different numbers for a and b ?
- Given the statement, $(3^a)^b = 3^{30}$. Find two numbers for a and b that satisfy the equation. Can you find different numbers for a and b ?
- Make up your own problem that requires two Properties of Exponents to simplify. Be sure to show your answer.
- Make up your own problem that requires two different properties than the ones you used in number 9. Be sure to show your answer.
- On Tuesday, you invited 2 friends to your party. On Wednesday, each of these friends invited 2 other friends. This pattern continued Thursday and Friday. How many people were invited on Friday? Write the answer as a power. How many people were invited in all? Explain your reasoning.

8.1e Class Activity: Zero and Negative Exponents

1. Complete the table below. (Hint: Use the patterns in the Powers of 10 section to help with the Powers of 2 section.)



POWERS of 10				POWERS of 2	
Words	Number (as decimals)	Number (as Fractions)	Number (with Powers)	Number	Number (with Powers)
a. One Million	1,000,000	$\frac{1,000,000}{1}$	10^6	64	2^6
b.	100,000	$\frac{100,000}{1}$		32	
c.				16	
d.		$\frac{1,000}{1}$		8	
e. One hundred	100		10^2		2^2
f.	10	$\frac{10}{1}$	10^1		
g.	1			1	
h.	.1	$\frac{1}{10}$	$\frac{1}{10^1}$ or 10^{-1}		
i.		$\frac{1}{100}$			
j.	.001				
k. One ten thousandth					
l.		$\frac{1}{100,000}$			
m.					

- Complete this sentence: Any number with a zero exponent is...
- Explain what happens to the size of the numbers as you move up the column from 10^1 .
- Explain what happens to the size of the numbers as you move down the column from 10^1 .
- Write 5^{-2} as a fraction. Write x^{-6} as a fraction.

Zero Exponent Property

Quotient	Expanded Form	Simplified	Thus...
$\frac{x^5}{x^5}$	$\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x}$	$\frac{1}{1} = 1$	$\frac{x^5}{x^5} = 1$
$\frac{4^3}{4^3}$			
$\frac{(ab)^2}{(ab)^2}$			

Quotient	Simplify using the Quotient Rule	Thus...	Zero Exponent Property
$\frac{x^5}{x^5}$	$x^{5-5} = x^0$	$\frac{x^5}{x^5} = x^0$	Since $\frac{x^5}{x^5} = x^0$ and $\frac{x^5}{x^5} = 1$, then $x^0 = 1$
$\frac{4^3}{4^3}$			
$\frac{(ab)^2}{(ab)^2}$			

Algebraic Rule for a Zero Exponent:

$$x^0$$

Simplify each expression:

a. a^0

b. $(240)^0$

c. $\left(\frac{ab}{c}\right)^0$

d. $\left(\frac{5^2x^3y^2}{z}\right)^0$

e. $\left(\frac{a^0b^4}{c}\right)^2$

f. $(3x^3y^4x^0y)^3$

Negative Exponent Rule

Quotient	Column 1 rewritten in expanded form or as repeated multiplication.	Column 1 simplified using the Quotient Rule.	Compare your answers; one written as a fraction and the other in exponent form.
$\frac{3^3}{3^5}$	$\frac{3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{3^2} = \frac{1}{9}$	$3^{3-5} = 3^{-2}$	$\frac{1}{3^2} = 3^{-2}$
$\frac{a^4}{a^7}$			
$\frac{a^3b^6}{a^6b^{10}}$			
$\frac{4^{15}a^{30}}{4^{20}a^{50}}$	<i>This is too long to expand, find a short cut.</i>		

Explain what a negative exponent means:

Algebraic Rule for Negative Exponents:

$$a^{-n}$$

Simplify each expression:

a. 4^{-1}

b. x^{-2}

c. $x^{-3}y^{-4}$

d. $a^{-3}b^4$

e. $\frac{1}{a^{-5}}$

f. $\frac{a^2bc^{-3}}{a^3b^{-4}}$

g. $4n^{-2} \cdot 3n^3$

h. $\frac{m^5n}{m^2n^3}$

i. $(m^{-3})^2$

1. Simplify each of the following and order them from least to greatest.

$$(-6)^4 \quad (-6)^0 \quad -6^4 \quad (-6)^{-4}$$

2. What is the difference between $-r^3$, r^{-3} and $(-r)^3$?

3. What is the difference between $6t^{-2}$ and $(6t)^{-2}$?

4. What is the difference between $\left(\frac{5}{3}\right)^{-3}$ and $\frac{5^{-3}}{3}$?

8.1e Homework: Zero Exponent and Negative Exponents

Directions: Simplify each expression. The simplified expression should not include any negative exponents.

#	Expression	Expanded Form	Simplified Solution
1.	$m^4 \cdot 2m^{-3}$		
2.	$4r^{-3}r^2$		
3.	$\frac{c^4d}{cd^4}$		
4.	$\frac{3w^3}{21w^5}$		
5.	$\frac{xyz^2}{x^2yz}$		
6.	$(4x^0)^4$		
7.	$\frac{4x^0y^{-2}z^{-3}}{4xy^{-1}}$		

Directions: Use the properties of exponents to simplify each expression.

8. $\frac{x^{-3}}{x}$

9. $(x^0)^2$

10. 3^{-4}

11. $(x^2)^0$

12. $2x^{-3}y^{-3} \cdot 2x^{-1}y^3$

13. $(2x^2)^{-4}$

14. $(4r^0)^4$

15. $(4xy)^{-1}$

16. $\frac{r^3}{2r^3}$

17. $\frac{3m^{-4}}{m^3}$

18. $\frac{m^4}{2m^4}$

19. $\frac{x^{-1}}{4x^4}$

8.1f Class Activity: Properties of Exponents Game and Mixed Practice

Directions: Complete each exponent property by filling in the space.

1. $a^b \cdot a^c = a^{\text{---}}$

2. $\frac{a^b}{a^c} = a^{\text{---}}$

3. $(a^b)^c = a^{\text{---}}$

4. $(ab)^c = a^{\text{---}}b^{\text{---}}$

5. $\left(\frac{a}{b}\right)^c = \frac{a^{\text{---}}}{b^{\text{---}}}$

6. $a^0 = \text{---}$

7. $a^{-1} = \text{---}$

Once you have received the exponent puzzle from your teacher assemble the puzzle so that each expression touches or lines up with its simplified form.

8.1f Homework: Properties of Exponents Mixed Practice

Directions: Simplify each expression. Assume that no denominator is equal to zero.

1. $x^3 \cdot x^5$

2. $a^{15} \cdot a^{11}$

3. $3y^8 \cdot 2y^2$

4. $5j^4(-9j^5)$

5. $(x^5)^2$

6. $(a^3)^6$

7. $(h^4)^3$

8. $[(k^5)^2]^3$

9. $(xy)^7$

10. $(4gz)^2$

11. $(-2a^4wy^4)^3$

12. $-3(km^4)^4$

13. $\frac{d^8}{d^4}$

14. $\frac{t^9}{t^3}$

15. $\frac{a^5b^3}{a^2d}$

16. $\frac{x^3y^2z}{x^2y^2}$

17. $\left(\frac{4}{5}\right)^2$

18. $\left(\frac{2}{3}\right)^4$

19. $\left(\frac{x}{3}\right)^3$

20. $\left(\frac{c}{b}\right)^{15}$

21. x^{-4}

22. $\frac{3}{c^{-2}}$

23. $\frac{s^3}{s^{-4}}$

24. $\frac{6p^{-2}}{p^2}$

25. 5^0

26. $\left[\left(\frac{33y^{17}z}{12a^{115}b}\right)^5\right]^0$

27. $[(-2^3)^3]^2$

28. $(bc^3)(b^4c^3)$

29. $\frac{(u^{-3}v^3)^2}{(u^3v)^{-3}}$

30. $\frac{9a^2b^7c^3}{2a^5b^4c^5}$

31. $\left(\frac{1}{2}w^3\right)^2 (w^4)^2$

32. $\left(\frac{-18x^0a^3}{6(x^{-2})(x^3a^2)}\right)^2$

8.1g Self-Assessment: Section 8.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Apply the properties of integer exponents to simplify algebraic and numerical expressions. <i>See sample problem #1</i>				

Sample problem #1

Use the properties of exponents to simplify each expression.

a. $\frac{3}{r^{-4}}$

b. $(xy^3)^2(x^2y)^4$

c. $\frac{9a^2b^4c^2}{3a^5bc^4}$

d. $\left(\frac{1}{2}r^2\right)^2 \cdot (s^0r)^3$

Section 8.2: Scientific Notation

Section Overview:

The first lesson in this section is a review of place value and powers of ten. In order to properly access and understand how scientific notation works students must have a solid foundation of place value and how powers of ten show the relationship between digits that are next to each other in a multi-digit number. They will use this foundation to compare numbers written as a single digit times an integer power of ten by a scale factor. After reviewing place value students learn to change numbers between standard form and scientific notation in order to estimate very large and very small quantities. Students also learn how to operate with numbers in scientific notation so they can compare and express how many times as much one number is to another. As they are problem solving with scientific notation they must choose units of appropriate size for measurements of very large or very small quantities. Finally, students will use scientific notation to solve in a variety of contexts that require the use of very large and very small numbers.

Concepts and Skills to master:

By the end of this section students should be able to:

1. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
2. Convert a number between scientific notation and standard form.
3. Perform operations with numbers expressed in scientific notation.
4. Choose units of appropriate size for measurements of very large or very small quantities.
5. Interpret scientific notation that has been generated by technology.
6. Use scientific notation to problem solve with really small and really large numbers.

8.2a Class Activity: Place Value and Powers of Ten

- Fill in the top row of the place value chart below with the name of the place value that each number digit given refers to. See page 27 if you need a reminder.

1	5	8	8	4	6	3	1	0	8	.	7	3	4	1	1
			10^6					10^1						10^{-4}	

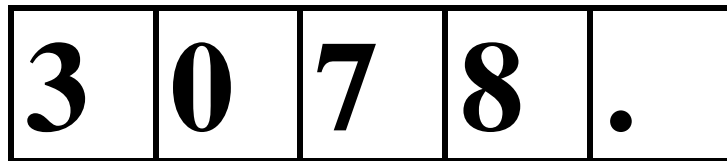
- Fill in the power of ten in the last row in the table above that corresponds with each place value. Some of them have been filled in for you.
- What is the value of the 6 in the chart above?
- What is the value of 7 in the chart above?
- Write each number given below as a single digit times an integer power of 10. The first one has been done for you.

a. $30,000 = 3 \times 10^4$	b. 700	c. 0.0005
d. 0.0000003	e. 8,000,000,000,000	f. 2

6. Estimate each number by writing it as a single digit times an integer power of 10.

a. The land area of Canada is 499,000,000 square kilometers.	b. An adult blue whale can eat 41,000,000 krill in one day.	c. The distance between Pluto and Earth is 4,670,000,000 miles.
d. The diameter of the average human body cell is 0.00012 meters.	e. A sample of sand from a beach has 21,450,000 grains of sand.	f. The width of the diameter of a piece of fishing line is 0.000017 meters.

7. Rearrange **all** the digits and decimal below to build a number with the given conditions. If needed write these numbers on a separate piece of paper and cut them out to rearrange them.



- Build the largest number
- Build the smallest number
- Build a number less than 8
- Build a different number less than 7
- Build a number between 70 and 80
- Build a number than rounds to 70
- Build a number that is between 7000 and 8000
- Build a number that is closest to 3
- Build a number that is between 0.7 and 0.8

8. Fill in the missing entries in the tables below. You may have to find and follow a pattern.

0.004	×	10	=	0.04
0.04	×		=	0.4
	×	10	=	4
4	×		=	
40	×	10	=	
400	×			4,000

1280	×	$\frac{1}{10}$	=	
	×	$\frac{1}{10}$	=	12.8
12.8	×		=	1.28
1.28	×		=	
0.128	×	$\frac{1}{10}$	=	
	×	$\frac{1}{10}$		0.00128

9. Use the tables above to fill in the missing words in the statement below.

In a multi-digit number, a digit in one place represents _____ times as much as it represents in the place to its right and _____ of what it represents in the place to its left.

10. Explain the relationship between the two 3's in the number 533,271.

11. Daxton's Candy (*adapted from an Illustrative Mathematics Task*)

- Daxton has a digital scale. He puts a Marshmallow Peep on the scale and it reads 6.2 grams. How much would you expect 10 Marshmallow Peeps to weigh? Why?
- Daxton takes the marshmallows off the scale. He then puts on 10 jellybeans and the scale reads 12.0 grams. How much would you expect 1 jellybean to weigh? Why?
- Daxton then takes off the jellybeans and puts on 10 cinnamon bunnies. The scale reads 88.2 grams. How much would you expect 1,000 cinnamon bunnies to weigh? Why?
- Estimate how many jellybeans equal one cinnamon bunny.

12. If one package of cereal costs \$2.46, then,
- 10 will cost _____
 - 100 will cost _____
 - 1,000 will cost _____
 - 1/10 of the package will cost _____
 - 1/100 of the package will cost _____

13. Ten thousand is how many times bigger than 100? (Hint: Remember the statement you completed on the previous page.)

14. Now write 100 and 10,000 as a single digit times an integer power of ten.

Use the powers of ten to determine how many times bigger 10,000 is than 100.

15. Write each number as a single digit times an integer power of ten to determine how many times bigger one is compared to the other.

a. 1,000,000 and 100	b. 10,000 and 10	c. 0.0001 and 0.000001
d. 0.001 and 0.00000001	e. 10 and 0.01	f. 200,000 and 2,000

16. Write each number as a single digit times an integer power of ten to determine how many times bigger one is compared to the other.

a. 60,000,000 and 30,000	b. 70,000 and 200	c. 0.0004 and 0.0000003
d. How many hundreds are in a thousand?	e. How many thousands are in a million?	f. How many pennies are in \$100?

If needed estimate each number given in the problems below by writing it in the form of a single digit times an integer power of 10 and use your estimates to approximate each answer.

17. Company A's net profit for the year was \$323,000. Company B's net profit for the year was \$49,500,000. Approximately how many times larger is Company B's profit than Company A's profit?

18. A species of bacteria is 0.00013 decimeters long. A virus is 0.000000012 decimeters long. Approximately how many times longer is the bacteria than the virus?

19. The E. coli bacterium is about 0.0000005 meters wide. A hair is about 0.000017 meters wide. Approximately how many times longer the hair is than the bacteria?

20. The mass of the earth is about 6×10^{24} kilograms. The mass of Mercury is about 3×10^{23} kilograms. Approximately how many times larger is the mass of Earth than the mass of Mercury?

8.2a Homework: Place Value and Powers of Ten

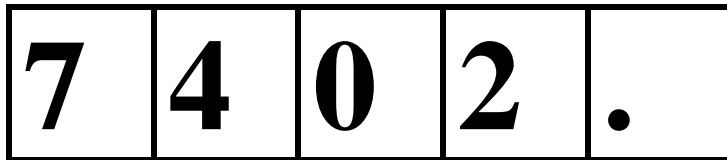
1. Write each number given below as a single digit times an integer power of 10.

a. 400,000	b. 70	c. 0.0008
d. 0.000002	e. 90,000,000,000,000	f. 0.4

2. Estimate each number by writing it as a single digit times an integer power of 10.

a. The land area of Australia is 2,967,892 square kilometers.	b. The cornea of an eye is 0.00058 meters thick.	c. The distance between Jupiter and Earth is 365,000,000 miles.
d. A large earthquake slowed the rotation of the Earth, making the length of a day 0.00000268 seconds shorter.	e. An average cell phone has 918,440 germs on it.	f. The thickness of a piece of paper is 0.001076 meters

3. Rearrange **all** the digits and decimal below to build a number with the given conditions. If needed write these numbers on a separate piece of paper and cut them out to rearrange them.



- Build the largest number
- Build the smallest number
- Build a number less than 7
- Build a different number less than 6
- Build a number between 70 and 80
- Build a number than rounds to 70
- Build a number that is between 7000 and 8000
- Build a number that is closest to 4
- Build a number that is between 0.7 and 0.8

4. At the store 4 pounds of salmon cost \$48, how much would 40 pounds of the same salmon cost?

5. Fill in the blanks.

a. $\$12.50 = \underline{\hspace{2cm}} \times \1.25

b. $\$125 = \underline{\hspace{2cm}} \times \1.25

c. $\$1,250 = \underline{\hspace{2cm}} \times \1.25

6. The following problem is given to Miranda:

$$23.10 \times 100$$

She states that since she is multiplying by 100 she simply must add two zeros to 23.10. Explain the error in Miranda's thinking and explain how to find the correct answer without using a calculator or long multiplication.

7. Write each number as a single digit times an integer power of ten to determine how many times bigger one is compared to the other.

a. 8,000,000 and 300	b. How many ten dollar bills are in \$100,000?	c. 0.007 and 0.000005
d. 9,000,000 and 300	e. 70,000 and 200	f. How many millions are in a trillion?

If needed estimate each number given in the problems below by writing it in the form of a single digit times an integer power of 10 and use your estimates to approximate each answer.

8. Website A gets 5,000,000 hits in one day. Website B gets 4,000 hits in one day. Approximately how many times more hits does Website A get than Website B?

9. Picoplankton can be as small as 0.00002 centimeters. Microplankton can be as small as 0.002 centimeters. Approximately how many times larger are Microplankton than Picoplankton?

10. The population of the United States is estimated to be 3×10^8 while the population of the Earth is estimated to be 7×10^9 . Approximately how many times larger is the population of the Earth compared to the population of the United States.

8.2b Class Activity: Scientific Notation Part 1

Think about the following question: Why do you text?



Most of us would agree that texting is a fast and efficient way of communicating. In fact, texting allows us to abbreviate many common phrases. Mathematicians and scientists have a way of expressing really large and really small numbers in a fast and efficient way; it is called **Scientific Notation**. Just like texting allows you to communicate quickly, scientific notation is a special way of writing a number that would otherwise be tedious to write if it were left in standard form.

The four expressions written below represent the same number. Write the number in **Standard Form** on a sheet of paper or mini-white board.

$$\begin{aligned} 500,000 \times 10^{-1} \\ 0.5 \times 10^5 \\ 5 \times 10^4 \\ 5,000 \times 10 \end{aligned}$$

Which of the expressions given below is greater?

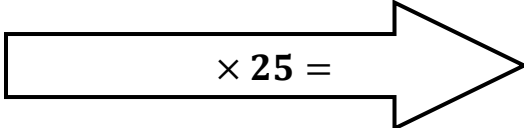
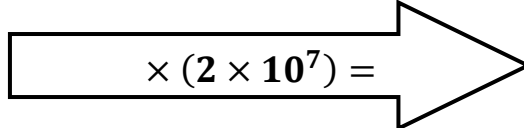
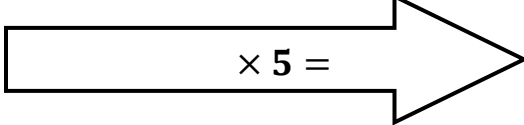
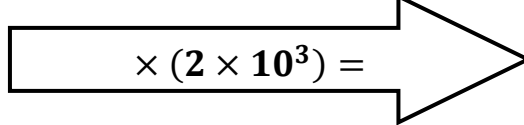


$$\begin{aligned} 6 \times 10^{-5} \\ 3 \times 10^{-2} \end{aligned}$$

Matching Activity

(Adapted from a MARS task found at: <http://map.mathshell.org/materials/download.php?fileid=1221>)

Cut out the cards and arrows given below. Work with a partner to match the number written in standard form with the number given in scientific notation. Do not worry about the Object and Arrow Cards right now. If there appears to be no match then write the corresponding number on the blank card to make a match. Lay your cards next to each other on your desk and be ready to defend and discuss your answers.

Standard Form	Scientific Notation	Object
0.0001 m	1×10^{-4}	Thickness of a sheet of paper
0.006 m		Length of an ant
0.15 m	1.5×10^{-1} m	Length of a pencil
	2×10^0 m	Height of the average NBA basketball player
20 m	2.0×10^1 m	Height of a Red Maple tree
60 m	6.0×10^1 m	Wingspan of a Boeing 777 aircraft
300 m	3.00×10^2 m	Length of a cruise ship
8,000 m	8×10^3 m	Height of a mountain
400,000,000 m	4×10^8 m	Distance to the moon from Earth.
 $\times 25 =$	 $\times (2 \times 10^7) =$	
 $\times 5 =$	 $\times (2 \times 10^3) =$	

8.2b Homework: Scientific Notation Part 1

1. The table below includes numbers written in standard form or scientific notation. Change the numbers written in scientific notation into standard form and vice versa. Use a calculator if needed.

Scientific Notation		Standard Form		Scientific Notation		Standard Form	
Calculator Notation	Exponent Notation	Standard Number		Calculator Notation	Exponent Notation	Standard Number	
Follow the Pattern							
a.	10^0			2×10^0			
b.	10^1			2×10^1			
c.	10^2	10^2		2×10^2	2×10^2		
d.			1,000				2,000
e.			10,000				
Watch for Patterns							
f.	$4 \cdot 10^3$	4×10^3		$4.2 \cdot 10^3$			
g.	$6 \cdot 10^5$			$6.9 \cdot 10^5$			
h.	$7 \cdot 10^8$			$7.12 \cdot 10^8$			
i.	$8.1 \cdot 10^3$			$8.1 \cdot 10^4$			81,000
j.		4×10^9	4,000,000,000				

2. From the table above, write two things you learned about scientific notation.
3. Complete the following statements:
- In scientific notation, as the exponent power goes up by 1, the standard number's decimal is...
 - In scientific notation, as the exponent power goes down by 1, the standard number's decimal is...

8.2c Class Activity: Scientific Notation Part 2

Recall the definition for scientific notation

A number that is in Scientific Notation takes on the form $a \times 10^n$ where a is called the *significant figure* and $1 \leq a < 10$ and n is an integer. The number after the \times , or 10^n , is called the *order of magnitude*.

1. Change these LARGE scientific notation numbers to standard notation and vice versa. Make up a number for the blank cells.

Scientific Notation	Standard Notation	Scientific Notation	Standard Notation
a. 6.345×10^8		e.	5,320
b. 8.04×10^4		f.	420,000
c. 4.26×10^5		g.	9,040,000,000
d.		h.	

2. Now try these SMALL numbers. See if you can figure out the method (one example is given). Make up a number for the blank cells.

Scientific Notation	Standard Notation	Scientific Notation	Standard Notation
Example: 3.2×10^{-3}	0.0032	Example: 5.4×10^{-6}	0.0000054
a. 4.2×10^{-8}		e.	0.00075
b. 8.12×10^{-7}		f.	0.004005
c. 7.625×10^{-3}		g.	0.00000000092
d.		h.	

3. Express 4,532,344 in scientific notation with 3 significant figures.
4. Express 0.00045323 in scientific notation with 2 significant figures.
5. Type the following into a calculator: 5,555,555,555 multiplied by 5,555,555,555. What does the answer say?

Some calculators can give you answers in scientific notation. Other calculators have different ways of displaying scientific notation. One way they can display scientific notation is $3.08E19$. This means 3.08×10^{19} .


6. Write this number in standard form.

7. A calculator gives you an answer of $5.025E-3$, write this number in scientific notation and standard form.

8. A calculator gives you an answer of $9.22E8$. Write this number in scientific notation and standard form.

9. Enter the following problems into your calculator, write the answer in scientific notation and standard form. Express your answer with three significant figures.
 - a. $(3 \times 10^5) + (5.45 \times 10^6)$
 - b. $(3.2 \times 10^{-2}) - (5.4 \times 10^2)$
 - c. $(2 \times 10^8)(1.4 \times 10^{-3})$



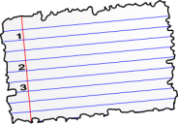




10. Explain why the numbers 402.2×10^{21} and 0.217×10^4 are not written in scientific notation.

11. Observe the numbers given below, if the number is written in scientific notation circle it. If it is not written in scientific notation change it to scientific notation. You will need to think about how many spaces you will have to move the decimal and how that will affect the exponent. 

a. 348×10^8	b. 0.004026×10^9	c. 0.00742×10^{-3}
d. 45.5×10^{-6}	e. 6.05×10^4	f. 3.03554×10^{-7}

12. As of September 2014 Facebook was worth \$2,000,000,000. Write this number in scientific notation.
13. The diameter of a human hair is 0.000099 meters long. Write this number in scientific notation.
14. A computer at a radio station stores all of the station's music digitally. The computer can display the amount of time it will take to play through its entire library of music. The DJ can choose if she wants to display this total amount of playing time in seconds, minutes, hours, and years. The radio station has about 7,000 songs on the computer that have an average playing time of 3 minutes for each song.
- Calculate the total amount of music in minutes that is on the radio station's computer. Write this number in scientific notation.
 - If the D.J. is planning a playlist for the entire week, should she display the total amount of time in seconds, minutes, hours, days, or years? Convert the playing time into your desired unit of time.
15. The mass of a snowflake is approximately 0.000003 kilograms.
- Write this number in scientific notation.
 - If you are only concerned about the mass of one snowflake circle the unit below that would best represent this quantity. Convert the mass of the snowflake to your chosen unit of measurement.
- | | | |
|------------|-------|-----------|
| Milligrams | Grams | Kilograms |
|------------|-------|-----------|
- Suppose there are approximately 1,000,000 snowflakes in one giant snowball. What unit should you choose to represent the weight of the snowball? Find the mass of the snowball with your chosen unit.
 - A snowplow is removing snow from a parking lot and dumping it into a dump truck. What unit of measurement would be most appropriate to represent the weight of the snow in the truck?
16. A seafloor spreads at a rate of 10 centimeters per year. If you collect data on the spread of the sea floor each week what unit of measurement would be most appropriate to use? Convert the rate at which the seafloor spreads to your chosen unit of measurement.
- | | | |
|-------------|-------------|--------|
| Millimeters | Centimeters | Meters |
|-------------|-------------|--------|

17. Change each number below to scientific notation then fill in the blank with the best unit of measure from the column to the right.

a. The E. coli bacteria has a width of 0.0005 _____.		millimeters kilometers meters
b. The acceleration of a bullet is 1,700,000 _____.		meters/second ² nanometers/second ² miles/second ²
c. The thickness of a piece of paper is 0.1 _____.		feet millimeters meters
d. The mass of a dust particle is 0.753 _____.		nanograms grams decagrams
e. The consumption of cereal in the United States is 1,350,000,000 _____.		nanograms centigrams kilograms
f. The net worth of the richest person in the United States is 46,000,000,000 _____.		pennies dollars nickels
g. The size of a drop of water is .002083 _____.		pounds ounces tons

8.2c Homework: Scientific Notation

1. Change these LARGE scientific notation numbers to standard notation and vice versa. **Make a number up for the blank cells.**

Scientific Notation	Standard Notation	Scientific Notation	Standard Notation
a. 1×10^{12}		e.	4,560
b. 9.3×10^6		f.	1,220,000
c. 7.832×10^{10}		g.	1,405,000,000
d.		h.	

2. Now try these SMALL numbers. **Make a number up for the blank cells.**

Scientific Notation	Standard Notation	Scientific Notation	Standard Notation
a. 5×10^{-4}		e.	0.0065
b. 6.8×10^{-7}		f.	0.005005
c. 3.065×10^{-8}		g.	0.00000000709
d.		h.	

3. Change the numbers below into scientific notation.

a. -0.00036

b. 0.00036

c. $36,000$

d. $-36,000$

4. Express the numbers below in scientific notation with 3 significant figures.

a. $4,651,284$

b. 0.0005643411

5. A calculator gives you an answer of $4.02E-6$, write this number in scientific notation and standard form.

6. A calculator gives you an answer of $2.21E7$, write this number in scientific notation and standard form.

7. Enter the following problems into your calculator, write the answer in scientific notation and standard form. Express your answer with three significant figures.

a. $(2 \times 10^4) + (1.35 \times 10^7)$	b. $(3.2 \times 10^{-8}) - (5.4 \times 10^{-9})$	c. $(2 \times 10^{11})(1.4 \times 10^{-3})$
---	--	---

8. The nucleus of a cell has a diameter of 1 micrometer that is equivalent to 0.000001 meters. Change this number to scientific notation.

9. The length of a DNA nucleotide building block is about 1 nanometer that is 0.000000001 meters. Change this number to scientific notation.

10. Teenagers spend \$13 billion on clothing each year. Change this number to scientific notation. (Go back and look at your place value chart if you don't know how many zeros a billion has.)

11. A bakery is making cakes for a huge weeklong city celebration. The recipe for each cake calls for 96 grams of sugar. Each cake serves 12 people and the city plans on serving 1500 slices of cake per day for 7 days.

a. How many total cakes does the bakery need to make?

b. If the bakery wants to know how much sugar to purchase for the entire event choose the best unit of measurement that would be the most appropriate to use. Find the amount of sugar needed based on the measurement you chose.

Grams

Kilograms

Tons

c. Rosa is very health conscious and wants to know how much sugar is in her piece of cake. Determine the amount of sugar in one piece of cake and label your answer with the appropriate unit of measure.

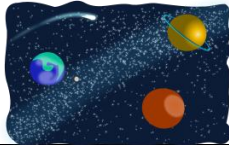





Extension: The diameter of an electron is 2.85×10^{-15} kilometers. If you are only concerned about the diameter of one electron circle the unit below that would best represent this quantity. Convert the diameter of the electron to your chosen unit of measurement.

Nanometers

Meters

Kilometers

12. Change each number below to scientific notation then fill in the blank with the best unit of appropriate size from the column to the right.

<p>a. The diameter of the Milky Way is 100,000_____.</p>		<p>feet miles light years</p>
<p>b. The wavelength of the shortest electromagnetic waves is 0.01_____.</p>		<p>meters decimeters millimeters</p>
<p>c. The speed of a Banana Slug is 0.00023_____.</p>		<p>meters/second kilometers/second miles/second</p>
<p>d. The area of the Antarctic Icecaps is 34,000,000_____.</p>		<p>millimeters² kilometers² inches²</p>
<p>e. The mass of a train is 72,200,000_____.</p>		<p>grams centigrams kilograms</p>
<p>f. The world's petroleum production is 3,214,000,000,000_____.</p>		<p>cups milliliters liters</p>

8.2d Class Activity: Multiplying and Dividing with Scientific Notation

In a previous section you were asked how many millions are in a trillion. Scientific notation can help you answer this question with ease.

1. Begin by writing these two numbers in standard form and then changing them to scientific notation.

	Standard Form	Scientific Notation
One Million		
One Trillion		

2. What operation should you use if you want to compare these numbers? (Hint: Remember it is asking how many millions are in a trillion.)
3. Write this problem out with the correct operation using scientific notation.

When numbers are written in scientific notation the problem above can be solved rather quickly. The problems below will help you practice the skills you will need to do this. You will return to the problem above on the next page.

4. Discuss with a partner what properties of exponents you will use to help simplify the problems below. Use these properties to simplify each expression.

a. $10^4 \times 10^3$	b. $10^{-3} \times 10^5$	c. $\frac{10^6}{10^3}$	d. $10^4 \div 10^6$
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5. Discuss the multiplication problem $(5 \times 3)(2 \times 8)$ with your class. Write your thoughts below.

6. Rewrite this problem $(5.1 \times 10^5)(6.8 \times 10^3)$ like the problem above (group the powers of 10 together). Then solve the problem (use exponent properties) and write the solution.

7. Use the same method to evaluate the problems below.

a. $(6.9 \times 10^2)(3.5 \times 10^5)$ Rewrite the problem: Problem solution:	b. Solve the problem: $(1.9 \times 10^3)(2.4 \times 10^6) =$	c. Solve the problem: $\frac{(7.2 \times 10^5)}{(3.6 \times 10^2)} =$
--	---	--

8. Find each product or quotient. Write your answer in scientific notation.

a. $\frac{2.3958 \times 10^3}{1.98 \times 10^7}$	b. $(7 \times 10^5)(3.5 \times 10^{-3})$	c. $\frac{3.006 \times 10^8}{7.3 \times 10^3}$
d. What is 3 millionths multiplied by 7 ten-thousandths?	e. $(3.1 \times 10^{-3}) \times 562.1$	f. How much is 40% of 140 million?
g. $\frac{30}{1.2 \times 10^5}$	h. $(5 \times 10^5)(0.4)$	i. What percent of (1.3×10^6) is (6.5×10^5) ?

9. Return back to the problem at the beginning of the section. If we want to figure out how many millions are in a trillion what operation will help us achieve this?

a. Use the method discovered above to perform this operation.

b. Now try it to find out how many thousands are in a trillion.

Use scientific notation to answer each question

10. In the world, approximately 1,146,000,000 people speak Chinese as their first language, while, 341,000,000 people speak English as their first language. Approximately how many times more people speak Chinese than English as their first language?

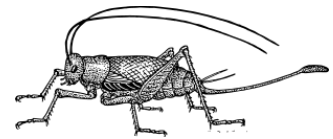
11. The thickness of a dollar bill is .010922 cm. The thickness of a dime is .135 cm. How many times thicker is a dime compare to a dollar bill?

12. A millipede's leg is 4.23×10^{-3} cm long.

a. How long is the millipede's leg in standard form?

b. Despite its name a millipede does not really have 1000 legs. If it did, what would the length be if you could line up all the legs of a 1,000 leg millipede end to end?

13. A cricket weighs 3.88×10^{-2} ounces. How many crickets are in a pound(a pound has 16 ounces)?



14. Determine the thickness of one sheet of paper in this book. Show your work and solution using scientific notation. Express your answer in units of appropriate size.

8.2d Homework: Multiplying and Dividing with Scientific Notation

1. Write each number in scientific notation.

a. 0.0006033×10

b. 0.000142×10^{-4}

c. 322×10^5

d. 13.5×10^{-7}

2. Find the product or quotient for the following. Negative exponents are acceptable.

a. $10^{-4} \times 10^2$

b. $10^{-5} \times 10^{-2}$

c. $10^3 \div 10^5$

d. $10^4 \div 10^{-2}$

3. Find each product or quotient. Write your answer in scientific notation.

a. $(7.2 \times 10^{-4}) \times (2.8 \times 10^{-3})$	b. $\frac{2.35 \times 10^8}{4.3 \times 10^3}$	c. $(8.4 \times 10^6) \times (1.3 \times 10^6)$
d. $\frac{3.1748 \times 10^4}{2.07 \times 10^8}$	e. $(5 \times 10^6)(4.5 \times 10^{-4})$	f. $\frac{1.005 \times 10^7}{6.3 \times 10^2}$
g. What is 4 millionths multiplied by 5 ten-thousandths?	h. $(4.2 \times 10^{-3}) \times 44,462.1$	i. How much is 30% of 170 million?

j. $\frac{3.15 \times 10^{-8}}{6.1 \times 10^2}$	k. $\frac{1.3 \times 10^{-4}}{0.3 \times 10^{-1}}$	l. 145,000 is what percent of 4 million?
--	--	--

4. In a class action lawsuit, 4,000 claimants were offered an \$800 million settlement. How much is that per claimant? Change the numbers into scientific notation to calculate.



5. A cable company earned \$125 million in one year. The next year they earned \$312.5 million dollars. Estimate how many times bigger their profit was the second year compared to the first year.
6. There are about 6.022×10^{23} atoms of hydrogen in a mole of hydrogen. How many hydrogen atoms are in 3.5×10^3 moles of hydrogen?
7. During the year 2013 approximately 7.07×10^9 pennies were minted (made by the U.S. Mint). In the year 2000 approximately 1.43×10^{10} were minted. Estimate how many times more pennies were minted in the year 2000 compared to the year 2013. Give a possible explanation for the decline.

6. You might be wondering what to do if the numbers do not have the same order of magnitude. Write down your ideas of how you might be able to add or subtract these numbers. Be ready to share your ideas with the class.

To add or subtract numbers in scientific notation:

1. Make sure they have the same exponent or order of magnitude. If they don't, move the decimal so they do.
2. Add or subtract the significant figures and keep the order of magnitude the same.
3. Write your final answer in scientific notation.

$$(a \times 10^n) + (b \times 10^n) = (a + b) \times 10^n$$

$$(a \times 10^n) - (b \times 10^n) = (a - b) \times 10^n$$

Try it out with the problems given below.

7. Find each sum or difference. Write your answer in scientific notation.

a. $(4.12 \times 10^6) + (3.94 \times 10^4)$	b. $(4.23 \times 10^3) - (9.56 \times 10^2)$	c. $(3.4 \times 10^{-3}) + (4.57 \times 10^{-2})$
d. $(3.67 \times 10^3) - (1.6 \times 10^{-1})$	e. $(8.41 \times 10^{-5}) - (7.9 \times 10^{-6})$	f. $(6.91 \times 10^{-2}) + (2.4 \times 10^2)$

Problem Solving (use scientific notation where possible)

8. The earth is 9.3×10^7 miles from the sun. Pluto is 3.67×10^9 miles from the sun. How far is it to Pluto from Earth? (Hint: Draw and label a picture.)

9. Pretend a new planet has been found in the far reaches of the universe.
 - a. You know the earth is 9.3×10^7 miles from the sun and the planet you are interested in is 7.3×10^{12} miles beyond the sun in the opposite direction of the earth. What is the distance to the planet from Earth?(Hint: Draw and label a picture)

 - b. Using the distance you found above and the fact that light travels at 5.88×10^{12} miles in one light year. Determine how many light years it will take for light to travel to this planet from Earth.

8.2e Homework: More Operations in Scientific Notation

1. Find each sum or difference. Write your answer in scientific notation.

a. $(2.3 \times 10^3) + (6.2 \times 10^3)$

b. $(9.8 \times 10^2) + (2.72 \times 10^4)$

c. $0.456 + (2.3 \times 10^5)$

d. $(7.23 \times 10^7) - (6.08 \times 10^6)$

e. $(2.3 \times 10^5) - (2.01 \times 10^6)$

f. $(8.9 \times 10^{-7}) + (9.6 \times 10^{-6})$

g. What is ten thousand plus 125,000?

h. What is the difference between 4 hundredths and 8 ten thousandths?

i. $(1.6 \times 10^{-4}) - (9.6 \times 10^{-3})$

2. The areas of 4 major oceans on the Earth are shown in the table below. Estimate how many square miles the oceans cover all together.

Ocean	Area (sq miles)
Arctic	5.44×10^6
Atlantic	3.18×10^7
Indian	2.89×10^7
Pacific	6.40×10^7

3. Estimate how many more square miles the Atlantic Ocean covers than the Arctic Ocean.

4. The surface area of the earth is 19.69×10^7 square miles. Find the percentage of Earth that is covered by the oceans listed above.

5. The mass of the Sun is about 1.98×10^{30} kg. The mass of the Earth is about 5.97×10^{24} kg. Estimate how many times bigger the mass of the Sun is than the mass of the Earth.

6. A neutron has a mass of 1.67×10^{-27} kg and an electron has a mass of 9.11×10^{-31} kg. Determine how many times smaller the mass of the electron is than the mass of the neutron.


For numbers 6 and 7 order the numbers from least to greatest.

6. -2.3×10^4 , 5.6×10^{-1} , -1.6×10^{-4}

7. -4.3×10^{-3} , -1.5×10^{-4} , 7.4×10^{-4}

8. Write one million in as many ways as you can.

9. To continue working with very large numbers, problem solve to answer the following questions. Be prepared to explain your problem solving process and solution.

Section 1	Section 2
<p>a. How long is a million days in years?</p> <p>b. How long is a million days in hours?</p> <p>c. How far is a million inches in miles?</p> <p>d. If you laid a million one-dollar bills end to end, how far would they reach?</p>	<p>e. At one time, McDonald's had sold more than a billion hamburgers (far more now). If it were possible to eat a hamburger every minute of every day (day and night) without stopping, how long would it take to eat a billion hamburgers? Express your answer in appropriate units of time.</p> 

8.2f Homework: Matching, Ordering, and Problem Solving with Scientific Notation

1. In the table below, sort the numbers given in the first column into the correct cells to help you order the numbers.

Numbers -4.57×10^2 7.36×10^2 -1.403×10^{-3} 4.65×10^7 3.44×10^{-3} -5.21×10^2 3.44×10^{-2} -1.14×10^1	Positive Numbers	Numbers Greater than 1	Greatest
	3.44×10^{-3}	Numbers between 0 and 1	
		Negative Numbers	Numbers between 0 and -1
			Numbers Less than -1

For numbers 2 and 3 order the numbers from least to greatest.

2. -4.3×10^4 , 4.2×10^{-1} , 4.6×10^{-4}

3. 1.4×10^{-4} , -2.3×10^{-2} , -1.5×10^4

As you work on the problems below, try to think about how you might use scientific notation to help you. Be prepared to explain your methods and solutions.

4. Calculate the following in relationship to your age on your next birthday. Write your answer in scientific notation.

a. How many days have you been alive?	b. How many hours have you been alive?
c. How many minutes have you been alive?	d. How many seconds have you been alive?

Extension:

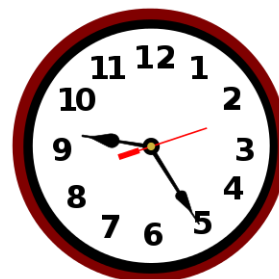
Counting one number per second how long does it take to count to...

- a. ...a million in minutes?

- b. ...a million in hours?

- c. ...a million in days?

- d. ...a million in weeks?



8.2g Homework: Problem Solving with Scientific Notation

Task 1: Gasoline

Gas 'N' Go Convenience Stores claim that 10% of Utahans fuel up at their stores each week. Decide whether their claim is true using the following information. Explain your answer.

- There are about 2.85×10^6 people in Utah.
- There are 2.18×10^2 *Gas 'N' Go* stores in Utah.
- Each stations serves gasoline to about 1.2×10^3 people each week.



Task 2: Time

Many chemical and physical changes happen in extremely small periods of time. For that reason the following vocabulary is used.

Scientific Notation

1 microsecond = 1000 nanoseconds

1 millisecond = 1000 microseconds

1 second = 1000 milliseconds

- Rewrite each of these terms using scientific notation (use the space given above).
- How many nanoseconds are in a millisecond?
- How many nanoseconds are in second?
- How many nanoseconds are in a hour?

Extension:

- I can download a byte of information in a nanosecond. How long will it take to download a typical book (250 megabytes)? Express your answer in appropriate measures of time.
- How long will it take to download the Library of Congress (containing 35 million books)? Express your answer in appropriate measures of time.

8.2h Self-Assessment: Section 8.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. <i>See sample problem #1</i>				
2. Convert a number between scientific notation and standard form. <i>See sample problem #2</i>				
3. Perform operations with numbers expressed in scientific notation. <i>See sample problem #3</i>				
4. Choose units of appropriate size for measurements of very large or very small quantities. <i>See sample problem #4</i>				
5. Interpret scientific notation that has been generated by technology. <i>See sample problem #5</i>				
6. Use scientific notation to problem solve with really small and really large numbers. <i>See sample problem #6</i>				

Sample Problem #1

Every day there is an estimated 329,000 smart phones bought in the United States.¹ Every day there is an estimated 12,000 smart phones lost or stolen in the United States.² Approximately how many times more smart phones are bought than are lost or stolen.

Sample Problem #2

Change the numbers below into scientific notation.

a. 3,450,000,000

b. 0.00000000455

Change the number given below into standard form.

c. 6.03×10^8

d. 1.2×10^{-6}

Sample Problem #3

Perform the indicated operation for each problem below.

a. $(3.13 \times 10^8) + (2.9 \times 10^9)$

b. $2.54 \times 10^{-4} - 3.2 \times 10^{-5}$

c. $(3 \times 10^6)(5.6 \times 10^{-8})$

d. $\frac{1.0004 \times 10^8}{7.2 \times 10^2}$

Sample Problem #4

Fill in the blank with a unit of appropriate size from the column to the right.

a. The mass of trash produced by New York City in one day is 1.2×10^7 _____.	kilograms nanograms grams
b. The period of the sun's orbit around the galaxy is 2.4×10^8 _____.	seconds hours years
c. The area of the Earth's land surface is 1.49×10^8 _____.	millimeters ² meters ² kilometers ²

1-<http://appleinsider.com/articles/14/02/20/apples-iphone-led-2013-us-consumer-smartphone-sales-with-45-share---npd>,
<http://www.latimes.com/business/technology/la-fi-tn-45-million-smartphones-lost-stolen-2013-20140417-story.html>,
<http://techcrunch.com/2013/09/19/gartner-102b-app-store-downloads-globally-in-2013-26b-in-sales-17-from-in-app-purchases/>

Section 8.3: Volume of Cylinders, Cones, and Spheres

Section Overview:

Throughout this section, students are solving real-world and mathematical problems involving volumes of cylinders, cones, and spheres. Students begin by deriving the volume of a cylinder, relying on their knowledge from previous grades that the volume of a right three-dimensional object can be found by taking the area of its base and multiplying it by the height. Students then use the formula for the volume of a cylinder to arrive at the formulas for the volumes of a cone and sphere. Using concrete models of these three-dimensional objects, students physically compare the volume of a cone to the volume of a cylinder. Students then manipulate the formula for the volume of a cylinder to reflect these differences, arriving at the formula for the volume of a cone. They use a similar process to derive the formula for the volume of a sphere. Once students understand where these formulas come from, they apply them to solve real-world problems, knowing when and how to use the formulas.

Concepts and Skills to be mastered:

By the end of this section students should be able to:

1. Find the volume of a cylinder, cone, and sphere given a radius and height.
2. Find a missing measurement (height, radius, or diameter) for a cylinder, cone, or sphere given the volume.
3. Use the formulas for the volumes of cylinders, cones, and spheres to solve a variety of real-world problems.

8.3a Class Activity: Wet or Dry *(This activity is optional)*



We have been discussing exponents throughout this chapter. You have learned how to simplify expressions with exponents in them and have looked at how expressing numbers in scientific notation can better help us deal with numbers that are really big and really small. Exponents are also used to find the volume of a three-dimensional object.

1. Describe what volume is. Compare it to finding perimeter or area.

To help us better understand how important it is to know how to find the volume of a three-dimensional object do the following activity.

2. Choose two different sizes of cylindrical cans to use for this activity. Measure the diameter and the height of each can **in centimeters**.

Can 1: Diameter _____ Height _____

Can 2: Diameter _____ Height _____

3. As a group determine the volume of each can. Show your work below or explain how you found the volume of your cans. Make sure that your units are correct. Once you have found the volume in cubic centimeters change your answer to millimeters. (Hint: One cubic centimeter is the same as one milliliter.)

Can 1

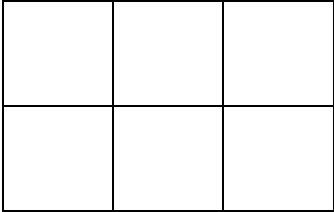
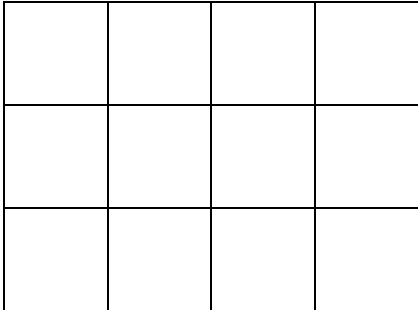
Can 2

Select one of your cans and bring it up to the teacher with your calculation for the volume of the can. Also, select one member of the team to test your calculations.


4. Which can did your team choose and why did you choose this can?
5. How close were your calculations to the actual volume of the can?
6. What would you do differently if you could recalculate the volume of your can?

8.3b Class Activity: Volume of Cylinders

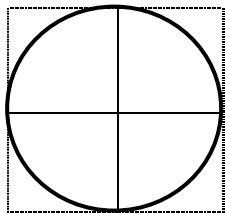
1. Gunner just started his summer job doing swimming pool maintenance. He has a variety of things to do for each pool. For each item below fill in the missing measurement in the space provided for each pool.
 - a. He needs to build a fence around each of the swimming pools below. If each unit represents one meter determine how much fencing he needs for each pool. Write your answer below each pool in the appropriate spot.
 - b. Gunner now has to cover each pool. Determine how much material he will need to cover each pool. Write your answer below each pool in the appropriate spot.
 - c. After Gunner has put up a fence and knows how much material he needs to cover the pools he needs to fill the pools back up with water. Determine how much water he would need to fill each pool to a depth of one meter. Write your answer below each pool in the appropriate spot.
 - d. Now determine of much water he would need to fill each pool to a depth of 2 meters. Continue filling in the chart to 10 meters deep for each pool.

Pool #1	Pool #2
	
Perimeter:	Perimeter:
Area:	Area:
1 meter deep volume:	1 meter deep volume:
2 meter deep volume:	2 meter deep volume:
3 meter deep volume:	3 meter deep volume:
4 meter deep volume:	4 meter deep volume:
10 meter deep volume:	10 meter deep volume:

2. Describe how to find the volume of the pool for any given depth.

3. Explain how the formula $V=Bh$ helps you find the volume. 

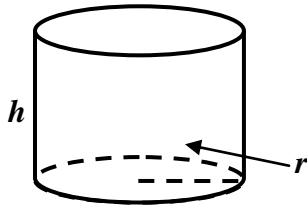
4. Gunner has one more pool to work on. Use what you know about the formula above to fill in the missing information for Pool #3. Recall that each unit represents 1 meter.

<p>Pool #3</p> <div style="text-align: center;">  </div>	<p>Perimeter:</p> <p>Area:</p> <p>1 meter deep volume:</p> <p>2 meter deep volume:</p> <p>3 meter deep volume:</p> <p>4 meter deep volume:</p> <p>10 meter deep volume:</p>
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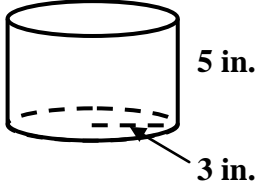

5. What type of three-dimensional object is Pool #4?

6. Use the picture given below to describe how to find the volume of a Cylinder. Be sure to describe each part of the formula and how it relates to the formula $V = Bh$.

A cylinder is a solid obtained by taking a circle in a plane (called the base) and drawing it out in a direction perpendicular to the base for a distance h (called the height).



Directions: Find the volume for each cylinder described below. If needed draw and label a picture.

<p>7.</p> <div style="text-align: center;">  </div>	<p>8.</p> <div style="text-align: center;">  </div>
--	---

<p>9. Cylinder with a Radius = 21 mm and Height = 19 mm.</p>	<p>10. Cylinder with a Diameter = 8.8 cm and Height = 9 cm.</p>
--	---

Directions: Find the missing measurement for each cylinder described below.

<p>11. The volume of a cylinder is 117.1 cubic feet, and its height is 15 ft. Find the diameter of the base of the cylinder.</p>	<p>12. The volume of a cylinder is 4,224.8 cubic millimeters, it has a diameter of 16.4 mm, find the height of the cylinder.</p> <p>Extension: Find the circumference of the base of the cylinder.</p>
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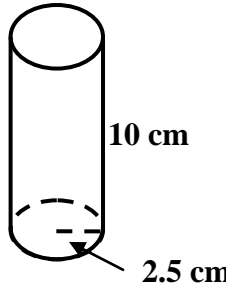
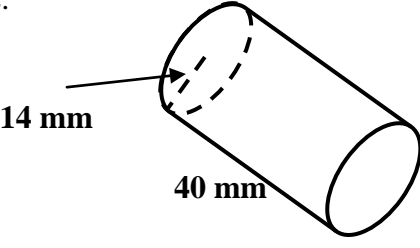
Directions: For each problem given below draw and label a picture that describes each cylinder. Then solve the problem.

13. An ice cream company wants to package a pint of ice cream in a circular cylinder that is 4 inches high. A pint is 16 fluid ounces and 1 fluid ounce is 1.8 cubic inches. What does the radius of the base circle have to be?

14. For a science project, Hassan put a can out to collect rainwater. The can was 11 inches tall and had a diameter of 8 inches. If it rained exactly 20 cubic inches each day, how many days did it take to fill the can?

8.3b Homework: Volume of Cylinders

Directions: Find the volume for each cylinder described below. If needed draw and label a picture.

1. 	2. 
3. Cylinder with a radius of 2 ft and a height of 7 ft.	4. Cylinder with a diameter of 2.7 m and a height of 30 m.

Find the missing measurement for each cylinder described below.

5. The volume of a cylinder is 63.6 cubic inches, and its height is 9 inches. Find the diameter of the base of the cylinder. Extension: Find the circumference of the base of the cylinder.	6. The volume of a cylinder is 8,685.9 cubic ft, it has a diameter of 19.2 ft, find the height of the cylinder.
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Directions: For each problem given below draw and label a picture that describes each cylinder. Then solve the problem.

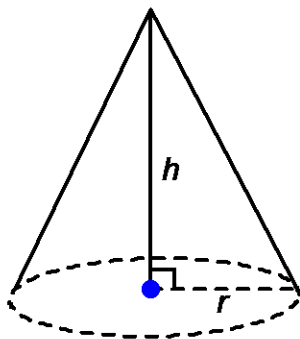
7. What is the volume of Keisha's thermos if it has a radius of 2.5 in at the opening and 10 in for a height?

8. Mr. Riley bought 2 cans of paint to paint his garage. Each can had a radius of 5.5 inches and a height of 8 inches. How many cubic inches of paint did he buy in all?

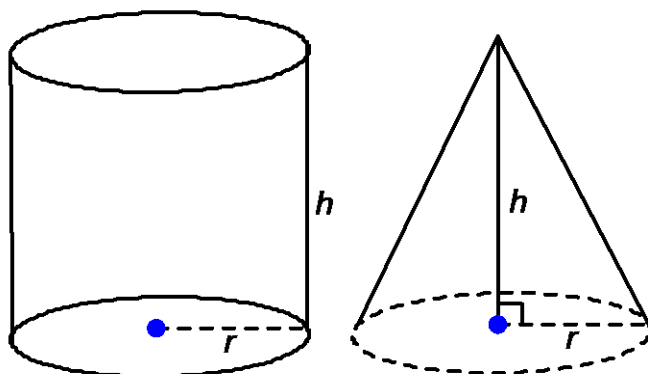
8.3c Class Activity: Volume of Cones



Recall from seventh grade, that a cone is a three-dimensional figure with a circular base. A curved surface connects the base and the vertex.

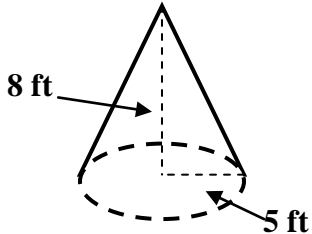
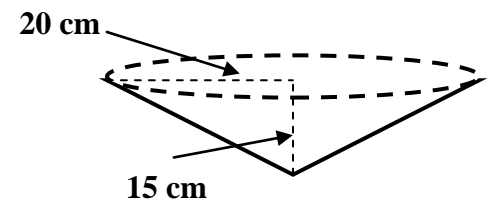


The cylinder and cone given below have the same height and their bases are congruent.



1. Predict how the volume of the cone compares to the volume of the cylinder.
2. If you fill the cone with water or other filling material, predict how many cones of water will fit into the cylinder.
3. Now try it. How many cones fit into the cylinder?
4. About what fraction of the cylinder is filled by the volume of one cone?
5. Manipulate the equation for the volume of the cylinder to show the volume of the cone.
6. Explain in your own words how the volume of a cone compares to the volume of a cylinder. Describe the parts of the formula for the volume of a cone. Write this formula below the cone in the picture above.

Directions: Find the volume for each cone described below. If needed draw and label a picture.

<p>7.</p>  <p>The volume of the cone is</p>	<p>8.</p>  <p>The volume of the cone is</p>
<p>9. A cone with a radius of 8.4 feet and a height of 5.5 feet.</p>	<p>10. A cone with a diameter of 9 meters and a height of 4.2 meters.</p>

Directions: Find the missing measurement for each cylinder described below. Round your answer to the nearest tenth.

<p>11. The volume of a cone is 122.8 cubic inches, and its height is 4.5 inches. Find the diameter of the base of the cone.</p>	<p>12. The volume of a cone is 188.5 cubic ft, it has a diameter of 12 ft, find the height of the cylinder.</p>
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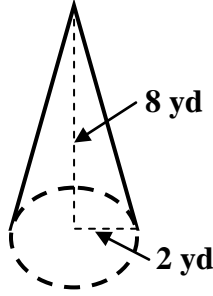
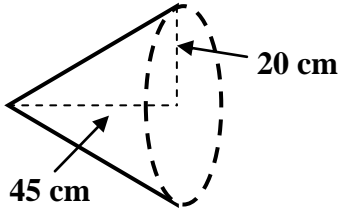
For each problem given below draw and label and picture that describes each cylinder. Then solve the problem.

13. Salt and sand mixtures are often used on icy roads. When the mixture is dumped from a truck into the staging area, it forms a cone-shaped mound with a diameter of 10 feet and a height of 6 feet. What is the volume of the salt-sand mixture?

14. A glass in the shape of a cone has a diameter of 8 cm. If the glass has a volume of 200 ml (or 200 cubic centimeters), what is the greatest depth that a liquid can be poured into the glass? Explain.

8.3c Homework: Volume of Cones

Directions: Find the volume for each cone described below. If needed draw and label a picture.

1. 	2. 
3. A cone with a radius of 40 feet and a height of 100 feet.	4. A cone with a diameter of 4.2 meters and a height of 5 meters.

Directions: Find the missing measurement for each cone described below.

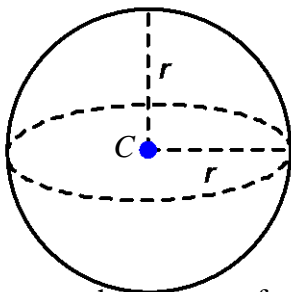
5. The volume of a cone is 37.7 cubic inches, and its height is 4 inches. Find the diameter of the base of the cone.	6. The volume of a cone is 628.3 cubic ft, it has a diameter of 20 ft, find the height of the cone.
--	---

Directions: For each problem given below draw and label a picture that describes each cylinder. Then solve the problem.

7. The American Heritage Center at the University of Wyoming is a conical building. If the height is 77 feet, and the area of the base is about 38,000 square feet, find the volume of air that the heating and cooling systems would have to accommodate.
8. A stalactite, a geological formation, in the Endless Caverns in Virginia is cone-shaped. It is 4 feet long and has a diameter at its base of 1.5 feet.
 - a. Assuming that the stalactite forms a perfect cone, find the volume of the stalactite.
 - b. The stalactite is made of calcium carbonate, which weighs 131 pounds per cubic foot. What is the weight of the stalactite?

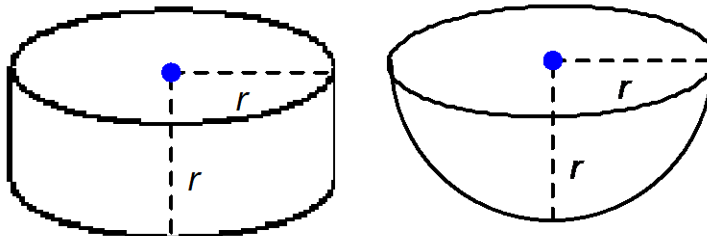
8.3d Class Activity Volume of Spheres

Recall that a sphere is a set of points in space that are a distance of r away from a point C , called the center of the sphere.



Just like you compared the volume of a cone to the volume of a cylinder to find the formula for the volume of a cone you are going to compare the volume of a sphere to the volume of a cylinder.

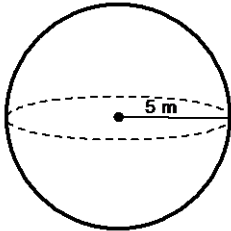
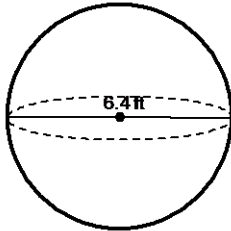
The cylinder and hemisphere given below have the same radius and the height of the cylinder is also the same as its radius.



1. Predict how the volume of the hemisphere compares to the volume of the cylinder. Which one holds more volume?
2. If you fill the hemisphere with water or other filling material, predict what fraction of the cylinder is filled by the volume of one hemisphere.
3. Now try it, what fraction of the cylinder is filled by the volume of one hemisphere?
4. Write down the formula for the volume of the cylinder below the cylinder, be sure to write your height in terms of the radius or r .

- Manipulate the equation for the volume of the cylinder to show the volume of the hemisphere.
- In number 10 you found the volume for a hemisphere. Adjust this formula to find the volume of a sphere.
- Explain in your own words how the volume of a sphere compares to the volume of a cylinder. Describe the parts of the formula for the volume of a sphere. Write this formula below the sphere in the picture on the previous page.

Directions: Find the volume for each sphere described below. If needed draw and label a picture.

<p>8.</p> 	<p>9.</p> 
<p>10. A sphere with a radius of 1.3 yds.</p>	<p>11. A sphere with a diameter of 25 inches</p>

Directions: Find the missing measurement for each sphere described below. Round your answer to the nearest tenth.

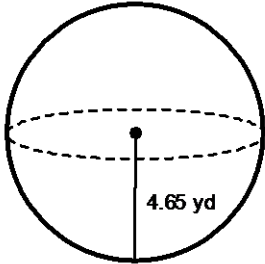
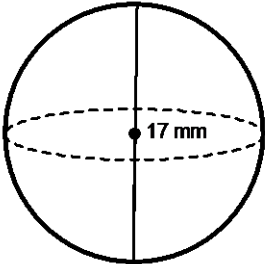
<p>12. The volume of a sphere is 6882.3 in^3; find the diameter of the sphere.</p>	<p>13. The volume of a sphere is 1436.8 ft^3; find the radius of the sphere.</p>
---	---

Directions: For each problem given below draw and label a picture that describes each sphere. Then solve the problem.

- If a golf ball has a diameter of 4.3 centimeters and a tennis ball has a diameter of 6.9 centimeters, find the difference between the volumes of the two balls.
- Kauri pours the water out of a cylindrical flower vase with a height of 5 inches and a radius of 4 inches into a spherical flower vase. The spherical vase has a radius of 4 inches. Will the water overflow? If so, by how much? If not, how much space is left in the spherical vase?

8.3d Homework: Volume of Spheres

Directions: Find the volume for each sphere described below. If needed draw and label a picture.

1. 	2. 
3. A sphere with a radius of 10 yards.	4. A sphere with a diameter of 60 inches

Directions: Find the missing measurement for each sphere described below.

5. The volume of a sphere is 113.1 cm^3 ; find the diameter of the sphere.	6. The volume of a sphere is 4,188.8 cubic feet; find the radius of the sphere.
--	---

Directions: For each problem given below draw and label a picture that describes each sphere. Then solve the problem.

- The diameter of the moon is 3,476 kilometers. Approximate the volume of the moon.
- Find the volume of the empty space in a cylindrical tube of three tennis balls. The diameter of each ball is 2.5 inches. The cylinder is 2.5 inches in diameter and is 7.5 inches tall.

8.3e Class Activity: Volume of Cylinders, Cones, and Spheres



Task 1: Silos

A silo is a storage bin that is a cylinder with a hemisphere on top. A farmer has a silo with a base radius of 30 feet and a storage height of 100 feet. The “storage height” is the part which can be filled with grain - it is just the cylinder. A cubic foot of grain weighs 62 lbs.

- Draw and label a picture of the silo
- How many pounds of grain can the farmer store in the silo?
- How high (including the hemispherical top) is the silo?
- One thousand square feet of wheat produces 250 pounds of grain. The farmer’s wheat field is 3,500 feet by 20,000 feet. Is the silo large enough to hold the grain? By how much? Explain your answer.
- If the farmer decides to fill the silo all the way to the top of the hemisphere how many cubic feet of grain can he store?

Task 3: Pipes

Which will carry the most water? Explain your answer.

- Two pipes each 100 cm tall. One with a 3 cm radius and the other with a 4 cm radius
- One pipe that is also 100 cm tall with a 5 cm radius.

Task 4: Fruit

A cantaloupe a diameter of 23 centimeters and a Clementine orange has a diameter of 7 centimeters. Predict how many times bigger the cantaloupe is than the orange. Then calculate the volume of each fruit to determine how many times bigger the cantaloupe is than the orange.

8.3f Class Activity: Banana Splits

Materials: graph paper, string, rulers, pen or pencil, banana, ice cream scoop

Use any of the materials on your table to approximate the volume of your banana and one scoop of ice cream. Be prepared to show and explain all your methods and your results.

1. What is your estimate for the volume of the peeled banana (include units)? _____

Show how you found this volume.

2. What is your estimate for the volume of one scoop of ice cream (include units)? _____

Show how you found this volume.

3. Comment on each of the other groups' methods and results. Compare their strategies and their results to yours.

8.3g Self-Assessment: Section 8.3

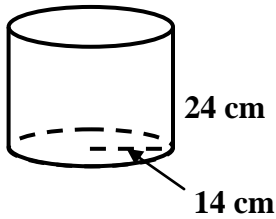
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Find the volume of a cylinder, cone, and sphere given a radius and height. <i>See sample problem #1</i>				
2. Find a missing measurement (height, radius, or diameter) for a cylinder, cone, or sphere given the volume. <i>See sample problem #2</i>				
3. Use the formulas for the volumes of cylinders, cones, and spheres to solve a variety of real-world problems. <i>See sample problem #3</i>				

Sample Problem #1

Find the volume for each object described below. Find the exact volume and the approximate volume rounded to the nearest hundredth.

- a. The cylinder pictured below.

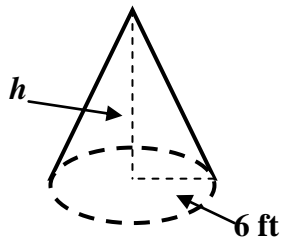


- b. A cone with a radius of 3 ft and a height of 10 ft.
- c. A glass tree ornament is a gold sphere. The diameter of the ornament is 4 inches.

Sample Problem #2

Find the missing measurement for each object described below. Draw and label a picture if needed.

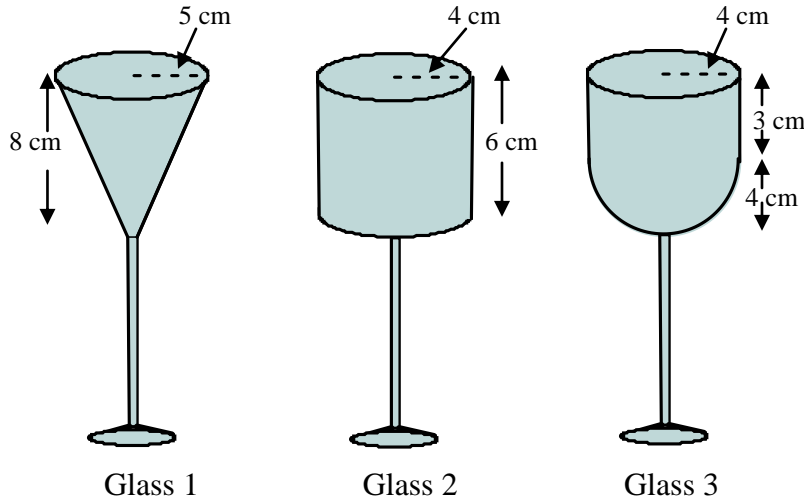
- a. The volume of a regular can of soda pop is approximately 23.7 in^3 . The height of the can is 4.83 inches. Find the diameter of the can.
- b. The volume of the cone below is approximately 377 ft^3 . Find the height of the cone.



- c. A sphere has a volume of 113.1 mm^3 . Find the radius of the sphere.

Sample Problem #3

Suzy is throwing a party and is choosing from the glasses below to serve her punch. Use the information below to answer the questions that follow.



- The shape of Glass 1 is a cone with a radius of 5 cm and a height of 8 cm.
 - The shape of Glass 2 is a cylinder with a radius of 4 cm and a height of 6 cm.
 - The shape of Glass 3 is a hemisphere with radius of 4 cm with a cylinder on top of it with a radius of 4 cm and a height of 3 cm.
- a. Suzy wants to choose the glass that has the smallest volume so that she doesn't have to use as much punch. Find the volume of each glass to determine which glass she should choose.
- b. Suzy really wants to use the cylinder shaped glass. What would the approximate height of the cylinder shaped glass need to be to hold the same amount of punch as the cone shaped glass. Would this be practical?

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Chapter 9 Geometry: Transformations, Congruence, and Similarity

Utah Core Standard(s):

- Verify experimentally the properties of rotations, reflections, and translations: (8.G.1)
 - a) Lines are taken to lines, and line segments to line segments of the same length.
 - b) Angles are taken to angles of the same measure.
 - c) Parallel lines are taken to parallel lines.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (8.G.2)
- Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (8.G.3)
- Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (8.G.4)

Academic Vocabulary: transformation, translation, reflection, rotation, rigid motion, image, pre-image, corresponding vertices, corresponding segments, corresponding angles, corresponding parts, coordinate rule, perpendicular bisector, line of reflection, slope, horizontal line, vertical line, clockwise, counterclockwise, center of rotation, angle of rotation, origin, concentric circles, congruent, dilation, center of dilation, scale factor, similar

Chapter Overview:

In this chapter, students explore and verify the properties of translations, reflections, rotations, and dilations. Students learn about the different types of rigid motion (translations, reflections, and rotations), execute them, and write coordinate rules to describe them. They describe the effects of these rigid motions on two-dimensional figures. Students then use this knowledge to determine whether one figure is congruent to another, understanding that two figures are congruent if there is a sequence of rigid motions that takes one figure onto the other. Then, students study dilations, again exploring and verifying the properties of dilations experimentally. They describe and execute dilations. They use this knowledge to determine whether one figure is similar to another, understanding that a two-dimensional figure is similar to another if there is a sequence of rigid motions and dilations that takes one figure onto the other.

Connections to Content:

Prior Knowledge: Up to this point, students have worked with two-dimensional geometric figures, solving real-world and mathematical problems involving perimeter and area. They have classified two-dimensional figures based on their properties. In 7th grade, students scaled figures. Students will rely on work with function and slope from previous chapters in this text in order to investigate the properties of the different transformations and to write coordinate rules to describe transformations. Students also use the skill of writing the equation of a line in order to write the equation for a line of reflection. Students have also been exposed to dilations in Chapter 2 using the properties of dilations to prove that the slope of a line is the same between any two distinct points on a non-vertical line and to derive the equation of a line.

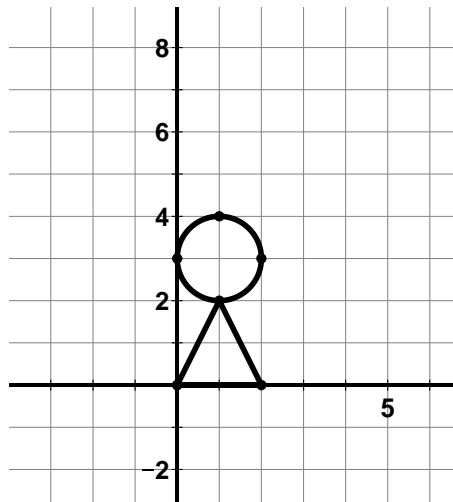
Future Knowledge: In subsequent courses, students will expand on this knowledge, explaining how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Similarly, they will use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

MATHEMATICAL PRACTICE STANDARDS:

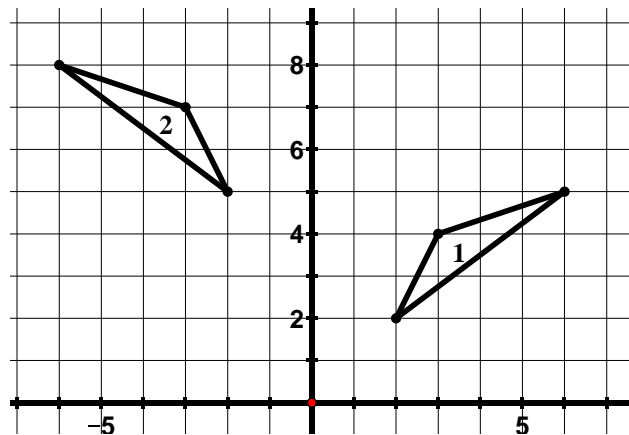


Make sense of problems and persevere in solving them.

Where would you put your center of dilation to enlarge the picture below by a factor of 2 and put the image entirely in the second quadrant? Mark your center of dilation with an O and then do the dilation.



Describe a sequence of rigid motions that would carry triangle 1 onto triangle 2.

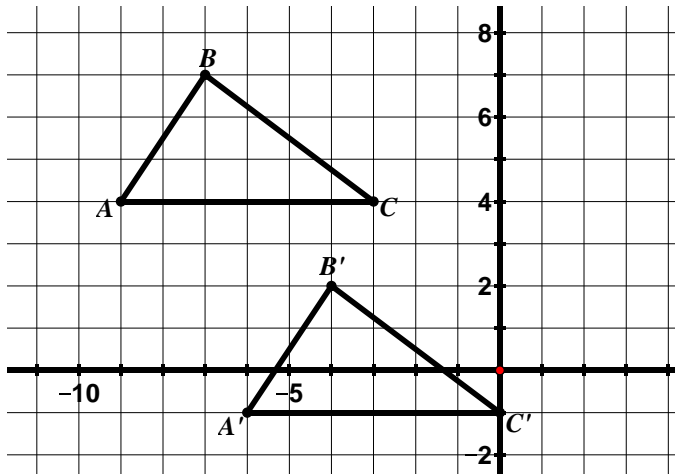


Throughout this chapter, students will see problems with multiple correct answers. For example, in the first problem above, there are many different places to put the center of dilation in order to meet the constraints specified in the problem. Students must use their knowledge of the properties of dilations as an entry point to solving the problem. In the second problem, there are many different sequences of rigid motion that will carry triangle 1 onto triangle 2. Students will have the opportunity to consider the different approaches taken by others, compare the approaches, and identify correspondences between the different approaches.



Reason abstractly and quantitatively.

Write a coordinate rule to describe the translation below.



Throughout the chapter, students write coordinate rules to describe transformations. They also perform transformations described by a coordinate rule.

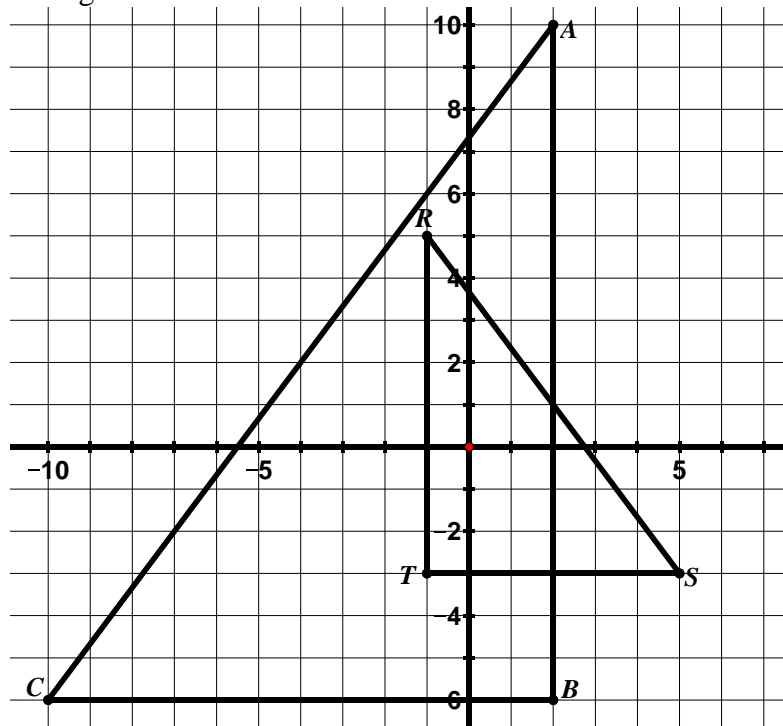
Determine the coordinate rule for a 90° rotation clockwise about the origin. Connect this rule to the slopes of perpendicular lines.

Students are also asked to explain why and how slopes of corresponding segments change under a given transformation, connecting the coordinate rule to the slope change. In this example, students are able to see that a rotation of a figure 90° clockwise can be described by the following coordinate rule $(x, y) \rightarrow (y, -x)$ helping them to understand why the slopes of perpendicular lines are opposite reciprocals of each other.



Construct viable arguments and critique the reasoning of others.

The triangles below are similar.



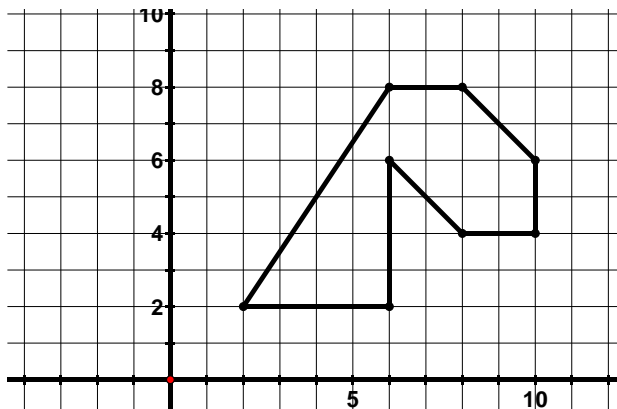
List the sequence of transformations that verifies the similarity of the two figures.

Write a similarity statement for the triangles.

Students will construct an argument that verifies the similarity of these two figures based on their understanding of the definition of similarity in terms of transformational geometry, that is, they must identify a sequence of rigid motions and dilations that takes one figure onto the other. There are different sequences that will accomplish this. Students will justify the sequence they have arrived at and communicate this to classmates. They will also have the opportunity to consider alternative sequences of others and decide whether these sequences do in fact verify the similarity of the two triangles. They will also have the opportunity to identify correspondences between the different sequences.

Animators need your help coming up with the coordinate rule that would reduce the size of the dragon's head to half its size when it enters a cave.

1. Write your proposed coordinate rule in the table below.
2. Write the new coordinates for your rule.
3. Graph the new coordinates.



Model with mathematics.

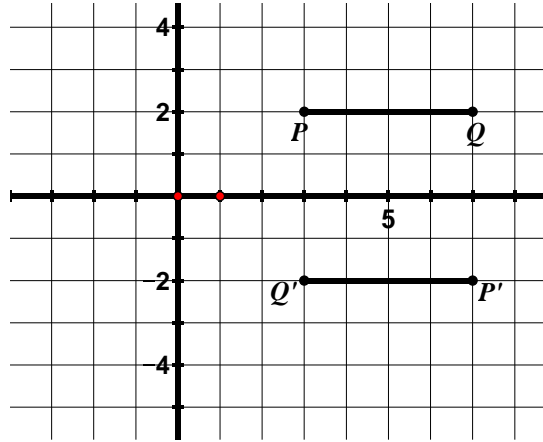
In this problem, students determine how to scale the figure shown above (i.e. make it a different size while maintaining its shape). Scaling is something we see and use constantly in the world around us. Many professionals such as architects and computer animators rely on scaling techniques in order to create scaled down versions of real-life objects.





Use appropriate tools strategically.

Find the angle of rotation (including direction of rotation) and center of rotation for the rotation shown below.

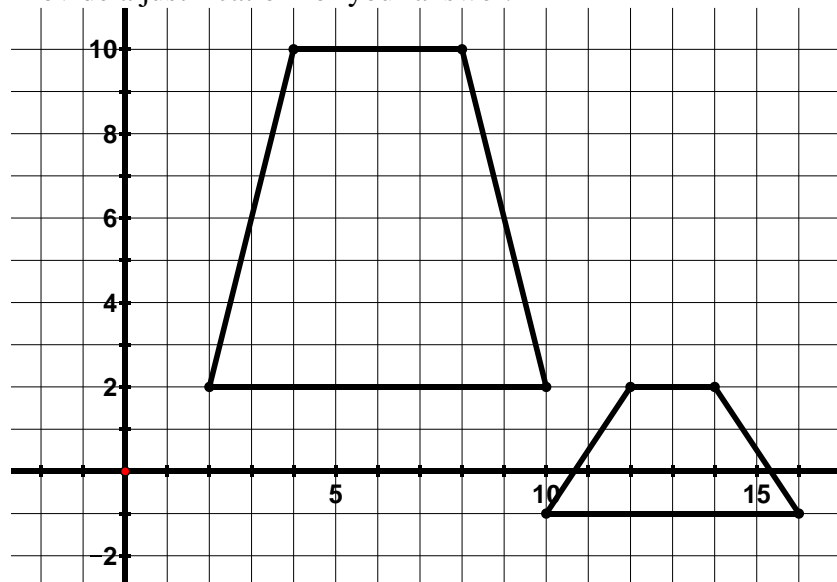


Students use a variety of tools in this chapter. These tools include straight edge, patty (or tracing) paper, compass, protractor, additional graph paper, colored pencils, and dynamic geometry software. In the problem above, one way to find the center of rotation is to trace the figures on patty paper and fold their paper so that P lines up with P' and fold a second time so that Q lines up with Q'. The intersection of these folds is the center of dilation. They can also use the patty paper to find the angle of rotation.



Attend to precision.

Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.

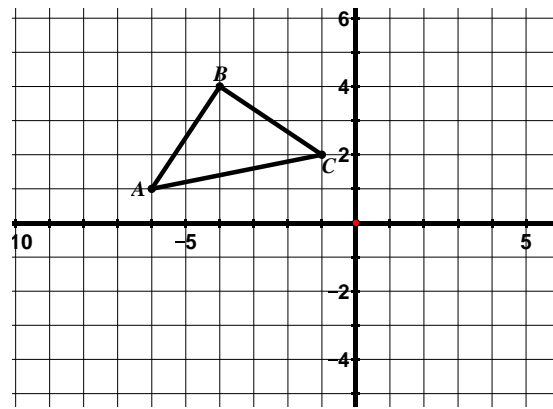


In order to answer this question, students must be clear about their understanding of what establishes congruence and similarity between two figures and they must be able to clearly communicate this to others.



Look for and make use of structure.

Reflect $\triangle ABC$ across the x -axis and label the image.



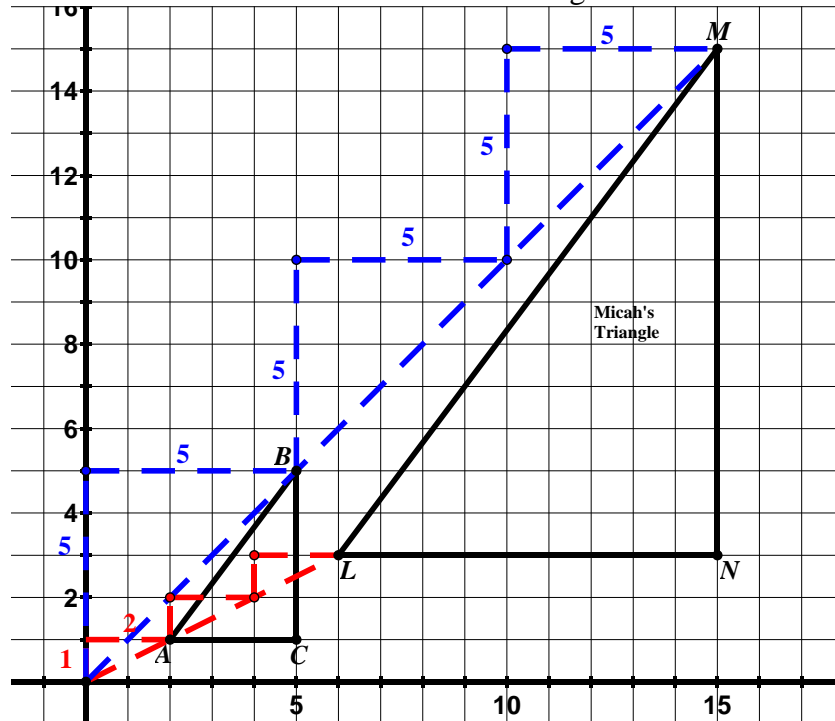
Write a coordinate rule to describe this reflection. Compare the slopes of the corresponding segments of the image and pre-image. What do you notice about the slopes? How does this connect to the coordinate rule $(x, y) \rightarrow (x, -y)$?

In this problem, students determine a coordinate rule to describe a reflection across the x -axis. In doing so, students examine the structure of the ordered pairs, realizing that under a reflection across the x -axis, the x -coordinates remain unchanged while the y -coordinates change sign. Following this, students use the coordinate rule to explain the effect this transformation has on the slope of a segment.



Look for and express regularity in repeated reasoning.

The figure below shows a triangle that has been dilated with a scale factor of 3 and a center of dilation at the origin.

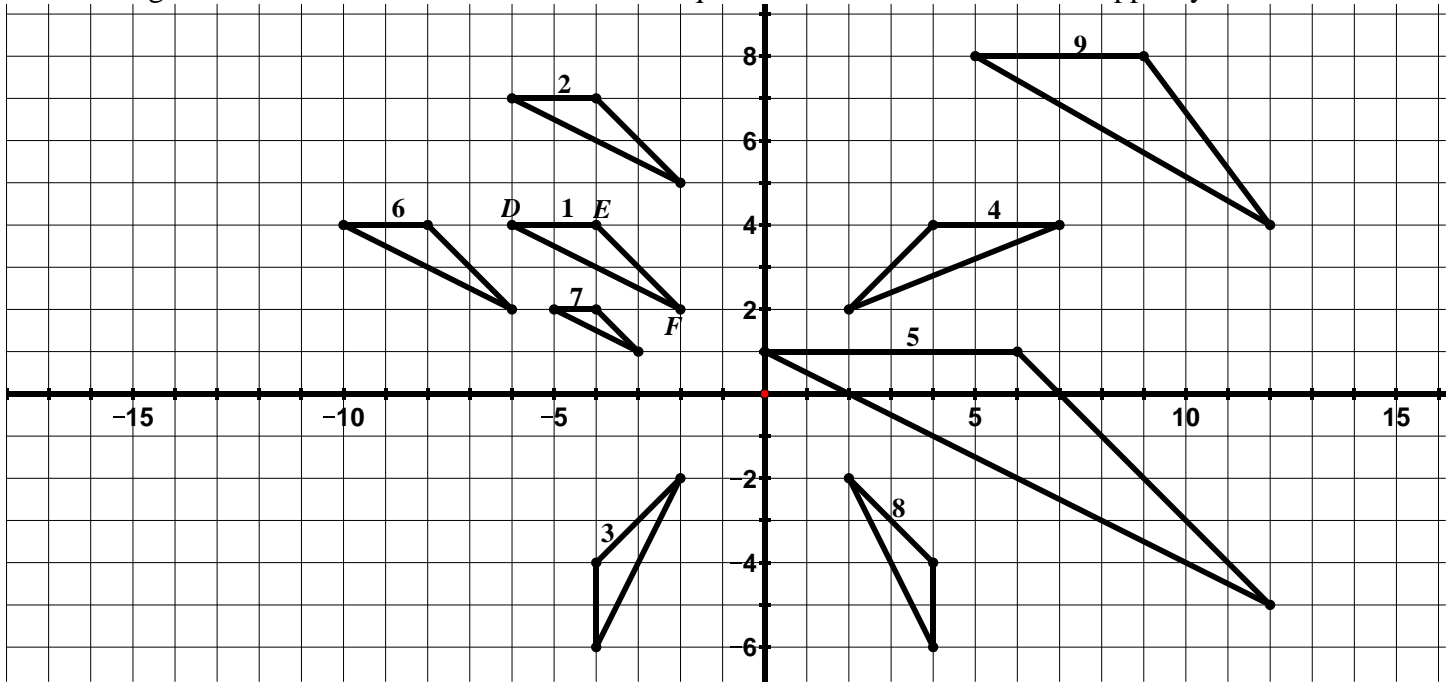


As students solve problems throughout the chapter, they use ideas about slope over and over again to discover the properties of rigid motions and dilations. They also use ideas about slope to translate, reflect, rotate, and dilate figures. In the picture above, Micah is using slope triangles to dilate $\triangle ABC$ with a scale factor of 3 and center of dilation at the origin.

9.0 Anchor Problem: Congruence and Similarity



Directions: Determine whether the triangles pictured below are **congruent** to $\triangle DEF$, **similar** to $\triangle DEF$, or neither congruent nor similar to $\triangle DEF$. Describe a sequence of transformations that support your claims.



9.1 Rigid Motion and Congruence

Section Overview:

In this section, students study the different types of rigid motion: translations, reflections, and rotations. Students begin their study of rigid motion with translations. Students describe translations that have taken place, both in words and with a coordinate rule. Students execute various translations, given a coordinate rule. Then, students summarize the properties of a translation based on the work they have done. These first few lessons also introduce students to some of the vocabulary used in transformational geometry. Next, students turn to reflections, again discovering the properties of reflections (including those over horizontal/vertical lines and the lines $y = x$ and $y = -x$). They write coordinate rules for reflections, connecting these rules to the slopes of the corresponding segments in the image and pre-image. Lastly, students draw lines of reflection and write the equations for these lines. Students then study rotations, with the emphasis on rotations of 90° increments. Students describe the properties of rotations and use these properties to solve problems. They start with rotations where the center of rotation is at the origin, again describing and executing rotations. Then, students study rotations where the center of rotation is not at the origin. Throughout the study of translations, reflections, and rotations, students articulate which properties hold for all of the rigid motions and which are specific to a given rigid motion. Students also perform a sequence of rigid motions and identify sequences of rigid motions that carry one figure to another. Students will then apply this knowledge to determine if two figures are congruent, understanding that two figures are congruent if there is a sequence of rigid motions that takes one figure onto the other.

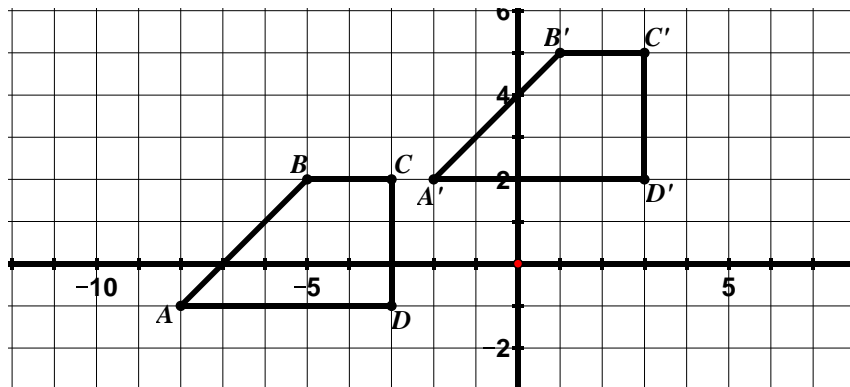
Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Given a pre-image and its image under a translation, describe the translation in words and using a coordinate rule.
2. Perform a translation of a figure given a coordinate rule.
3. Describe the properties of a translation and the effects a translation has on a figure and use this knowledge to solve problems.
4. Given a pre-image and its image under a reflection, describe the reflection in words and using a coordinate rule.
5. Perform a reflection of a figure given a line of reflection.
6. Describe the properties of a reflection and the effects a reflection has on a figure and use this knowledge to solve problems.
7. Find a reflection line for a given reflection and write the equation of the reflection line.
8. Given a pre-image and its image under a rotation, describe the rotation in words and using a coordinate rule (coordinate rule for rotations centered at the origin only).
9. Perform a rotation of a figure given a center of rotation, an angle of rotation, and a direction.
10. Describe the properties of a rotation and the effects a rotation has on a figure and use this knowledge to solve problems.
11. Connect ideas about slopes of perpendicular lines and rotations.
12. Understand what it means for two figures to be congruent.
13. Determine if two figures are congruent based on the definition of congruence.
14. Given two figures that are congruent, describe the sequence of transformations that exhibits the congruence between them.

9.1a Class Activity: Properties of Translations

1. In the grid below, $ABCD$ has been transformed to obtain $A'B'C'D'$.



$ABCD$ is called the **pre-image** and $A'B'C'D'$ is called the **image**. The **pre-image** is the figure prior to the transformation and the **image** is the figure after the transformation. A and A' , B and B' , C and C' , and D and D' are **corresponding vertices**.

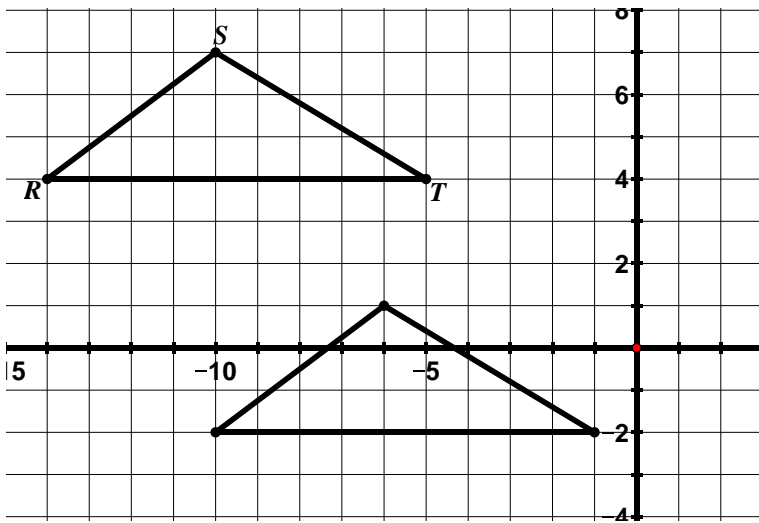
- c. In the table below, write the coordinates for the vertices of the pre-image and image.

Pre-Image	Image
A :	A' :
B :	B' :
C :	C' :
D :	D' :

- a. This type of transformation is called a **translation**. Describe in your own words the movement of a figure that has been translated.
- b. Show on the picture how you would move on the coordinate plane to get from A to A' , B to B' , C to C' , and D to D' .

- d. The coordinate rule for this translation is $(x, y) \rightarrow (x + 6, y + 3)$. Connect this notation to your answer for part b. and to the coordinates of corresponding vertices in the table.


2. In the grid below, $\triangle RST$ has been translated to obtain $\triangle R'S'T'$.




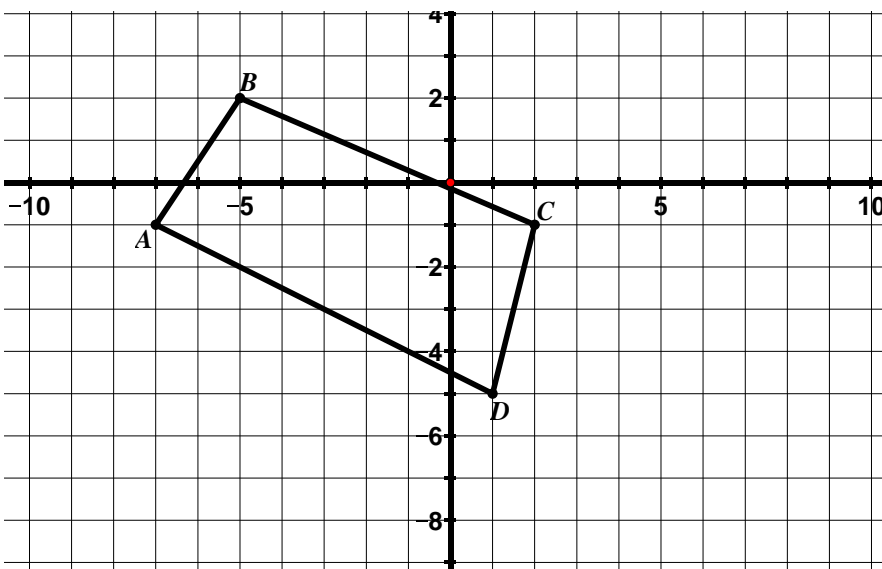
- Label the corresponding vertices of the image on the grid.
- Describe or show on the picture how you would move on the coordinate plane to get from the vertices in the pre-image to the corresponding vertices in the image.

c. In the table below, write the coordinates for the vertices of the pre-image and image.

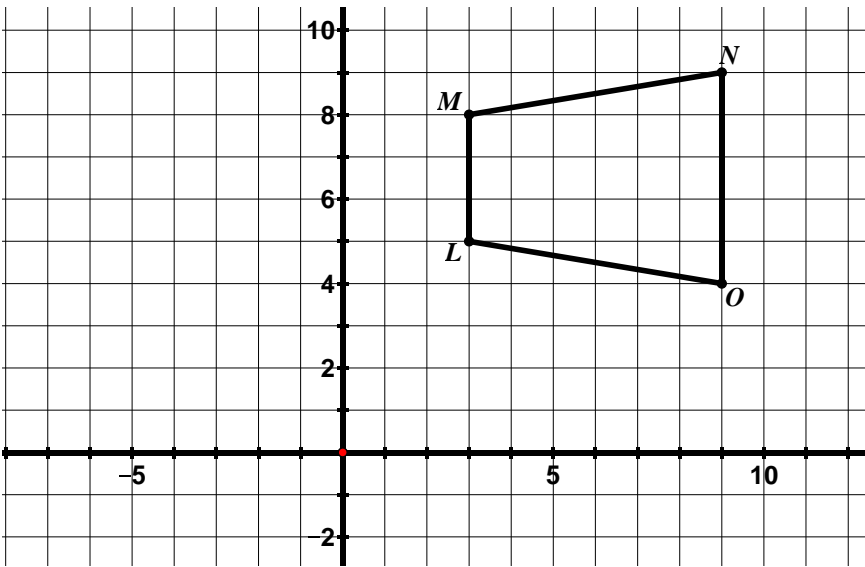
Pre-Image	Image
R:	R':
S:	S':
T:	T':

- Write the coordinate rule that describes this translation. 

3. Draw and label the image of the figure below for the translation $(x, y) \rightarrow (x + 5, y - 3)$ 



4. Draw and label the image of the figure below for the translation $(x, y) \rightarrow (x - 7, y)$



Determine the slopes for:

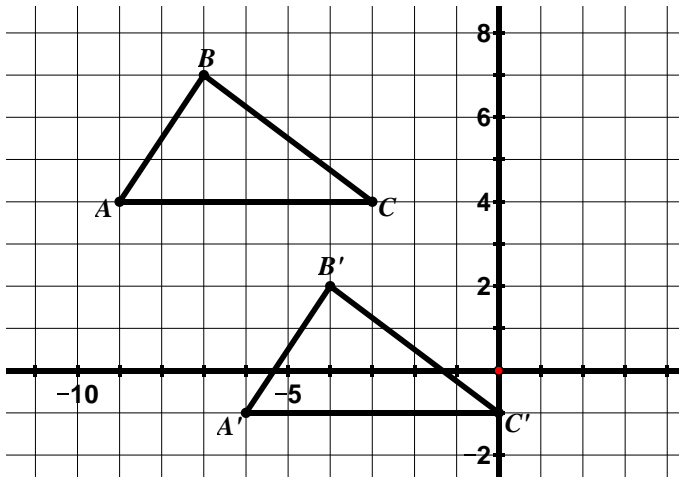
$$\overline{MN}: \quad \overline{M'N'}:$$

$$\overline{NO}: \quad \overline{N'O'}:$$

$$\overline{LO}: \quad \overline{L'O'}:$$

$$\overline{ML}: \quad \overline{M'L'}:$$

5. Write a coordinate rule to describe the translation below. _____



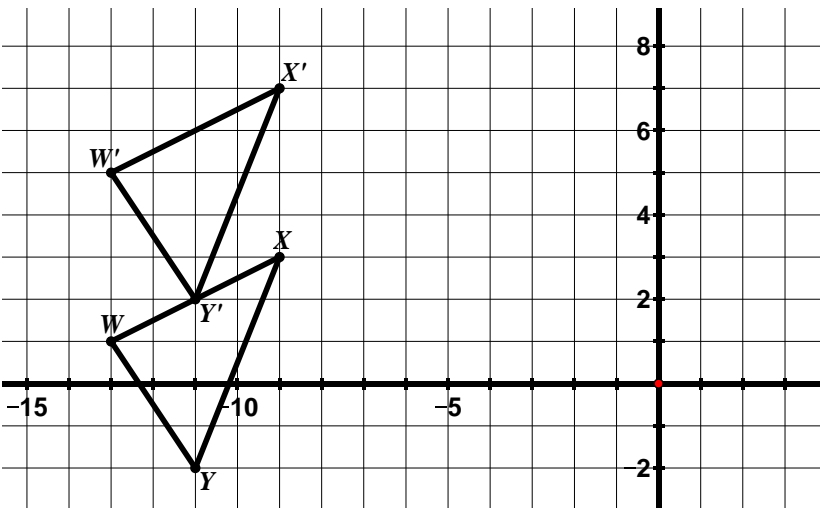
Determine the slopes for:

$$\overline{AB}: \quad \overline{A'B'}:$$

$$\overline{BC}: \quad \overline{B'C'}:$$

$$\overline{CA}: \quad \overline{C'A'}:$$

6. Write a coordinate rule to describe the translation below _____



Determine the slopes for:

$$\overline{WX}: \quad \overline{W'X'}:$$

$$\overline{XY}: \quad \overline{X'Y'}:$$

$$\overline{YW}: \quad \overline{Y'W'}:$$

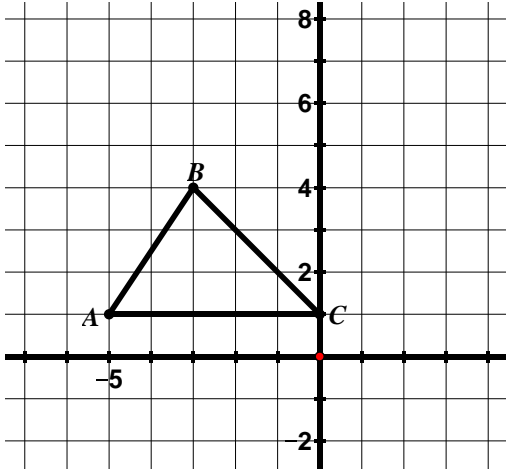
7. Use questions #1 – 6 to explore some **properties of translations** and write your observations below.



9.1a Homework: Properties of Translations

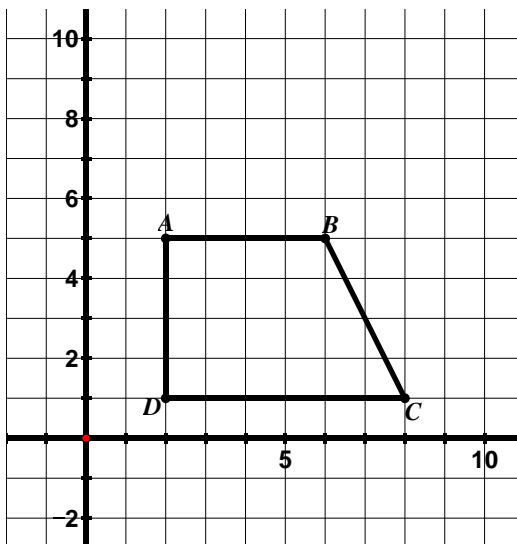
Directions: For #1 – 3, draw and label the image for the coordinate rule given. Then answer the questions.

1. Translate the figure below according to the rule $(x, y) \rightarrow (x + 3, y + 2)$ and label the image.

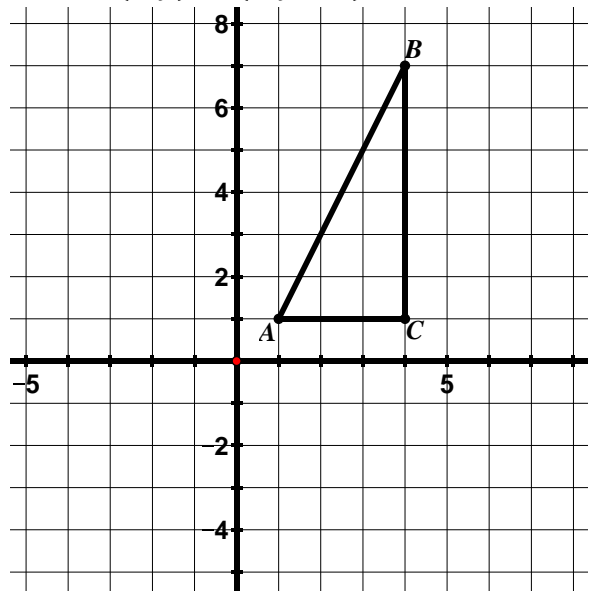


- If the slope of \overline{BC} is -1 , determine the slope of $\overline{B'C'}$ without doing any calculations.
 - If the length of \overline{BC} is $3\sqrt{2}$, determine the length of $\overline{B'C'}$ without doing any calculations.
 - Determine the slopes of \overline{AB} and of $\overline{A'B'}$. What do you notice about the slopes of **corresponding segments of a translated figure**?
- d. Using a ruler, draw a line connecting corresponding vertices in the image and pre-image (A to A' , B to B' , and C to C'). Find the slopes of $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. What do you notice about the slopes of the **segments connecting corresponding vertices** of the image and pre-image of a translated figure?

2. Translate the figure below according to the rule $(x, y) \rightarrow (x - 1, y + 5)$ and label the image.

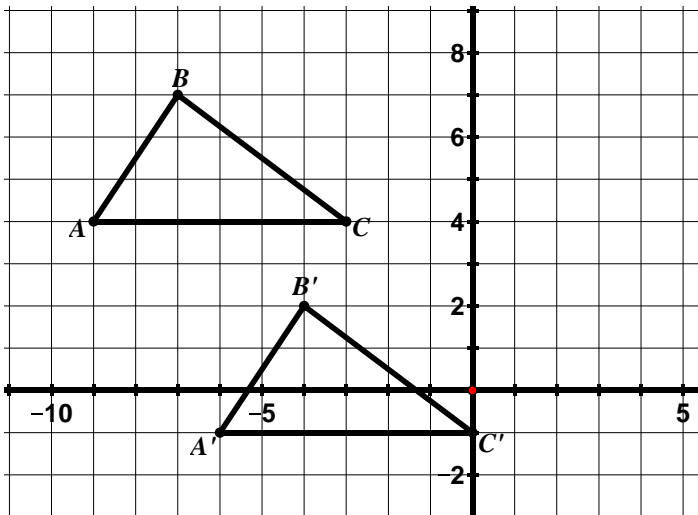


3. Translate the figure below according to the rule $(x, y) \rightarrow (x, y - 4)$ and label the image.



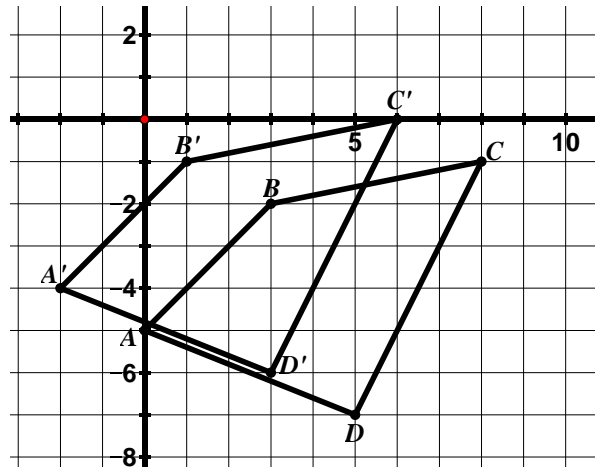
Directions: For #4 – 7, write a coordinate rule to describe the translation. Then answer the questions.

4. Coordinate Rule:

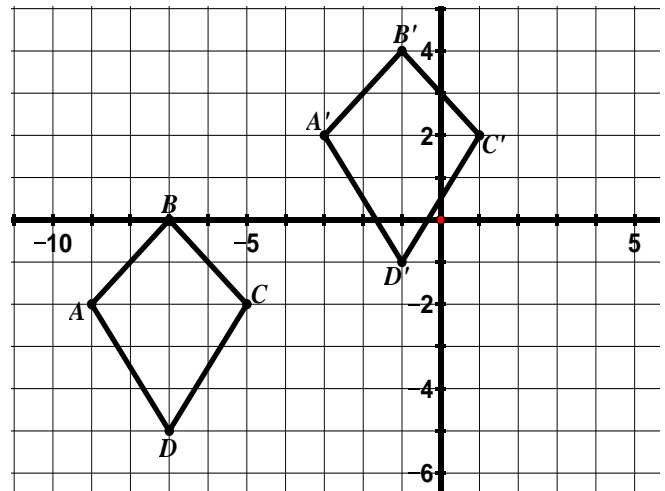


- The slope of $\overline{BB'}$ is $-\frac{5}{3}$. Name two other segments that also have a slope of $-\frac{5}{3}$.
- If the length of $\overline{BB'}$ is $\sqrt{34}$, determine the length of $\overline{CC'}$ without doing any calculations.
- Determine the length of \overline{AC} and of $\overline{A'C'}$.
- Determine the slope of \overline{AC} and of $\overline{A'C'}$.

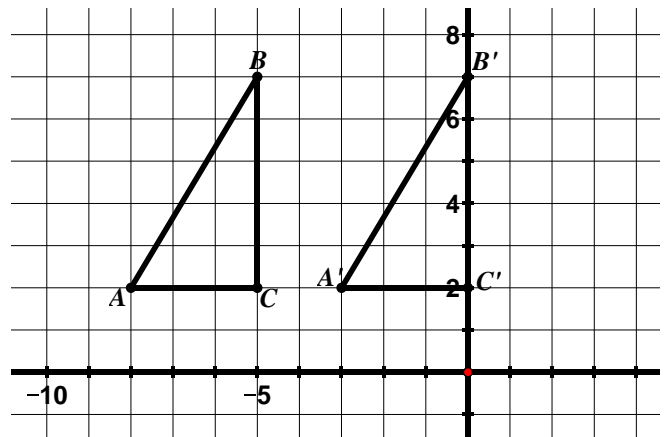
5. Coordinate Rule:



6. Coordinate Rule:

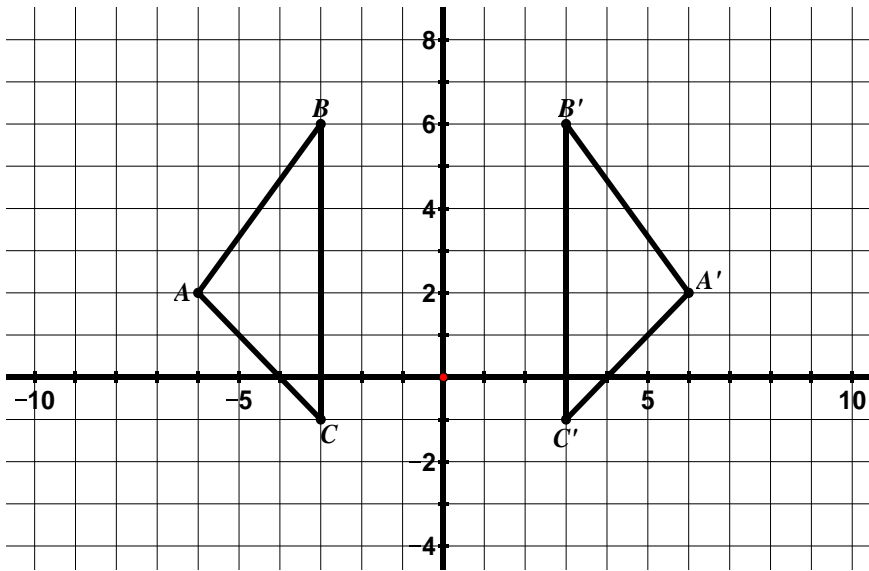


7. Coordinate Rule:



9.1b Class Activity: Properties of Reflections

1. In the grid below, $\triangle ABC$ has been **reflected** over the y -axis to obtain $\triangle A'B'C'$.



a. Describe the movement of a figure that has been **reflected**.

b. In the table below, write the coordinates for the vertices of the pre-image and image.

Pre-Image	Image
A:	A':
B:	B':
C:	C':

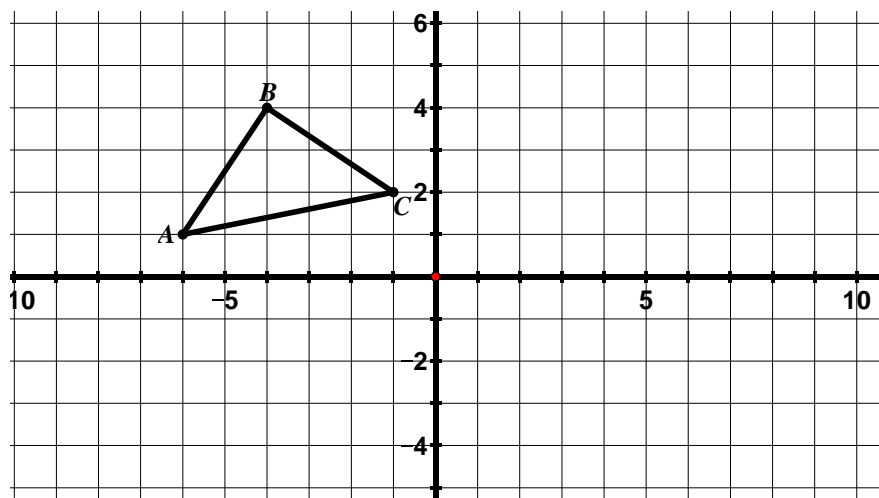
c. Write a coordinate rule to describe this reflection.

d. Will this coordinate rule hold true for any figure reflected over the y -axis? Why or why not?



Directions: Draw and label the image of each figure for the reflection given. Then, answer the questions.

2. Reflect $\triangle ABC$ across the x -axis and label the image.




a. In the table below, write the coordinates for the vertices of the pre-image and image.

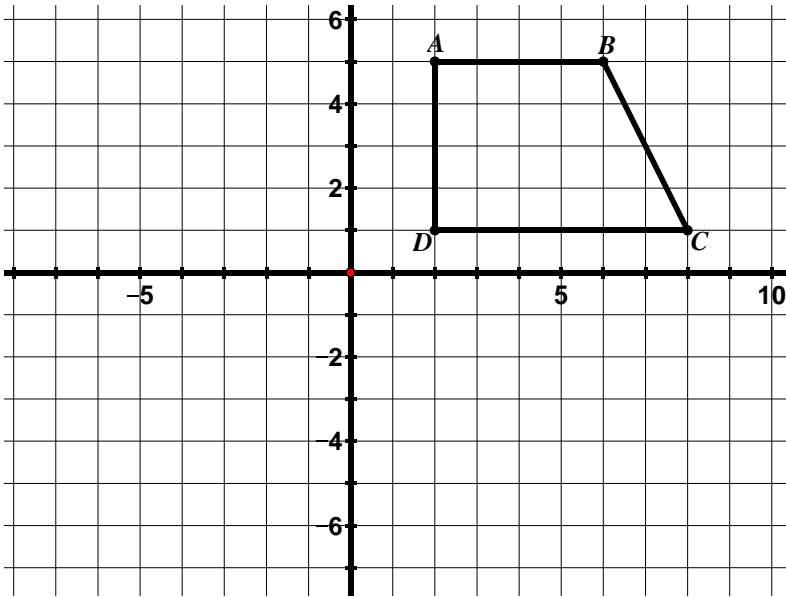
Pre-Image	Image
A:	A':
B:	B':
C:	C':

b. Write a coordinate rule to describe this reflection.

c. Will this coordinate rule hold true for any figure reflected over the x -axis? Why or why not?

3. Use questions #1 – 2 to explore some **properties of reflections**.
- Go back to problem #1. Draw a segment connecting B and B', A and A', and C and C'. Make at least two conjectures about the relationship between **the line of reflection and the segments connecting corresponding vertices** in the image and pre-image of a reflection.
 - Do your conjectures hold true in problem #2?
 - Go back to problem #1. For a *translation* we learned that corresponding segments are parallel (have the same slope). Is this property also true for reflections?
 - Now, go to problem #2. Find the slopes of the following segments:
 $\overline{AB} =$ $\overline{AC} =$ $\overline{BC} =$
 $\overline{A'B'} =$ $\overline{A'C'} =$ $\overline{B'C'} =$
 - Compare the slopes of the corresponding segments of the image and pre-image. What do you notice about the slopes? How does this connect to the coordinate rule $(x, y) \rightarrow (x, -y)$? 
 - Examine problems #1 and #2. What do you notice about the **lengths of corresponding segments** in the image and pre-image?

4. Reflect $ABCD$ across the line $y = -1$ and label the image.

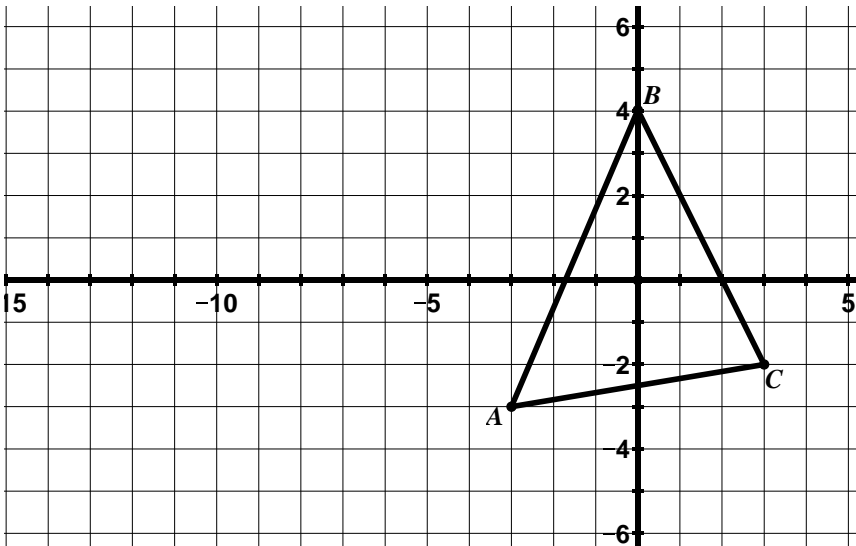


a. In the table below, write the coordinates for the vertices of the pre-image and image.

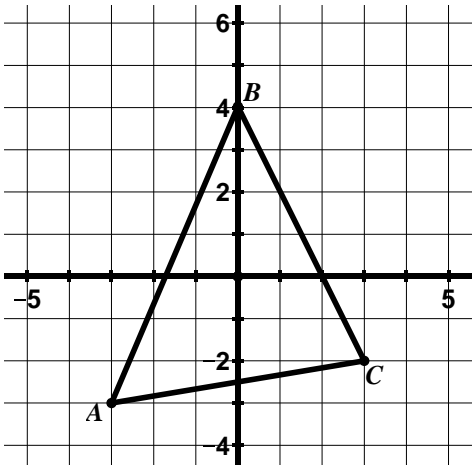
Pre-Image	Image
A:	A':
B:	B':
C:	C':
D:	D':

b. Write a coordinate rule to describe this reflection.

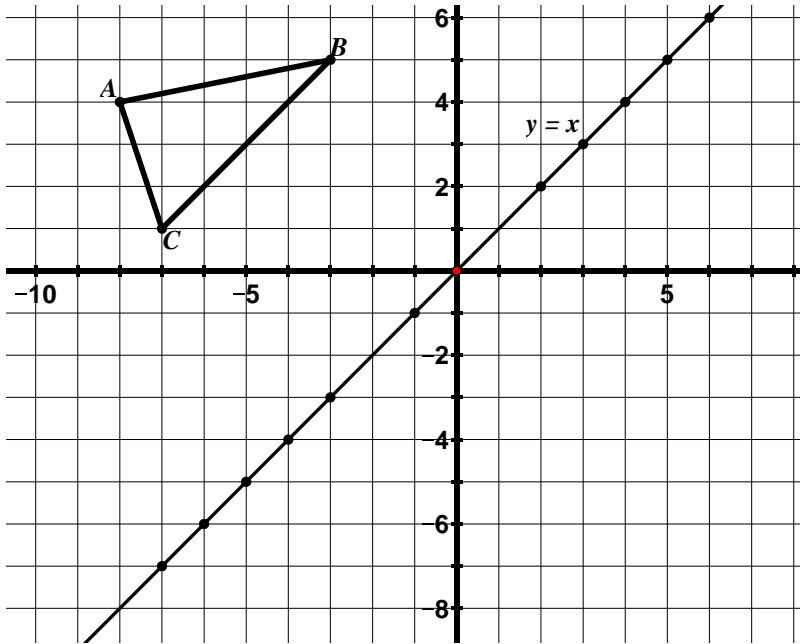
5. Reflect $\triangle ABC$ across the line $x = -5$ and label the image.



6. Reflect $\triangle ABC$ over the y -axis and label the image.



7. Reflect $\triangle ABC$ across the line $y = x$ and label the image.




- a. Describe the method you used to solve this problem. 

- b. In the table below, write the coordinates for the vertices of the pre-image and image.

Pre-Image	Image
A:	A':
B:	B':
C:	C':

- c. Write a coordinate rule to describe this reflection.
- d. Will this coordinate rule hold true for any figure reflected over the line $y = x$? Why or why not?

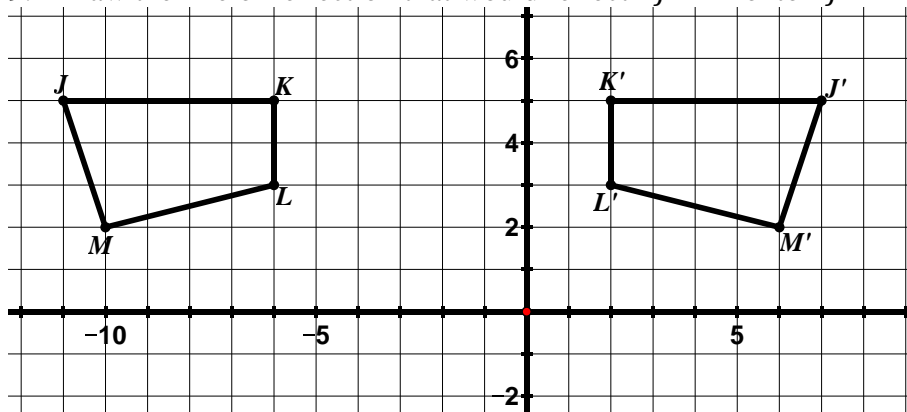
- e. Find the slopes of the following segments:
 $\overline{AB} =$ $\overline{AC} =$ $\overline{BC} =$
 $\overline{A'B'} =$ $\overline{A'C'} =$ $\overline{B'C'} =$
- f. Compare the slopes of the corresponding segments of the image and pre-image. What do you notice? How does this connect to the coordinate rule?
- g. **Bonus:** What is the coordinate rule for a figure reflected across the line $y = -x$?

8. The following table lists the properties of translations discovered in the previous lesson. Put a checkmark in the box if the property is also true for reflections. Add additional statements to the table that are only true for reflections. 

Properties of Translations	Also True for Reflections?
Segments connecting the corresponding vertices of the image and pre-image are the same length.	
Segments connecting the corresponding vertices of the image and pre-image are parallel to each other.	
*Corresponding segments in the image and pre-image are the same length.	
*Corresponding angles in the image and pre-image have the same measure.	
*Parallel lines in the pre-image remain parallel lines in the image.	
Corresponding segments in the image and pre-image have the same slope.	

Directions: For #9 – 11, draw the line of reflection that would reflect one figure onto the other. Then, write the equation for the line of reflection and the coordinate rule that describes the reflection.

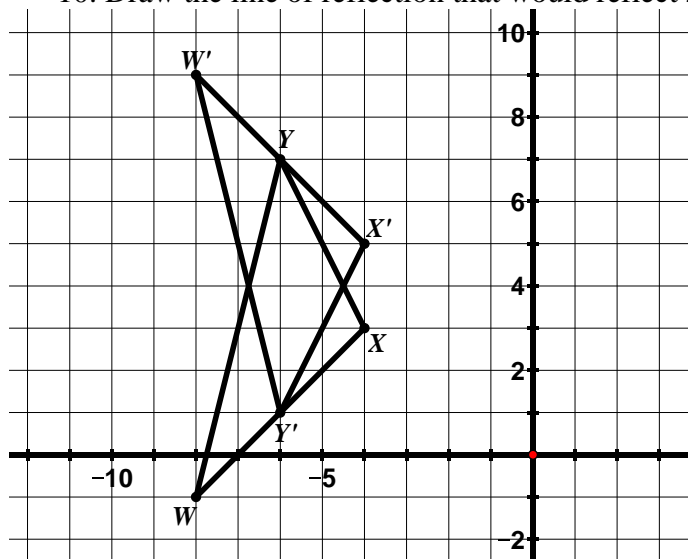
9. Draw the line of reflection that would reflect $\triangle JKL$ onto $\triangle J'K'L'M'$.



a. Write the equation for the line of reflection.

b. Write a coordinate rule for the reflection.

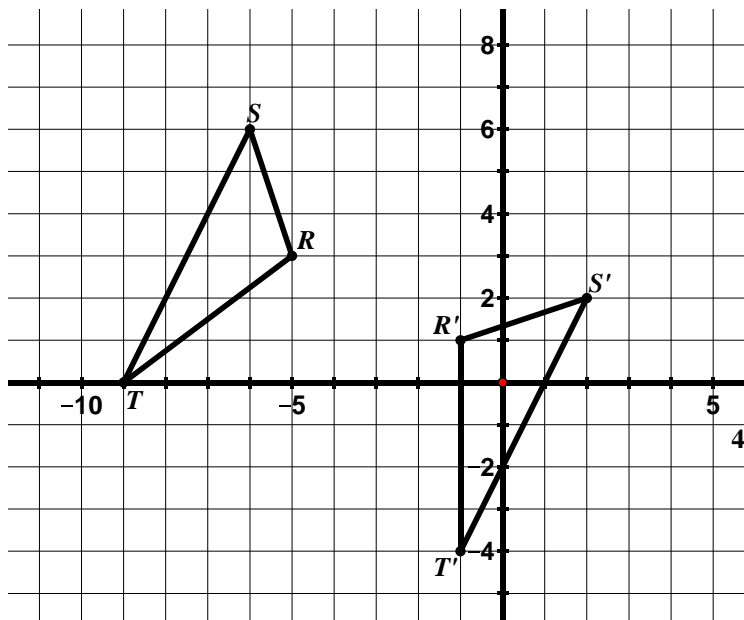
10. Draw the line of reflection that would reflect $\triangle WXY$ onto $\triangle W'X'Y'$.



a. Write the equation for the line of reflection.

b. Write a coordinate rule for the reflection.

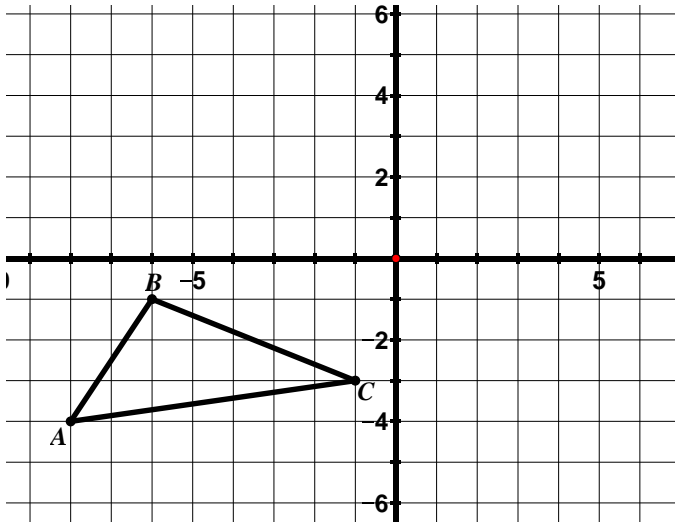
11. Draw the line of reflection that would reflect $\triangle RST$ onto $\triangle R'S'T'$.



a. Write the equation for the line of reflection.

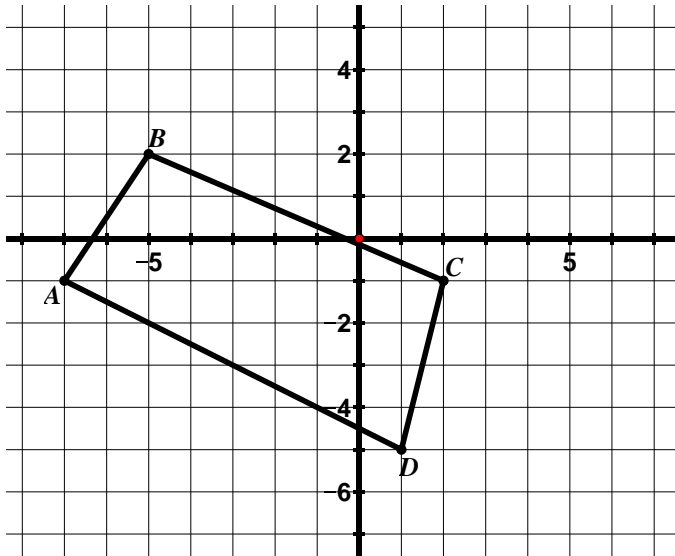
9.1b Homework: Properties of Reflections

1. Reflect $\triangle ABC$ across the x -axis and label the image.



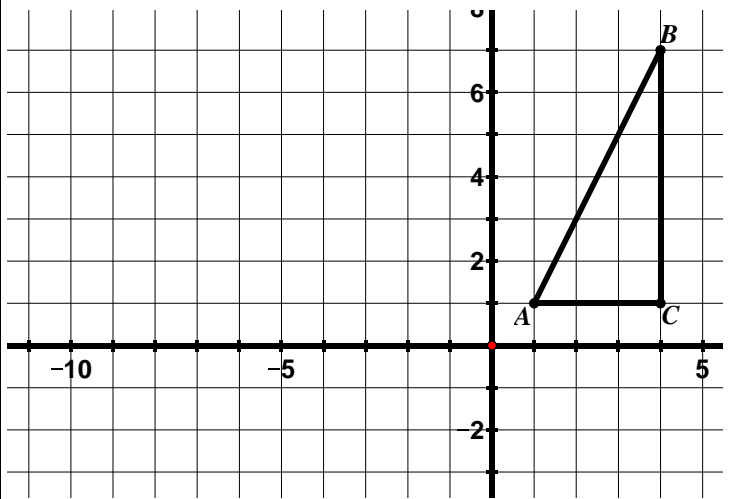
- a. Write a coordinate rule to represent this transformation.

2. Reflect $ABCD$ across the y -axis and label the image.



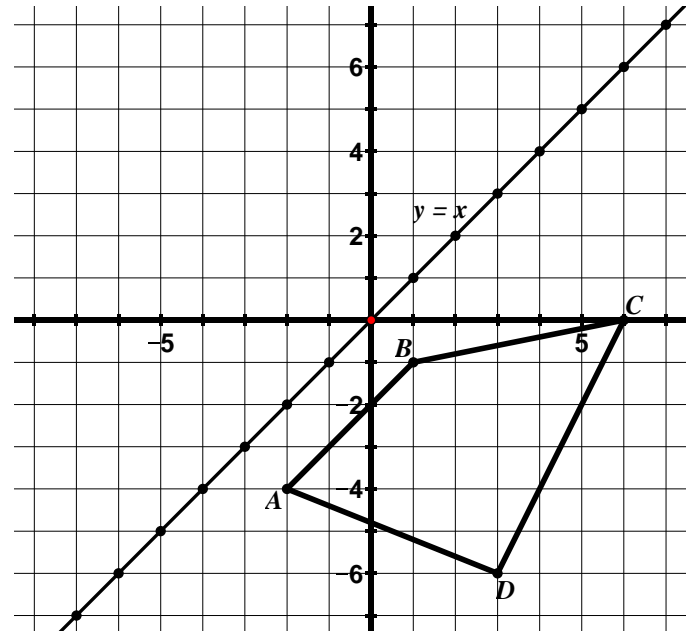
- a. Write a coordinate rule to represent this transformation.

3. Reflect $\triangle ABC$ across the line $x = -3$ and label the image.



- a. Write a coordinate rule to represent this transformation.

4. Reflect $ABCD$ across the line $y = x$.



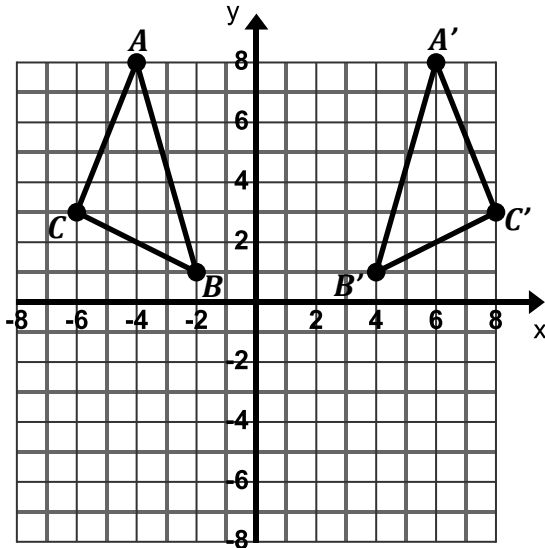
- a. Write a coordinate rule to represent this transformation.

5. For each of the following:

- Draw the line of reflection that would reflect the pre-image onto the image.
- Find the equation for the line of reflection.
- Write a coordinate rule to describe the reflection.

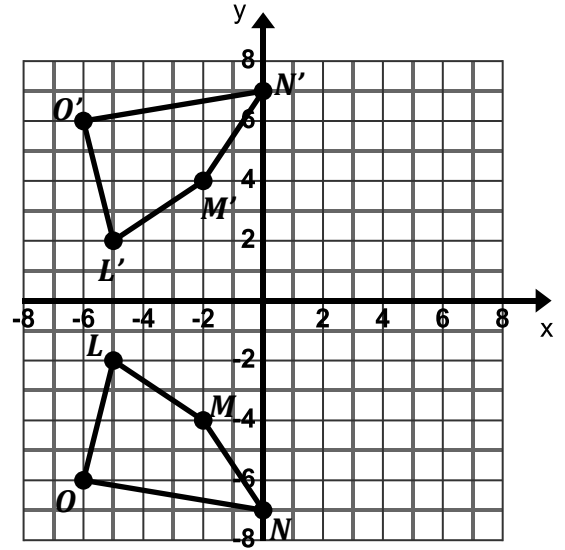
a. Equation of line of reflection: _____

Coordinate Rule: _____



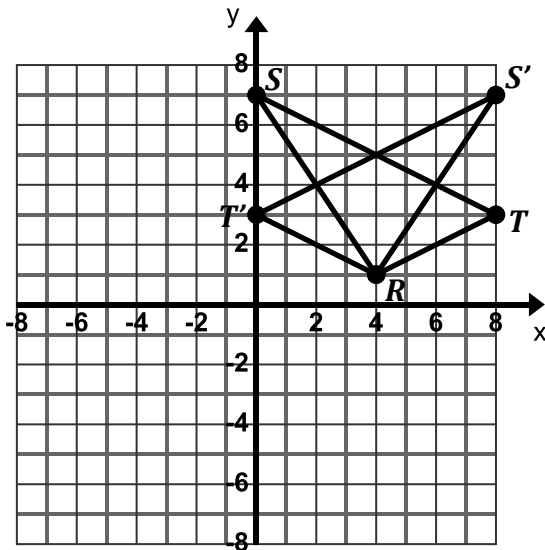
b. Equation of line of reflection: _____

Coordinate Rule: _____



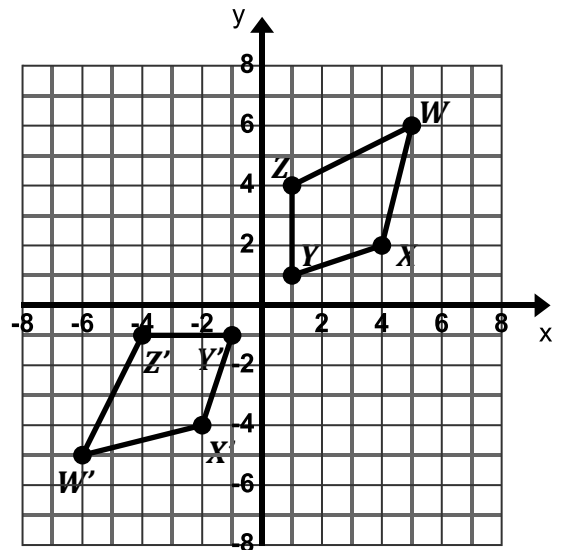
c. Equation of line of reflection: _____

Coordinate Rule: _____

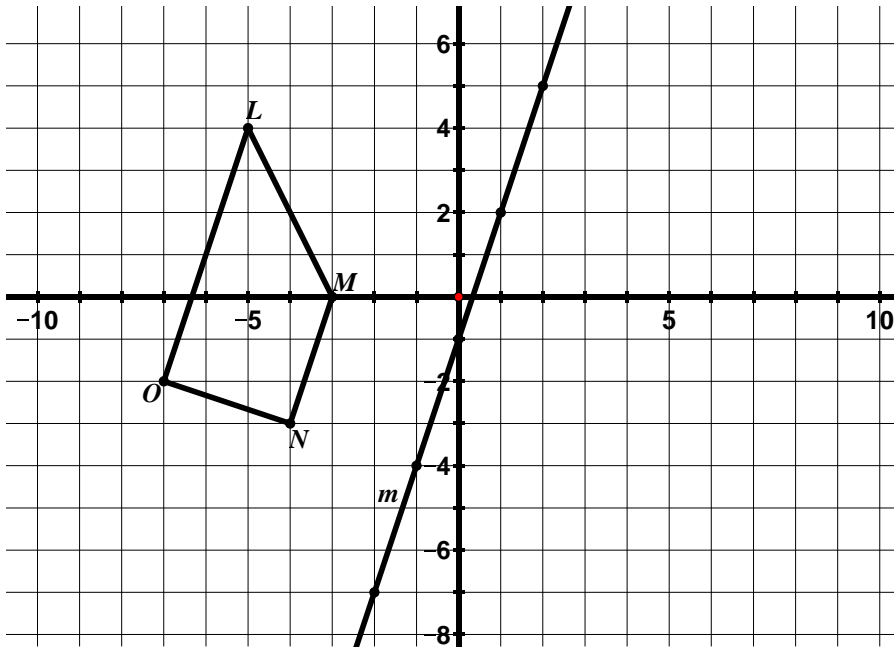


d. Equation of line of reflection: _____

Coordinate Rule: _____



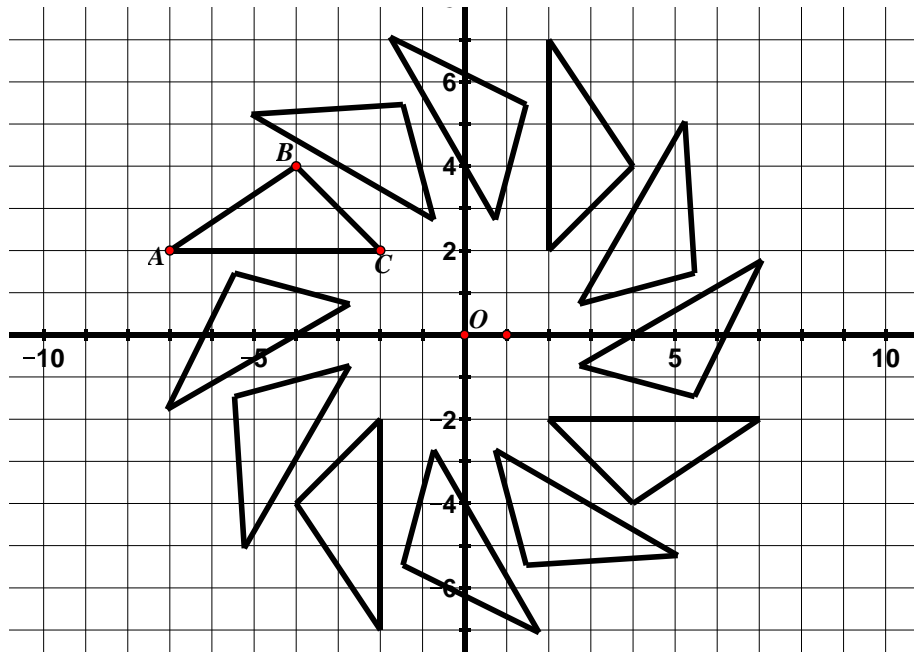
6. Given $LMNO$.




- What is the equation of the line of reflection?
- Based on the slope of the line of reflection, determine what the slope of the segments connecting corresponding points of the image and pre-image should be.
- Reflect $LMNO$ over the line m and label the image.
- What is the relationship between \overline{LO} and \overline{MN} before the transformation? What is the relationship between these two segments after the transformation? Use numerical evidence to support your answer.

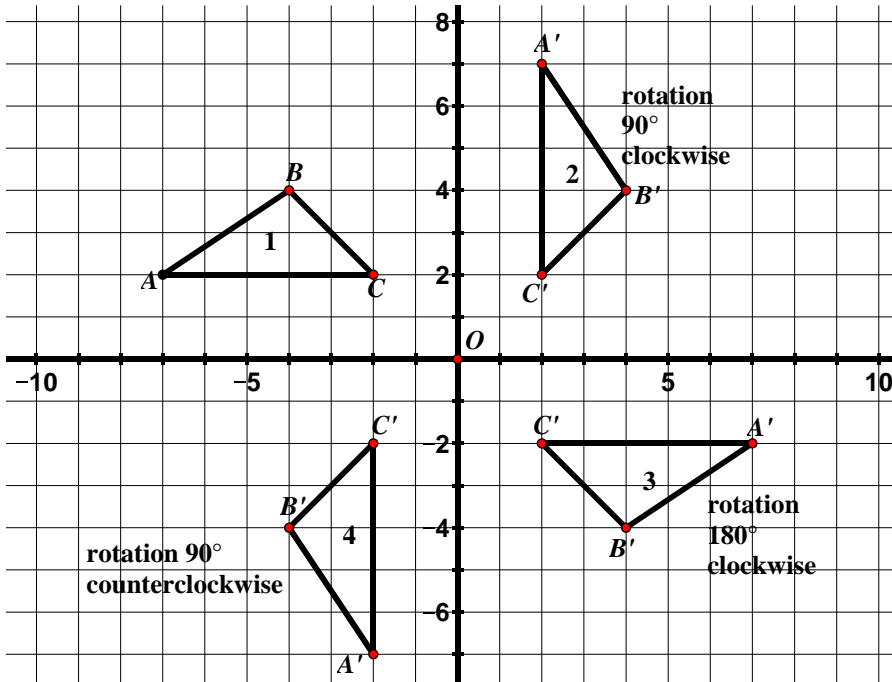
9.1c Class Activity: Properties of Rotations

1. In the grid below, $\triangle ABC$ has been **rotated** counterclockwise with the **center of rotation** at the origin O . This process was repeated several times to create the images shown.



- Using tracing paper, trace $\triangle ABC$ and the x -axis. Holding your pencil as an anchor on the origin, rotate the triangle counterclockwise to see how the images were created.
 - Label the corresponding vertices of the images of $\triangle ABC$.
- c.  Describe the relationship between C and its images to the **center of rotation** O . Do the same for A and its images. Does this relationship to the center of rotation hold true for B and its images?
- d. If there are 360° in one full rotation, determine the angle of rotation from one image to the next in the picture above.

2. The picture from the previous page was modified so that only the images that are increments of 90° rotations of the pre-image $\triangle ABC$ are shown. The center of rotation is the origin O .



- a. Verify using tracing paper that the descriptions of the rotations are accurate.

- b. The rotation from figure 1 to figure 4 has been described as a rotation 90° counterclockwise. How would you describe this rotation in the clockwise direction?

- c. Consider the rotation from Figure 1 to Figure 2, a rotation 90° clockwise. Find the slopes of the following segments:

$$\overline{AB} = \quad \overline{BC} = \quad \overline{AC} =$$

$$\overline{A'B'} = \quad \overline{B'C'} = \quad \overline{A'C'} =$$

- d. Use the slopes from the previous question to determine the relationship between corresponding segments in a 90° rotation.

- e. Which segments would you expect to be perpendicular in the rotation from Figure 1 to Figure 4, the rotation 90° counterclockwise? Use slope to support your answer.

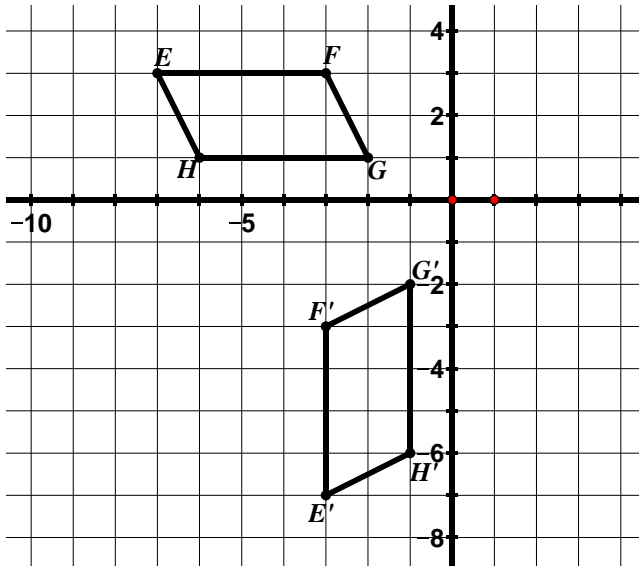
- f. Determine the coordinate rule for a 90° rotation clockwise about the origin. Connect this rule to the slopes of perpendicular lines. **n#**

- g. Determine the coordinate rule for a 90° rotation counterclockwise about the origin. Connect this rule to the slopes of perpendicular lines.

- h. Describe what happens in a 180° rotation of a figure. What is the relationship of the corresponding segments?

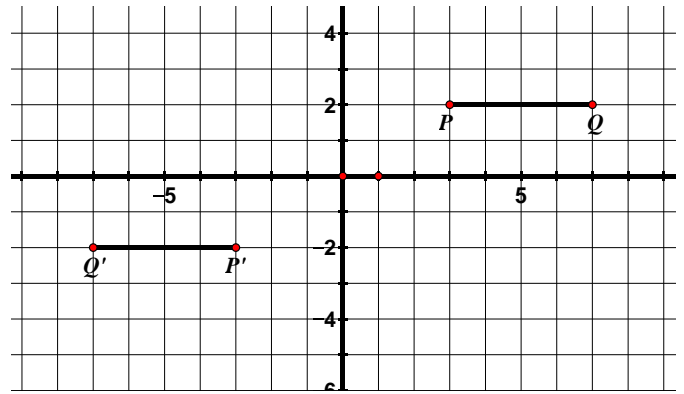
- i. Determine the coordinate rule for a rotation of 180° . Connect this rule to your answer for part h.

3. For the following rotation, the center of rotation is the origin.



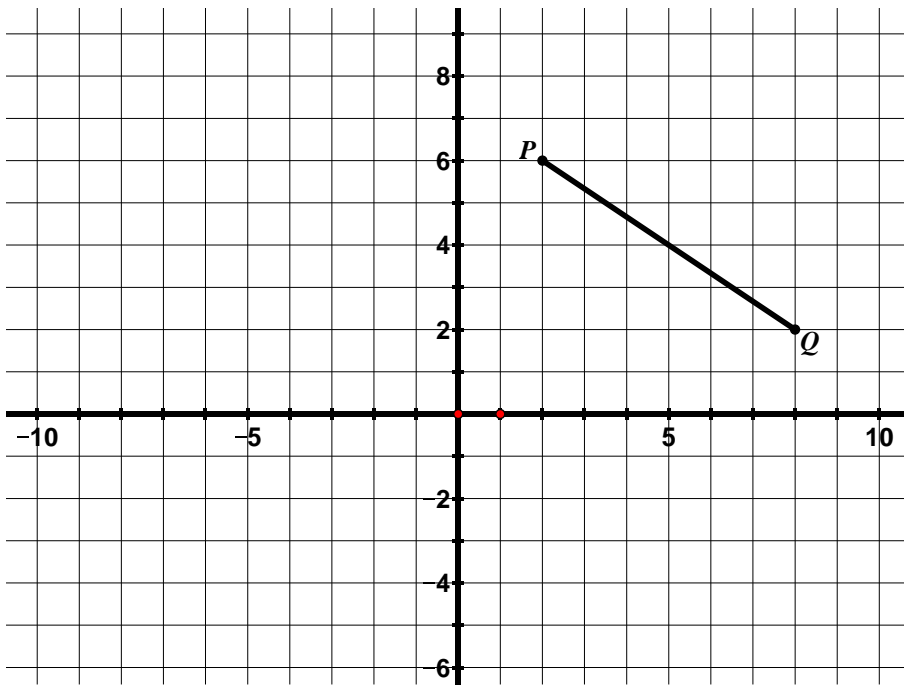
- a. Determine the angle of rotation. Be sure to also indicate a direction of rotation.
- b. If the slope of \overline{EH} is -2 , determine the slope of $\overline{E'H'}$ without doing any calculations.

4. For the following rotation, the center of rotation is the origin.




- a. Determine the angle of rotation. Be sure to also indicate a direction of rotation.

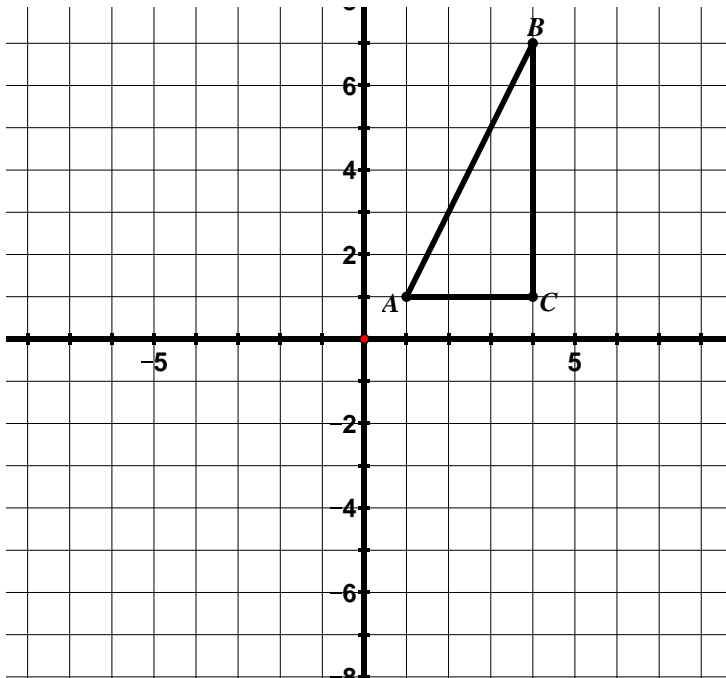
5. Rotate \overline{PQ} 90° counterclockwise with the center of rotation at the origin and label the image.



- How can you verify using slope that your image is in fact a 90° rotation?
- How can you verify using distance that the center of rotation is the origin?
- Use a compass to draw the concentric circles of this rotation.

d.  What do the concentric circles prove?

6. Rotate $\triangle ABC$ 180° counterclockwise with the center of rotation at the origin and label the image.

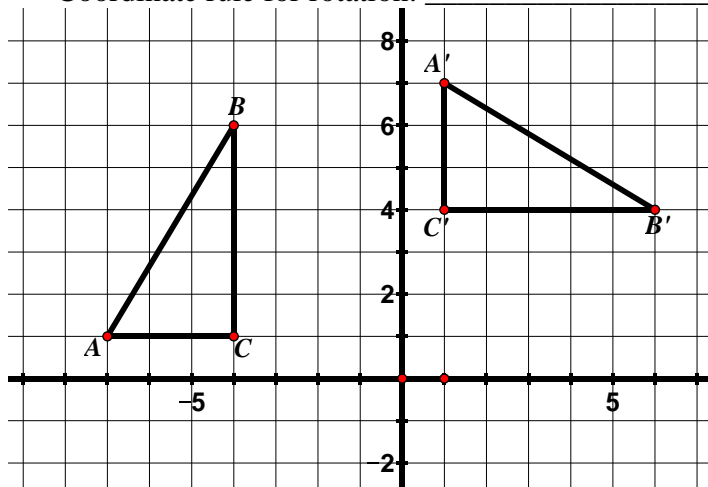


9.1c Homework: Properties of Rotations

Directions: For each of the following rotations, the center of rotation is the origin. Determine the angle of rotation (be sure to also indicate a direction of rotation). Write a coordinate rule for the transformation.

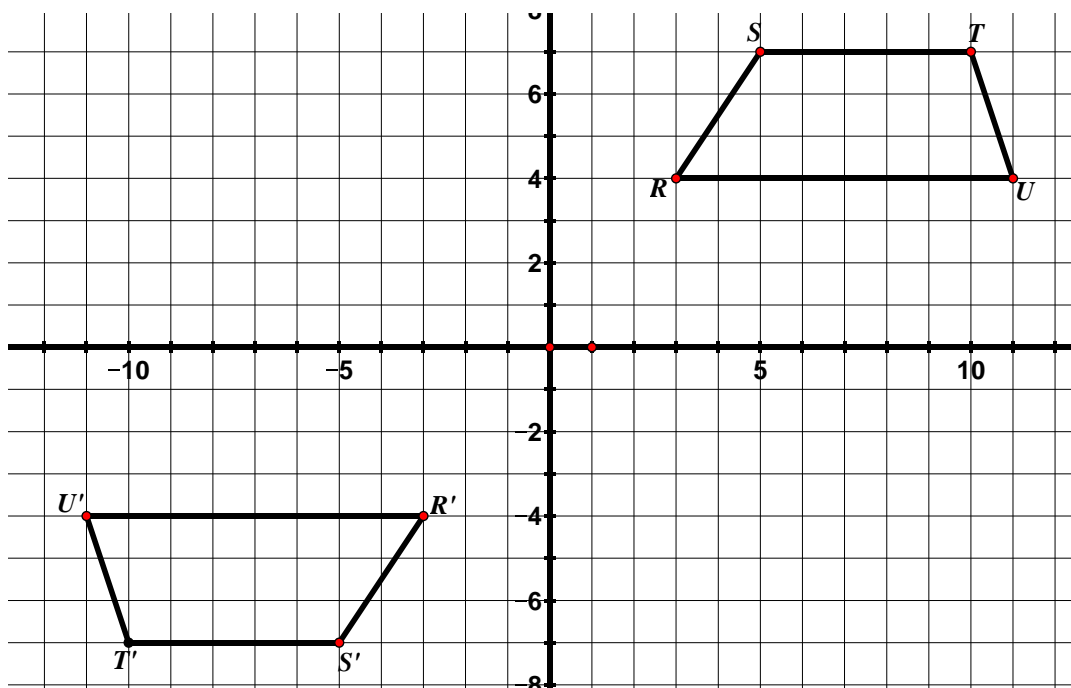
1. Angle of Rotation (including direction of rotation): _____

Coordinate rule for rotation: _____

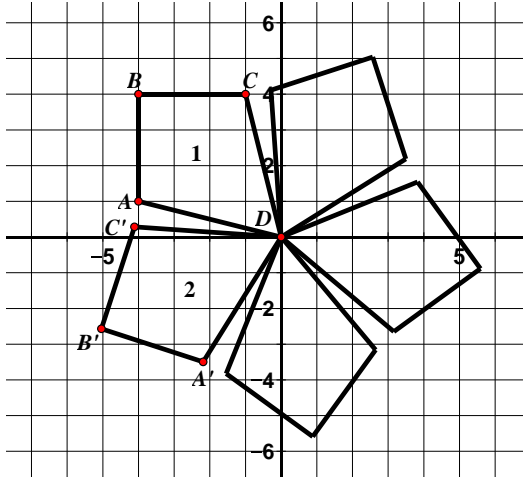


2. Angle of Rotation (including direction of rotation): _____

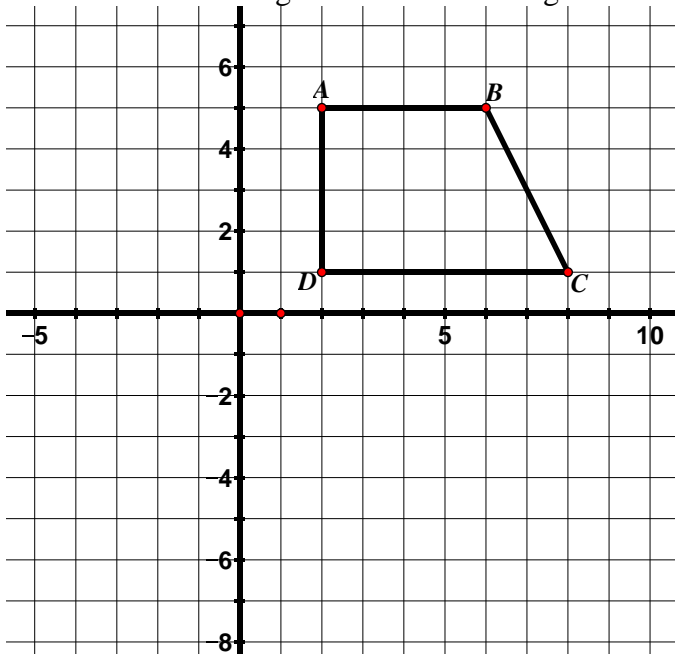
Coordinate rule for rotation: _____



3. Find the angle of rotation from Figure 1 to Figure 2. Be sure to include a direction.

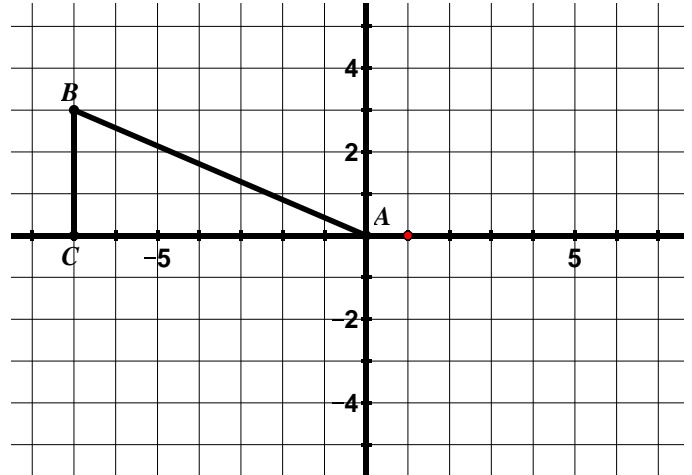


4. Rotate $ABCD$ 90° clockwise with the center of rotation at the origin and label the image.

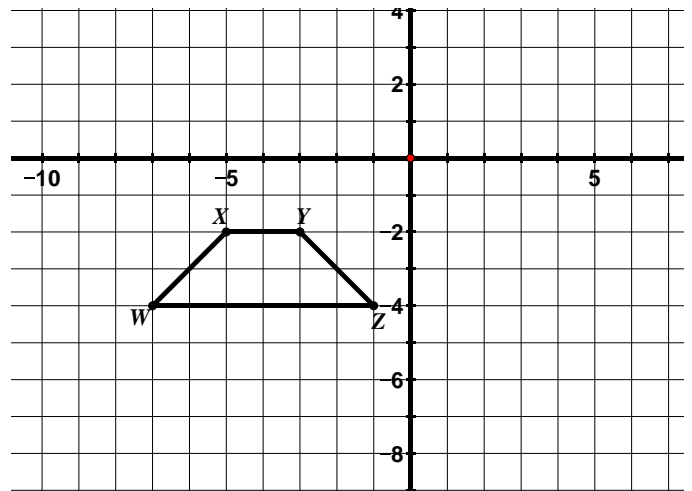


- How can you verify using slope that your image is in fact a 90° rotation?
- How can you verify using distance that the center of rotation is the origin?
- Write a coordinate rule for the rotation.

5. Rotate $\triangle ABC$ 180° clockwise with the center of rotation at the origin.



- Write a coordinate rule for the rotation.
 - Compare the slopes of the segments of the pre-image to the image.
6. Rotate $WXYZ$ 90° counterclockwise with the center of rotation at the origin.



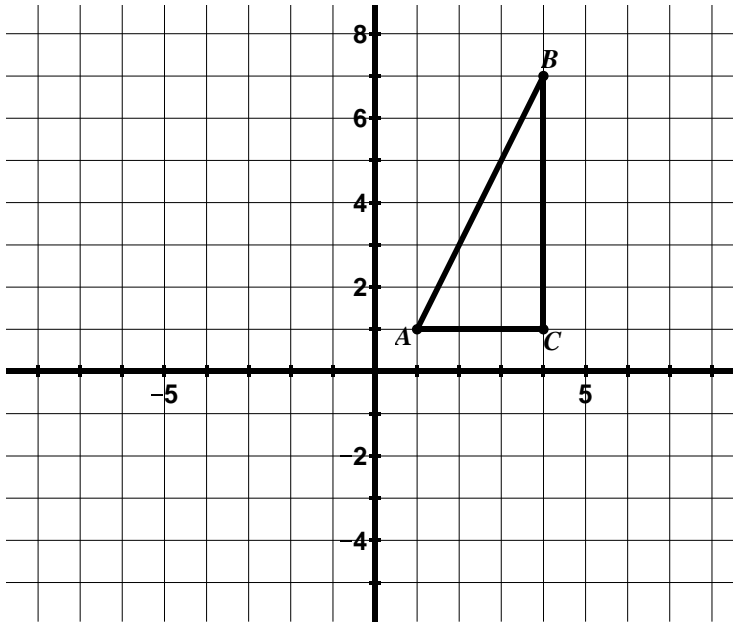
- Compare the slopes of the segments of the pre-image to the image.


9.1d Class Activity: Properties of Rotations cont.

In our prior work with rotations, the center of rotation was always at the origin. Today, we will look at rotations where the center may not be at the origin.

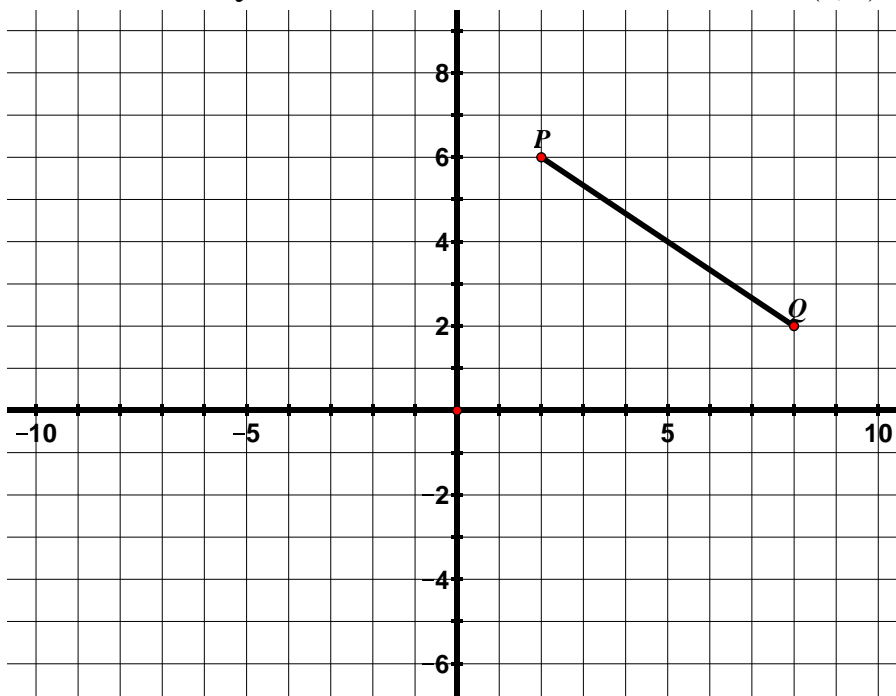
1. **Quiet Write:** In the space below, write everything you have learned about rotations so far.

2. Rotate $\triangle ABC$ 180° counterclockwise with the center of rotation at $(1, 1)$ and label the image.





a. How can you verify that your center of rotation is at $(1, 1)$? 

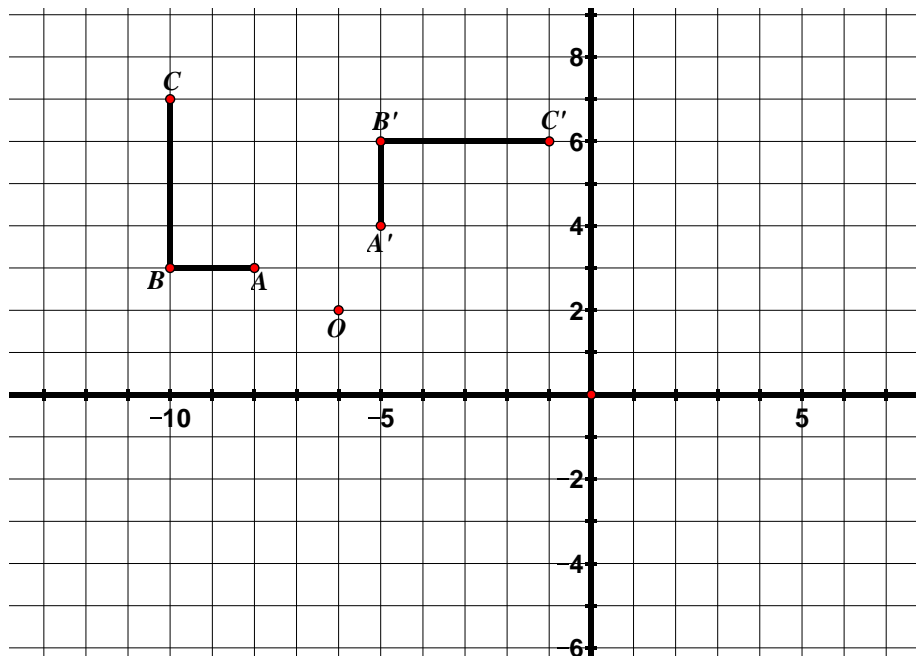
3. Rotate \overline{PQ} 90° clockwise with the center of rotation at $(0, 4)$.



- How can you verify using slope that your image is in fact a 90° rotation?
- How can you verify using distance that the center of rotation is at $(0, 4)$?

4. A teacher asked her students to determine the center of rotation and angle of rotation for the rotation

shown below.  

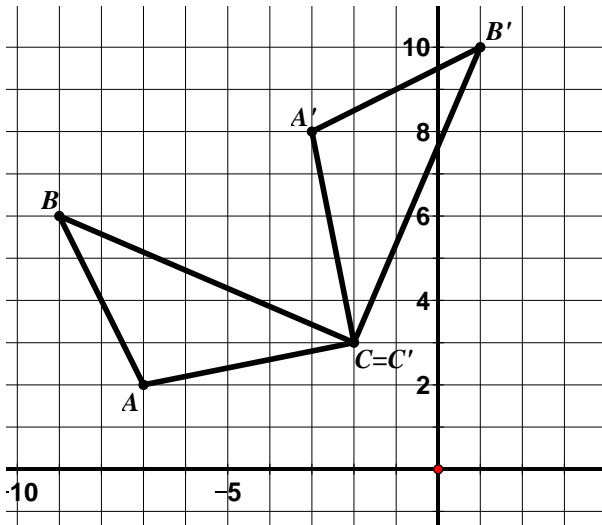


- Aisha described the rotation as a rotation 90° clockwise with the center at $O (-6, 2)$. Do you agree with Aisha? Use the properties of rotations and numerical evidence to support your answer.

Directions: For #5 – 7, find the angle of rotation (including the direction) and the center of rotation.

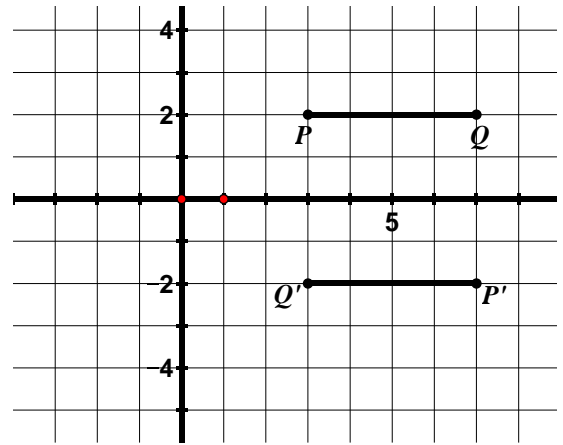
5. Angle of Rotation (including direction of rotation): _____

Center of Rotation: _____



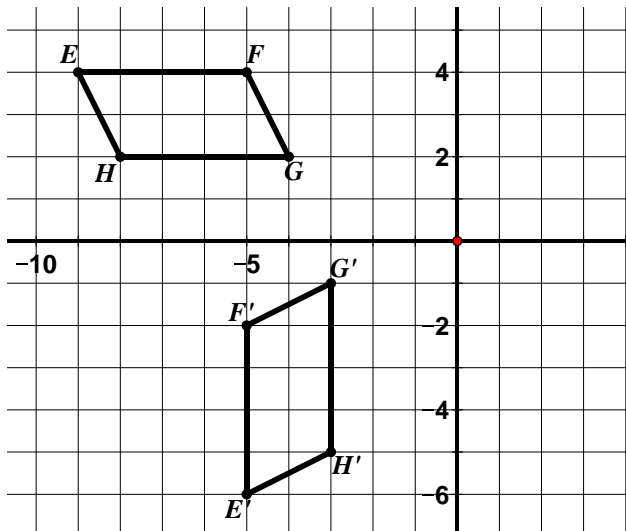
7. Angle of Rotation (including direction of rotation): _____

Center of Rotation: _____



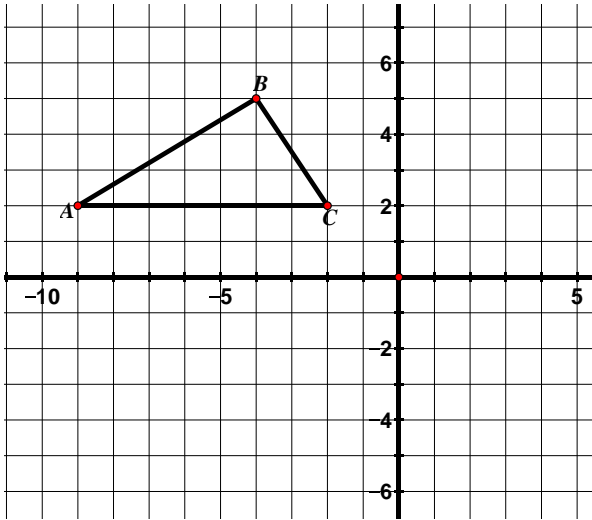
6. Angle of Rotation (including direction of rotation): _____

Center of Rotation: _____

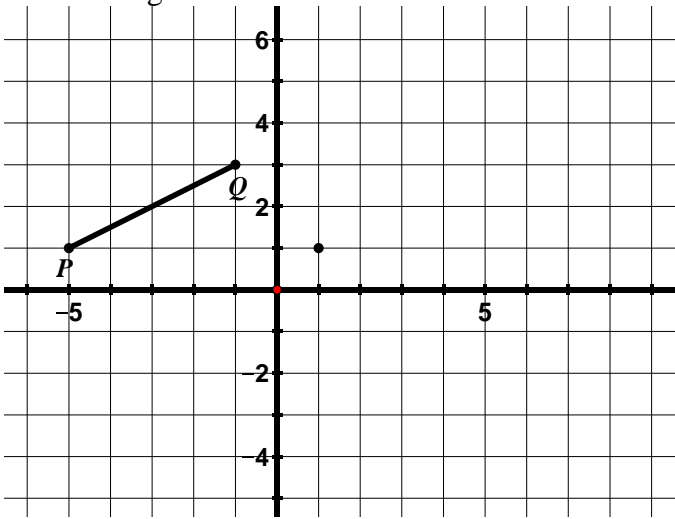


9.1d Homework: Properties of Rotations cont.

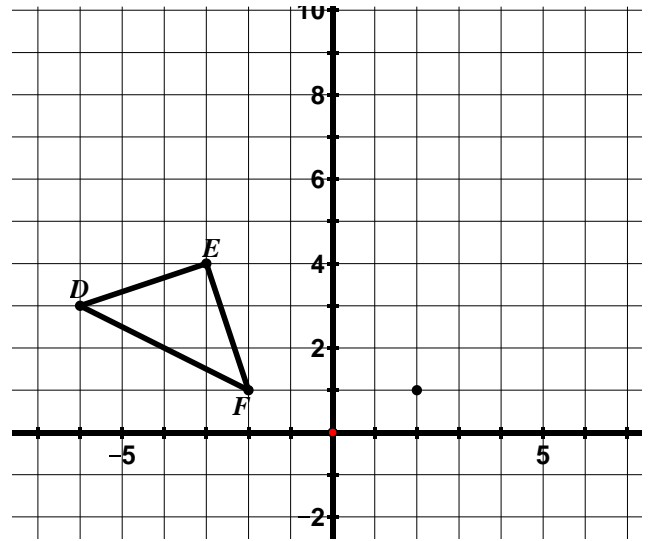
1. Rotate $\triangle ABC$ 90° counterclockwise about C and label the image.



2. Rotate PQ 180° clockwise about $(1, 1)$ and label the image.



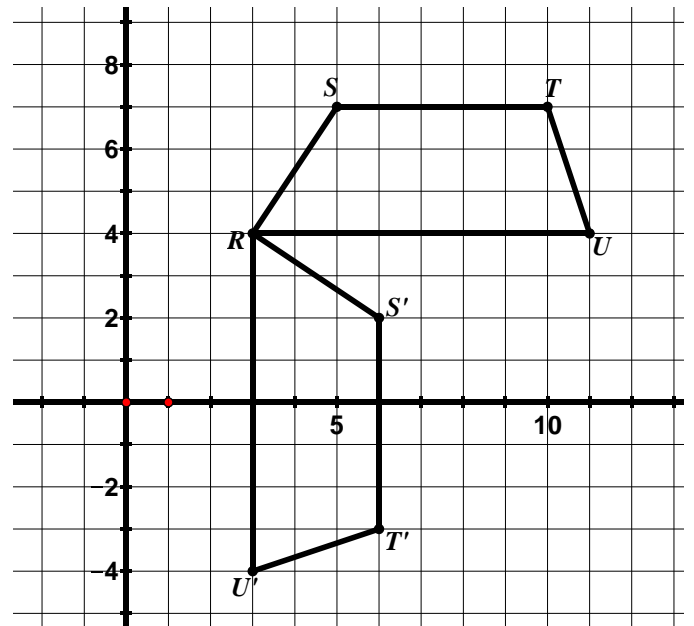
3. Rotate $\triangle DEF$ 90° clockwise about $(2, 1)$ and label the image.



Directions: For #4 – 6, find the angle of rotation (including the direction) and the center of rotation.

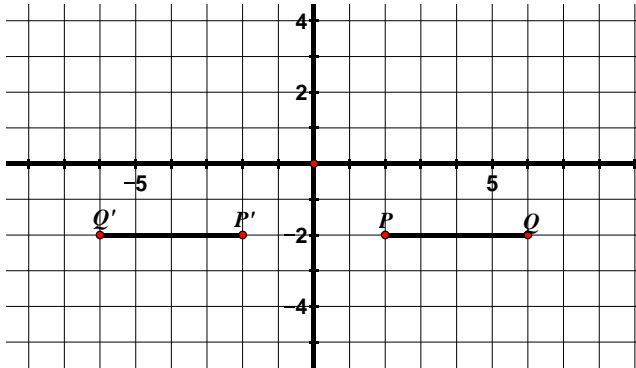
4. Angle of Rotation (including direction of rotation): _____

Center of Rotation: _____



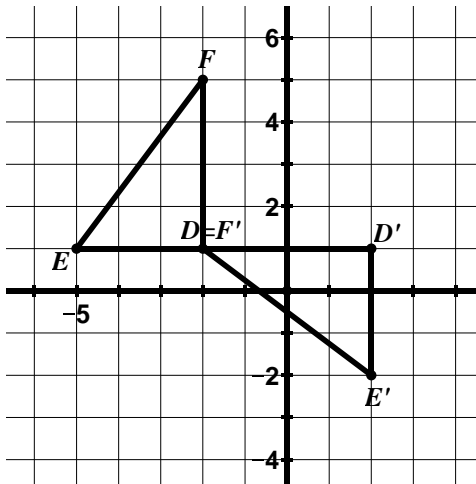
5. Angle of Rotation (including direction of rotation): _____

Center of Rotation: _____

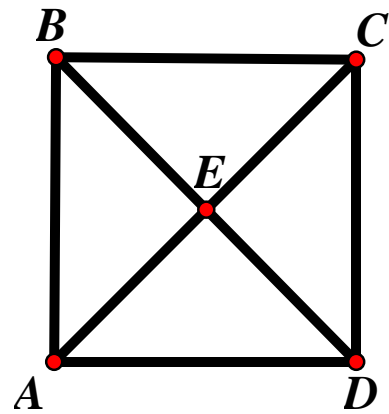


6. Angle of Rotation (including direction of rotation): _____

Center of Rotation: _____



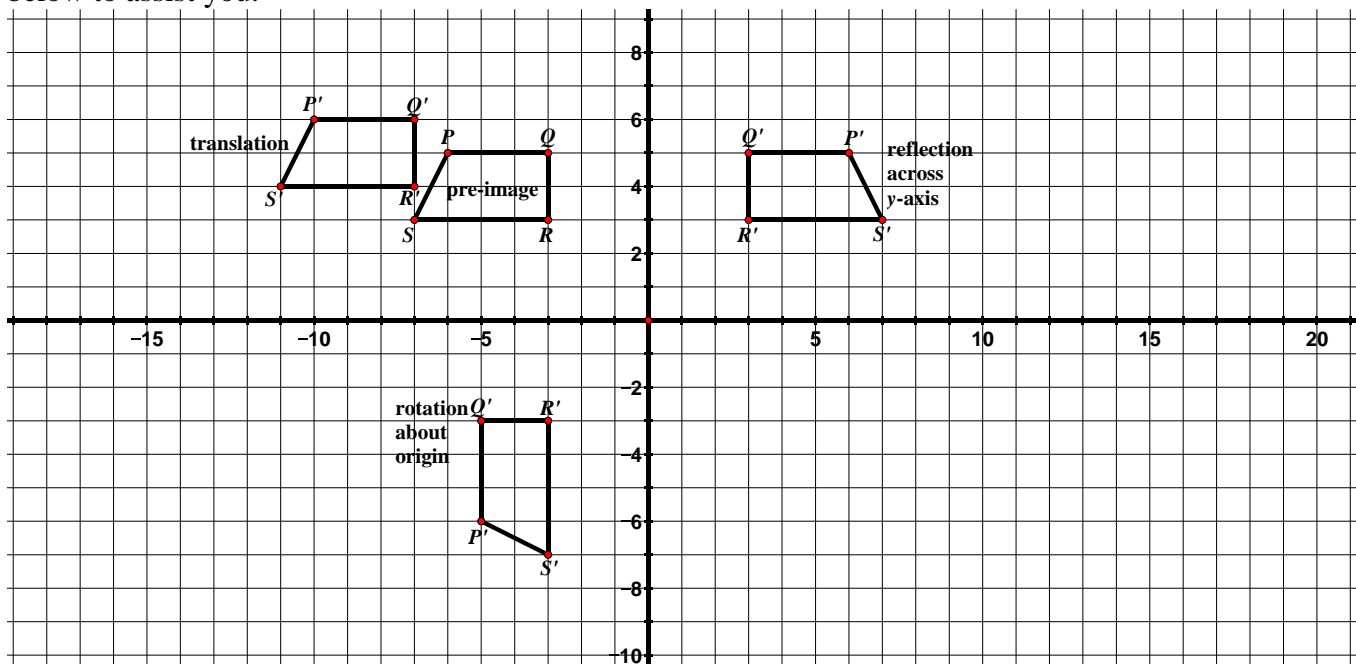
7. $ABCD$ is a square.
- What is the image of B under a 90° rotation counterclockwise about C ?
 - What is the image of B under a 180° rotation about E ?
 - Name three different rotations for which the image of A is C .



9.1e Class Activity: Congruence

The following phrases and words are properties or descriptions of one or more of the transformations we have studied so far: translation, reflection, and rotation. Determine which type of transformation(s) the statements describe and write your answer(s) on the line. An example of each type of transformation has been provided

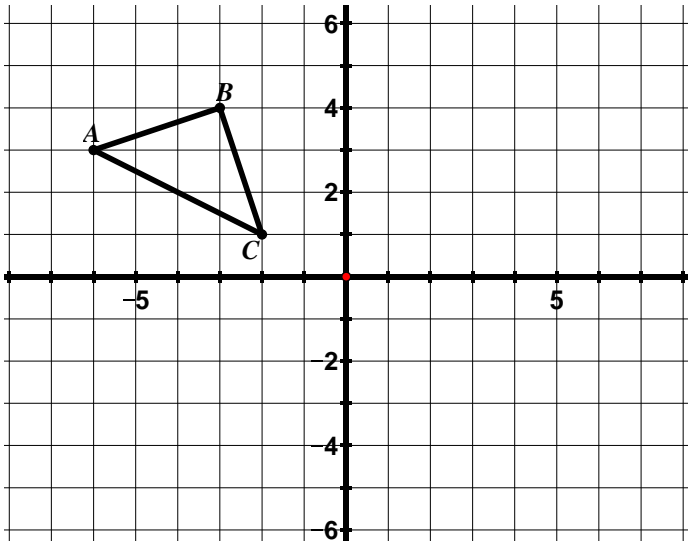
below to assist you. 



Property/Description	Type of Transformation(s)
Flip	
Slide	
Turn	
Image has the same orientation as pre-image.	
Specified by a figure, a center of rotation, and an angle of rotation.	
Specified by a figure and a line of reflection.	
Specified by a figure, a distance, and a direction.	
Segments connecting corresponding vertices of image and pre-image are the same length.	
Corresponding image and pre-image vertices lie on the same circle.	
Segments connecting corresponding vertices of image and pre-image are parallel to each other.	
Line of reflection is the perpendicular bisector of all segments connecting corresponding vertices of the image and pre-image.	
Concentric circles	
Orientation of the figure does not change.	
The slopes of corresponding segments may change.	
Corresponding segments in the image and pre-image are the same length.	
Corresponding angles in the image and pre-image have the same measure.	
Parallel lines in the pre-image remain parallel lines in the image.	

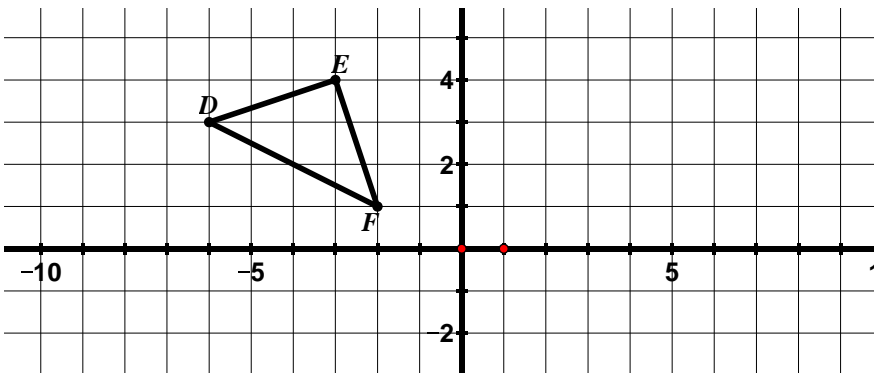
In the examples we have studied so far, we have only performed one transformation on a figure. We can also perform more than one transformation on a figure. In the following problems, you will perform a **sequence of transformations** on a figure.

1. $\triangle ABC$ has been plotted below.



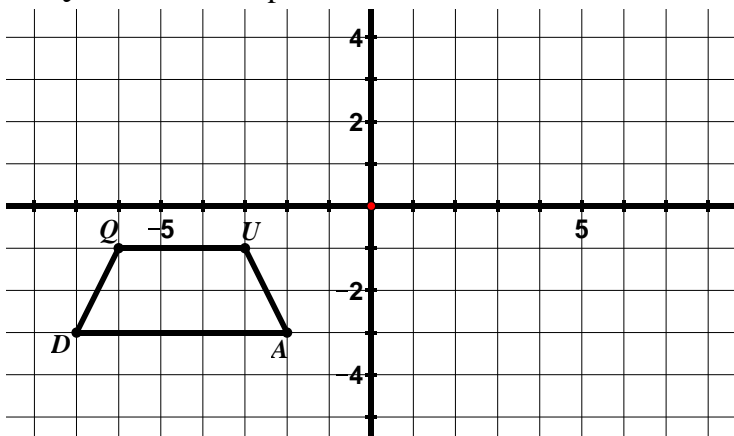
- Reflect $\triangle ABC$ over the y -axis and label the image $\triangle A'B'C'$.
- Reflect $\triangle A'B'C'$ over the x -axis and label the image $\triangle A''B''C''$.
- What **one** transformation is the same as this double reflection?

2. $\triangle DEF$ has been plotted below.



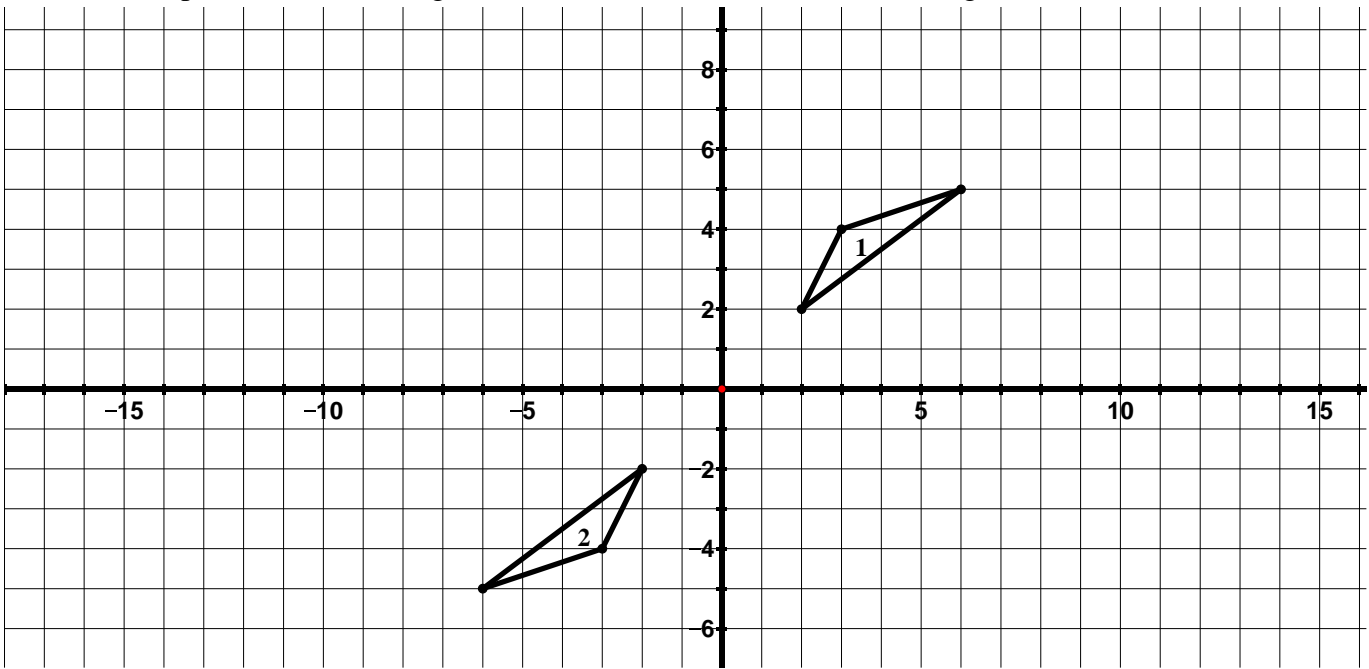
- Reflect $\triangle DEF$ over the line $x = 1$ and label the image $\triangle D'E'F'$.
- Reflect $\triangle D'E'F'$ over the y -axis and label the image $\triangle D''E''F''$.
- What **one** transformation is the same as this double reflection?
- Write a coordinate rule for the transformation of $\triangle DEF$ to $\triangle D''E''F''$.

3. $QUAD$ has been plotted below.



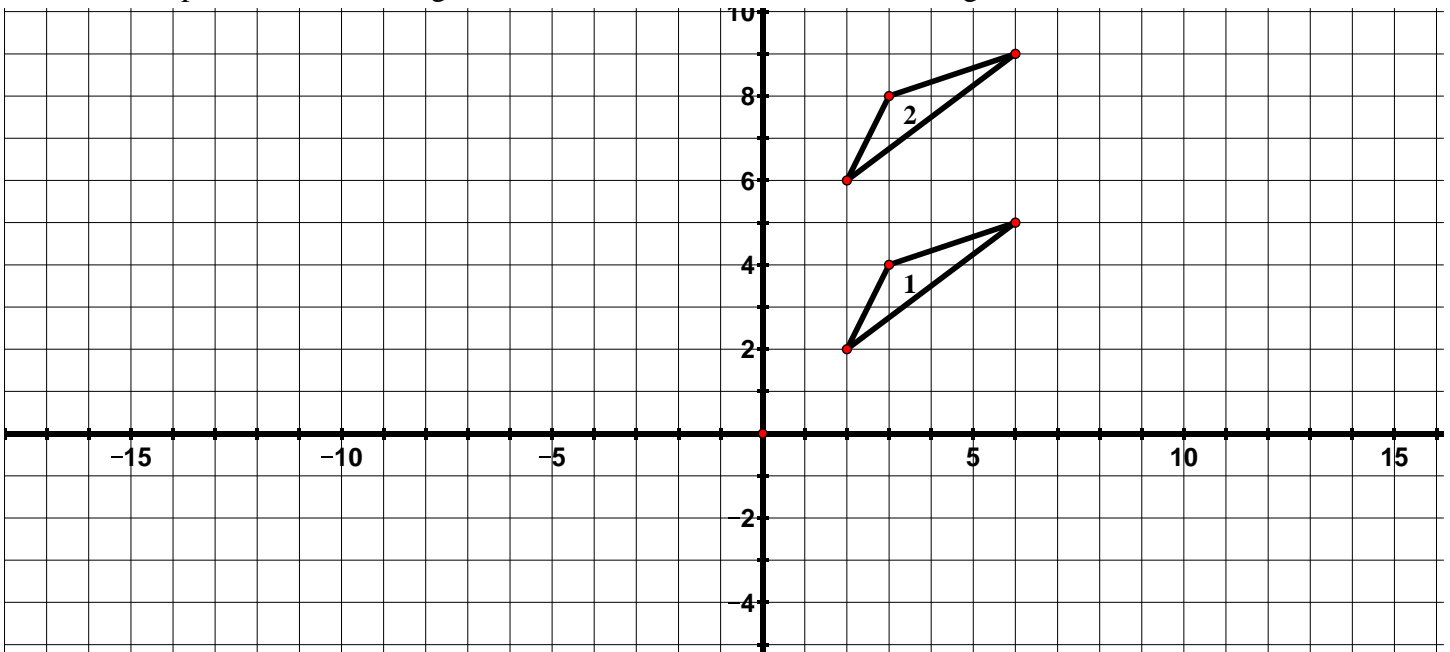
- Reflect $QUAD$ over the x -axis and label the image $Q'U'A'D'$.
- Translate $Q'U'A'D'$ according to the rule $(x, y) \rightarrow (x + 9, y)$ label the image $Q''U''A''D''$.

4. In the picture below, triangle 1 has been transformed to obtain triangle 2.



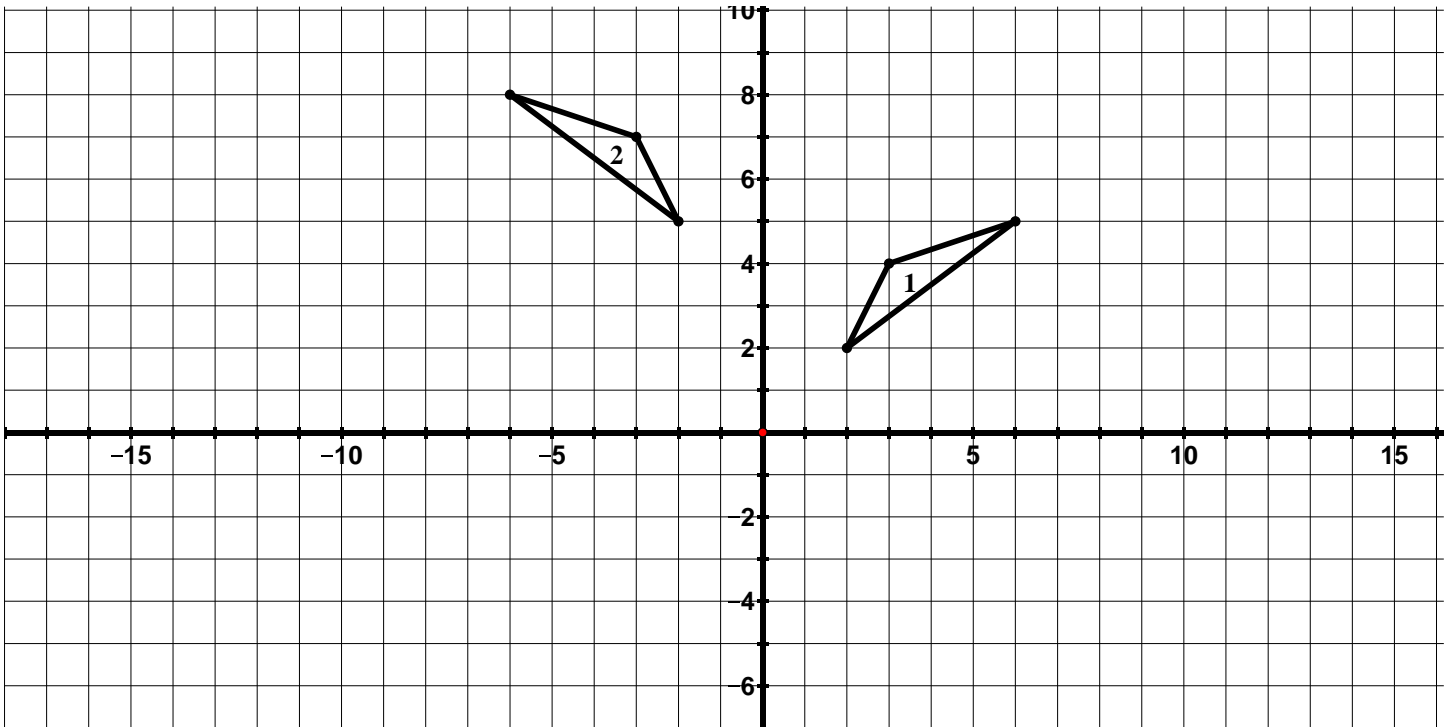
- a. Which **two** transformations in succession would carry triangle 1 onto triangle 2?
- b. Which **one** transformation would carry triangle 1 onto triangle 2?

5. In the picture below, triangle 1 has been transformed to obtain triangle 2.

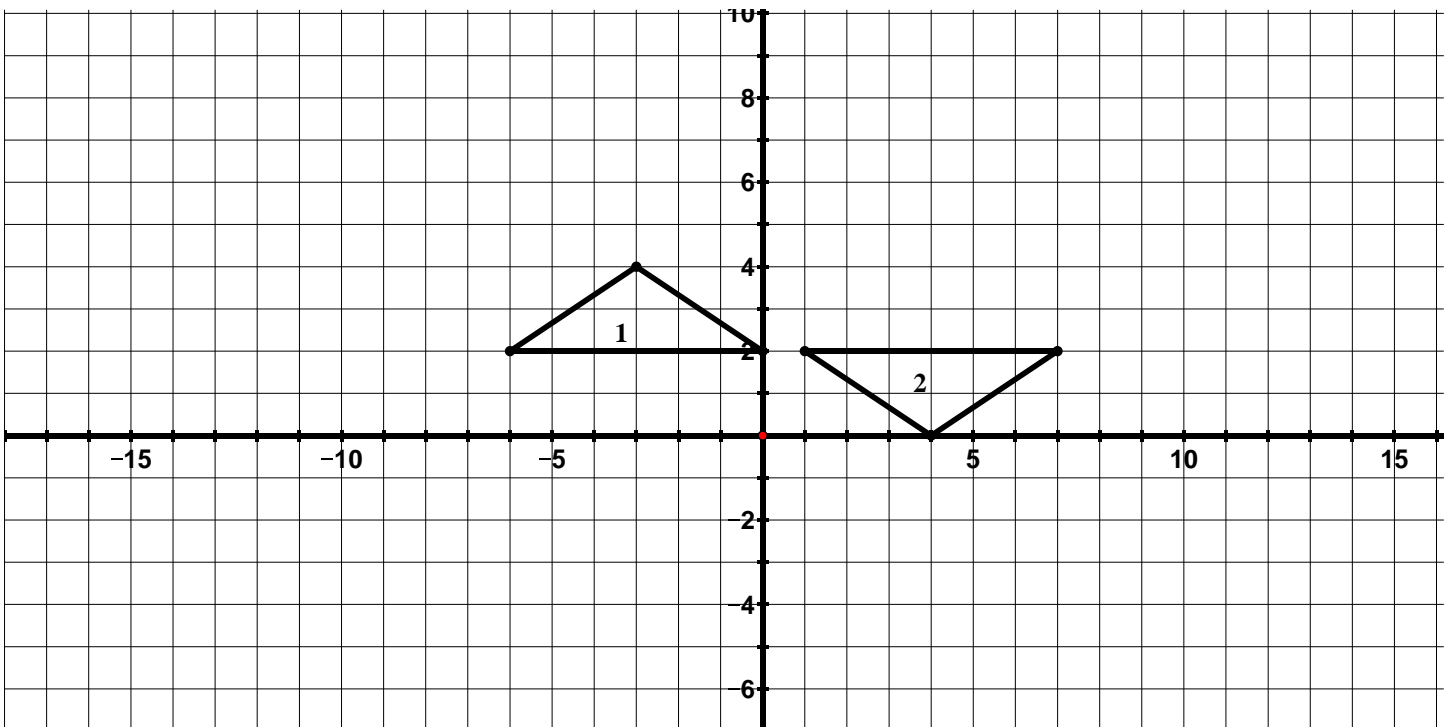


- a. Which **two** transformations in succession would carry triangle 1 onto triangle 2?
- b. Which **one** transformation would carry triangle 1 onto triangle 2?

6. Describe a transformation or sequence of transformations that would carry triangle 1 onto triangle 2.

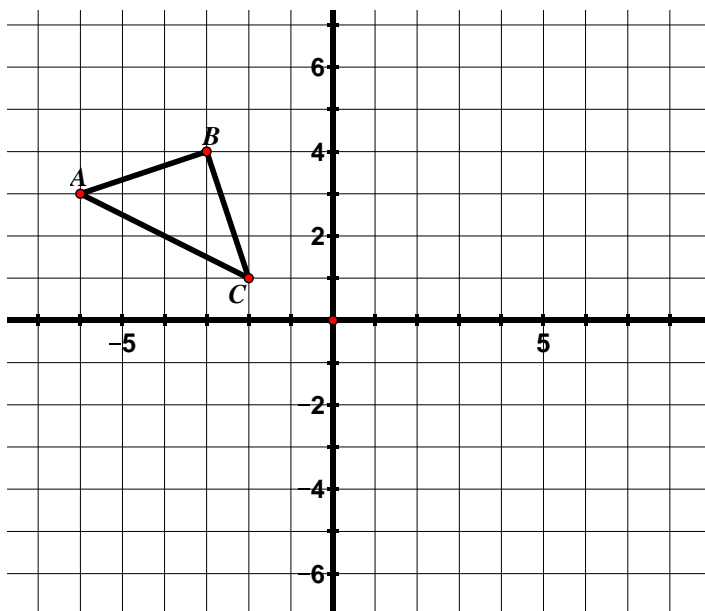



7. Describe a transformation or sequence of transformations that would carry triangle 1 onto triangle 2.



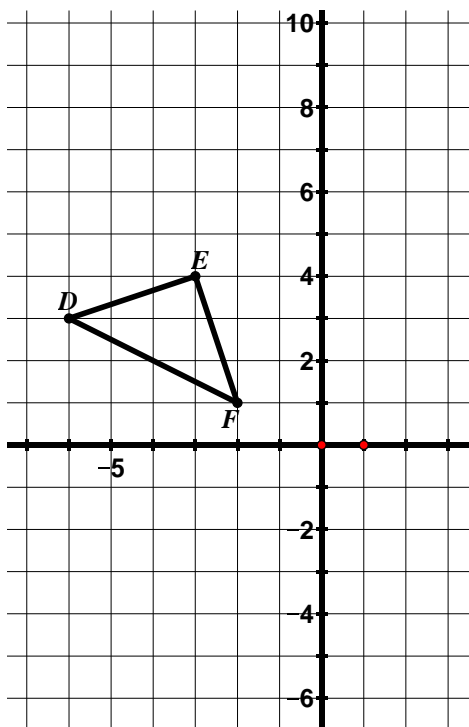
9.1e Homework: Congruence


1. $\triangle ABC$ has been plotted below.



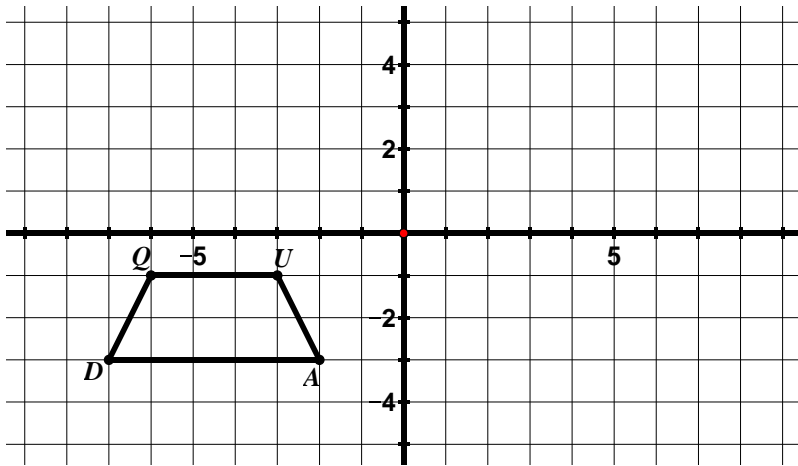
- Reflect $\triangle ABC$ over the x -axis and label the image $A'B'C'$.
- Reflect $\triangle A'B'C'$ over the y -axis and label the image $A''B''C''$.
- What **one** transformation is the same as this double reflection?
- In #1 of your class work, we performed a similar series of transformation; however we reflected over the y -axis first and then the x -axis. Compare these transformations. 


2. $\triangle DEF$ has been plotted below.



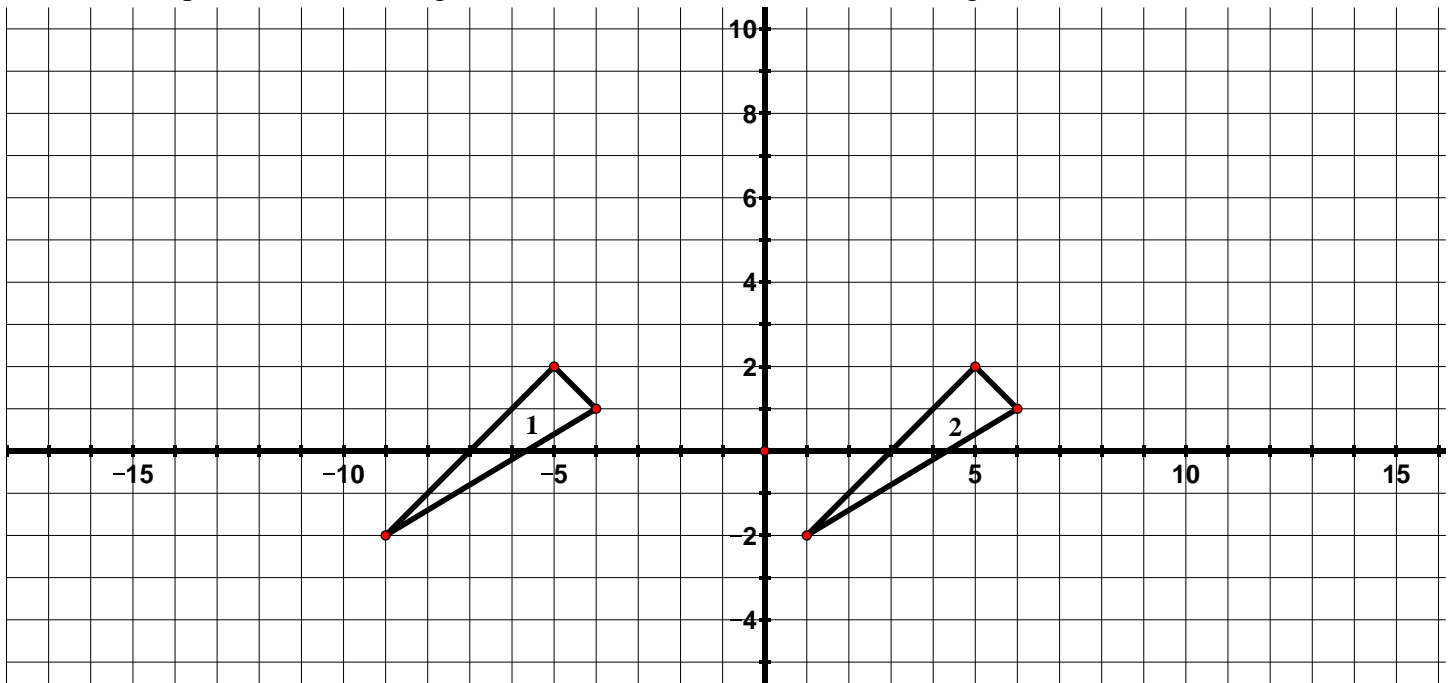
- Reflect $\triangle DEF$ over the line $y = -1$ and label the image $\triangle D'E'F'$.
- Reflect $\triangle D'E'F'$ over the line $y = 2$ and label the image $\triangle D''E''F''$.
- What **one** transformation is the same as this double reflection?
- Write a coordinate rule for the transformation of $\triangle DEF$ to $\triangle D''E''F''$.
- In #2 of your class work, we performed a similar series of transformation; however we reflected over vertical lines. Compare these transformations and write your observations below. 

3. *QUAD* has been plotted below.



- Translate *QUAD* according to the rule $(x, y) \rightarrow (x + 9, y)$.
- Reflect $Q'U'A'D'$ over the x -axis and label the image $Q''U''A''D''$.
- In #3 of your class work, we performed a similar series of transformation; however we did the reflection first and then the translation. Compare these transformations and write your observations below. 

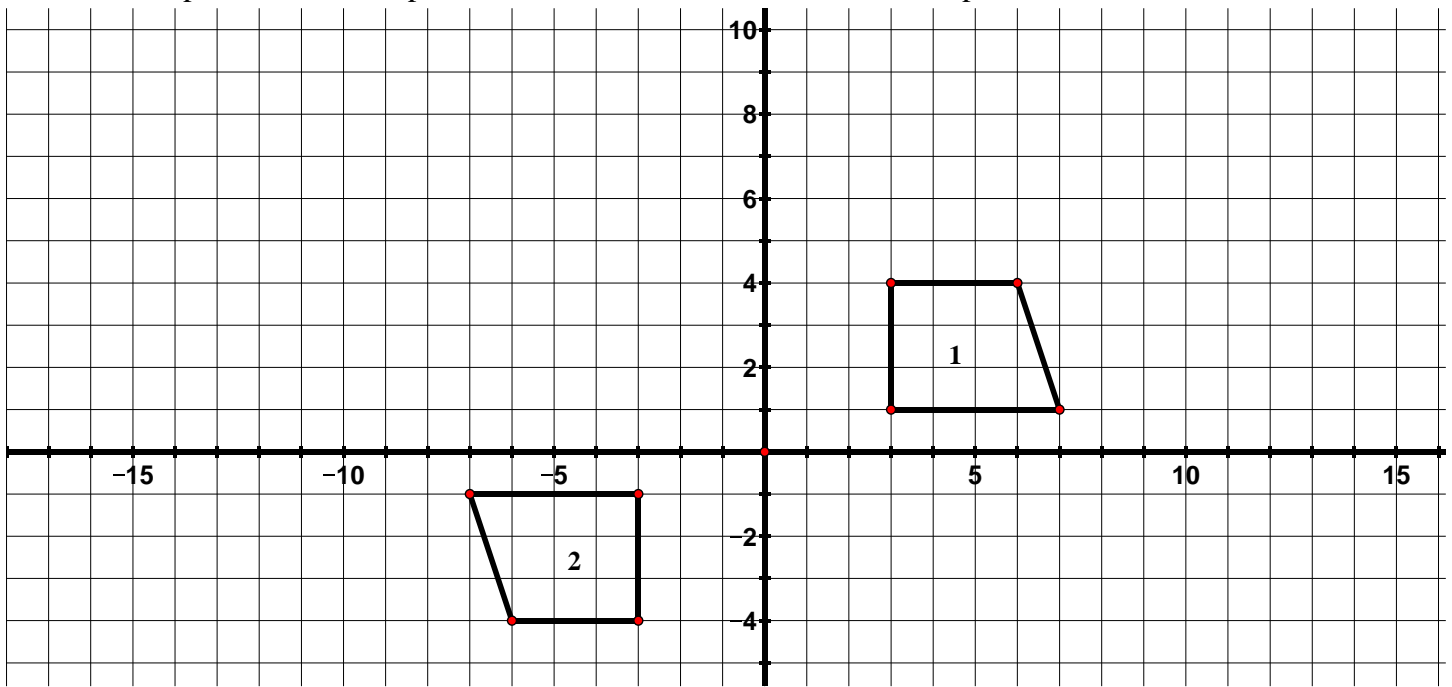
4. In the picture below, triangle 1 has been transformed to obtain triangle 2.



a. Which **two** transformations in succession would carry triangle 1 onto triangle 2?

b. Which **one** transformation would carry triangle 1 onto triangle 2?

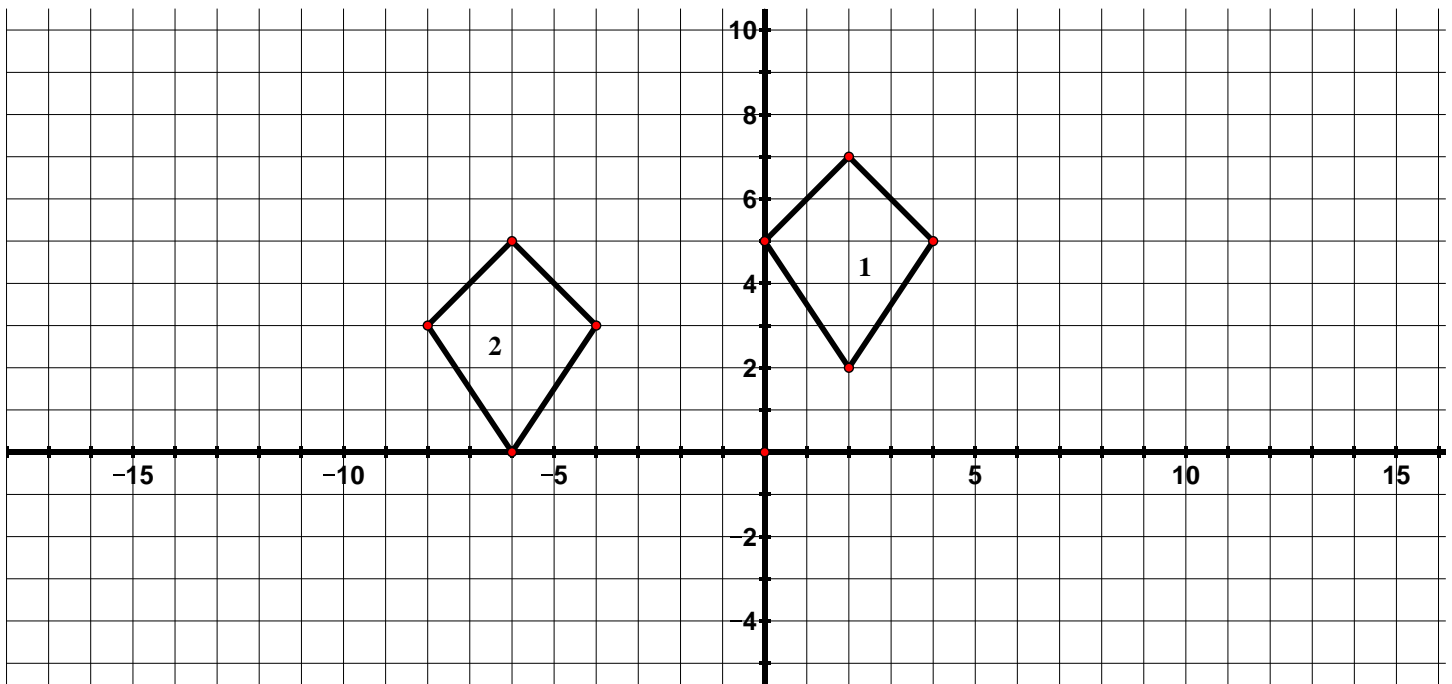
5. In the picture below, trapezoid 1 has been transformed to obtain trapezoid 2.



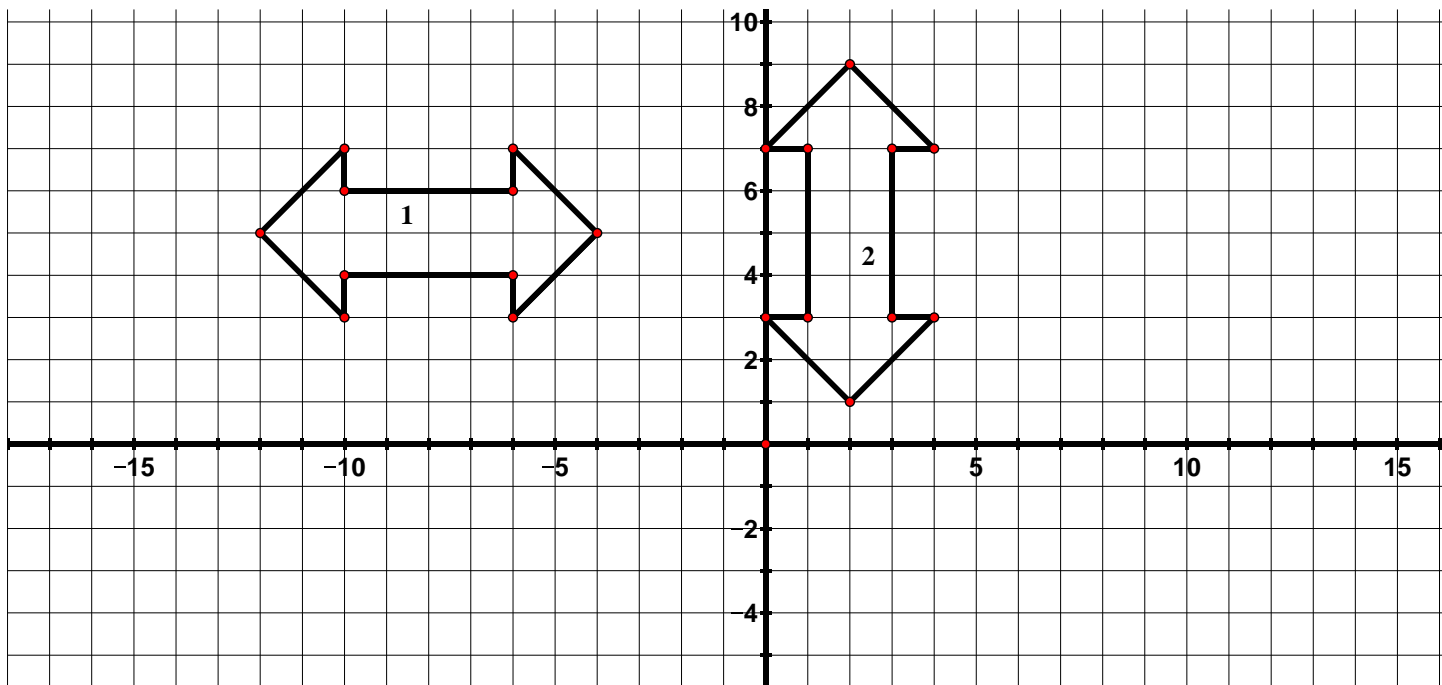
a. Which **two** transformations in succession would carry trapezoid 1 onto trapezoid 2?

b. Which **one** transformation would carry trapezoid 1 onto trapezoid 2?

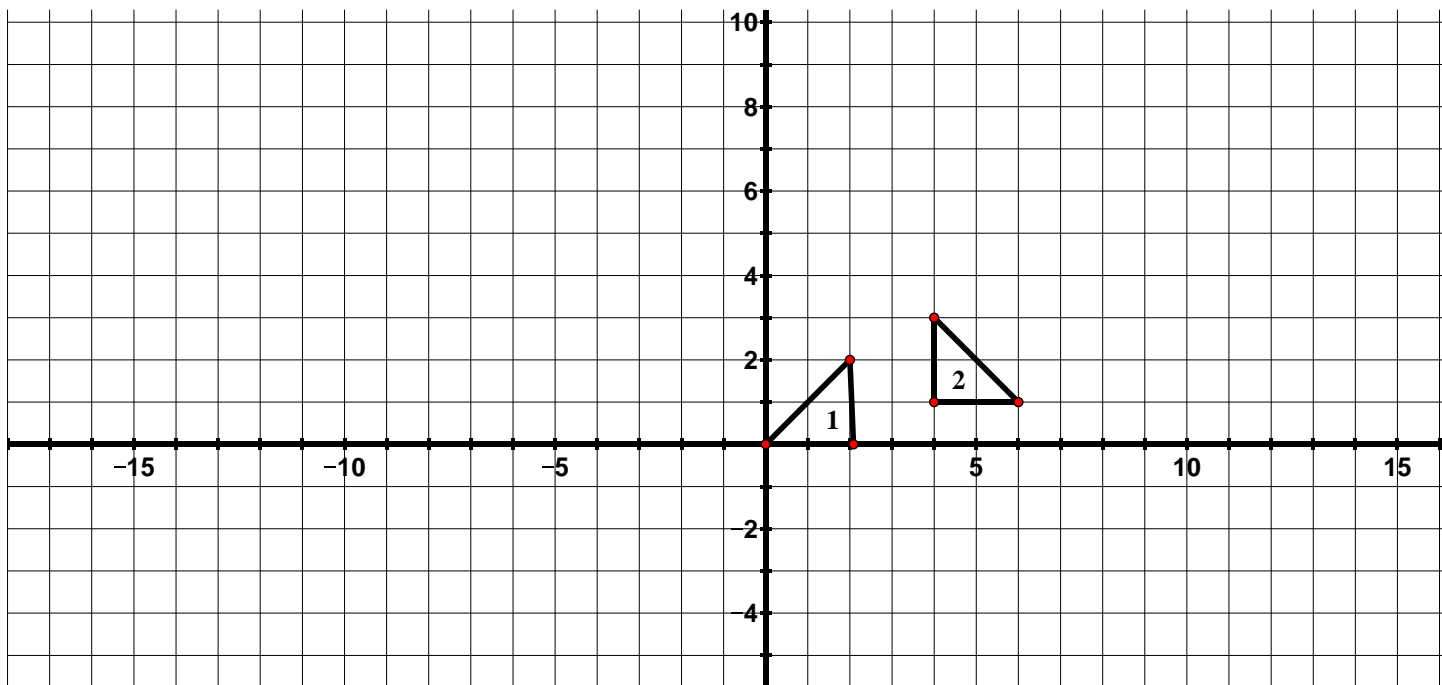
6. Describe a transformation or sequence of transformations that would carry figure 1 onto figure 2.



7. Describe a transformation or sequence of transformations that would carry figure 1 onto figure 2.

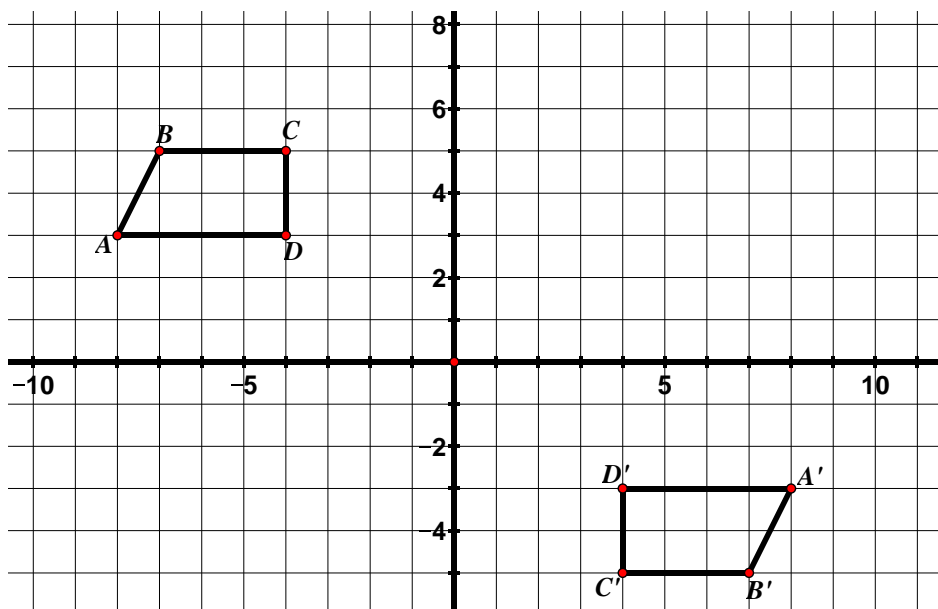


8. Describe a transformation or sequence of transformations that would carry triangle 1 onto triangle 2.



9.1f Class Activity: Congruence cont.

1. Observe the two figures below.



a. Describe the ways in which the figures are the same and the ways in which they are different.

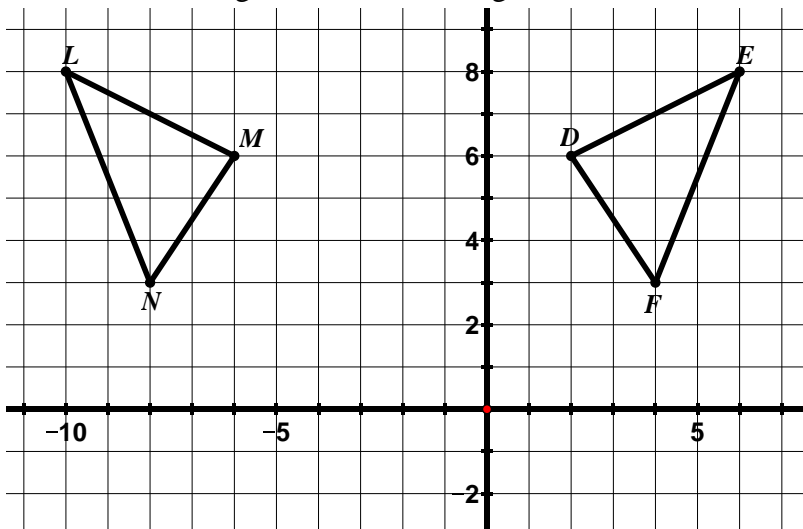
The two figures above are said to be **congruent**. In 7th grade, you learned that two figures are congruent if they have the **same shape** and are the **same size**. In 8th grade, we define congruence in terms of transformations. A two-dimensional figure is congruent to another if the second can be obtained from the first by a rigid motion (rotation, reflection, or translation) or a sequence of rigid motions. If you can move one of the figures using one of these transformations or a series of these transformations so that it fits exactly on the other one, the two figures are congruent.

b. In this case, there are several different transformations that will carry one figure onto the other. Describe one transformation (or sequence of transformations) that will carry $ABCD$ onto $A'B'C'D'$.

c. Can you think of a different transformation (or sequence of transformations) that will carry $ABCD$ onto $A'B'C'D'$?

d. A translation, reflection, and rotation are described as rigid motions. Describe in your own words what this means.

2. The two figures below are congruent.



a. Describe the transformation or sequence of transformations that will carry $\triangle LMN$ onto $\triangle EDF$.

b. Congruent figures have corresponding parts – their matching sides and angles. For example, in the figure above, \overline{LM} corresponds to \overline{ED} and $\angle D$ corresponds to $\angle M$. List the other corresponding parts below.

\overline{LN} corresponds to _____

$\angle E$ corresponds to _____

\overline{MN} corresponds to _____

$\angle F$ corresponds to _____

We can write a congruence statement for the two triangles. You can denote that two figures are congruent by using the symbol \cong and listing their vertices in corresponding order.

In the example above, we would write this symbolically as $\triangle LMN \cong \triangle EDF$. The order the vertices is written tells us which segments and angles are corresponding in the figures.

Corresponding parts of congruent figures are congruent (corresponding segments have the same length and corresponding angles have the same measure). We can show this symbolically in the following way:

$$\overline{LM} \cong \overline{ED}$$

$$\angle D \cong \angle M$$

$$\overline{LN} \cong \overline{EF}$$

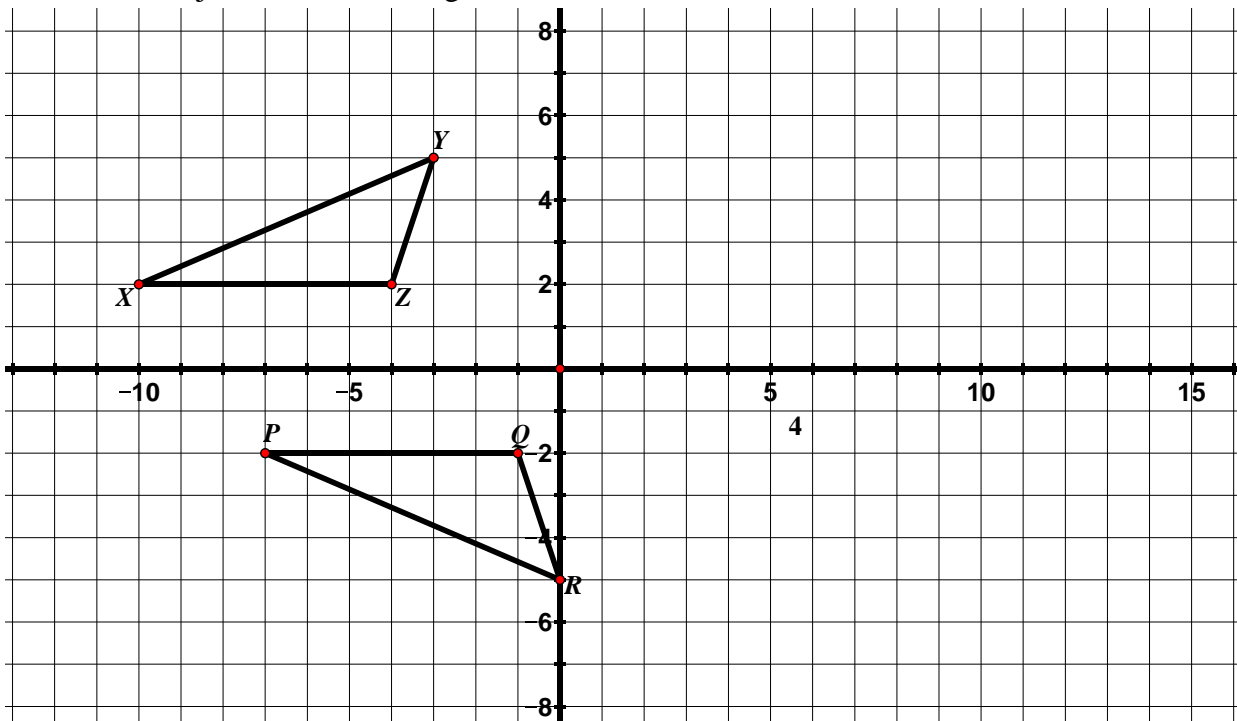
$$\angle E \cong \angle L$$

$$\overline{MN} \cong \overline{DF}$$


$$\angle F \cong \angle N$$

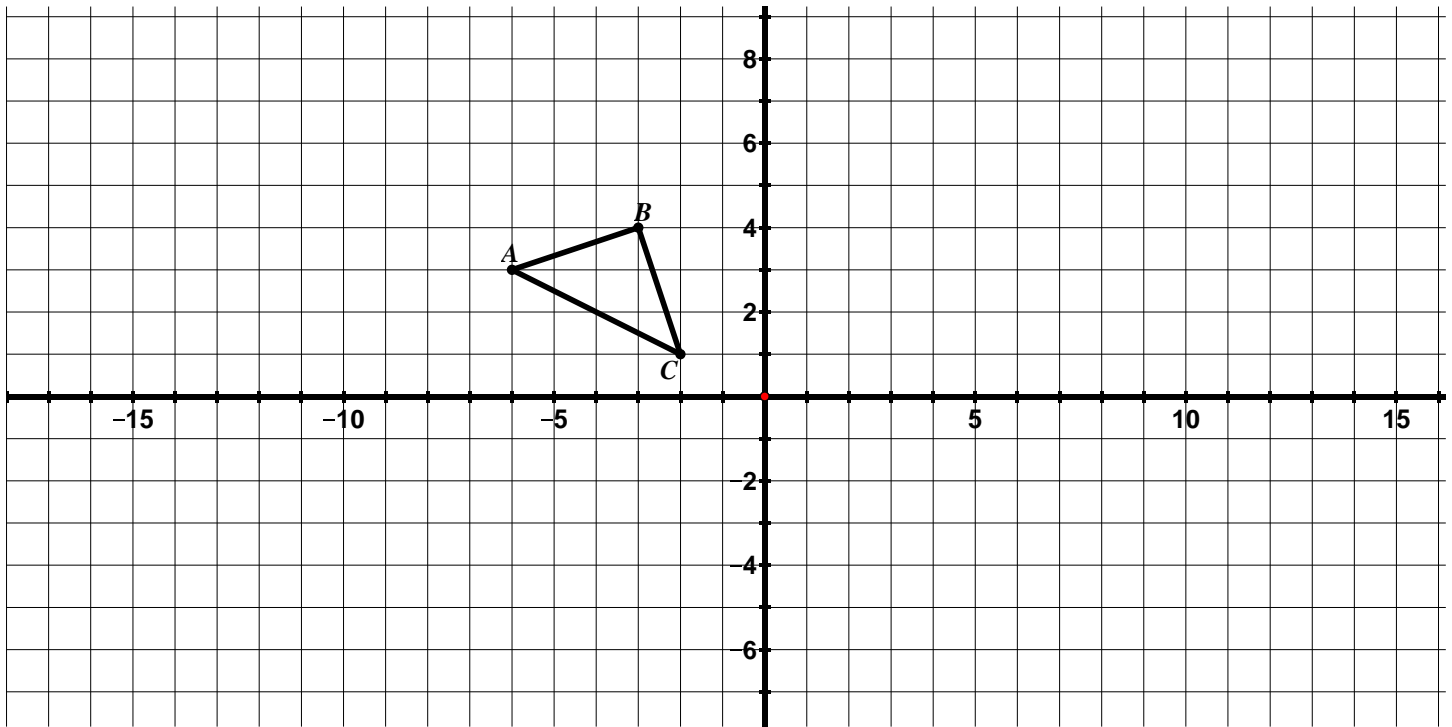
We can also annotate the diagram to show which parts are congruent. Do this on the diagram above.

3. The two objects below are congruent.



- Describe the transformation or sequence of transformations that will carry $\triangle XYZ$ onto $\triangle PRQ$.
- List the congruent corresponding parts.
- Write a congruence statement for the triangles.
- Annotate the diagram to show which parts are congruent.

4. Using $\triangle ABC$ in the diagram below as the pre-image, apply the following rules to $\triangle ABC$ and determine whether the resulting image is congruent to $\triangle ABC$. Always start with $\triangle ABC$ as your pre-image. 



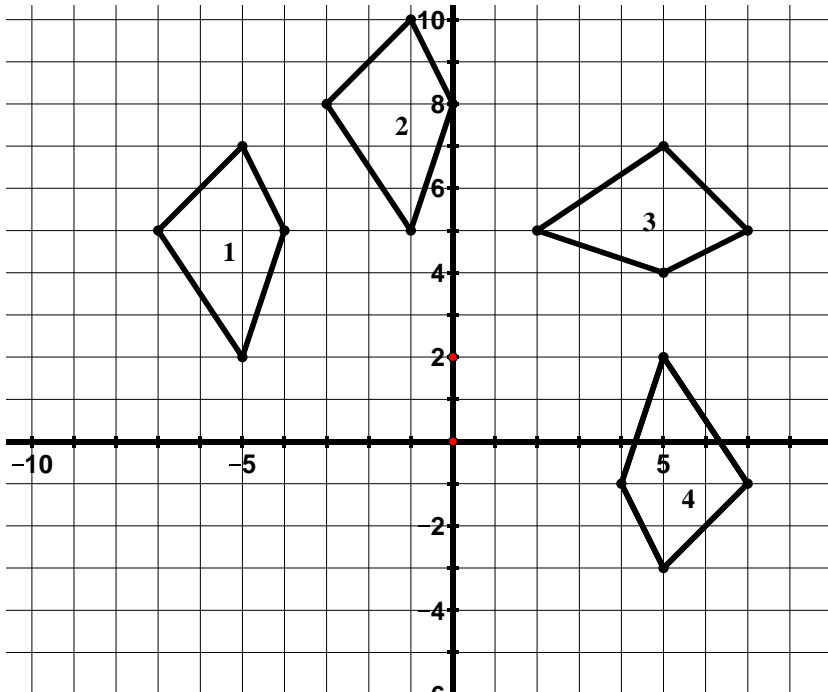
- $(x, y) \rightarrow (x, y + 7)$
 Is the resulting image congruent to $\triangle ABC$? Why or why not?
- $(x, y) \rightarrow (-x, y)$
 Is the resulting image congruent to $\triangle ABC$? Why or why not?
- $(x, y) \rightarrow (x, 2y)$
 Is the resulting image congruent to $\triangle ABC$? Why or why not?
- $(x, y) \rightarrow (2x, 2y)$
 Is the resulting image congruent to $\triangle ABC$? Why or why not?
- Write your own coordinate rule that is different than the ones above that would result in an image that **is congruent** to $\triangle ABC$. How do you know that the resulting image is congruent to $\triangle ABC$?
- Write your own coordinate rule that is different than the ones above that would result in an image that **is not congruent** to $\triangle ABC$. How do you know that the resulting image is not congruent to $\triangle ABC$?




5. Which of the following properties of a figure can change during a rigid motion? Explain.
- a. Interior angles
 - b. Slope of a side
 - c. Parallel lines in the pre-image
 - d. Orientation
 - e. Side lengths
 - f. Location in the plane
 - g. Perimeter
 - h. Area

9.1f Homework: Congruence cont.

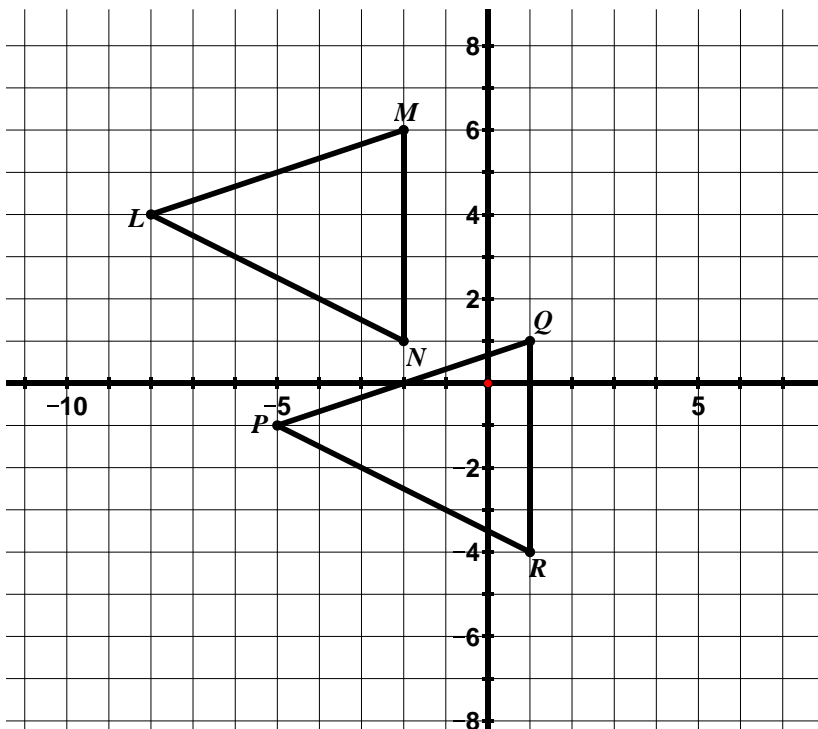
1. Jeff's teacher asked him to create 3 figures that were congruent to figure 1 in the picture below. Jeff created figures 2, 3, and 4.



- a. Use the definition of congruence to determine if Jeff's figures are congruent to figure 1. Explain your answers. 

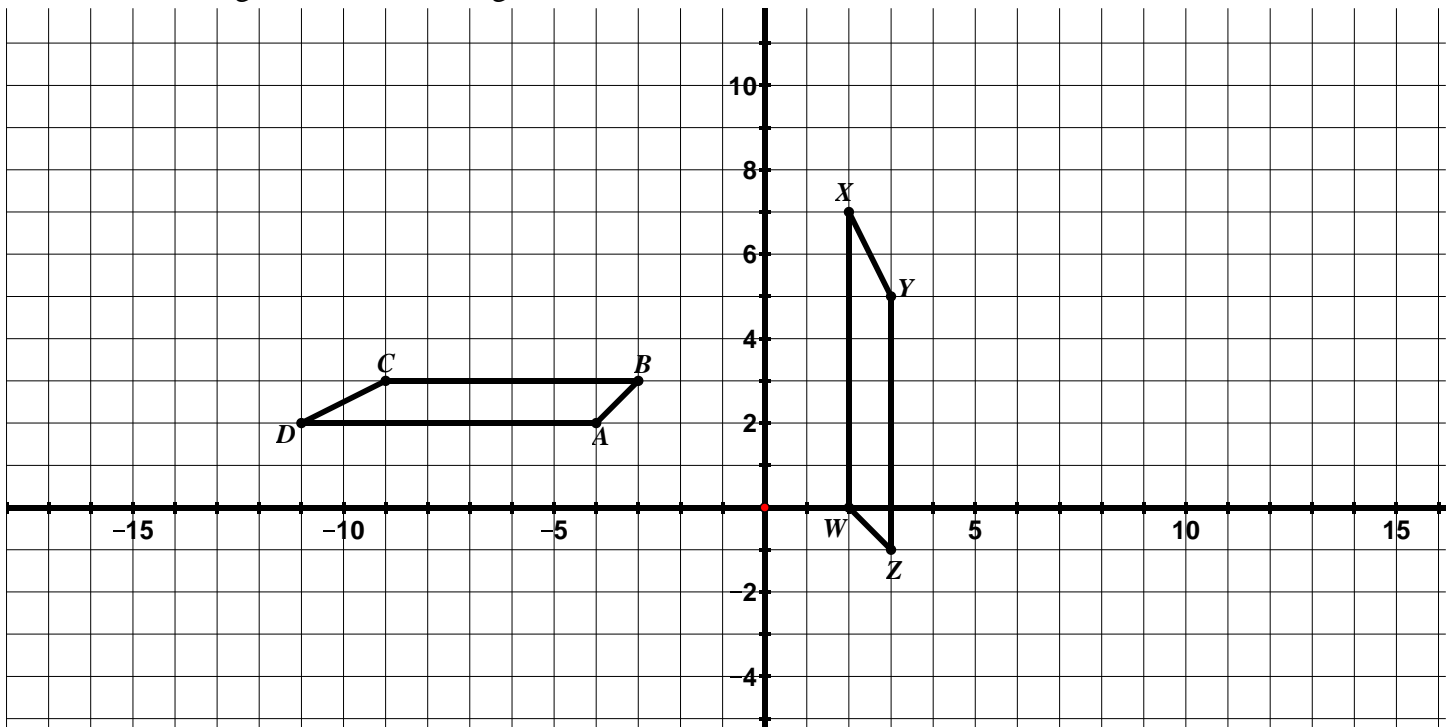
- b. Draw an additional figure that is congruent to figure 1. How do you know your figure is congruent to figure 1?

2. The two figures below are congruent.



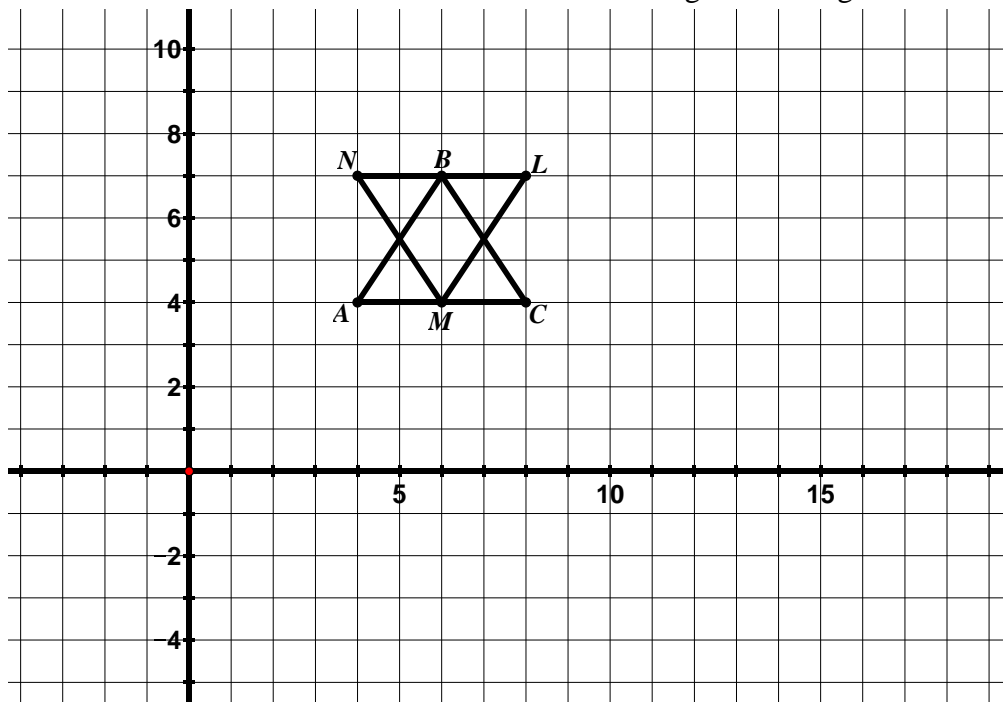
- a. Describe the transformation or sequence of transformations that will carry $\triangle LMN$ onto $\triangle PQR$.
- b. List the congruent corresponding parts.
- c. Write a congruence statement for the triangles.


3. The two figures below are congruent.



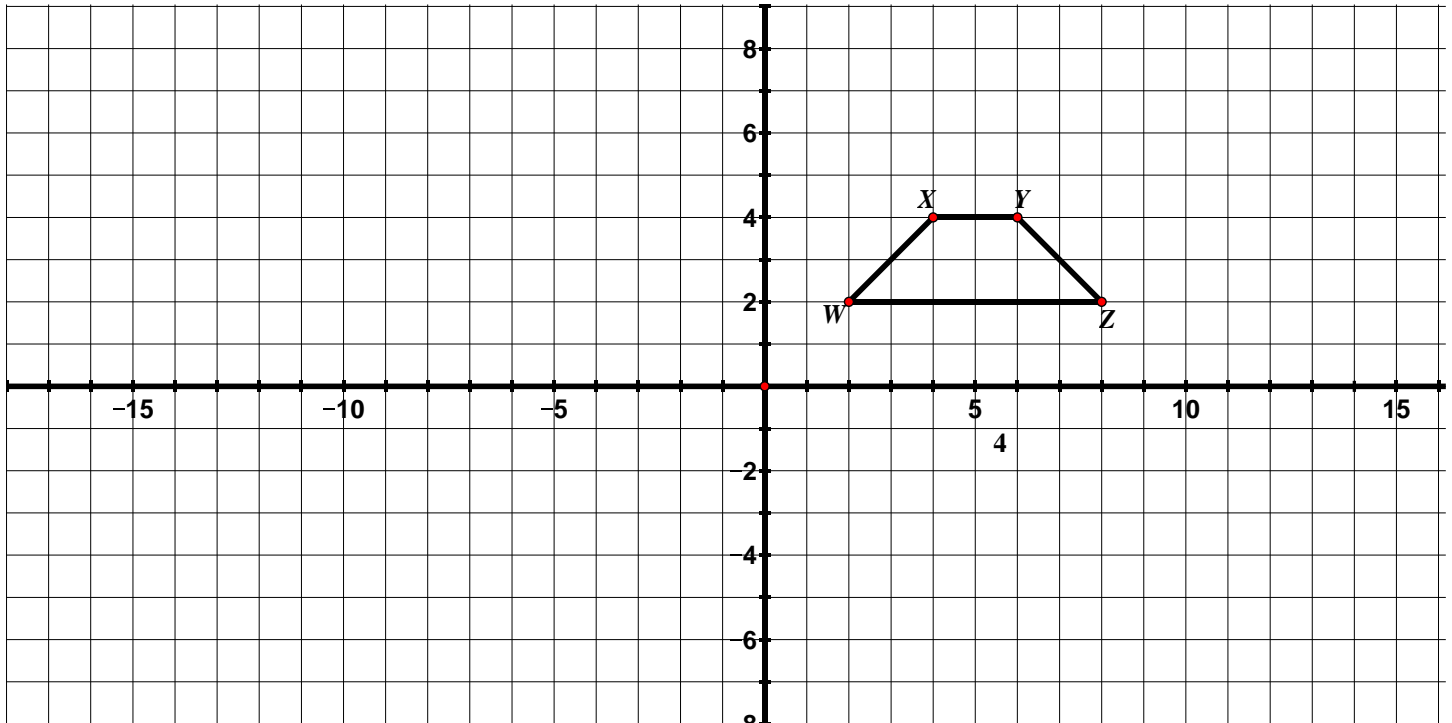
- a. Describe the transformation or sequence of transformations that will carry $ABCD$ onto $WXYZ$.
- b. Write a congruence statement for the parallelograms.

4. Consider $\triangle ABC$ and $\triangle LMN$ below. The two triangles are congruent.



- a. Prove that $\triangle ABC \cong \triangle LMN$. 

5. Using $WXYZ$ in the diagram below as the pre-image, apply the following rules to $WXYZ$ and determine whether the resulting image is congruent to $WXYZ$. Always start with $WXYZ$ as your pre-image.



- $(x, y) \rightarrow (x - 2, y + 1)$
 Is the resulting image congruent to $WXYZ$? Why or why not?
- $(x, y) \rightarrow (y, x)$
 Is the resulting image congruent to $WXYZ$? Why or why not?
- $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$
 Is the resulting image congruent to $WXYZ$? Why or why not?
- Write your own coordinate rule that is different than the ones above that would result in an image that **is congruent** to $WXYZ$. How do you know that the resulting image is congruent to $WXYZ$?
- Write your own coordinate rule that is different than the ones above that would result in an image that **is not congruent** to $WXYZ$. How do you know that the resulting image is not congruent to $WXYZ$?

9.1g Self-Assessment: Section 9.1

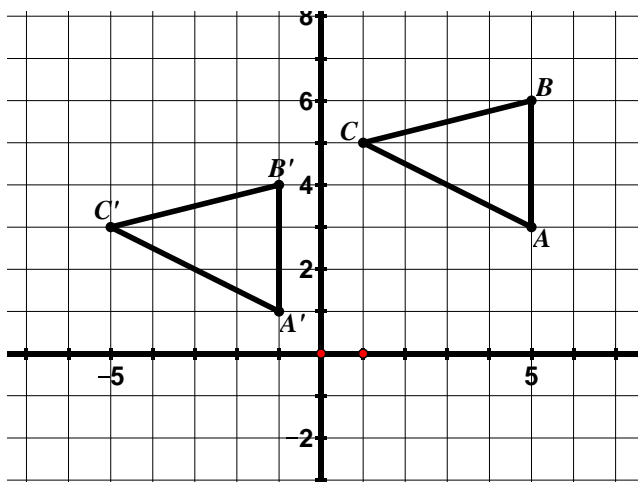
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Given a pre-image and its image under a translation, describe the translation in words and using a coordinate rule.				
2. Perform a translation of a figure given a coordinate rule.				
3. Describe the properties of a translation and the effects a translation has on a figure and use this knowledge to solve problems.				
4. Given a pre-image and its image under a reflection, describe the reflection in words and using a coordinate rule.				
5. Perform a reflection of a figure given a line of reflection.				
6. Describe the properties of a reflection and the effects a reflection has on a figure and use this knowledge to solve problems.				
7. Find a reflection line for a given reflection and write the equation of the reflection line.				
8. Given a pre-image and its image under a rotation, describe the rotation in words and using a				

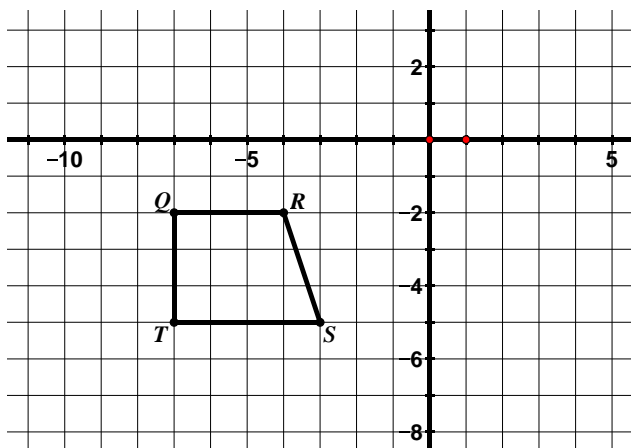
coordinate rule (coordinate rule for rotations centered at the origin only).				
9. Perform a rotation of a figure given a center of rotation, an angle of rotation, and a direction.				
10. Describe the properties of a rotation and the effects a rotation has on a figure and use this knowledge to solve problems.				
11. Connect ideas about slopes of perpendicular lines and rotations.				
12. Understand what it means for two figures to be congruent.				
13. Determine if two figures are congruent based on the definition of congruence.				
14. Given two figures that are congruent, describe the sequence of transformations that exhibits the congruence between them.				

Section 9.1 Sample Problems (For use with self-assessment)

1. Describe the following transformation in words and using a coordinate rule.

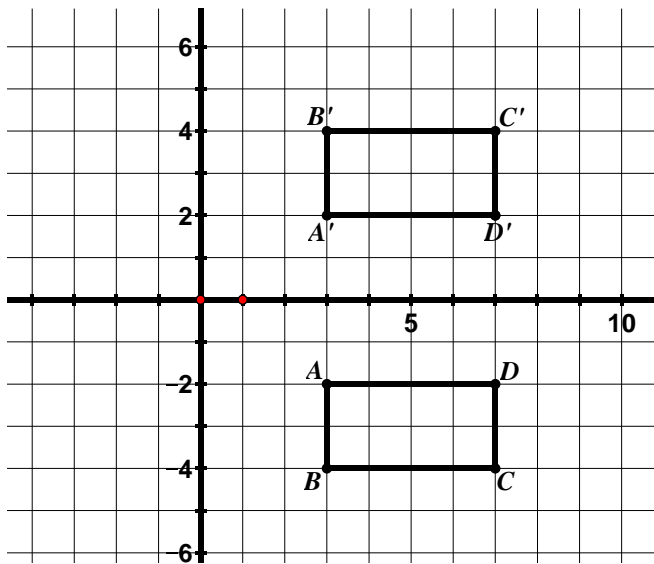


2. Translate $QRST$ according to the rule $(x, y) \rightarrow (x + 3, y - 1)$.

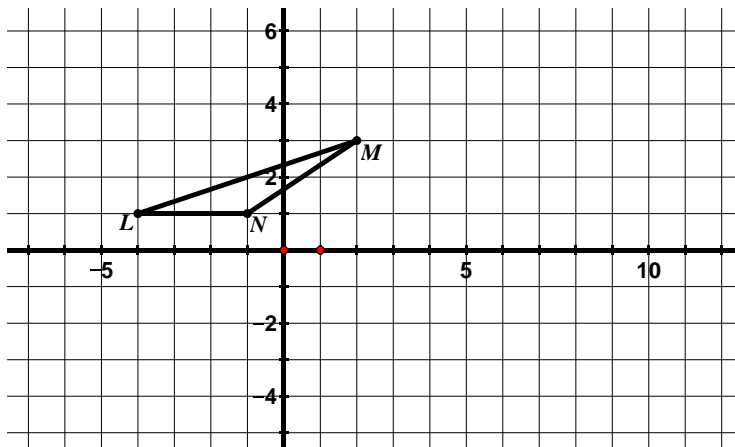


3. Which of the following properties are true about a figure that has been translated. Check all that apply.
- The slopes of corresponding segments are opposite reciprocals.
 - Segments connecting corresponding vertices of the image and pre-image are the same length.
 - The perimeter of the pre-image is smaller than the perimeter of the image.
 - Segments connecting corresponding vertices of the image and pre-image are parallel.
 - Corresponding vertices of the image and pre-image lie on the same circle.

4. Describe the following transformation in words and using a coordinate rule.

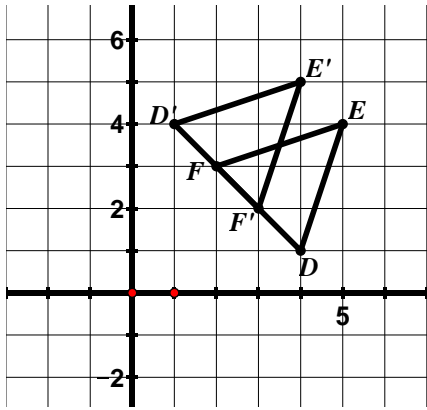


5. Reflect $\triangle LMN$ over the line $x = 2$.

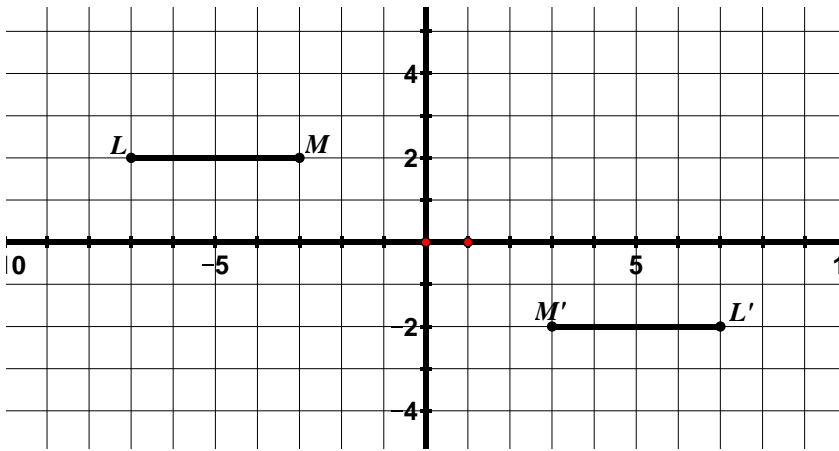


6. Which of the following properties are true about a figure that has been reflected. Check all that apply.
- The slopes of corresponding segments are the same.
 - Segments connecting corresponding vertices of the image and pre-image are the same length.
 - Segments connecting corresponding vertices of the image and pre-image are parallel.
 - The area of the pre-image is the same as the area of the image.
 - Corresponding vertices are equidistant from the line of reflection.
 - The measure of the interior angles of an object may change under a reflection.

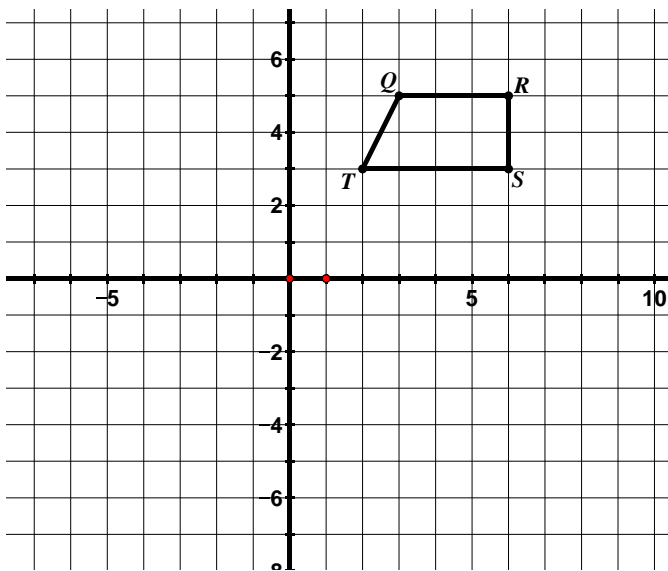
7. $\triangle EFD$ has been reflected to obtain $\triangle E'F'D'$. Write the equation for the line of reflection.



8. Describe the following rotation in words and using a coordinate rule.



9. Rotate $QRST$ 90° clockwise about the origin.



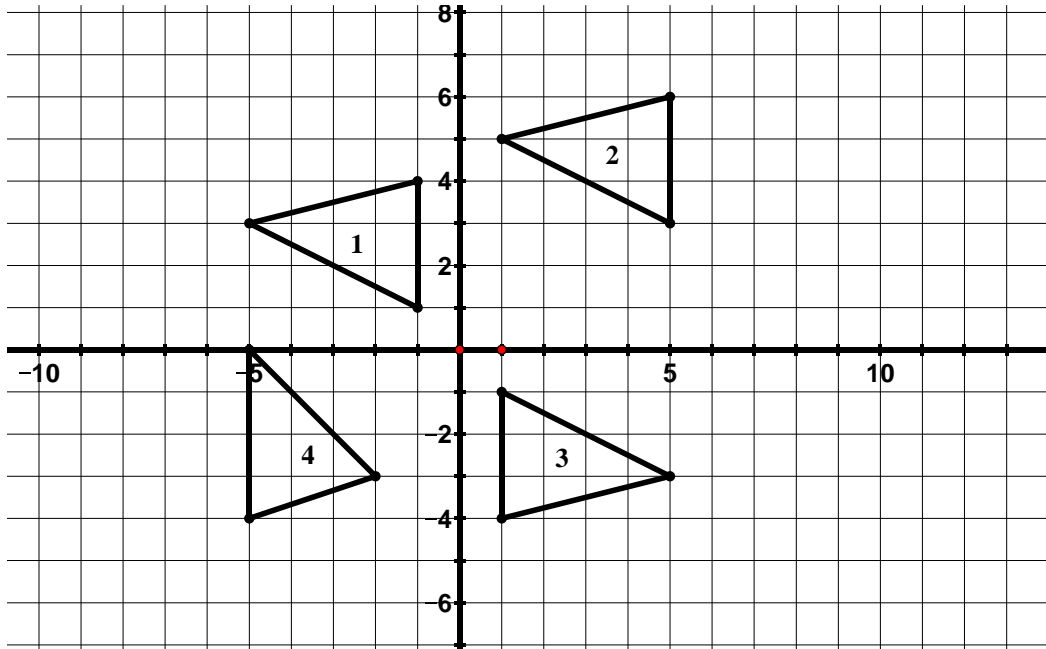
10. Which of the following properties are true about a figure that has been rotated 90° . Check all that apply.

- The slopes of corresponding segments are opposite reciprocals.
- Segments connecting corresponding vertices of the image and pre-image are the same length.
- Segments connecting corresponding vertices of the image and pre-image are parallel.
- Lines that are parallel in the pre-image are not necessarily parallel in the image.
- Corresponding segments are perpendicular.
- Corresponding vertices lie on the same circle.

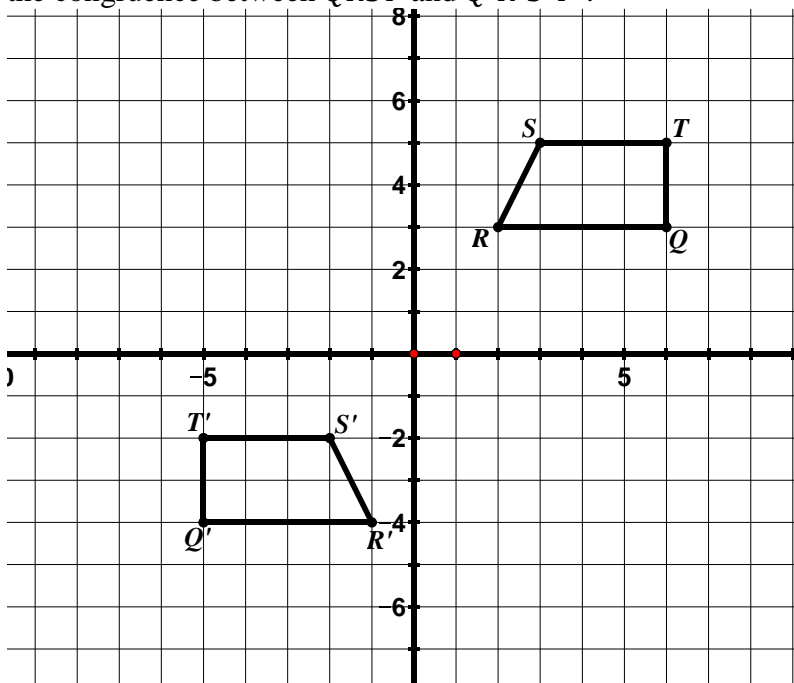
11. In a 90° rotation, what happens to the slopes of the corresponding segments of the image and pre-image? How does the coordinate rule show this? In a 180° rotation, what happens to the slopes of the corresponding segments of the image and pre-image? How does the coordinate rule show this?

12. In your own words, describe what it means for two figures to be congruent.

13. Which of the following triangles are congruent? Justify your answers.



14. $QRST$ is congruent to $Q'R'S'T'$. Describe a transformation or sequence of transformations that exhibits the congruence between $QRST$ and $Q'R'S'T'$.



Section 9.2 Dilations and Similarity

Section Overview:

Students start this section by applying different transformations (given by coordinate rules) to a figure to determine what these rules do to the shape and size of the figure. In this activity, they start to surface ideas about similarity. Students then begin to study dilations in detail. They perform dilations using the slope triangle method and scaling method when given a scale factor and center of dilation. Students determine the scale factor and center of dilation of two figures that have been dilated and write a coordinate rule to describe the dilation. Students continue to refer back to their work in order to describe the properties of dilations. Students will then apply this knowledge to determine if two figures are similar, understanding that a two-dimensional figure is similar to another if there is a sequence of rigid motions and dilations that takes one figure onto the other. In addition, students will be given two figures that are similar and asked to describe a sequence of transformations that exhibits the similarity between them.

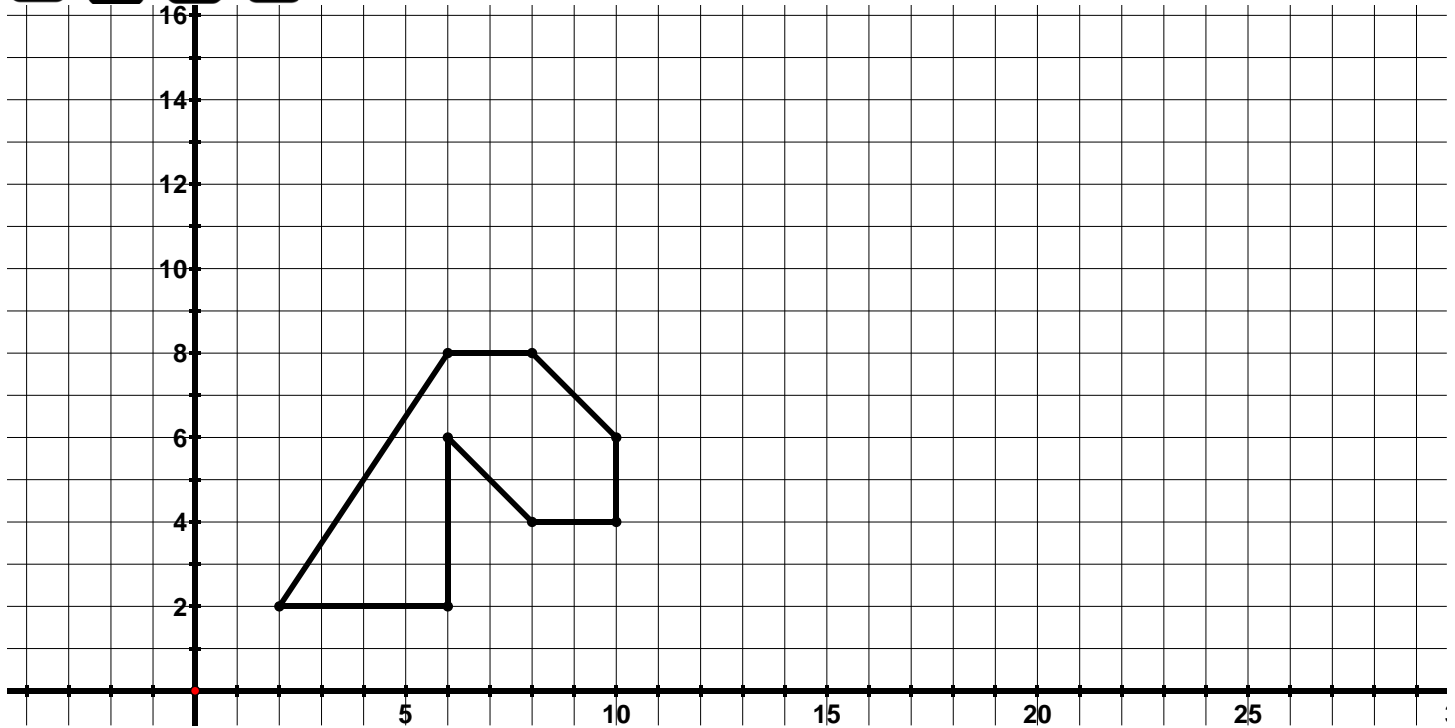
Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Describe the properties of a figure that has been dilated.
2. Perform a dilation given a scale factor and center of dilation.
3. Describe a dilation in words and using a coordinate rule.
4. Determine the center of dilation using the properties of dilations.
5. Understand what it means for two figures to be similar.
6. Determine if two figures are similar.
7. Given two figures that are similar, describe the sequence of transformations that exhibits the similarity between them.

9.2a Class Activity: Video Game Animation

Computer animators are working on designing the head of a dragon for a new video game. The picture below shows the original shape and size of the dragon's head. However, when the dragon eats a plant, the lengths of the sides of the dragon head double in size. If the dragon eats a cricket, the lengths of the sides of the dragon head triple in size. When the dragon enters a cave, the lengths of the sides shrink to half their original size.



Four different animators submitted the following proposals for how to double the lengths of the sides of the dragon's head when it eats a plant:

- Animator 1 said to apply the following rule $(x, y) \rightarrow (2x, y)$
- Animator 2 said to apply the following rule $(x, y) \rightarrow (x, 2y)$
- Animator 3 said to apply the following rule $(x, y) \rightarrow (x + 2, y + 2)$
- Animator 4 said to apply the following rule $(x, y) \rightarrow (2x, 2y)$

The chart below shows the coordinates of the dragon's head when it is its original size. Write the new coordinates for the dragon's head for the coordinate rules proposed by each of the animators. Then graph each of the animator's new dragon heads.

Original Size	$(2x, y)$	$(x, 2y)$	$(x + 2, y + 2)$	$(2x, 2y)$
(2, 2)				
(6, 8)				
(8, 8)				
(10, 6)				
(10, 4)				
(8, 4)				
(6, 6)				
(6, 2)				

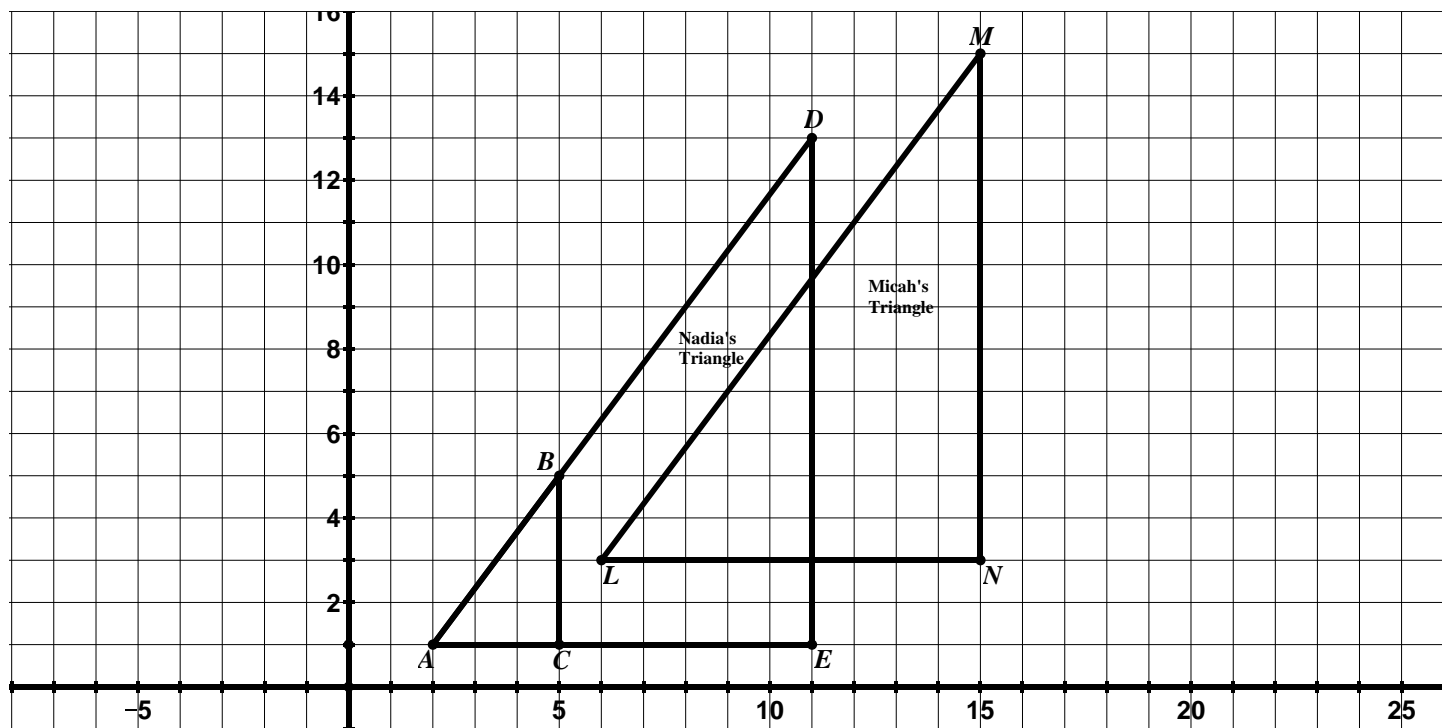
4. Which of the animator's rules results in a dragon that is the same shape as the original but whose side lengths are twice the size? What is the same about this dragon compared to the original dragon? What is different?
5. Describe what the coordinate rules of the other three animators do to the dragon's head.
6. The animators need your help coming up with the coordinate rule that would reduce the size of the dragon's head when it enters a cave (the lengths of the sides should be half their original size).
- Write your proposed coordinate rule in the table below.
 - Write the new coordinates for your rule.
 - Graph the new coordinates.

Original Size	Coordinate Rule:
(2, 2)	
(6, 8)	
(8, 8)	
(10, 6)	
(10, 4)	
(8, 4)	
(6, 6)	
(6, 2)	

- What is the same about this dragon compared to the original dragon? What is different?
- Write the coordinate rule that would triple the size of the dragon's head when it eats a cricket.

9.2b Class Activity: Properties of Dilations

1. Ms. Williams gave her students the grid shown below with $\triangle ABC$ graphed on it. She then asked her students to create a triangle that was the same shape as the original triangle but has sides lengths that are three times larger. Micah created $\triangle LMN$ shown below and Nadia created $\triangle ADE$ shown below.



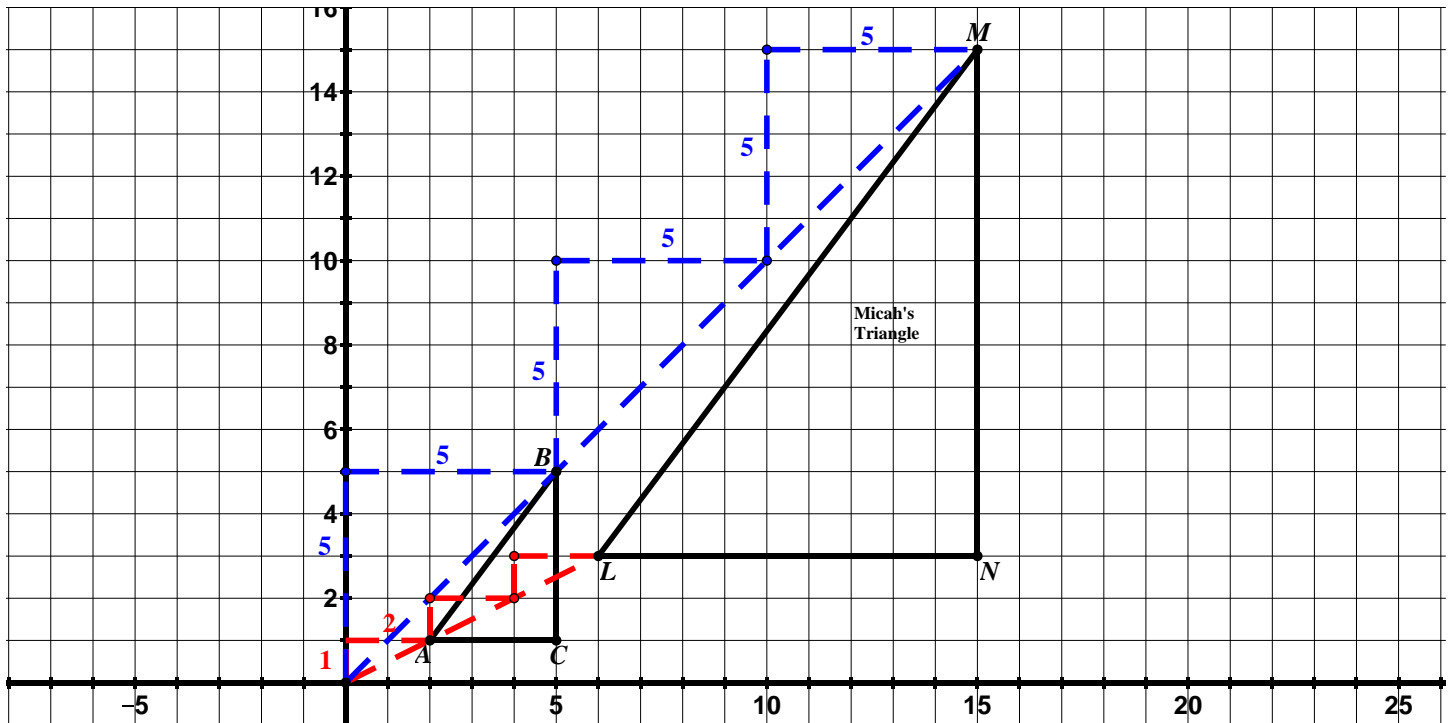
- a. The teacher asked the class who had done the assignment correctly, Nadia or Micah? Iya said they were both correct. Hendrix disagreed and said they could not both be correct because the triangles were not in the same location in the coordinate plane. Who do you agree with and why?



- b. The teacher asked Micah and Nadia to explain the methods they used to create the triangles.

Nadia's Method: Using a ruler, I slid C along the line containing the points A and C until my new segment was three times larger than \overline{AC} and labeled the new point E . I then used my ruler to slide B along the line containing the points A and B as shown below and labeled the new point D . Lastly, I checked to make sure \overline{DE} was 3 times larger than \overline{BC} and it was!

Micah's Method: I noticed that the slope of the line passing through the origin and A had a rise of 1 and a run of 2. Since I wanted the image to be three times larger than $\triangle ABC$, I placed the point that corresponds to A three slope triangles (with a rise of 1 and a run of 2) from the origin. I used the same method to plot the point that corresponds to B . The slope of the line passing through the origin and B has a rise of 5 and a run of 5. Again, I moved three slope triangles with a rise of 5 and a run of 5 from the origin and plotted M . The slope of the line passing through C and the origin has a rise of 1 and a run of 5. I moved three slope triangles with a rise of 1 and a run of 5 from the origin and plotted N .



- c. Compare the two methods used. What is the same about the resulting triangles? What is different? What accounts for the differences in the triangles?

In the previous example, Micah and Nadia both **dilated** $\triangle ABC$. A **dilation** is a transformation that produces an image that is the same shape as the original figure but the image is a different size.

Every dilation has a center of dilation and a scale factor. The **center of dilation** is a fixed point in the plane from which all points are expanded or contracted. The **scale factor** describes the size change from the original figure to the image. We use the letter r to represent scale factor. The dilation is an enlargement if the scale factor is greater than 1 and a reduction if the scale factor is between 0 and 1.

In the example on the previous page, the scale factor for both Nadia and Micah was 3; however Nadia's center of dilation was $A: (2, 1)$ while Micah's was the origin $(0, 0)$.

2. We will use the example on the previous page to examine some of the **properties of dilations**.
- a. Find the following ratios for Nadia's triangle:

$$\frac{AD}{AB} =$$

$$\frac{DE}{BC} =$$

$$\frac{AE}{AC} =$$

- b. Find the following ratios for Micah's triangle:

$$\frac{LM}{AB} =$$

$$\frac{MN}{BC} =$$

$$\frac{LN}{AC} =$$

- c. Complete the following sentence. Under a dilation, **the ratios of the image segments to the corresponding pre-image segments** are...
- d. Complete the following sentence. Under a dilation, **corresponding angles** are...
- e. Complete the following sentence. Under a dilation, **corresponding segments** are...
- f. Complete the following sentence. Under a dilation, **corresponding vertices**...
- g. Complete the following sentence. Under a dilation, **segments connecting corresponding vertices**...

- h. In the table below, list the coordinates of the corresponding vertices in Micah's dilation:

A:	L:
B:	M:
C:	N:

- i. Write a coordinate rule for Micah's dilation using the information in the table above.



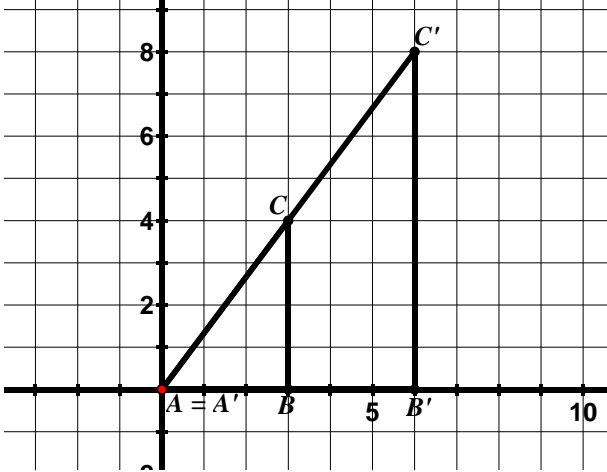
- j. In the table below, list the coordinates of the corresponding vertices in Nadia's dilation:

A:	A:
B:	D:
C:	E:

- k. Write a coordinate rule for Nadia's dilation using the information in the table above. Remember that Nadia's center of dilation is not at the origin. Think about how this shift off the origin will affect the coordinate rule.

Directions: In the following problems, one figure has been dilated to obtain a new figure. Determine the scale factor and center of dilation. Determine the coordinate rule for the dilation when asked.

3. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.

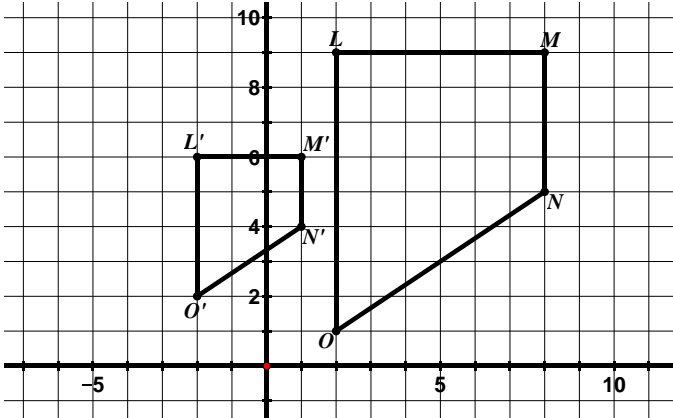


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

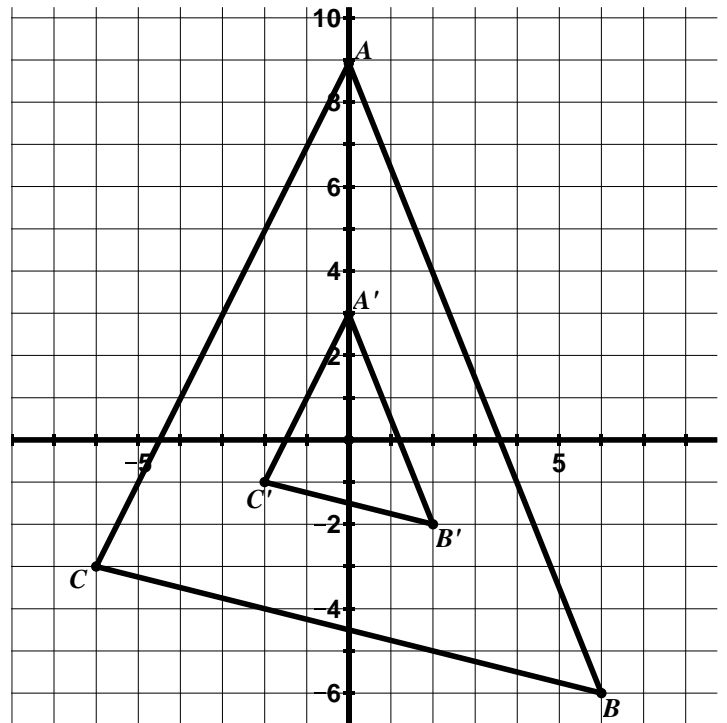
4. $LMNO$ has been dilated to obtain $L'M'N'O'$.



Scale Factor: _____

Center of Dilation: _____

5. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.

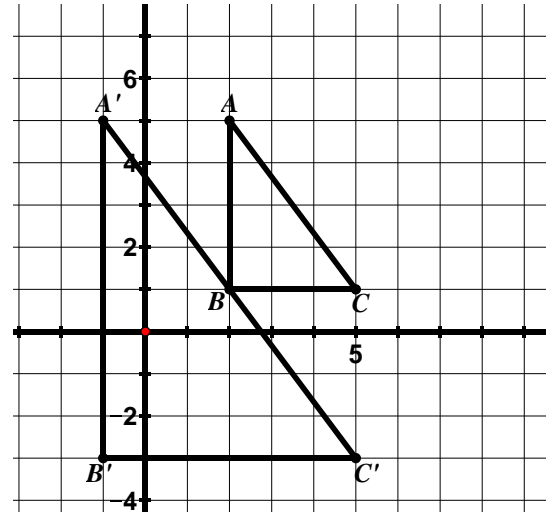


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

6. $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.



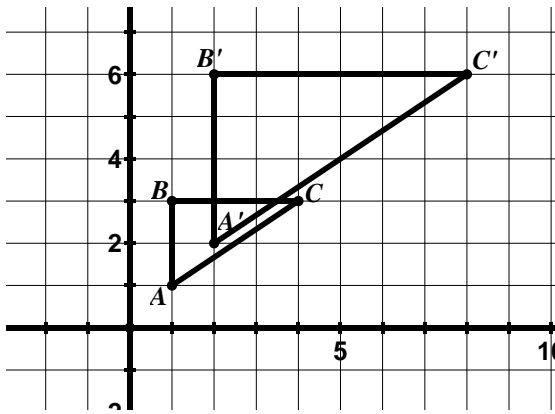
Scale Factor: _____

Center of Dilation: _____

9.2b Homework: Properties of Dilations

Directions: In the following problems, one figure has been dilated to obtain a new figure. Determine the scale factor and center of dilation. Determine the coordinate rule for the dilation when asked.

1. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.

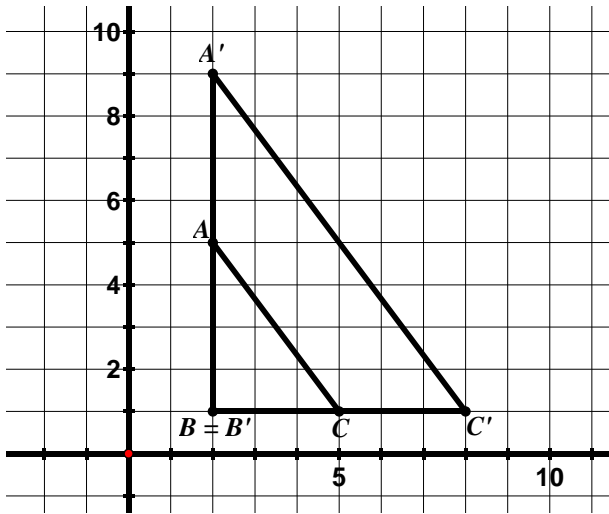


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

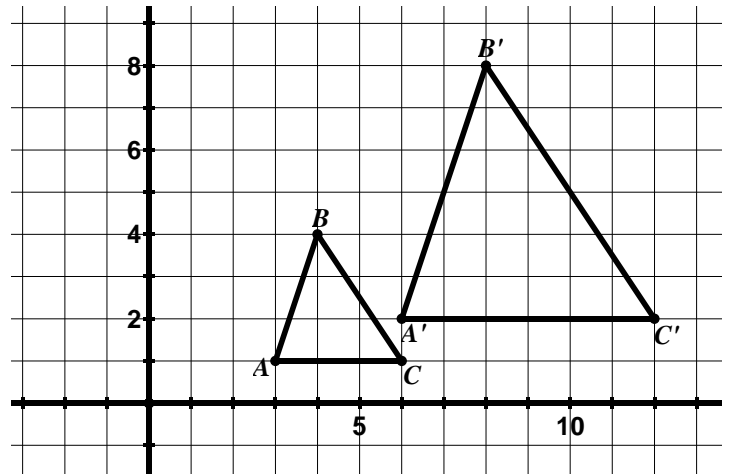
2. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.



Scale Factor: _____

Center of Dilation: _____

3. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.

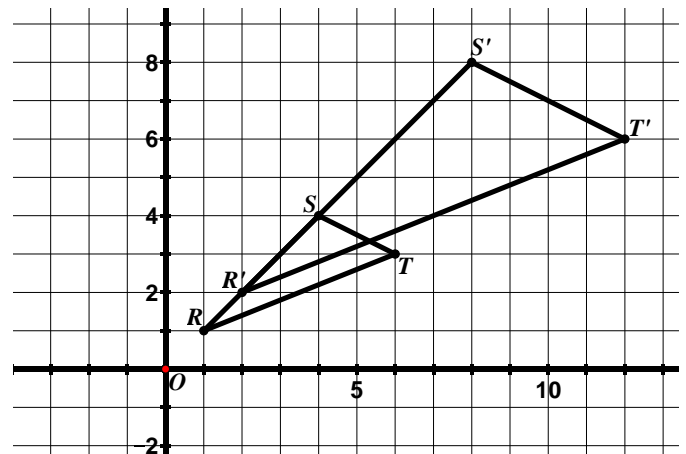


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

4. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.

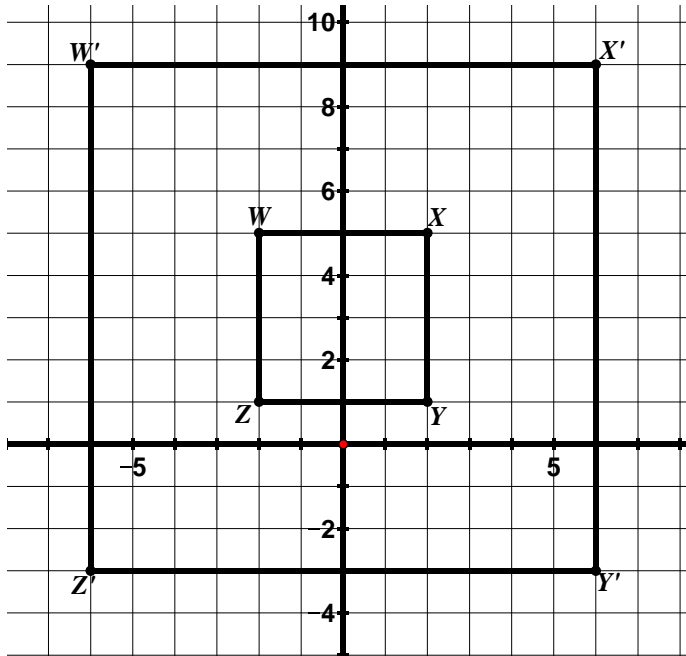


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

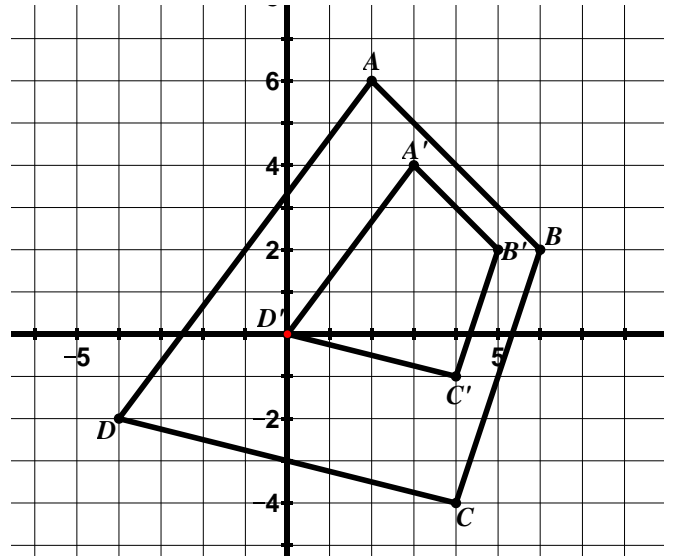
5. $WXYZ$ has been dilated to obtain $W'X'Y'Z'$.



Scale Factor: _____

Center of Dilation: _____

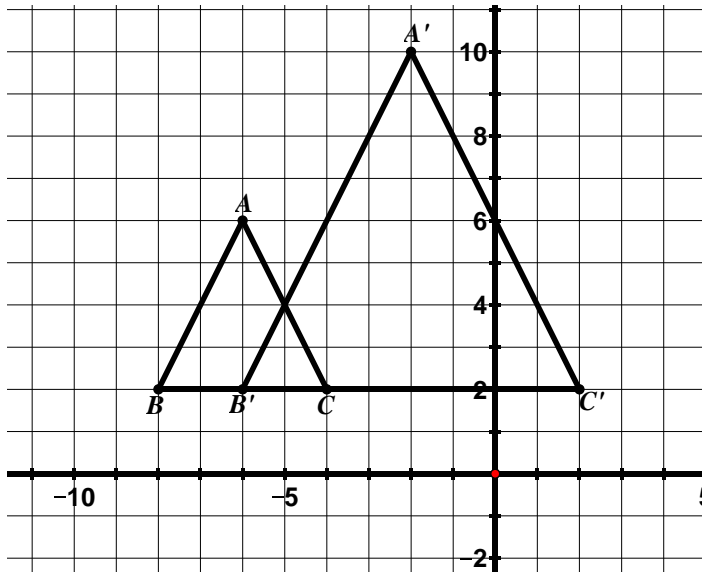
7. $ABCD$ has been dilated to obtain $A'B'C'D'$.



Scale Factor: _____

Center of Dilation: _____

6. $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.

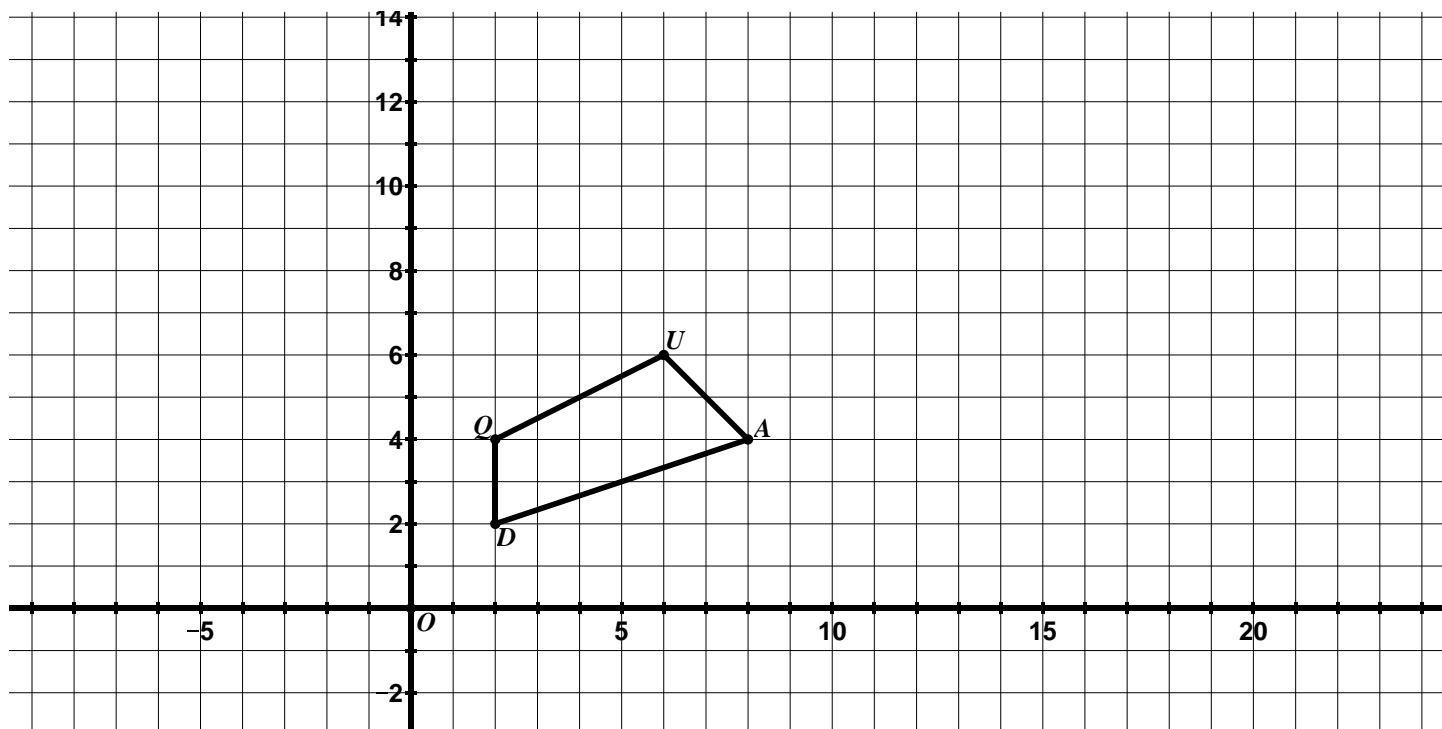


Scale Factor: _____

Center of Dilation: _____

9.2c Class Activity: Dilations cont.

1. $QUAD$ is graphed below.



- a. Create a new quadrilateral whose side lengths are two times larger than the side lengths of $QUAD$ with the center of dilation at the origin and label the image $Q'U'A'D'$. In the space below, describe the method you used to create your new quadrilateral.
- b. Based on what we have learned so far about dilations, what are some different ways you can verify that the side lengths of your new quadrilateral are in fact two times larger than the side lengths of the original?

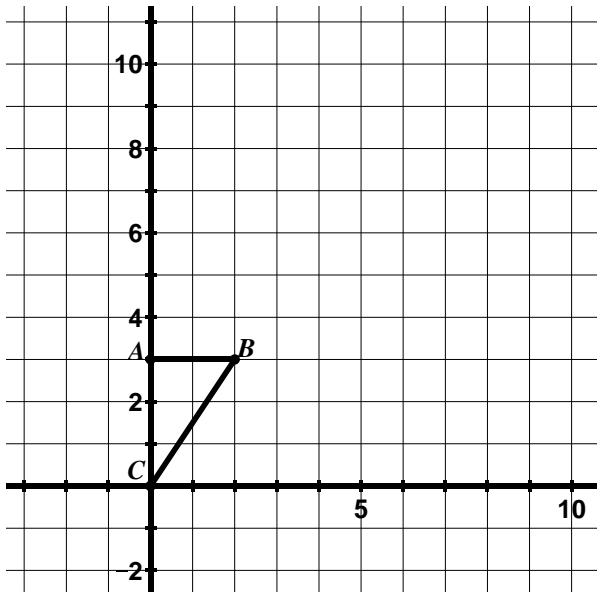


- c. This time, create a quadrilateral whose sides lengths are $\frac{1}{2}$ the size of the side lengths of $QUAD$ with the center of dilation at the origin and label the image $Q''U''A''D''$.

Directions: Find the image of each figure for a dilation with the given center and scale factor. Draw and label the image.

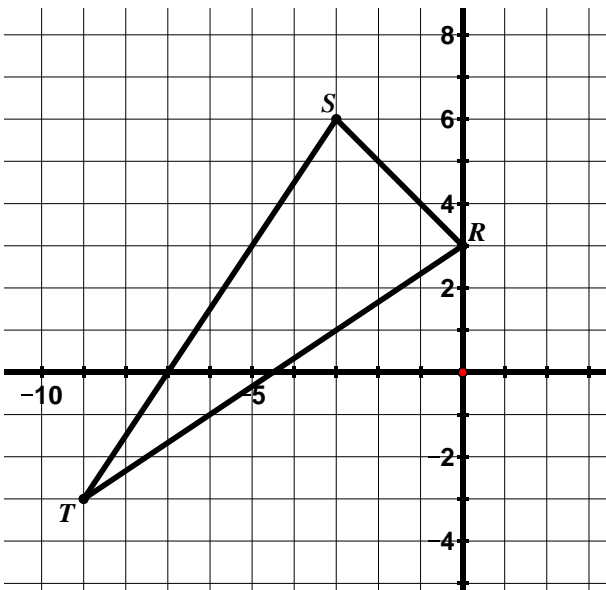
2. $r = 3$

Center of Dilation: C



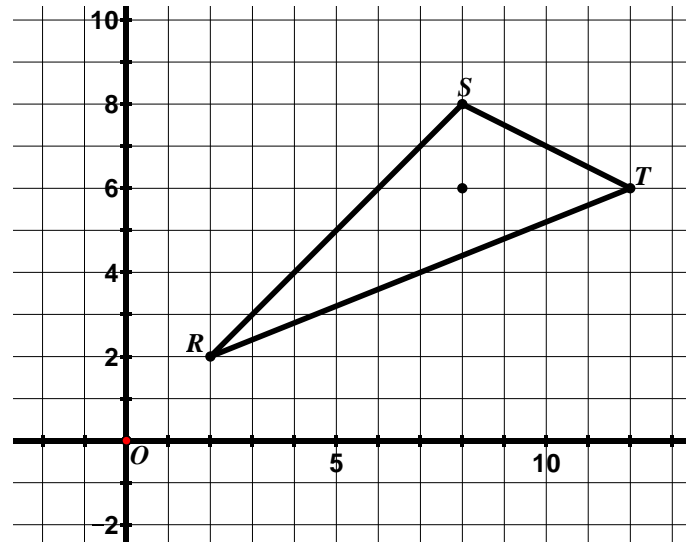
3. $r = \frac{1}{3}$

Center of Dilation: origin



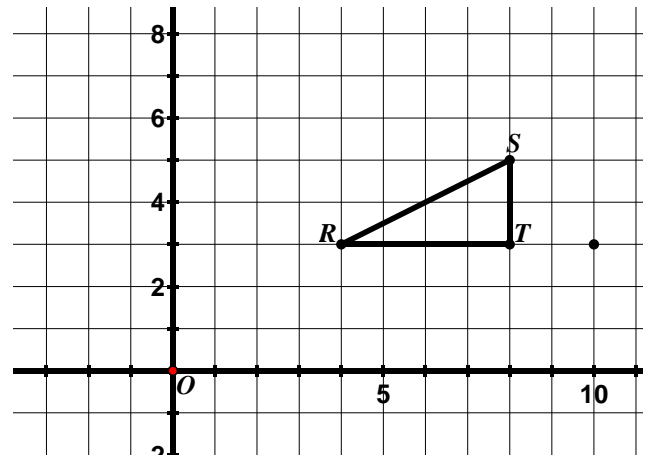
4. $r = \frac{1}{2}$

Center of Dilation: $(8, 6)$.



5. $r = 2$

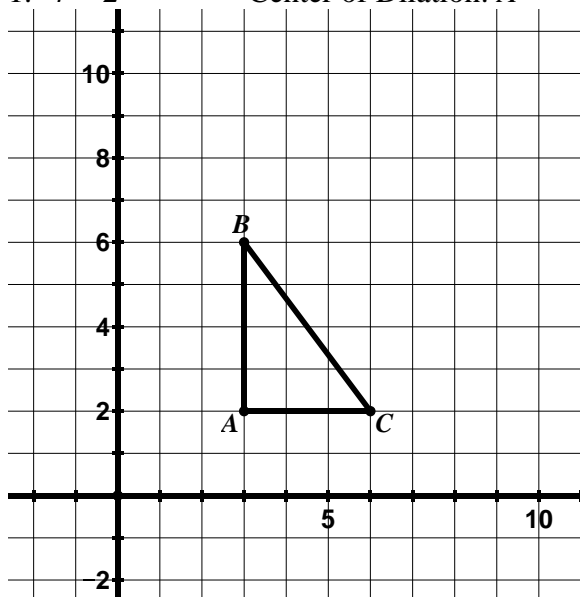
Center of Dilation: $(10, 3)$



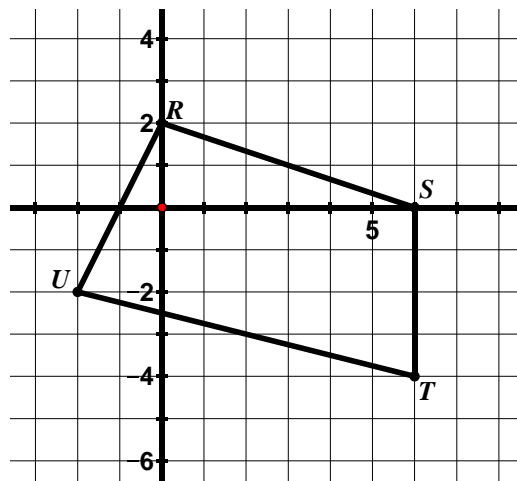
9.2c Homework: Dilations cont.

Directions: Find the image of each figure for a dilation with the given center and scale factor. Draw and label the image.

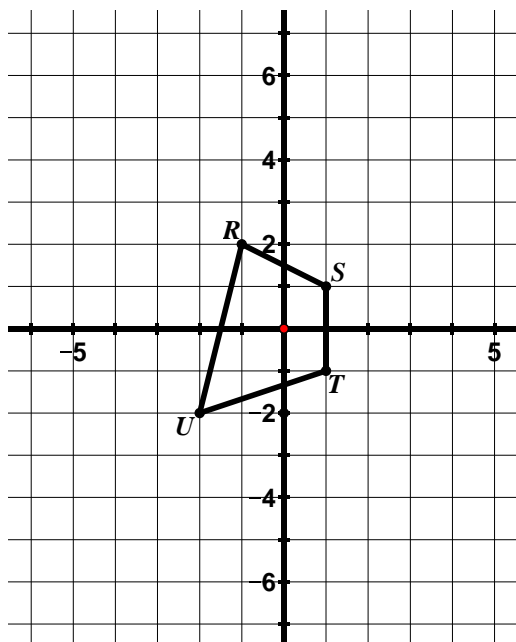
1. $r = 2$ Center of Dilation: A



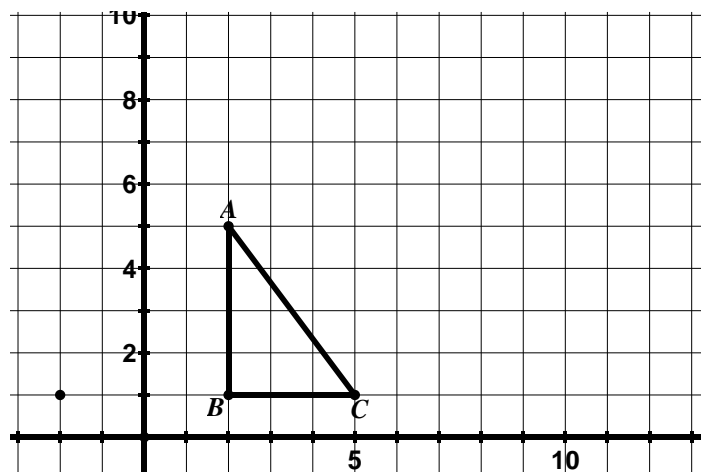
3. $r = \frac{1}{2}$ Center of Dilation: origin



2. $r = 3$ Center of Dilation: origin

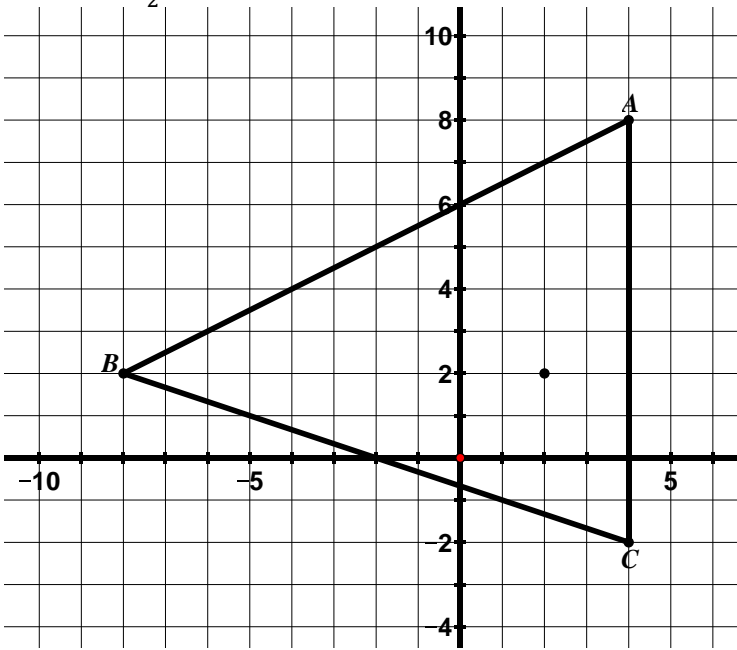


4. $r = 2$ Center of Dilation: $(-2, 1)$



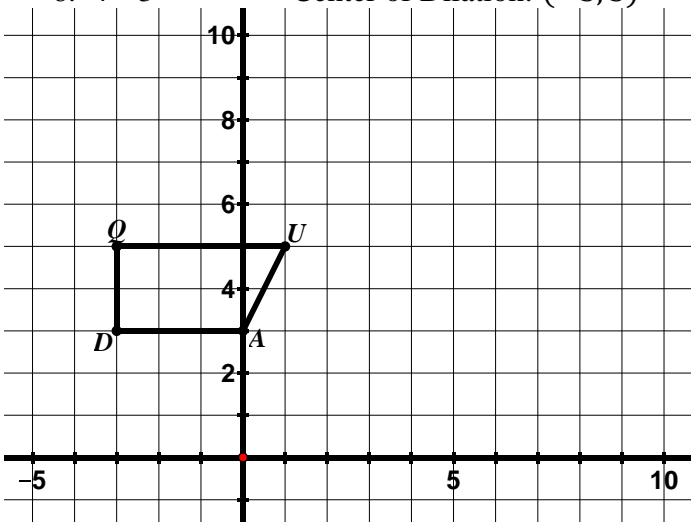
5. $r = \frac{1}{2}$

Center of Dilation: (2, 2)



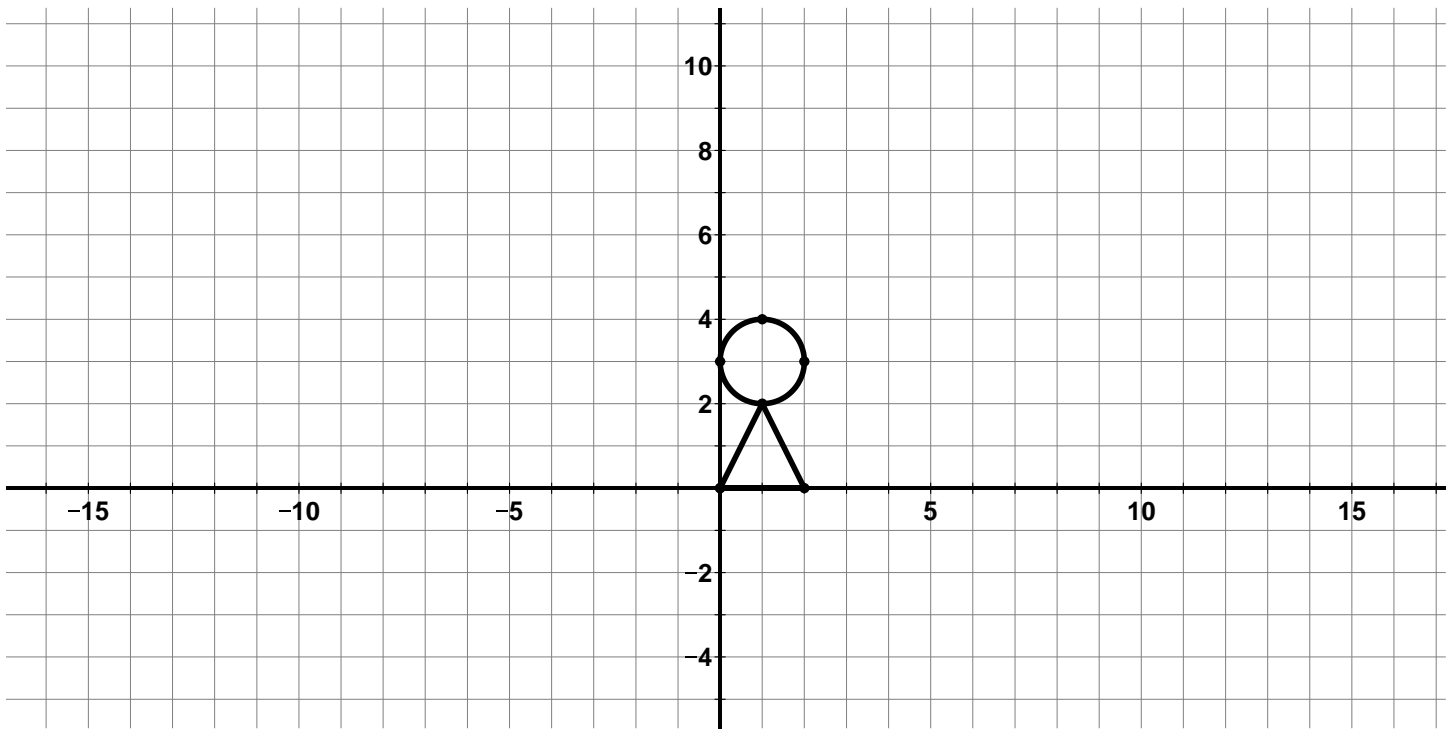
6. $r = 3$

Center of Dilation: (-3, 3)



9.2d Class Activity: Problem Solving with Dilations

1. Where would you put your center of dilation to enlarge the picture below by a factor of 2 and put the image entirely in the second quadrant? Mark your center of dilation with an O and then do the dilation.

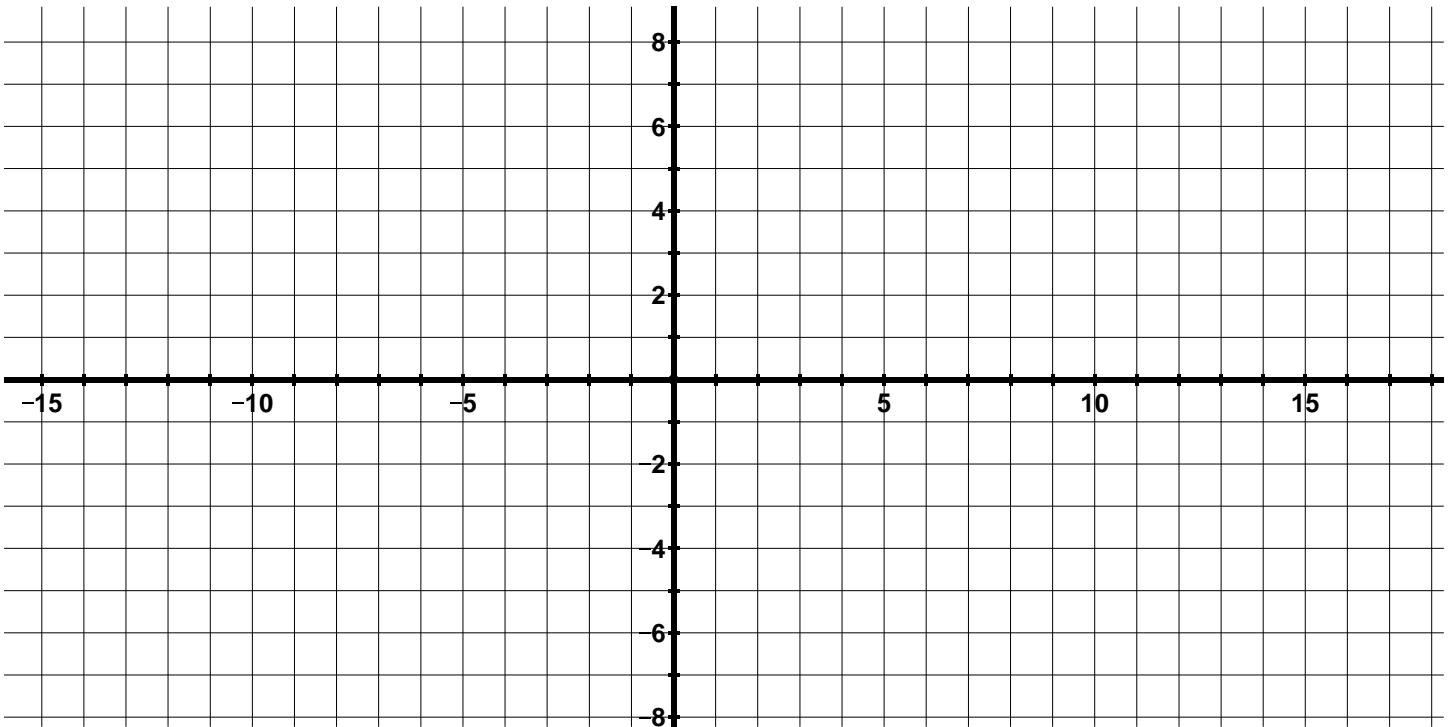


2. A dilation with the center of dilation at the origin maps $\triangle ABC$ to $\triangle A'B'C'$.

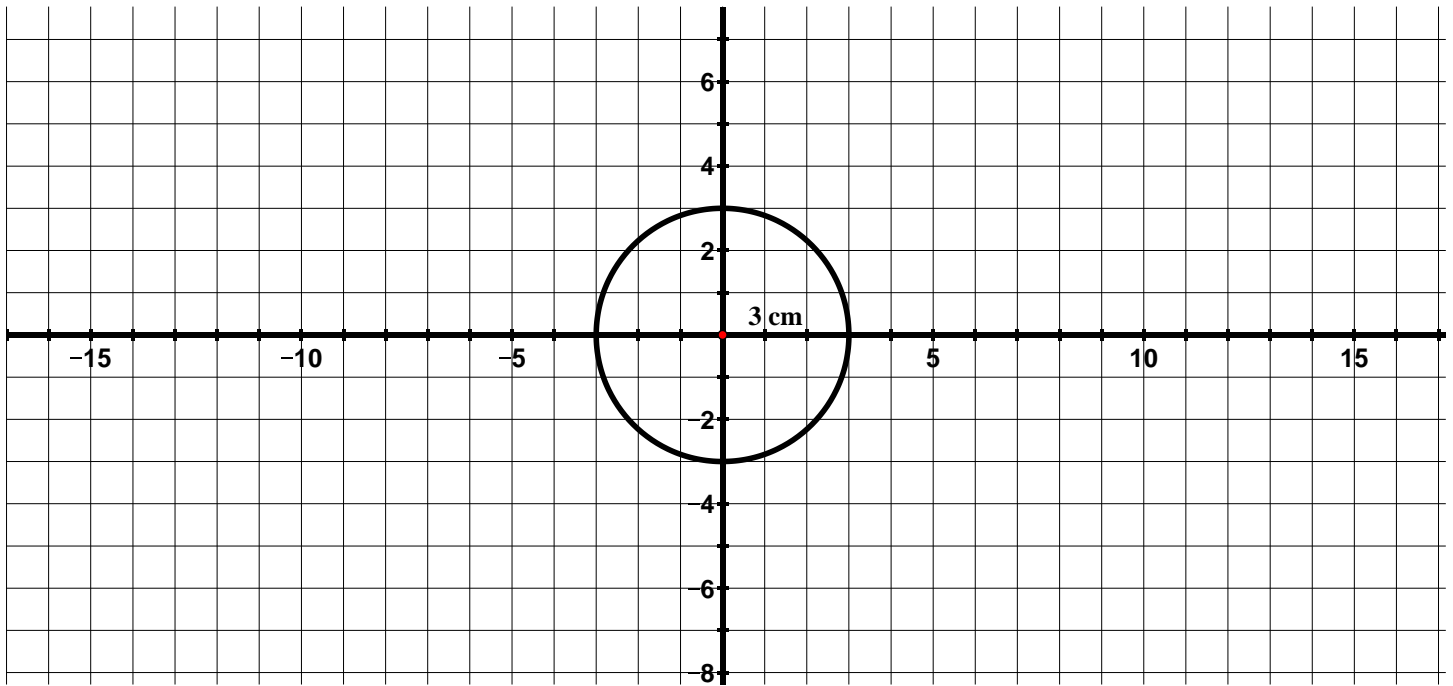
- If $AB = 3$ and $A'B' = 6$, what is the scale factor of the dilation?
- If $B'C' = 8$, what is the length of BC ?
- If $AC = 5$, what is the length of $A'C'$?
- If the slope of \overline{AB} is 0, what is the slope of $\overline{A'B'}$?
- If the slope of $\overline{A'C'}$ is $\frac{4}{3}$, what is the slope of \overline{AC} ?
- Create a picture of this dilation on the grid below using the information from parts $a - e$ and the additional pieces of information below. Remember that the center of dilation is the origin.



- The slope of CB is undefined
- A is at the origin



3. A circle with a radius of 3 cm is shown below.



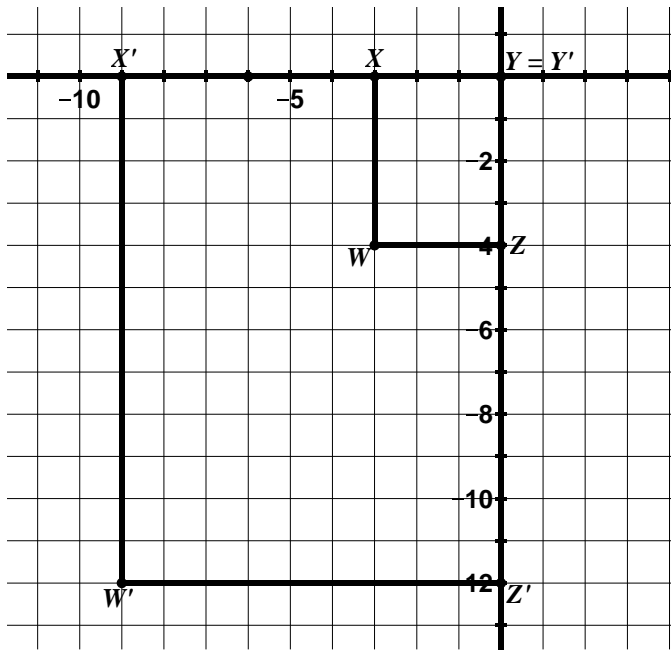
- a. Determine the length of the radius of a circle whose circumference would be twice as large as the circle pictured above.

- b. Determine the length of the radius of a circle whose area would be twice as large as the circle pictured above.

9.2d Homework: Review of Dilations

Directions: In the following problems, one figure has been dilated to obtain a new figure. Determine the scale factor and center of dilation. Determine the coordinate rule for the dilation when asked.

1. In the picture below, $WXYZ$ has been dilated to obtain $W'X'Y'Z'$.

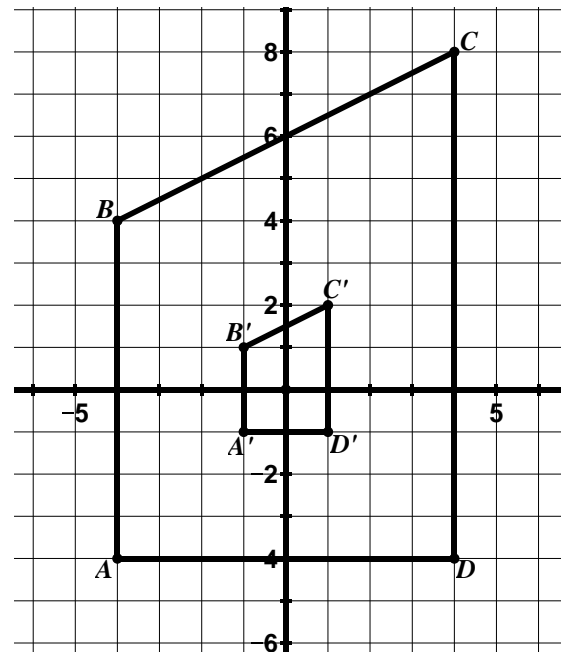


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

2. In the picture below, $ABCD$ has been dilated to obtain $A'B'C'D'$.

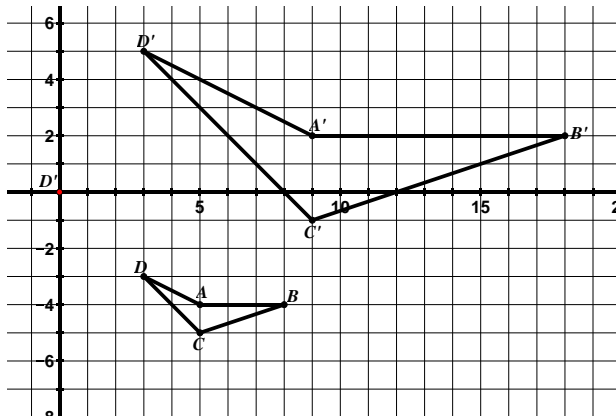


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

3. $ABCD$ has been dilated to obtain $A'B'C'D'$.



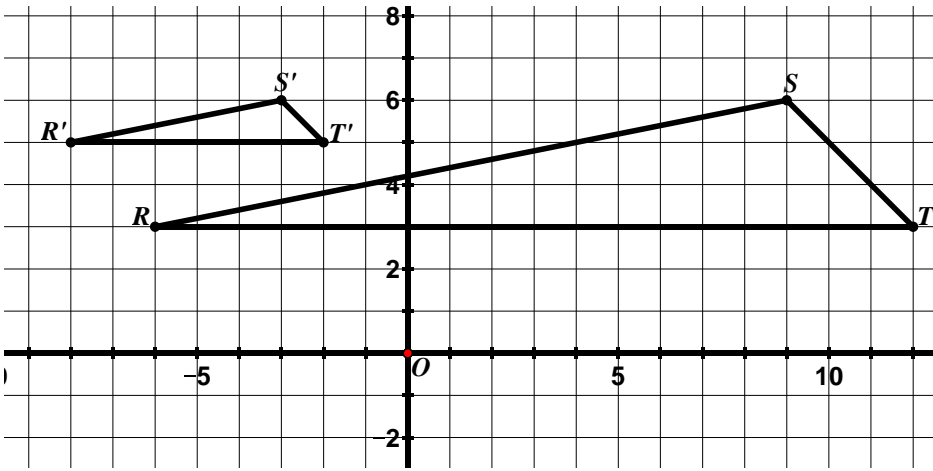
Scale Factor: _____

Center of Dilation: _____

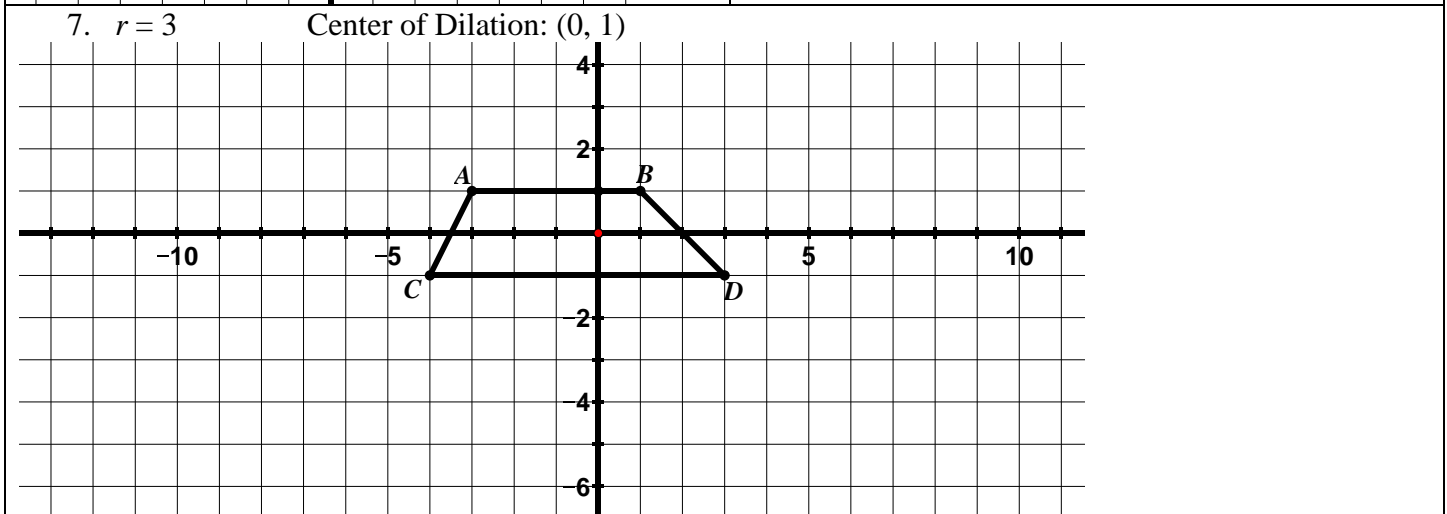
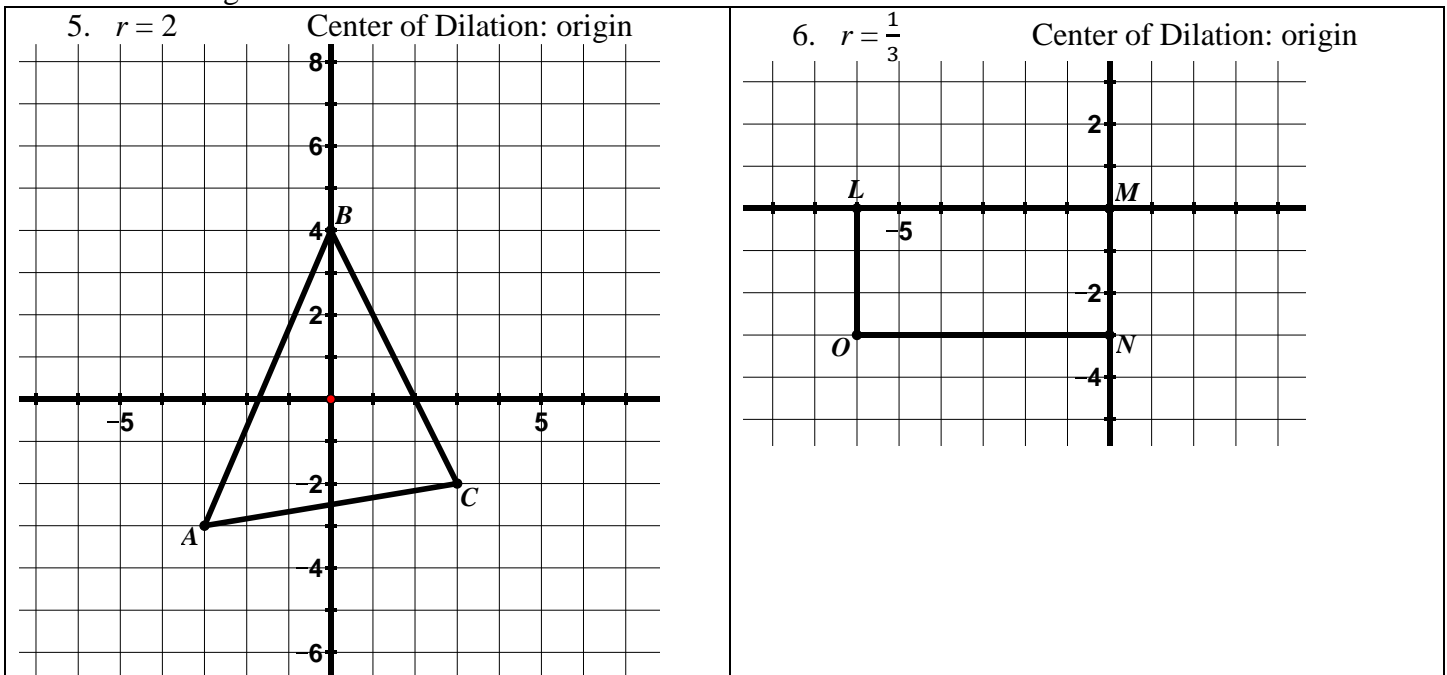
4. $\triangle RST$ has been dilated to obtain $\triangle R'S'T'$.

Scale Factor: _____

Center of Dilation: _____



Directions: For #5 – 7, find the image of each figure for a dilation with the given center and scale factor. Draw and label the image.



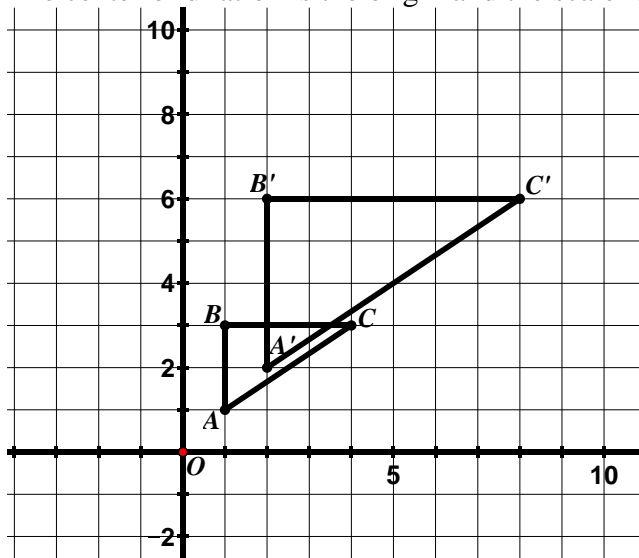
9.2e Class Activity: Similarity

In the first part of the chapter, we discussed congruence. Two figures are **congruent** if one can be obtained from the other by a rigid motion (rotation, reflection, or translation) or a sequence of rigid motions. If you can move one of the figures using one of these transformations or a series of these transformations so that it fits exactly on the other one, the two figures are congruent.

In this section we have seen problems where two figures are similar. In 7th grade, you learned that two figures are similar if they have the **same shape** – they may or may not be the same size. In 8th grade, we define similarity in terms of transformations. Two figures are said to be **similar** if there is a sequence of rigid motions and dilations that take one figure onto the other.

While studying dilations, we have learned that (1) a dilation creates a figure that is the same shape as the original figure but a different size, (2) the measure of corresponding angles is the same and (3) the ratios of corresponding sides are all the same. Since similar figures are produced by a dilation, these properties, as well as some others we observed, also hold true for similar figures.

Let's revisit a problem we have seen before. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$. The center of dilation is the origin and the scale factor is 2.



Because $\triangle A'B'C'$ was produced by a dilation of $\triangle ABC$, the two triangles are similar. We can write a similarity statement for the two triangles. You can denote that two figures are similar by using the symbol \sim and listing their vertices in corresponding order.

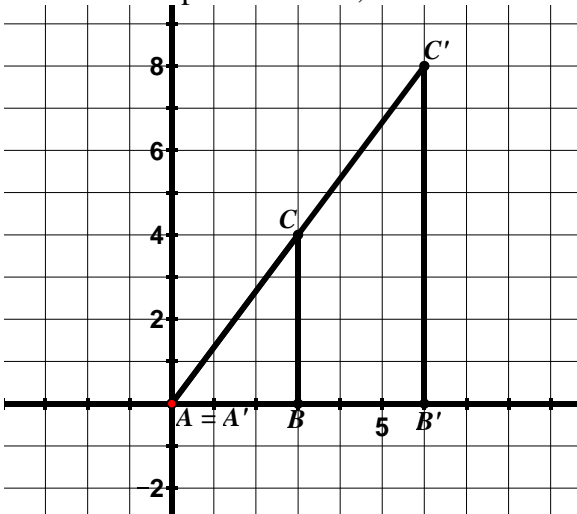
Write a similarity statement for the two triangles.

The order the vertices is written tells us which segments and angles are corresponding in the figures.

When two figures are similar, corresponding angles are congruent and corresponding sides are proportional. Write the congruence statements to represent this.

The ratio of the lengths of the corresponding sides is a similarity ratio. Write the similarity ratio for these two triangles.

1. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$. The center of dilation is the origin.



a. Write a similarity statement for the triangles.

b. Complete each statement:

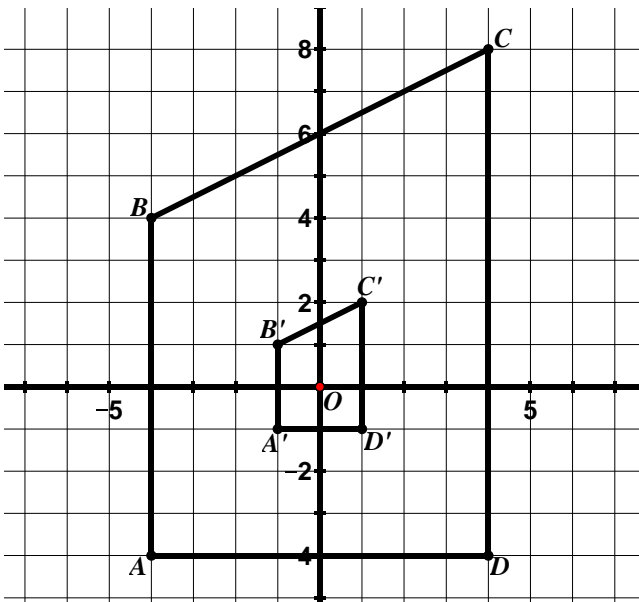
$$m\angle C \cong$$

$$\text{If } m\angle B = 90^\circ, \text{ then } m\angle B' =$$

$$\frac{A'C'}{AC} =$$

$$\frac{A'B'}{AB} = 2$$

2. In the picture below, $ABCD$ has been dilated to obtain $A'B'C'D'$. The center of dilation is the origin.



a. Write a similarity statement for the trapezoids.

b. Complete each statement.

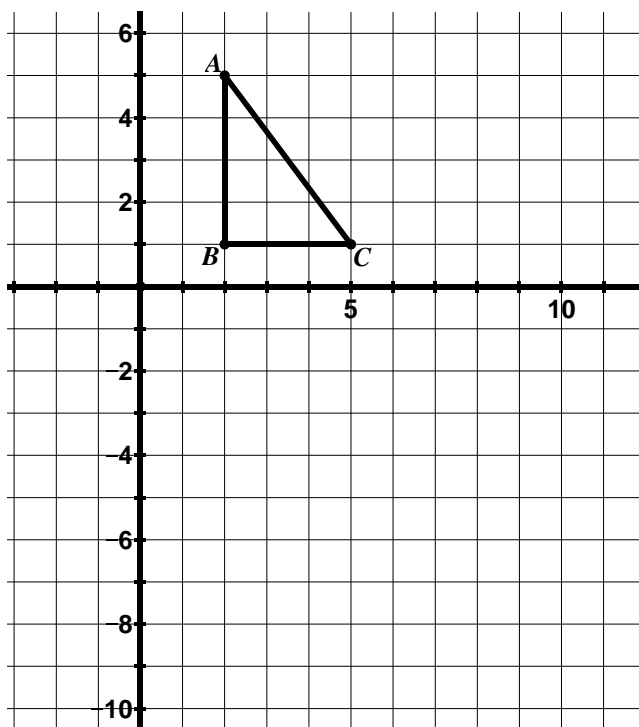
$$m\angle C \cong$$

$$\text{If } m\angle A = 90^\circ, \text{ then } m\angle A' =$$

$$\frac{A'D'}{AD} =$$

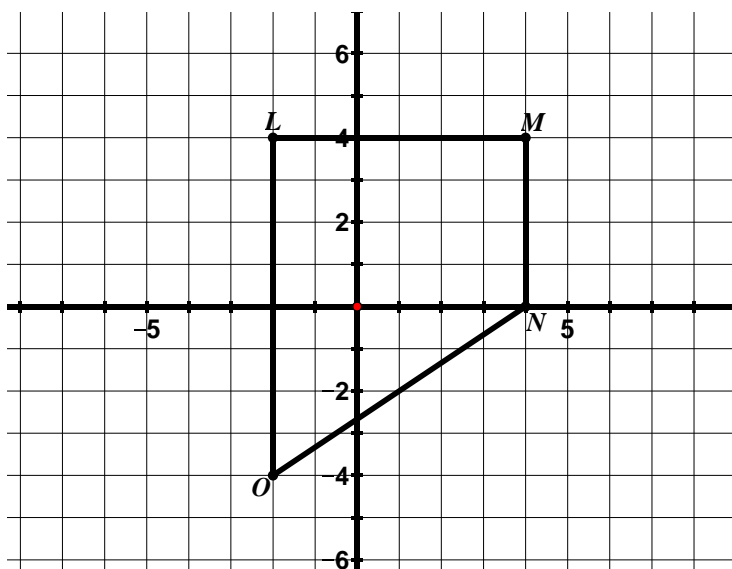
$$\frac{A'B'}{AB} = \frac{1}{4}$$

3. $\triangle ABC$ is graphed on the grid below.



- Reflect $\triangle ABC$ over the x -axis. Label the new triangle $\triangle A'B'C'$.
- Dilate $\triangle A'B'C'$ by a scale factor of 2 with the center of dilation at the origin. Label the new triangle $\triangle A''B''C''$.
- Write a statement that shows the relationship between $\triangle ABC$ and $\triangle A'B'C'$.
- Write a statement that shows the relationship between $\triangle A'B'C'$ and $\triangle A''B''C''$.
- List five other relationships between corresponding angles, sides, etc. that you know to be true based on the relationships in parts c and d.

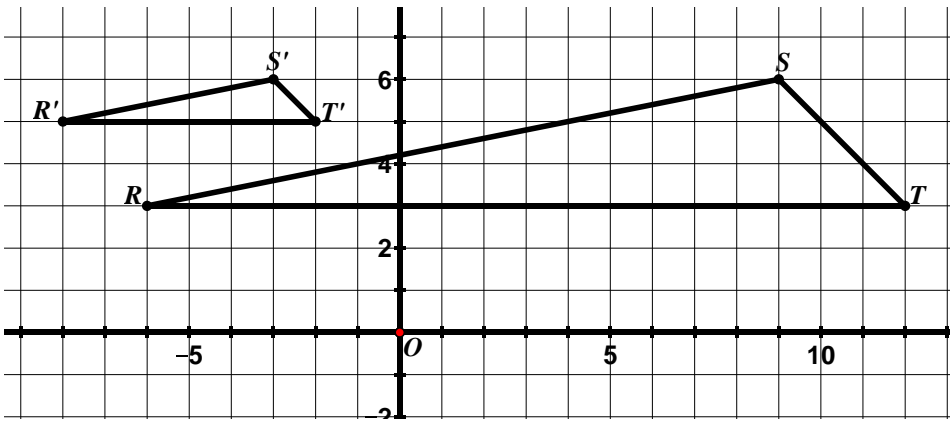
4. $LMNO$ is graphed on the grid below.



- Dilate $LMNO$ by a scale factor of $\frac{1}{2}$ with the center of dilation at the origin. Label the new quadrilateral $L'M'N'O'$.
- Translate $L'M'N'O'$ according to the rule $(x, y) \rightarrow (x - 6, y + 2)$. Label the new quadrilateral $L''M''N''O''$.
- Write a statement that shows the relationship between $LMNO$ and $L'M'N'O'$.
- Write a statement that shows the relationship between $L'M'N'O'$ and $L''M''N''O''$.
- List five other relationships between corresponding angles, sides, etc. that you know to be true based on the relationships in parts c and d.

9.2e Homework: Similarity

1. In the picture below $\triangle RST$ has been dilated to obtain $\triangle R'S'T'$.



a. Write a similarity statement for the triangles.

b. Complete each statement:

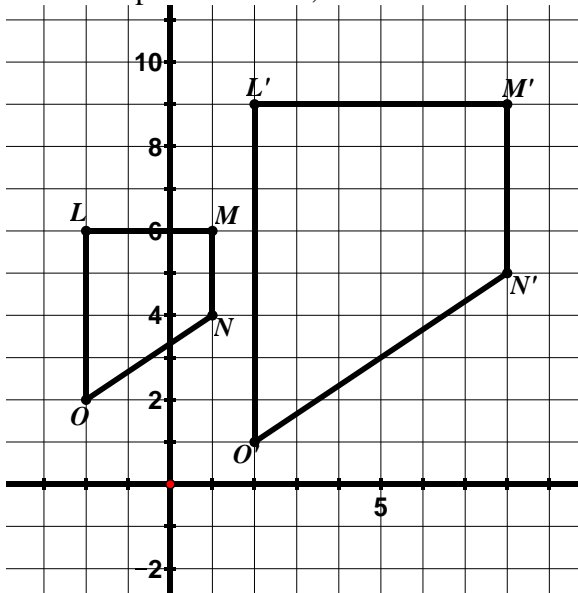
$$m\angle S \cong$$

If $m\angle T = 45^\circ$, then $m\angle T' =$

$$\frac{R'T'}{RT} =$$

$$\frac{R'S'}{RS} = \frac{1}{3}$$

2. In the picture below, $LMNO$ has been dilated to obtain $L'M'N'O'$. The center of dilation is the origin.



a. Write a similarity statement for the triangles.

b. Complete each statement.

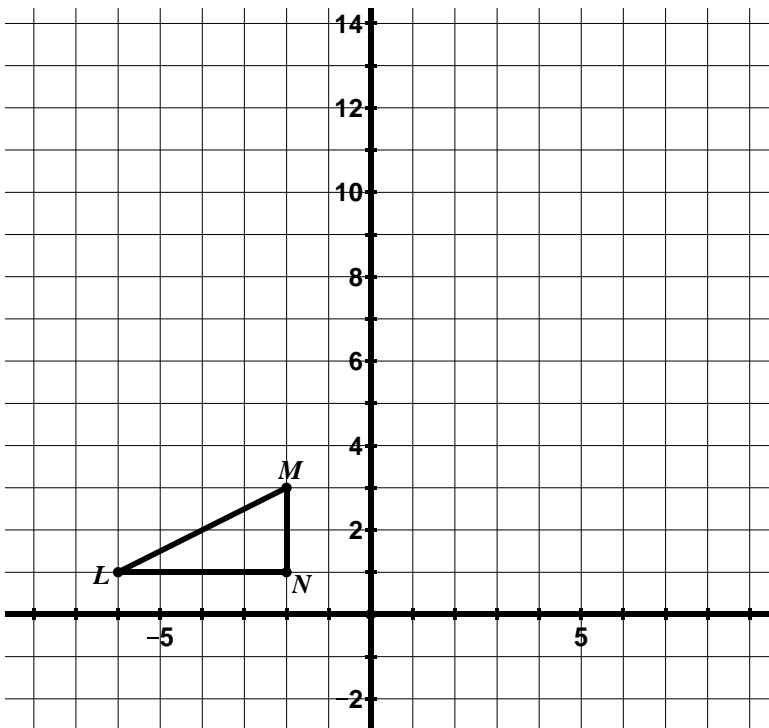
$$m\angle O \cong$$

If $m\angle L = 90^\circ$, then $m\angle L' =$

$$\frac{O'N'}{ON} =$$

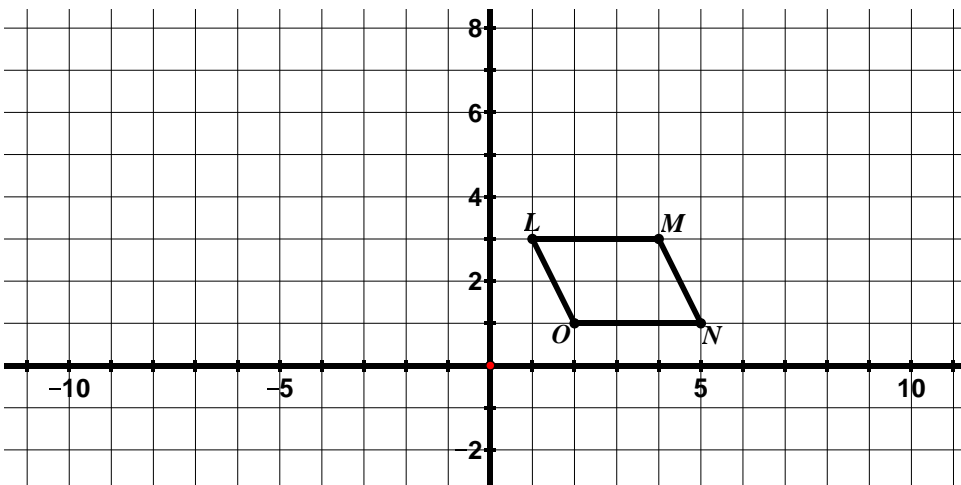
$$\frac{L'O'}{LO} = 2$$

3. $\triangle LMN$ is graphed on the grid below.



- Rotate $\triangle LMN$ 90° clockwise about the origin. Label the new triangle $\triangle L'M'N'$.
- Dilate $\triangle L'M'N'$ by a scale factor of 3 with the center of dilation at $(1, 2)$. Label the new triangle $\triangle L''M''N''$.
- Write a statement that shows the relationship between $\triangle LMN$ and $\triangle L'M'N'$.
- Write a statement that shows the relationship between $\triangle L'M'N'$ and $\triangle L''M''N''$.
- List five other relationships between corresponding angles, sides, etc. that you know to be true based on the relationships in parts c and d.

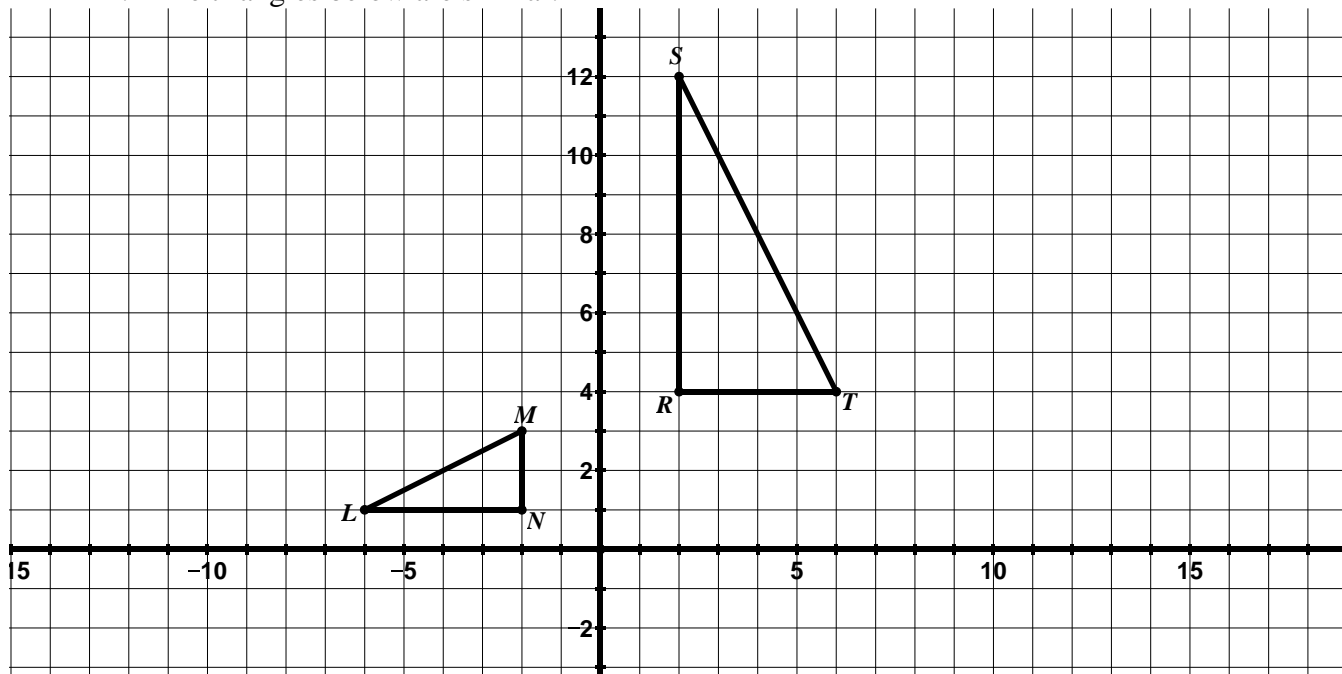
4. $LMNO$ is graphed on the grid below.



- Dilate $LMNO$ by a scale factor of 2 with the center of dilation at the origin. Label the new quadrilateral $L'M'N'O'$.
- Reflect $L'M'N'O'$ across the y -axis. Label the new quadrilateral $L''M''N''O''$.
- Write a statement that shows the relationship between $LMNO$ and $L'M'N'O'$.
- Write a statement that shows the relationship between $L'M'N'O'$ and $L''M''N''O''$.

9.2f Class Activity: Similarity cont.

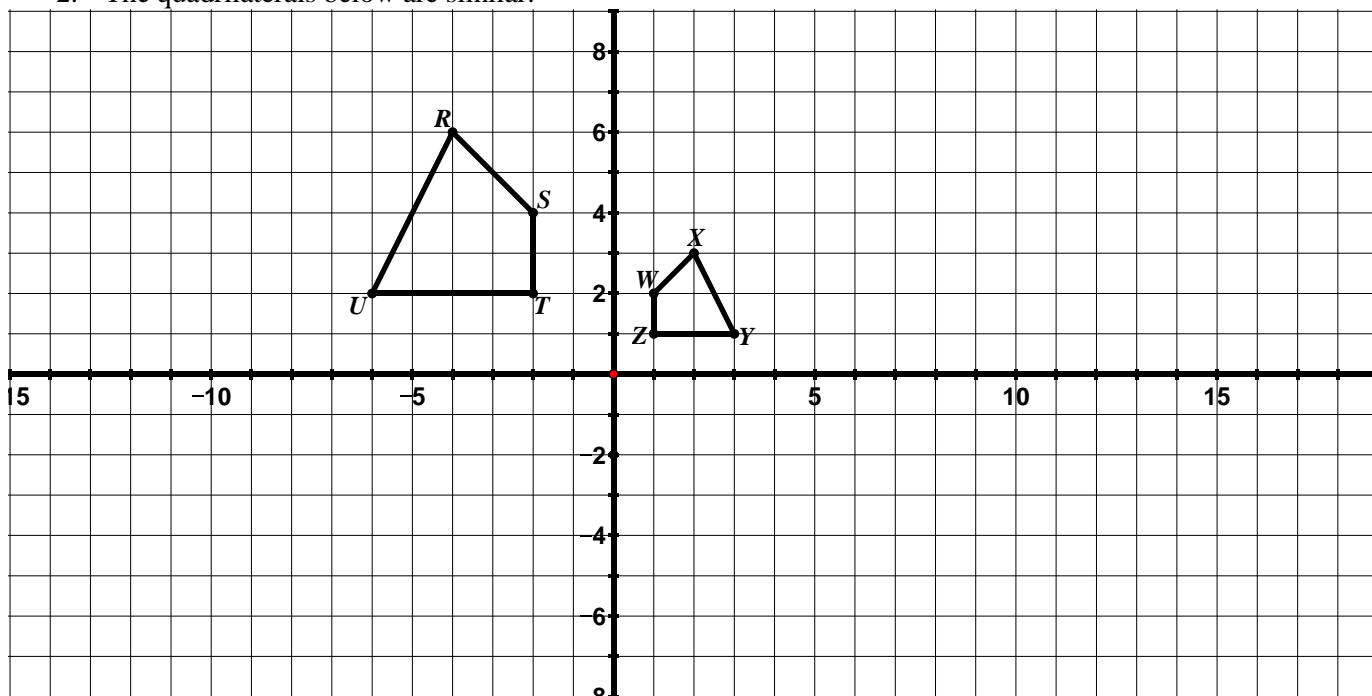
1. The triangles below are similar.



a. List the sequence of transformations that verifies the similarity of the two figures.

b. Write a similarity statement for the triangles.

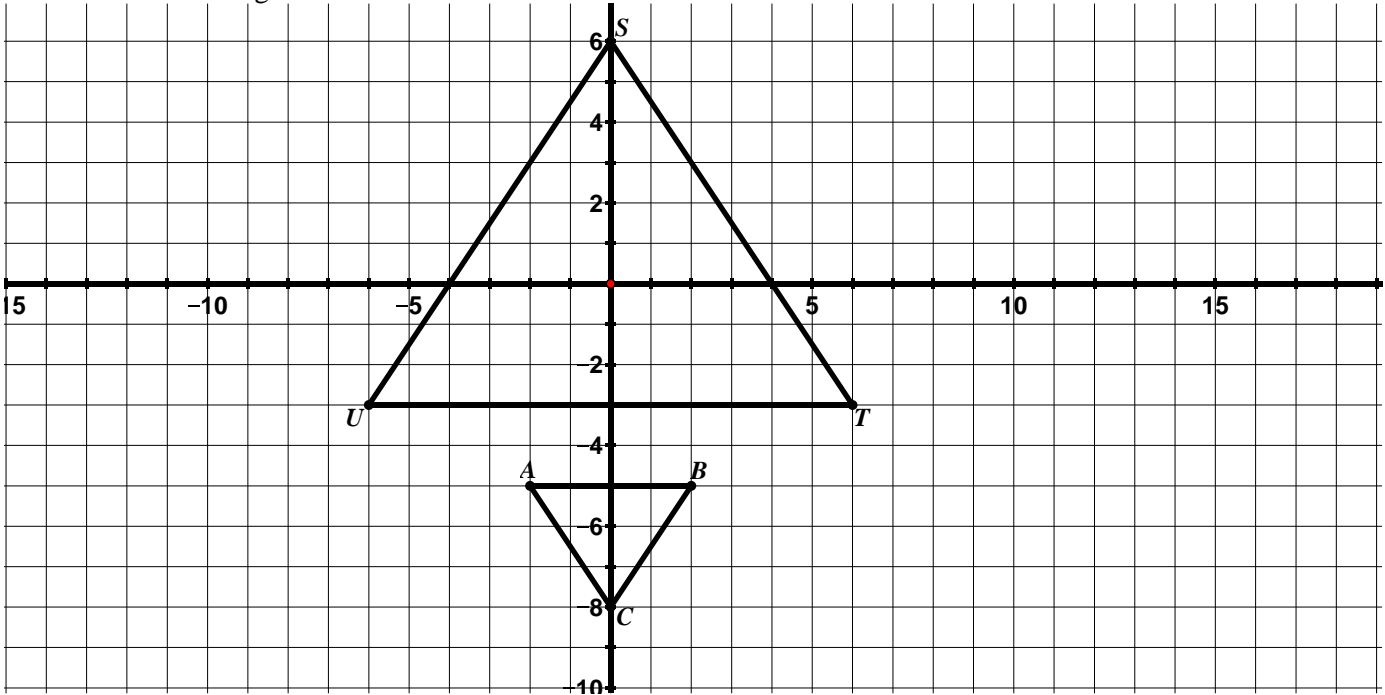
2. The quadrilaterals below are similar.



a. List the sequence of transformations that verifies the similarity of the two figures.

b. Write a similarity statement for the quadrilaterals.

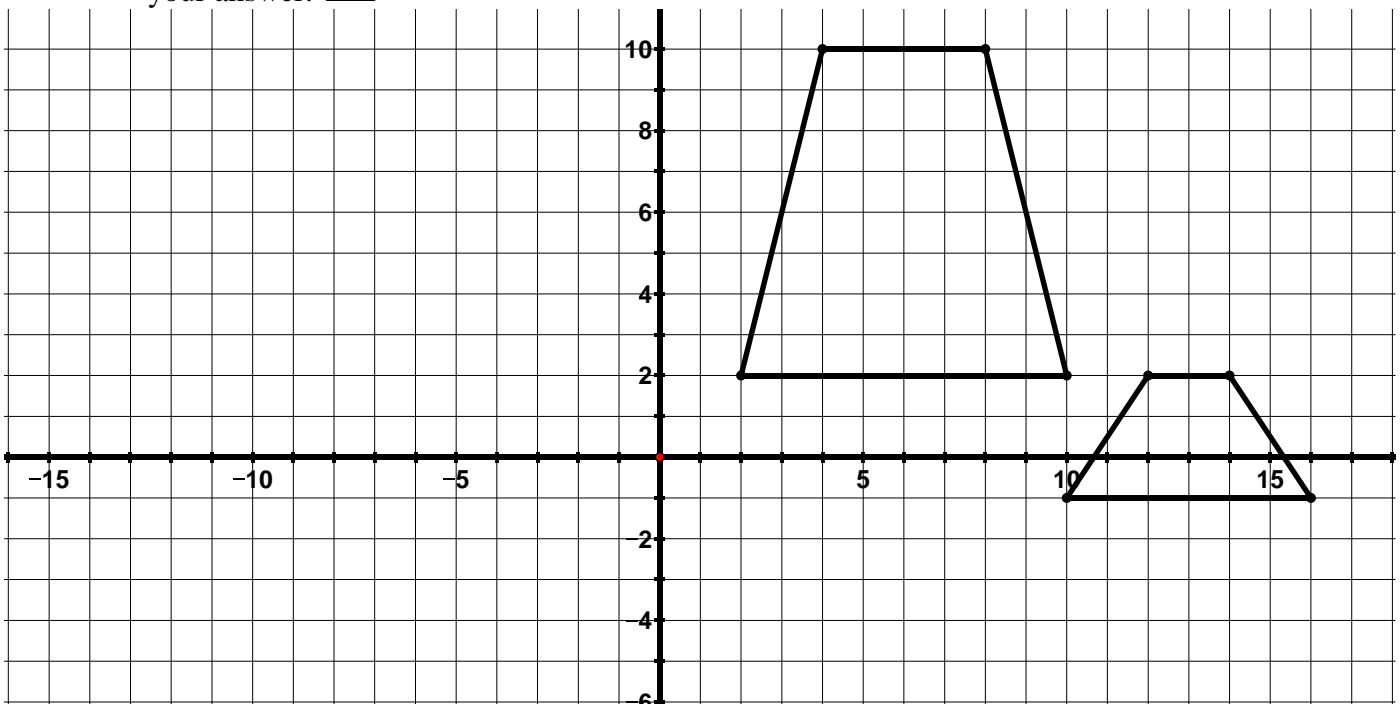
3. The triangles below are similar.



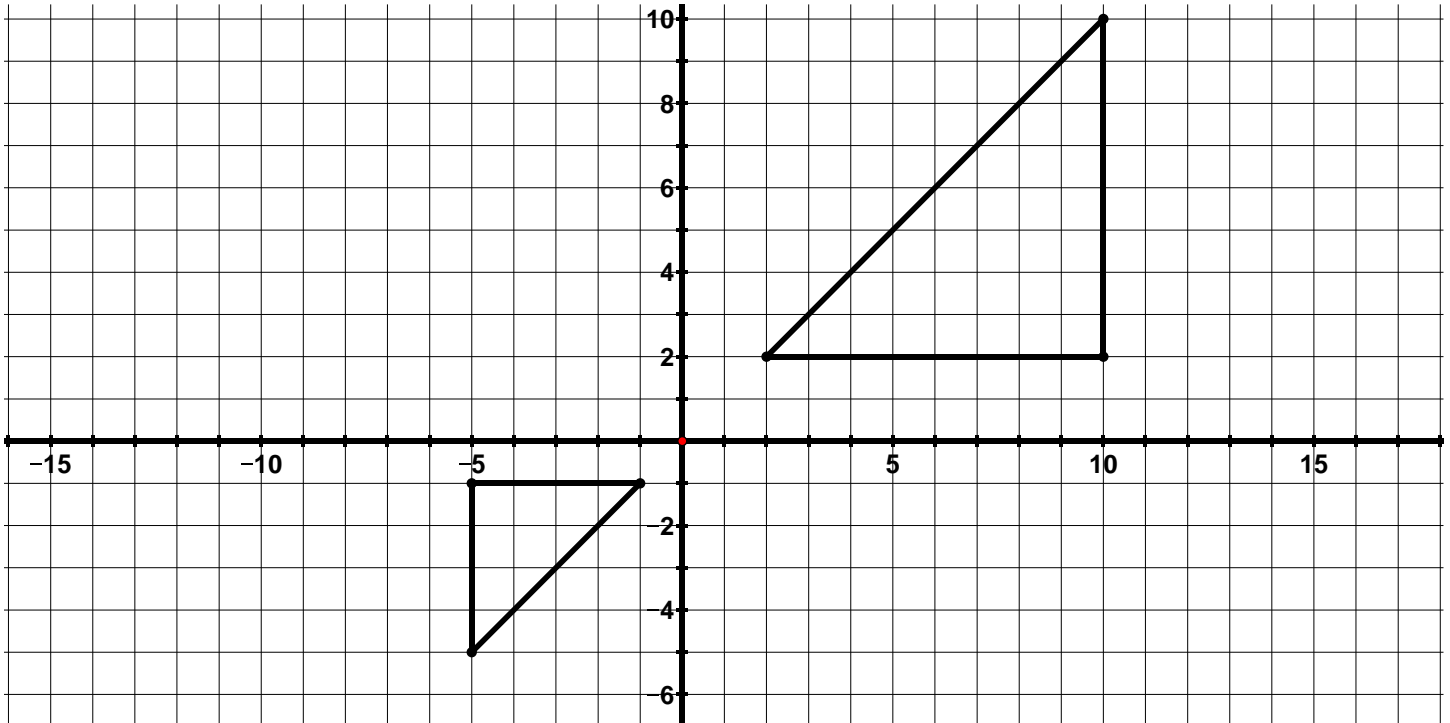
a. List the sequence of transformations that verifies the similarity of the two figures.

b. Write a similarity statement for the triangles.

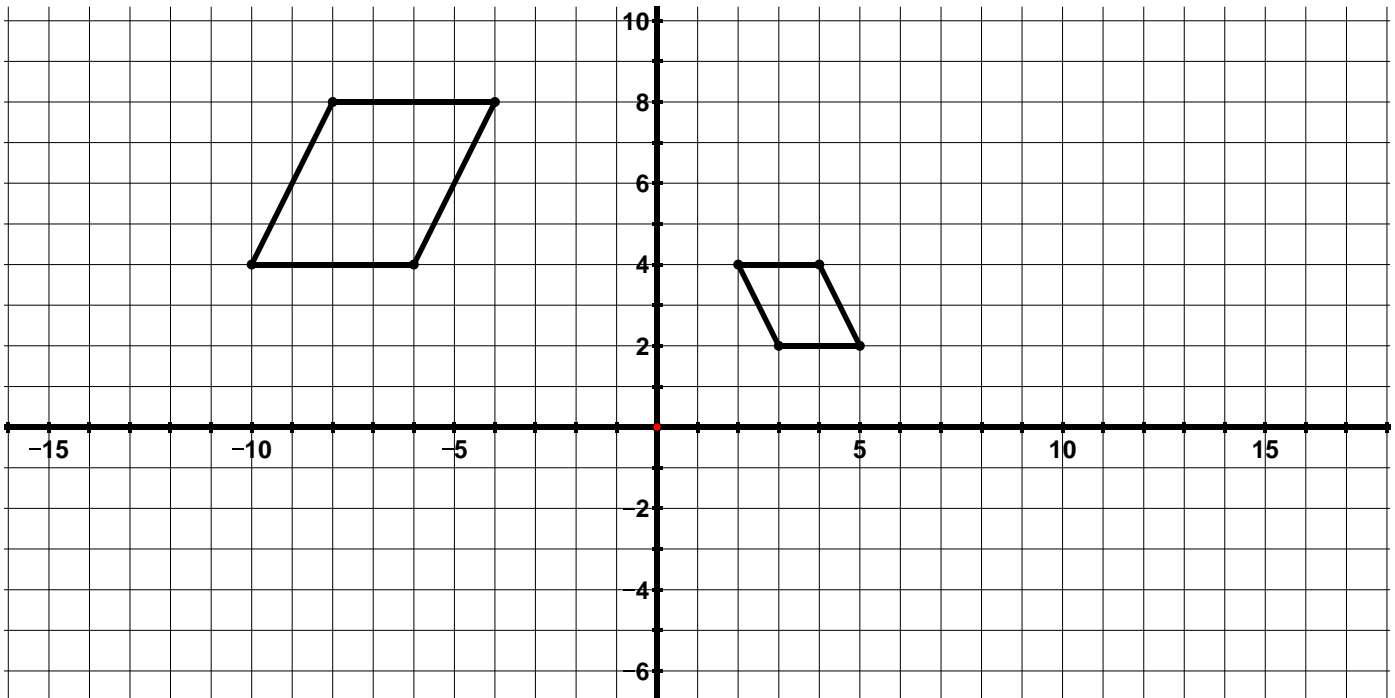
4. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



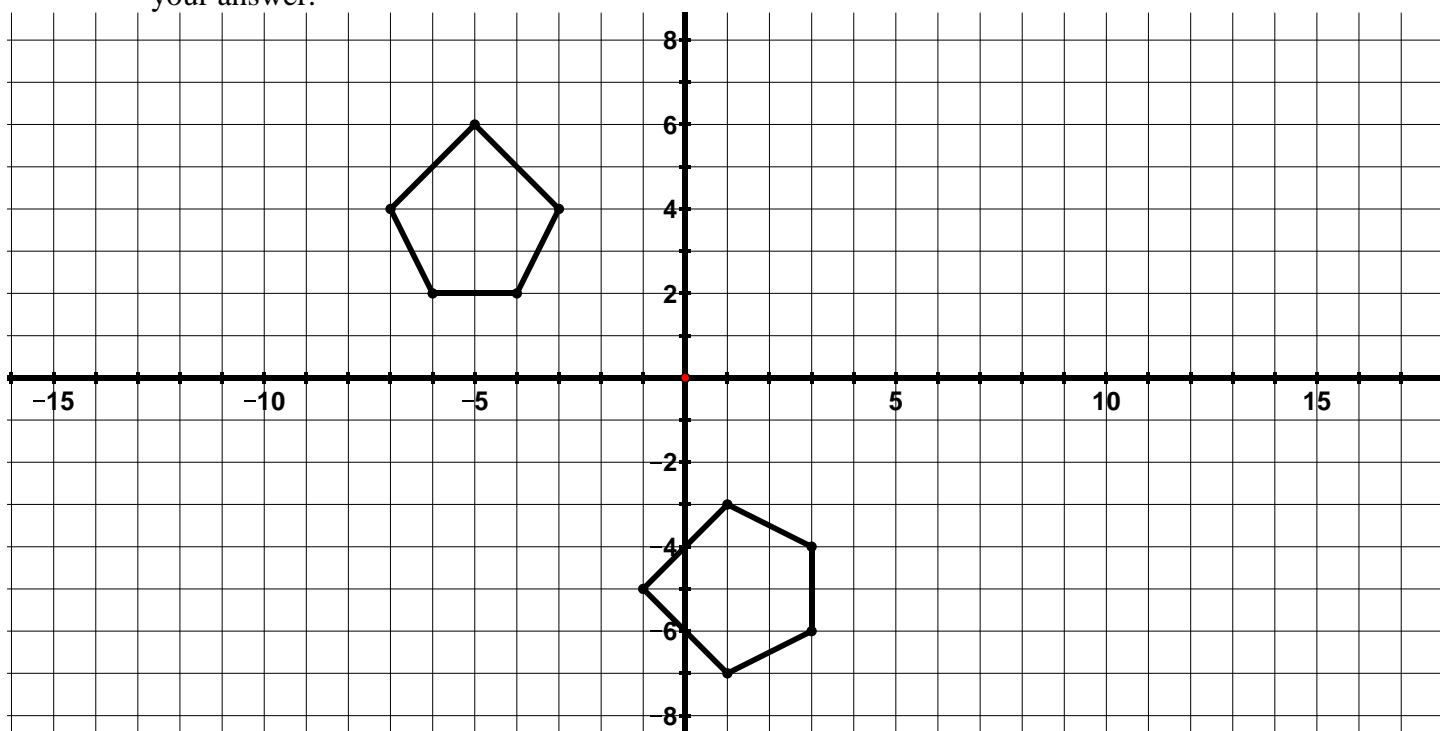
5. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.





6. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



7. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.

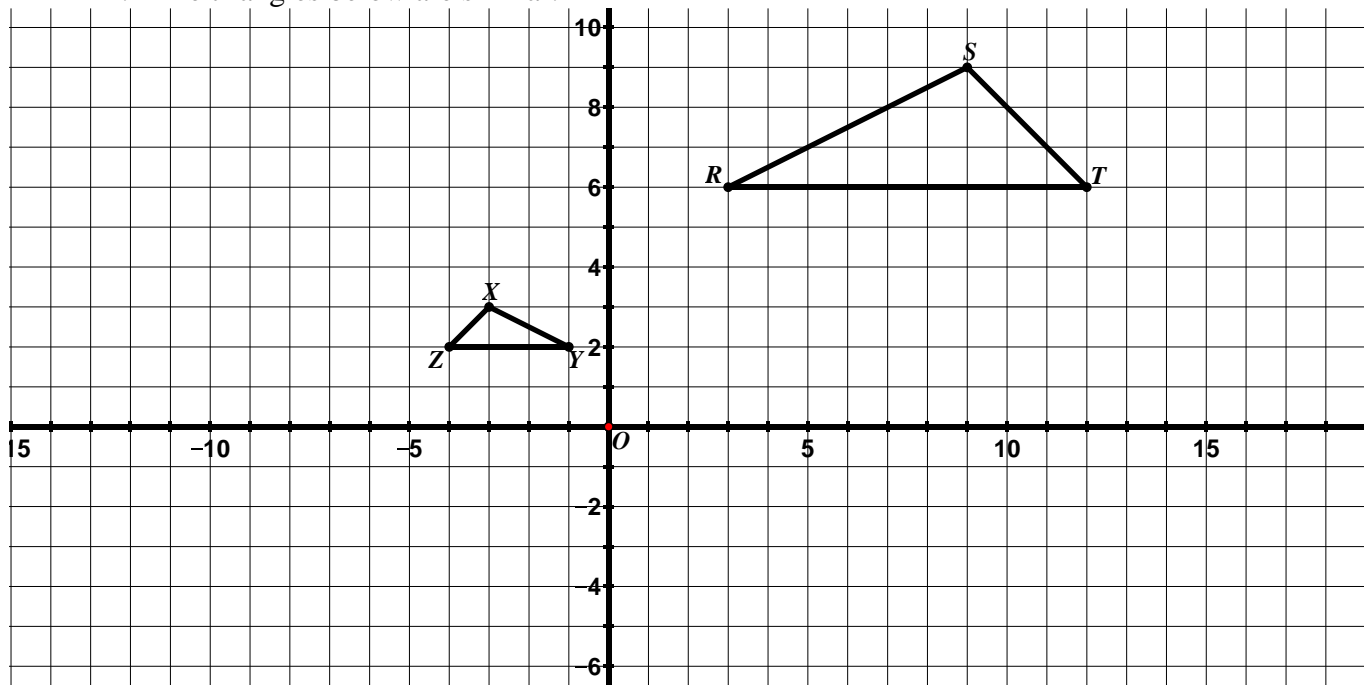


8. Determine whether the following transformations or sequence of transformations will result in similar figures, congruent figures, or neither. Provide a justification for your answer.  

- $(x, y) \rightarrow (x - 6, y + 2)$
- $(x, y) \rightarrow (-x, y)$ followed by $(x, y) \rightarrow (2x, 2y)$
- $(x, y) \rightarrow (2x, 3y)$ followed by a reflection across the x -axis
- $(x, y) \rightarrow (x + 5, y + 5)$ followed by a 90° rotation counterclockwise about the origin
- $(x, y) \rightarrow \left(\frac{1}{4}x, \frac{1}{4}y\right)$ followed by $(x, y) \rightarrow (y, x)$
- $(x, y) \rightarrow (x, y + 4)$ followed by a 180° rotation clockwise about the origin

9.2f Homework: Similarity cont.

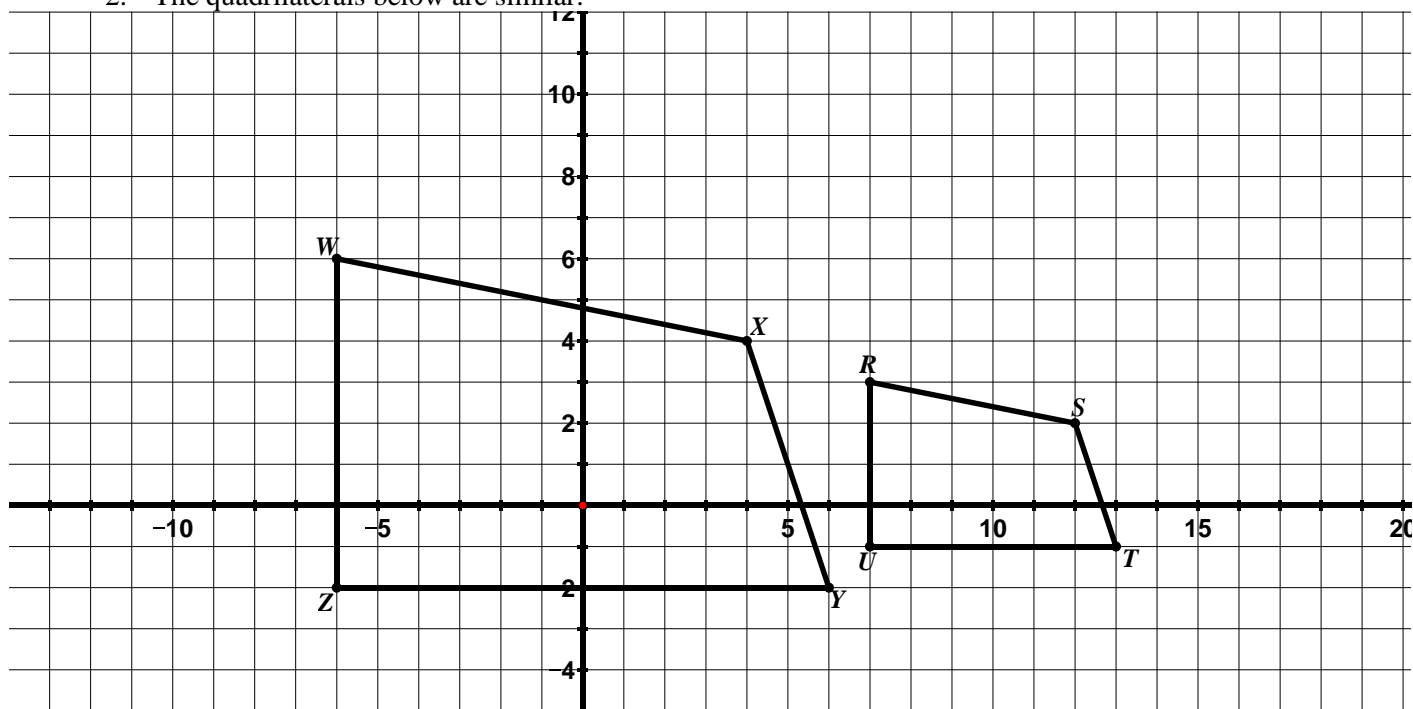
1. The triangles below are similar.



a. List the sequence of transformations that verifies the similarity of the two figures.

b. Write a similarity statement for the triangles.

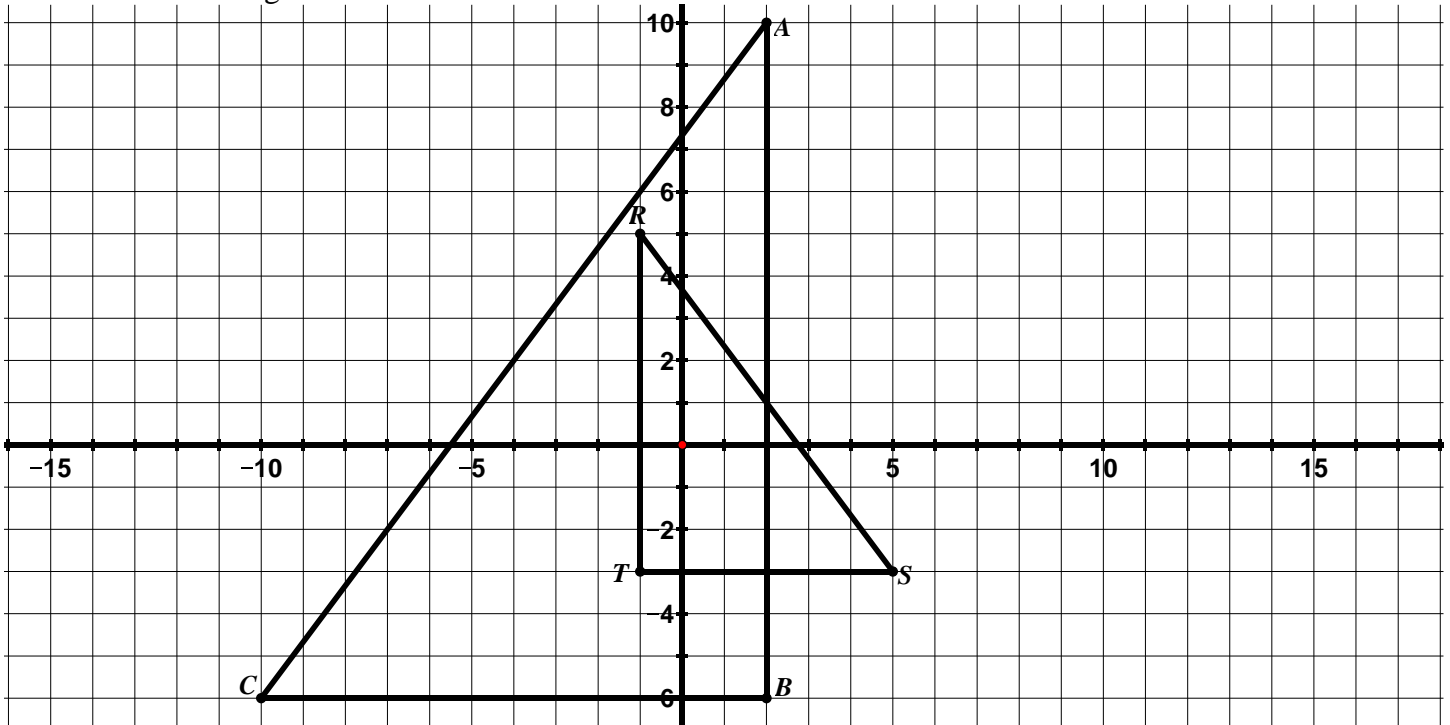
2. The quadrilaterals below are similar.



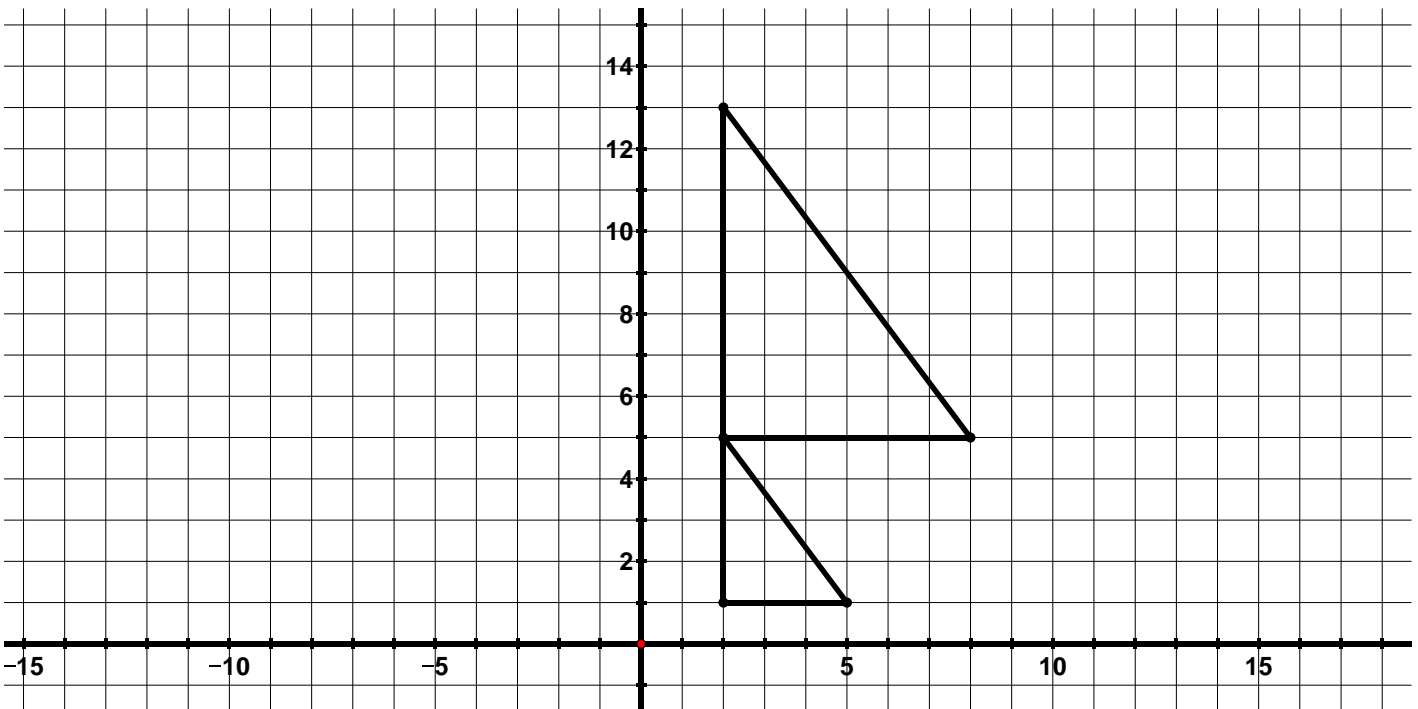
a. List the sequence of transformations that verifies the similarity of the two figures.

b. Write a similarity statement for the quadrilaterals.

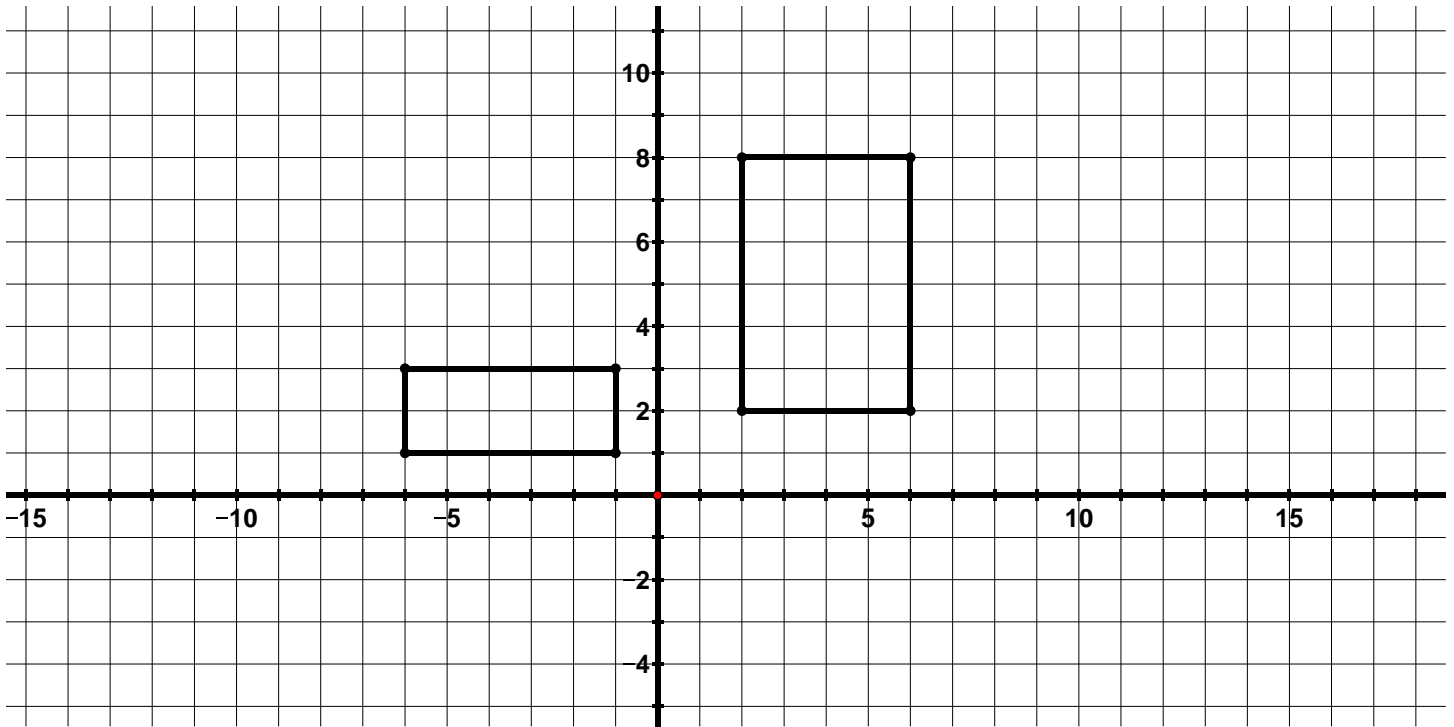
3. The triangles below are similar.



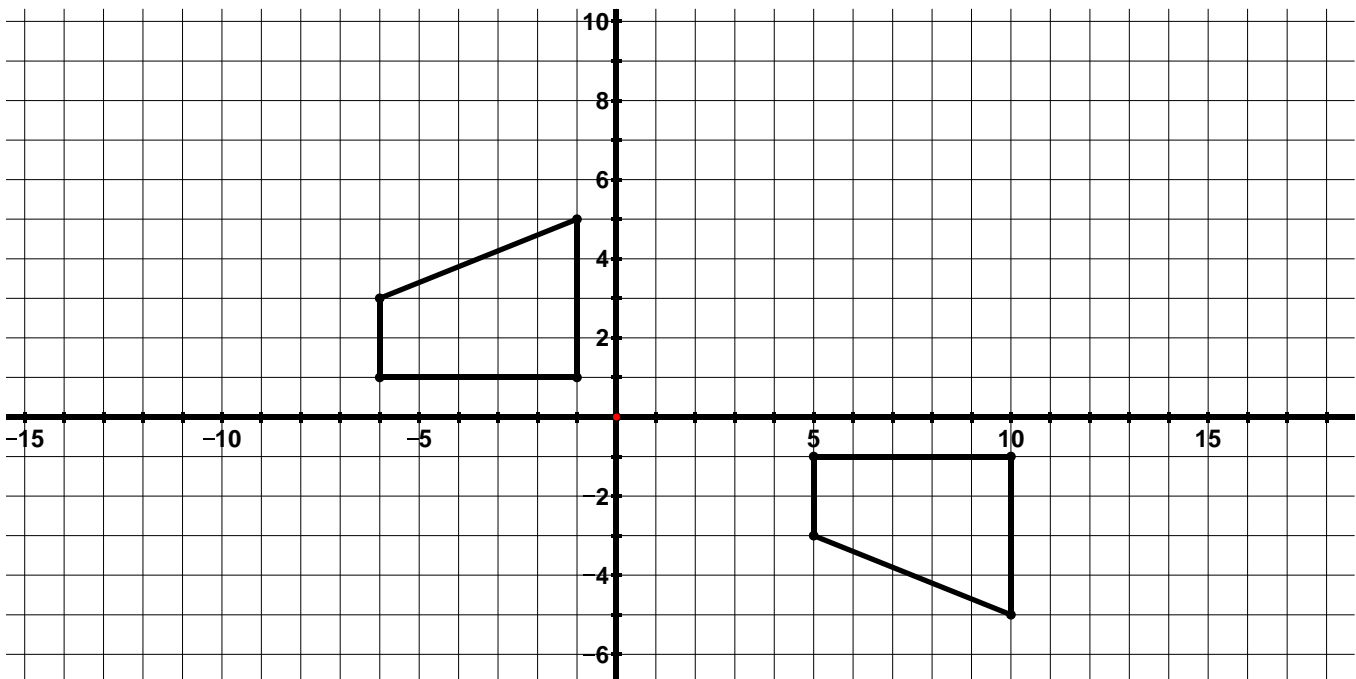
- a. List the sequence of transformations that verifies the similarity of the two figures.
 - b. Write a similarity statement for the triangles.
4. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



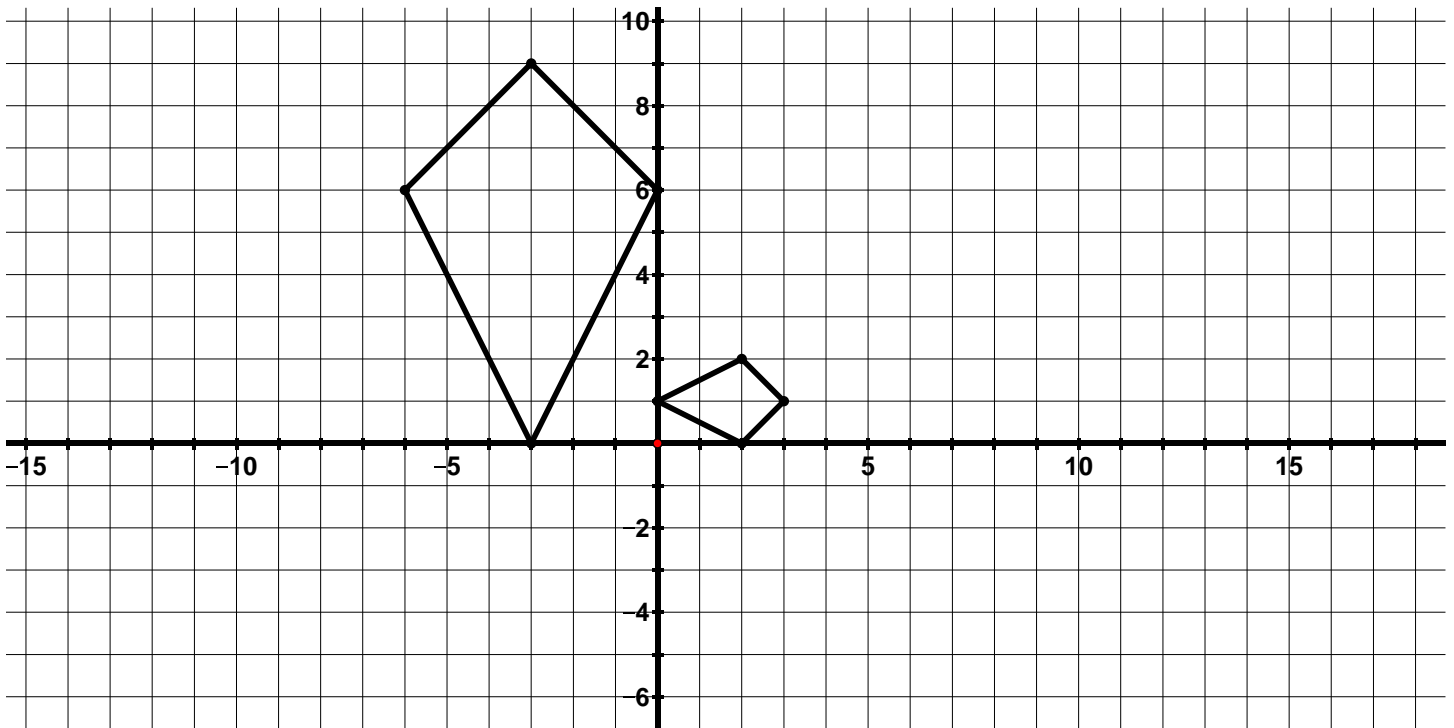
5. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



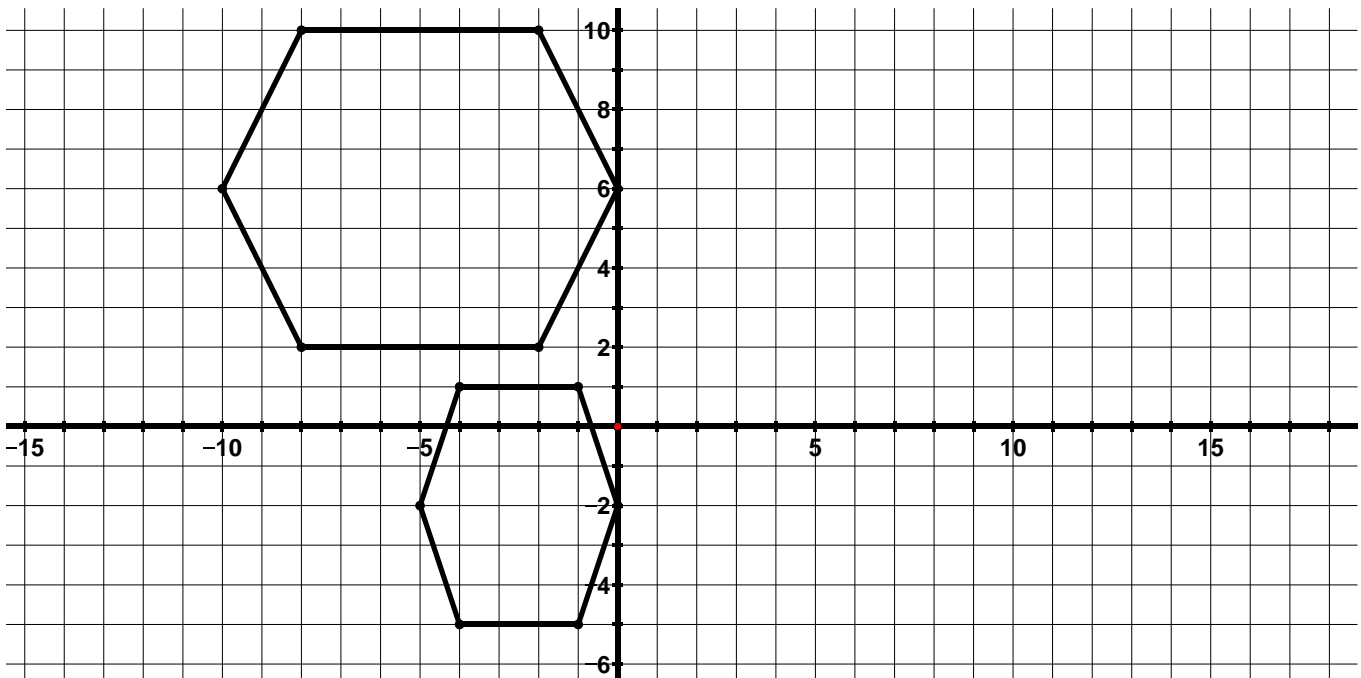
6. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



7. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



8. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



9. Determine whether the following transformations or sequence of transformations will result in similar figures, congruent figures, or neither.
- a. $(x, y) \rightarrow (x + 6, 6y)$
 - b. $(x, y) \rightarrow (x, -y)$ followed by a 270° rotation clockwise about the origin
 - c. $(x, y) \rightarrow (4x, 4y)$ followed by $(x, y - 2)$
 - d. A 180° rotation counterclockwise about the origin followed by $(x + 2, y + 2)$
 - e. $(x, y) \rightarrow (3x, x + y)$
 - f. $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$ followed by a reflection across the x -axis
 - g. Write your own transformation or sequence of transformations that will result in two figures that are congruent.
 - h. Write your own transformation or sequence of transformations that will result in two figures that are similar.
 - i. Write your own transformation or sequence of transformations that will result in two figures that are neither congruent nor similar.

9.2g Self-Assessment: Section 9.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

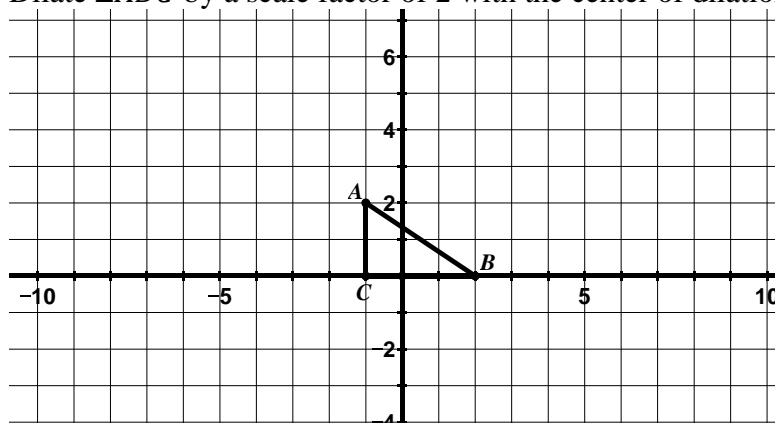
Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Describe the properties of a figure that has been dilated.				
2. Perform a dilation given a scale factor and center of dilation.				
3. Describe a dilation in words and using a coordinate rule.				
4. Determine the center of dilation using the properties of dilations.				
5. Understand what it means for two figures to be similar.				
6. Determine if two figures are similar.				
7. Given two figures that are similar, describe the sequence of transformations that exhibits the similarity between them.				

Section 9.2 Sample Problems (For use with self-assessment)

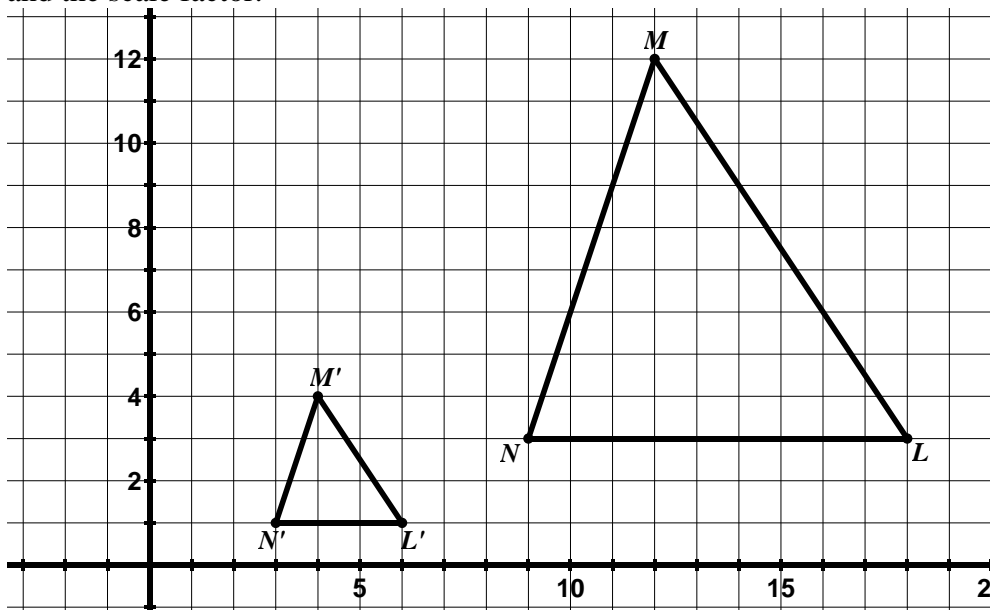
1. Which of the following properties are true about a figure that has only been dilated. Check all that apply.

- The ratios of the image segments to their corresponding pre-image segments are equal to the scale factor.
- Corresponding vertices lie on the same circle as the center of dilation.
- Corresponding vertices lie on the same line as the center of dilation.
- Corresponding segments are parallel.
- Corresponding segments are perpendicular.
- The area of the image is always the same as the area of the pre-image.

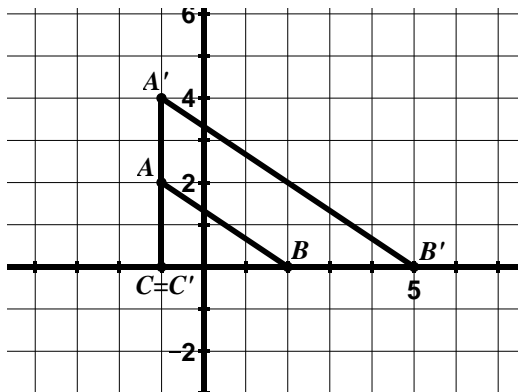
2. Dilate $\triangle ABC$ by a scale factor of 2 with the center of dilation at the origin.



3. Describe the dilation below in words and with a coordinate rule. Be sure to specify the center of dilation and the scale factor.

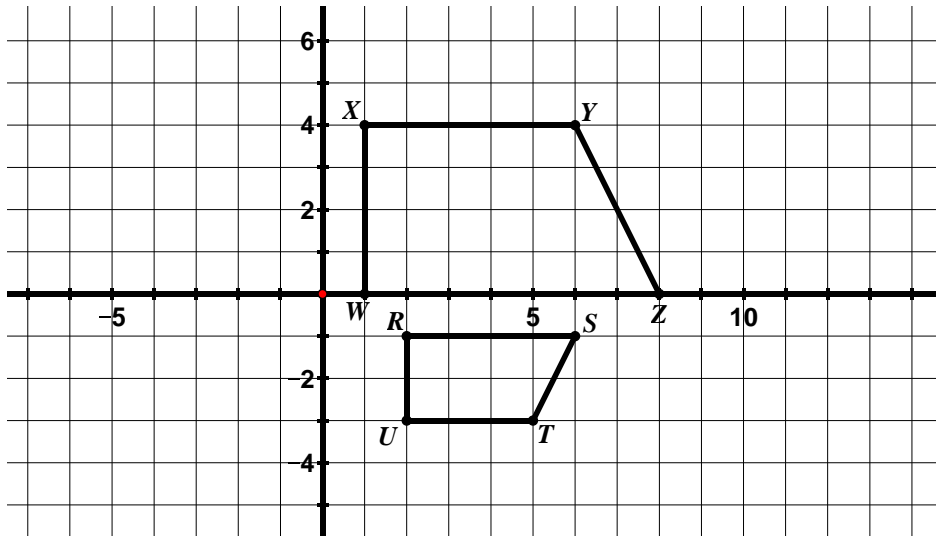


4. $\triangle ABC$ was dilated to produce $\triangle A'B'C'$. Determine the scale factor and center of dilation.

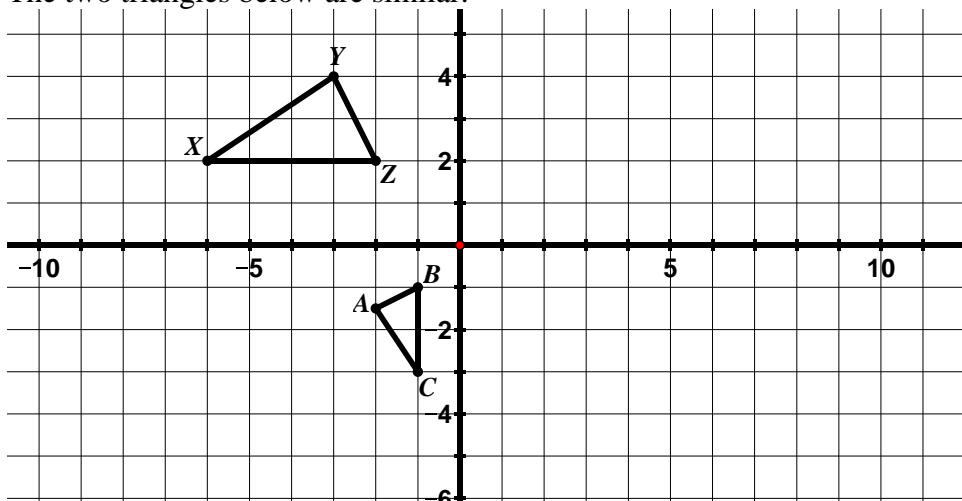


5. Describe in your own words what it means for two figures to be similar.

6. Are the parallelograms shown below similar? Provide a justification for your response.



7. The two triangles below are similar.



- Describe a sequence of transformations that verifies the similarity of the triangles.
- Write a similarity statement for the triangles.
- Determine which angles are congruent.
- Complete the following statements:

$$\frac{BC}{ZX} =$$

$$\frac{ZY}{BA} =$$

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Chapter 10 Geometry: Angles, Triangles, and Distance (3 weeks)

Utah Core Standard(s):

- Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. (8.G.5)
- Explain a proof of the Pythagorean Theorem and its converse. (8.G.6)
- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (8.G.7)
- Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (8.G.8)

Academic Vocabulary:

right triangle, right angle, congruent, leg, hypotenuse, Pythagorean Theorem, converse of Pythagorean Theorem, simplest radical form, Pythagorean triple, rectangular prism, cube, unit cube, distance formula, vertical angles, adjacent angles, straight angles, supplementary, congruent, parallel lines, \parallel , transversal, vertex, point of intersection, corresponding angles, alternate interior angles, alternate exterior angles, similar, angle-angle criterion for triangles

Chapter Overview:


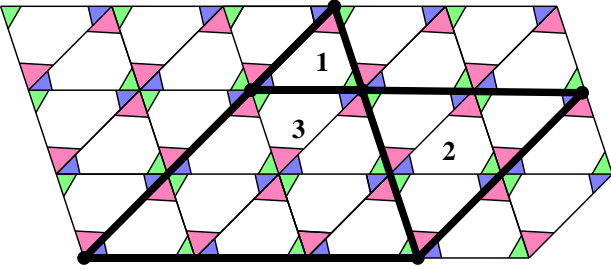


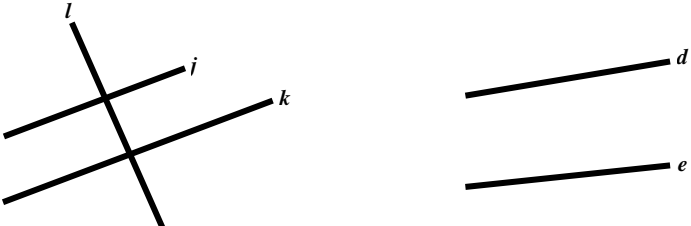
This chapter centers around several concepts and ideas related to angles and triangles. In the first section, students will study theorems about the angles in a triangle, the special angles formed when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. They will apply these theorems to solve problems. In Sections 2 and 3, students will study the Pythagorean Theorem and its converse and realize the usefulness of the Pythagorean Theorem in solving many real-world problems. In this chapter, we are referring to these theorems as a collection of facts. The focus in 8th grade is that students are able to observe these facts through examples, exploration, and concrete models. Students will explain why the theorems are true by constructing mathematical arguments, relying on knowledge acquired throughout the year, particularly the properties of rigid motion and dilations and the understanding of congruence and similarity. The explanations and arguments made by students will come in many different forms, including a bulleted list, a narrative paragraph, a diagram without words, and proof by example. They should give their arguments and explanations within their writing and speaking. The emphasis is on students starting to gain an understanding of what makes a good argument or explanation. Can they explain things in a number of different ways? Can they critique the reasoning of others? They should be asking themselves questions such as: What do I know? What is the question asking? Can I draw a model of the situation? Does my argument/explanation have a claim, evidence, and warrant? What is the connection? These practices engaged in by students set the foundation for a more formal study of proof in Secondary II.


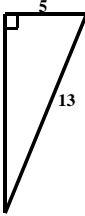

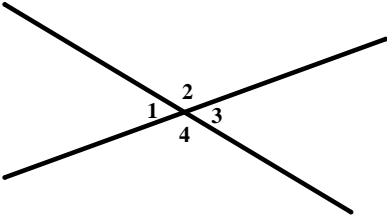

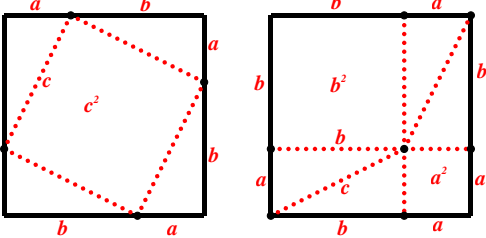
Connections to Content:


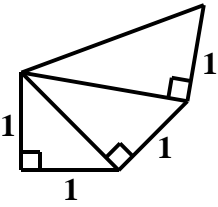

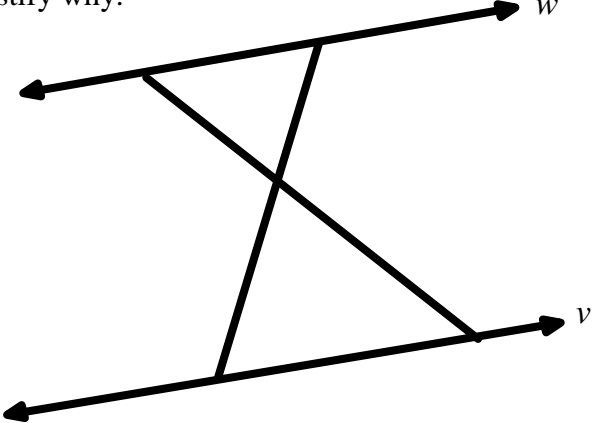
Prior Knowledge: In elementary grades, students have worked with geometric objects such as points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. They have also studied the different types of triangles (right, acute, and obtuse and equilateral, scalene, and isosceles). They have also learned and used facts about supplementary, complementary, vertical, and adjacent angles. In Chapter 9 of this text, students studied rigid motions and dilations and the definition of congruence and similarity.

Future Knowledge: In Secondary II, students will formally prove many of the theorems studied in this chapter about lines, angles, triangles, and similarity. They will also define trigonometric ratios and solve problems involving right triangles.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

	<p>Make sense of problems and persevere in solving them.</p>	 <p>What is the relationship between the triangles formed by the dark lines? Justify your answer. <i>Students will use the concrete model shown above in order to make arguments about several of the theorems studied in this chapter. They will also rely on their knowledge of rigid motions and dilations.</i></p>
	<p>Model with mathematics.</p>	<p>A new restaurant is putting in a wheelchair ramp. The landing from which people enter the restaurant is 2 feet higher than street level. Safety standards require that for every 1 foot of rise on a wheelchair ramp there must be a run of 12 feet. How long will the ramp have to be to meet safety standards? Round your answer to the nearest tenth. <i>Students will apply the Pythagorean Theorem in order to solve many real-world problems. They will have to analyze the situation to determine if the Pythagorean Theorem can be used to solve the problem, draw a picture of the situation, analyze givens and constraints, and understand what they are solving for.</i></p>
	<p>Construct viable arguments and critique the reasoning of others.</p>	<p>Suppose you are given two lines j and k in the picture below. You have been asked to determine whether the two lines are parallel. You start by drawing the transversal l through the two lines as shown below. Devise a strategy to determine whether the two lines are parallel using what you know about the properties of rigid motion. Next, use your strategy to determine whether lines d and e are parallel. Just saying they do not look parallel, is not a justification.</p>  <p><i>Throughout the chapter, students will observe theorems about angles and triangles by example, exploration, and concrete models. Students will construct mathematical arguments as to why the theorems are true, relying on knowledge acquired throughout the year, particularly the properties of rigid motion and dilations and the understanding of congruence and similarity. Students will begin to understand the necessary elements of what makes a good proof as outlined in the chapter overview.</i></p>

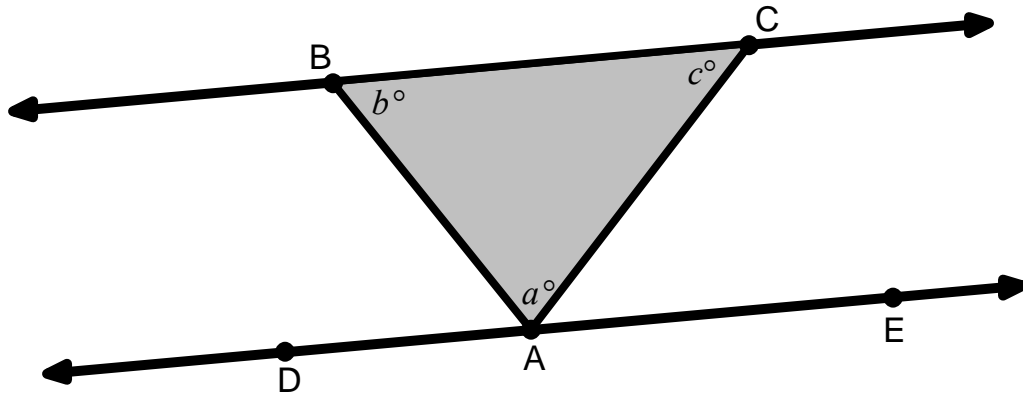
	<p>Attend to Precision</p>	<p>Find, Fix, and Justify: Megan was asked to solve for the unknown side length in the triangle below. Her work is shown below. She made a mistake when solving. Explain the mistake she made and then solve the problem correctly.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: left;"> <p><u>Megan's Solution:</u></p> $a^2 + b^2 = c^2$ $5^2 + 13^2 = c^2$ $25 + 169 = c^2$ $194 = c^2$ $\sqrt{194} = c$ </div> </div> <p><i>This problem requires that students are clear in their understanding of the Pythagorean Theorem and how to use it to solve for missing side lengths.</i></p>
	<p>Use appropriate tools strategically.</p>	<div style="text-align: center;">  </div> <p>Use ideas of rigid motion to prove that $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. <i>Students will rely heavily on the knowledge learned in Chapter 9 about rigid motions and congruence and dilations and similarity. This knowledge will be a tool they apply to understand and informally prove many of the theorems about angles, triangles, and similarity in this chapter.</i></p>
	<p>Reason abstractly and quantitatively.</p>	<div style="display: flex; justify-content: center; align-items: center;">  </div> <p>Using the picture above, prove that the sum of the areas of the squares along the two smaller sides of the right triangle equals the area of the square along the larger side of the triangle for <u>any</u> right triangle.</p> <p><i>Students first begin to study and understand the Pythagorean Theorem using concrete examples. Then, they move to an abstract proof of the Pythagorean Theorem to show that it holds true for any right triangle.</i></p>

	<p>Look for and express regularity in repeated reasoning.</p>	<p>Use the picture below to answer questions a) and b).</p>  <p>a. Find all the missing side lengths and label the picture with the answers. b. Using the picture above, devise a strategy for constructing a segment with a length of $\sqrt{5}$. Explain your strategy below.</p> <p><i>In this problem, students should start to notice that the hypotenuse of the new triangle will follow a pattern. This observation gives them a process for constructing any segment of length \sqrt{n} where n is a whole number.</i></p>
	<p>Look for and make use of structure.</p>	<p>Given that line $w \parallel$ line v, determine if the triangles given below are similar. If they are similar justify why.</p>  <p><i>In the problem above students must look at the geometric figure above and evaluate the information given to them. They are given that line $w \parallel$ line v. They must recognize that the two intersecting lines that form the triangles are transversals of the parallel lines. Students might do this by extending the transversals beyond the interior of the two parallel lines or by drawing an auxiliary line over these lines that extends beyond the parallel lines. Once they look at these lines as transversals they can use what they know about special angle relationships to determine congruence amongst angles within the triangles. As students view the structure of the intersecting lines their perspective shifts and they are able to derive more information about the figure.</i></p>

10.0 Anchor Problem: Reasoning with Angles of a Triangle and Rectangles

Part I

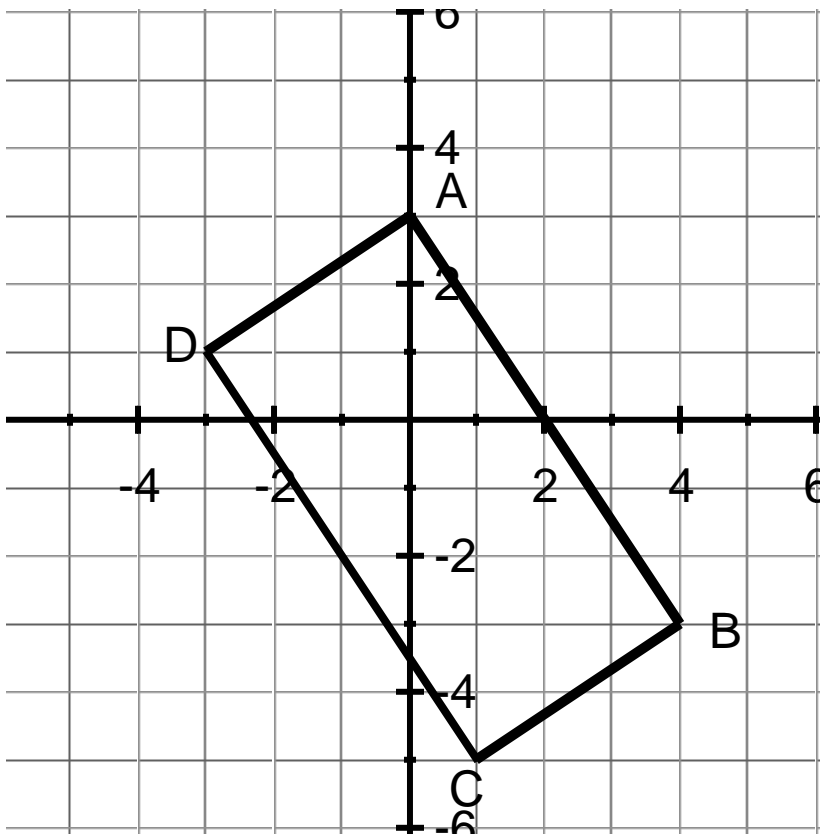
Given that $\overline{BC} \parallel \overline{DE}$ in the picture below, show that $a + b + c = 180^\circ$.



Part II

Pedro's teacher asks him to classify the quadrilateral below. He claims it is a rectangle. His teacher tells him to give a good argument and explanation. Help Pedro to support his claim using mathematical evidence.

Remember: Opposite sides of a rectangle have the same length and are parallel and the sides of a rectangle meet at right angles.



10.1 Angles and Triangles

Section Overview:

The focus of this section is on the development of geometric intuition through exploration with rigid motions and dilations. Through exploration, observation, and the use of concrete models, students will analyze facts about triangles and angles and use these facts to describe relationships in geometric figures. There will also be a focus on making sound mathematical explanations and arguments in order to verify theorems about angles and triangles and when explaining and justifying solutions to problems throughout the section.

Concepts and Skills to Master:

By the end of this section, students should be able to:

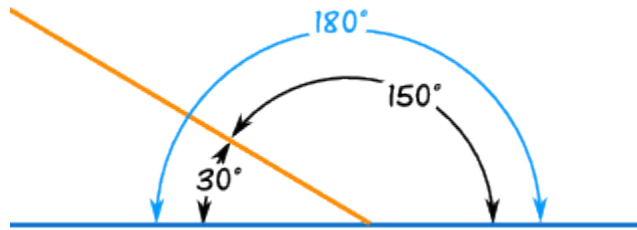
1. Know that straight angles sum to 180° and that vertical angles are congruent.
2. Know that the sum of the angles in a triangle is 180° . Understand that the measure of an exterior angle of a triangle is equal to the sum of the measures of the non-adjacent angles. Use these properties to find missing angle measures related to a triangle.
3. Determine the relationship between angles formed when a transversal intersects parallel lines. Use these relationships to find missing angle measures.
4. Determine whether two lines are parallel based on the angle measures when a transversal intersects the lines.
5. Understand and apply the angle-angle criterion to determine whether two triangles are similar.

10.1a Class Activity: Straight and Vertical Angles Review

In this section, you will observe and use several different geometric facts learned in previous grades. They will be denoted using bullets.

- **Angles that lie on the same line (straight angles) are supplementary.**

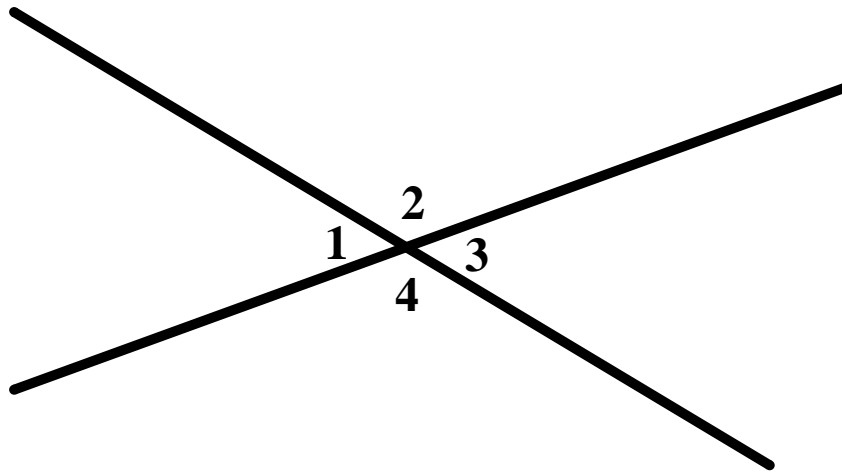
In 7th grade, you learned that a straight angle has a measure of 180° as shown below. Angles that sum to 180° are **supplementary**. In the picture below, 30° and 150° are supplementary and together they form a straight angle.



- **Vertical angles have the same measure.**

Vertical angles are the opposing angles formed by two intersecting lines.

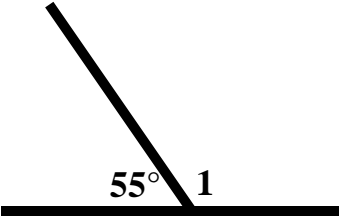
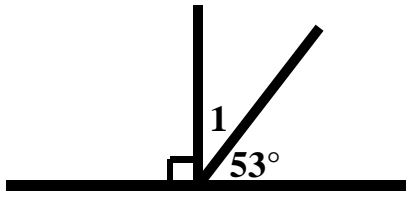
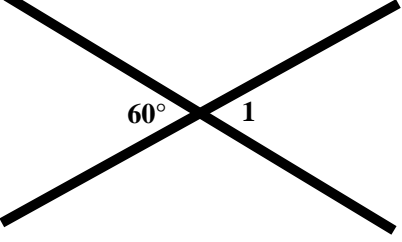
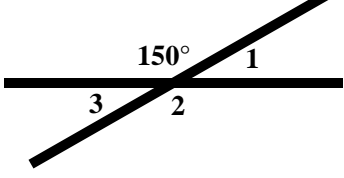
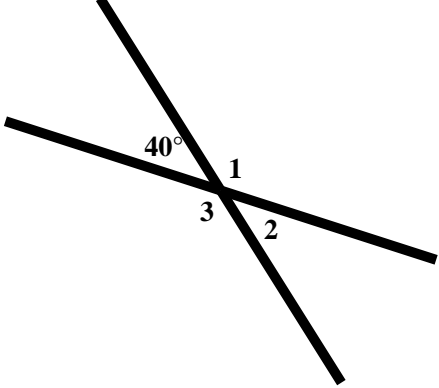
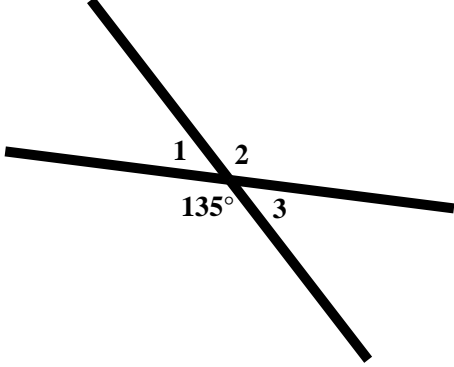
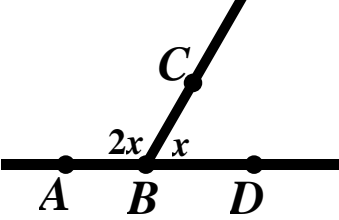
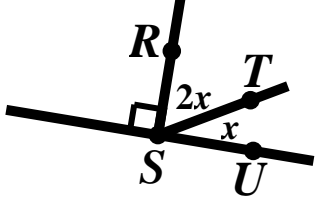
In the picture below, $\angle 1$ and $\angle 3$ are vertical angles and $\angle 2$ and $\angle 4$ are vertical angles.



1. Show that $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. (Hint: Think about ideas of rigid motion and straight angles.) 

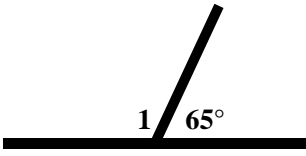
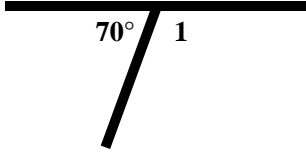
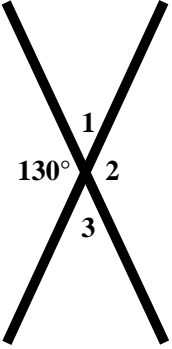
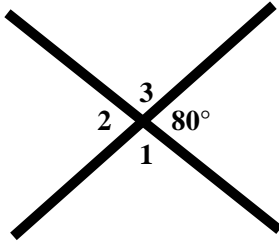
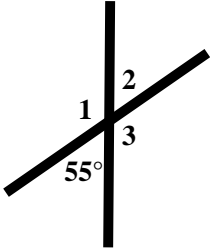
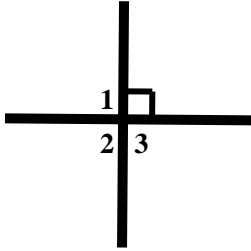
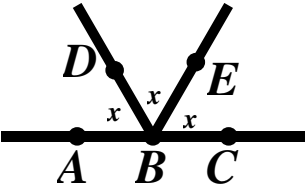
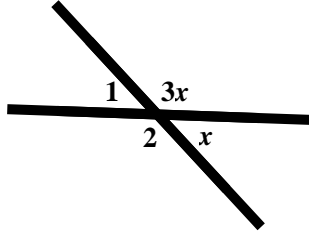
2. Which pairs of angles are supplementary in the picture above?

Review: Find the missing angle measures without the use of a protractor.

<p>3.</p>  <p>$m\angle 1 = \underline{\hspace{2cm}}$</p>	<p>4.</p>  <p>$m\angle 1 = \underline{\hspace{2cm}}$</p>
<p>5.</p>  <p>$m\angle 1 = \underline{\hspace{2cm}}$</p>	<p>6.</p>  <p>$m\angle 1 = \underline{\hspace{2cm}}$ $m\angle 2 = \underline{\hspace{2cm}}$</p> <p>$m\angle 3 = \underline{\hspace{2cm}}$</p>
<p>7.</p>  <p>$m\angle 1 = \underline{\hspace{2cm}}$ $m\angle 2 = \underline{\hspace{2cm}}$</p> <p>$m\angle 3 = \underline{\hspace{2cm}}$</p>	<p>8.</p>  <p>$m\angle 1 = \underline{\hspace{2cm}}$ $m\angle 2 = \underline{\hspace{2cm}}$</p> <p>$m\angle 3 = \underline{\hspace{2cm}}$</p>
<p>9.</p>  <p>$m\angle ABC = \underline{\hspace{2cm}}$</p> <p>$m\angle CBD = \underline{\hspace{2cm}}$</p>	<p>10.</p>  <p>$m\angle RST = \underline{\hspace{2cm}}$ $m\angle RSU = \underline{\hspace{2cm}}$</p> <p>$m\angle TSU = \underline{\hspace{2cm}}$</p>

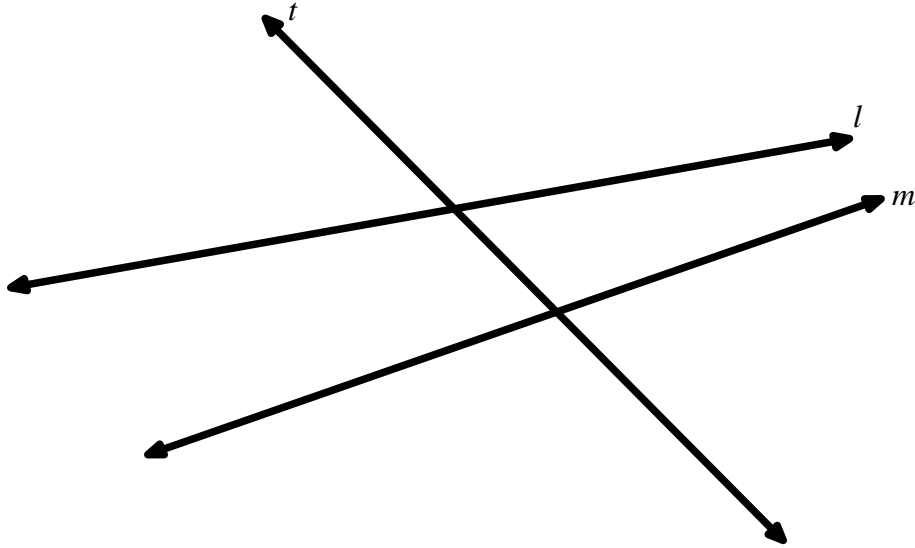
10.1a Homework: Straight and Vertical Angles Review

Review: Find the missing angle measures without the use of a protractor.

<p>1.</p>  <p>$m\angle 1 = \underline{\hspace{2cm}}$</p>	<p>2.</p>  <p>$m\angle 1 = \underline{\hspace{2cm}}$</p>
<p>3.</p>  <p>$m\angle 1 = \underline{\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}}$ $m\angle 3 = \underline{\hspace{1cm}}$</p>	<p>4.</p>  <p>$m\angle 1 = \underline{\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}}$ $m\angle 3 = \underline{\hspace{1cm}}$</p>
<p>5.</p>  <p>$m\angle 1 = \underline{\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}}$ $m\angle 3 = \underline{\hspace{1cm}}$</p>	<p>6.</p>  <p>$m\angle 1 = \underline{\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}}$ $m\angle 3 = \underline{\hspace{1cm}}$</p>
<p>7.</p>  <p>$m\angle ABD = \underline{\hspace{1cm}}$ $m\angle DBE = \underline{\hspace{1cm}}$</p> <p>$m\angle ABE = \underline{\hspace{1cm}}$ $m\angle ABC = \underline{\hspace{1cm}}$</p>	<p>8.</p>  <p>$m\angle 1 = \underline{\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}}$</p>

10.1b Class Activity: Special Angles Formed by Transversals

1. In the picture given below line l and line m are cut by a transversal line called t .



2. Define transversal in your own words. Draw another transversal for the two lines above and label it line r .

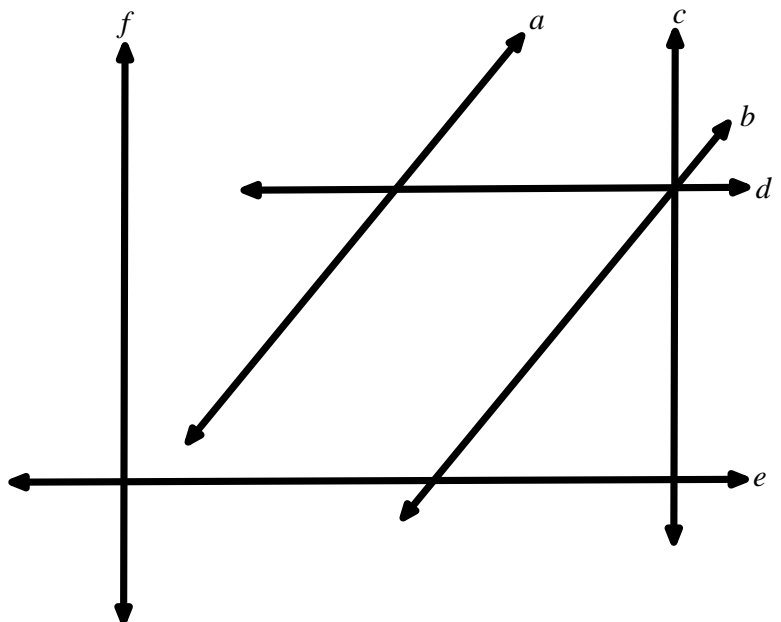
3. Some of the runways at a major airport are shown in the drawing below. Identify at least 2 sets of lines to which each line is a transversal.

a. line a

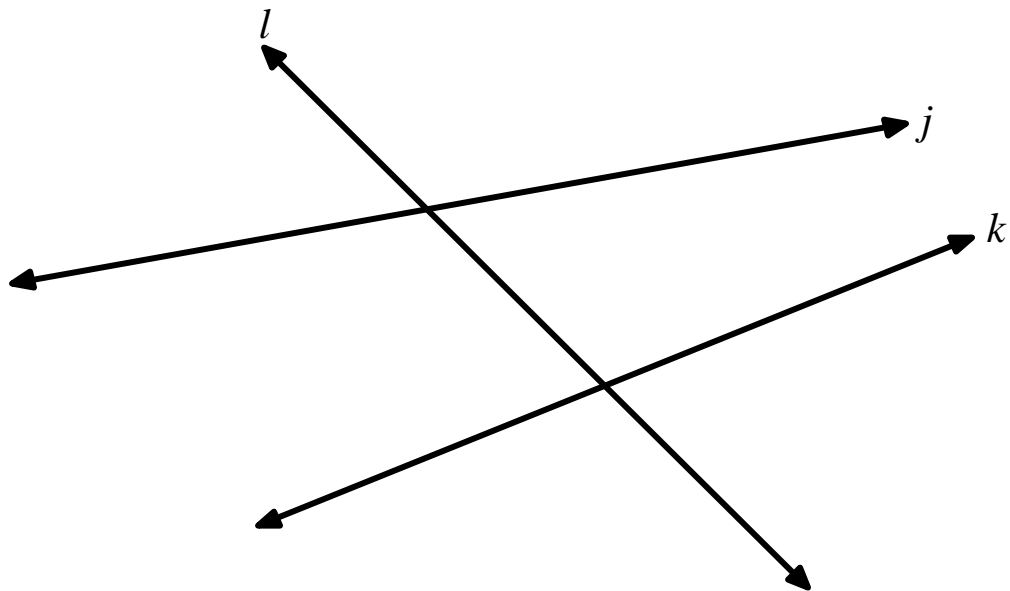
b. line b

c. line c

d. line e

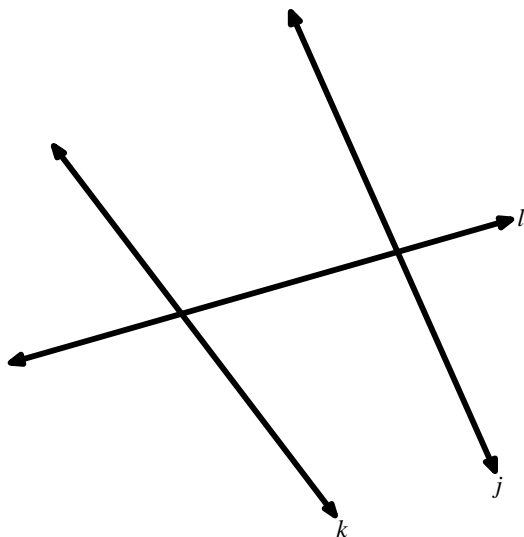
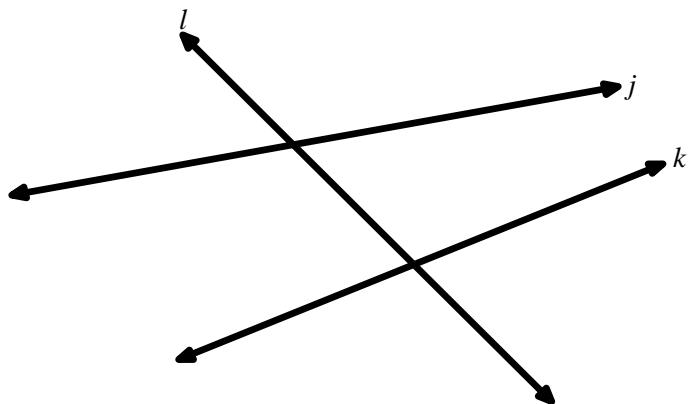


When two lines are intersected by a transversal there are special angle pairs that are formed. Use the angle names provided by your teacher to move the angle names around the picture below until you think you have found its correct location. Be ready to justify your reasoning. There will be several correct locations for each set of angle pairs and more than one term may fit at an angle.

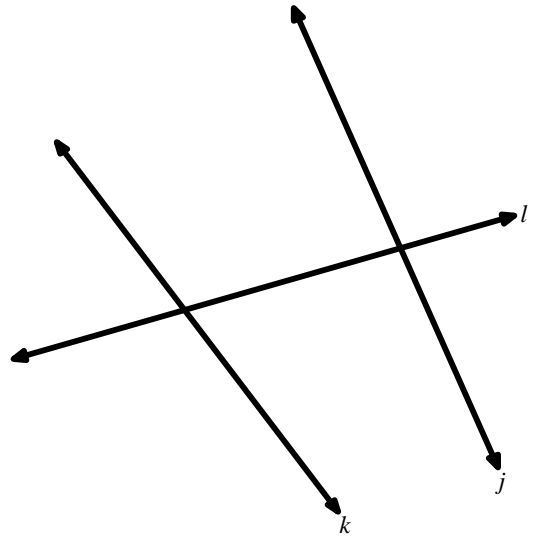
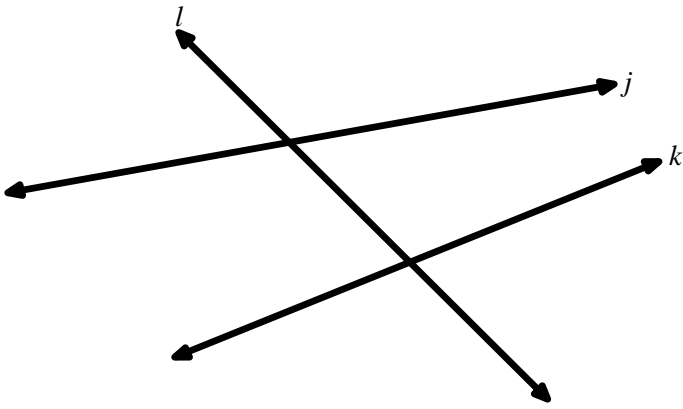


Directions: Color code the following sets of angles by coloring each set of angle pairs the same color. Find at least two sets of the special angles for each drawing.

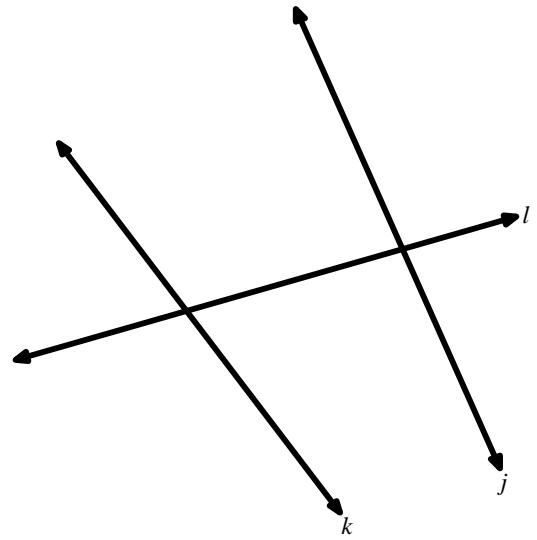
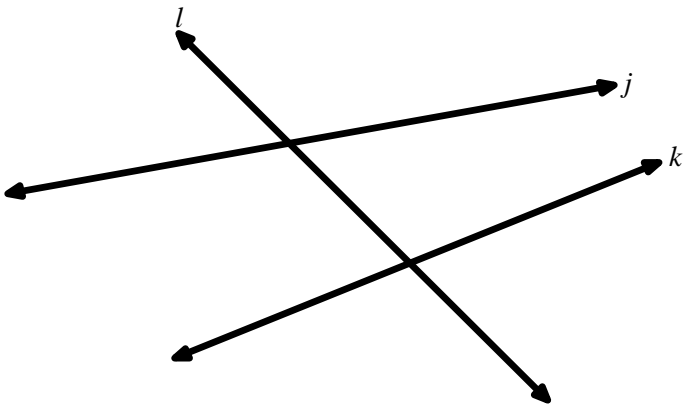
4. Alternate Exterior Angle Pairs



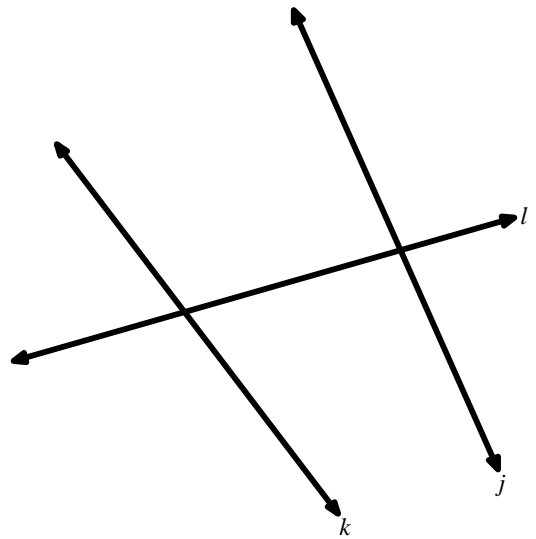
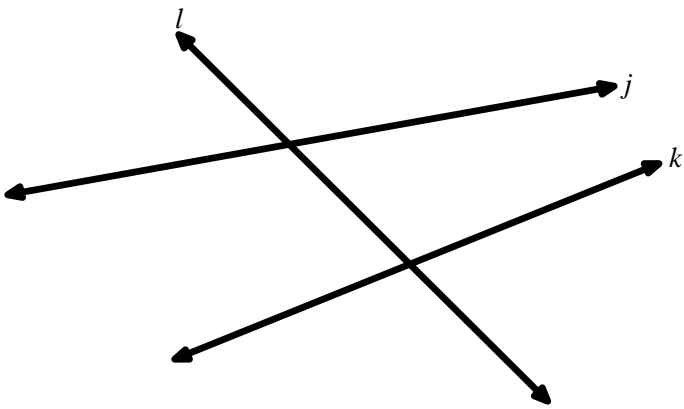
5. Alternate Interior Angle Pairs



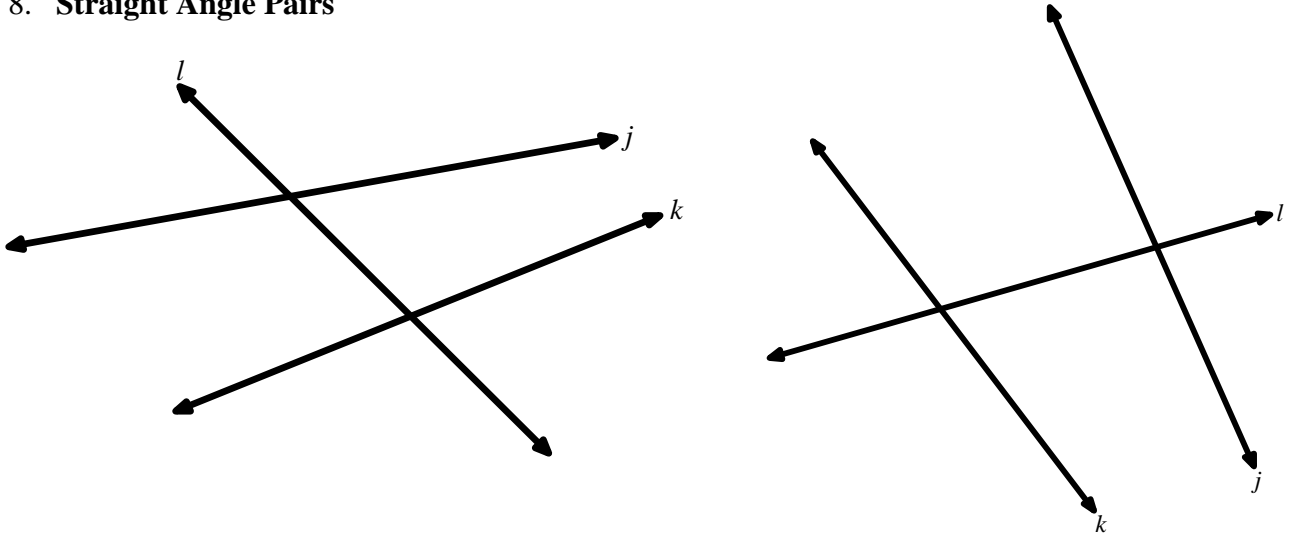
6. Corresponding Angle Pairs



7. Vertical Angle Pairs



8. Straight Angle Pairs



9. Refer to the figure below; identify the following pairs of angles as alternate exterior angles, alternate interior angles, corresponding angles, vertical angles, or straight angles.

a. $\angle 1$ and $\angle 8$

b. $\angle 12$ and $\angle 11$

c. $\angle 13$ and $\angle 21$

d. $\angle 14$ and $\angle 15$

e. $\angle 7$ and $\angle 14$

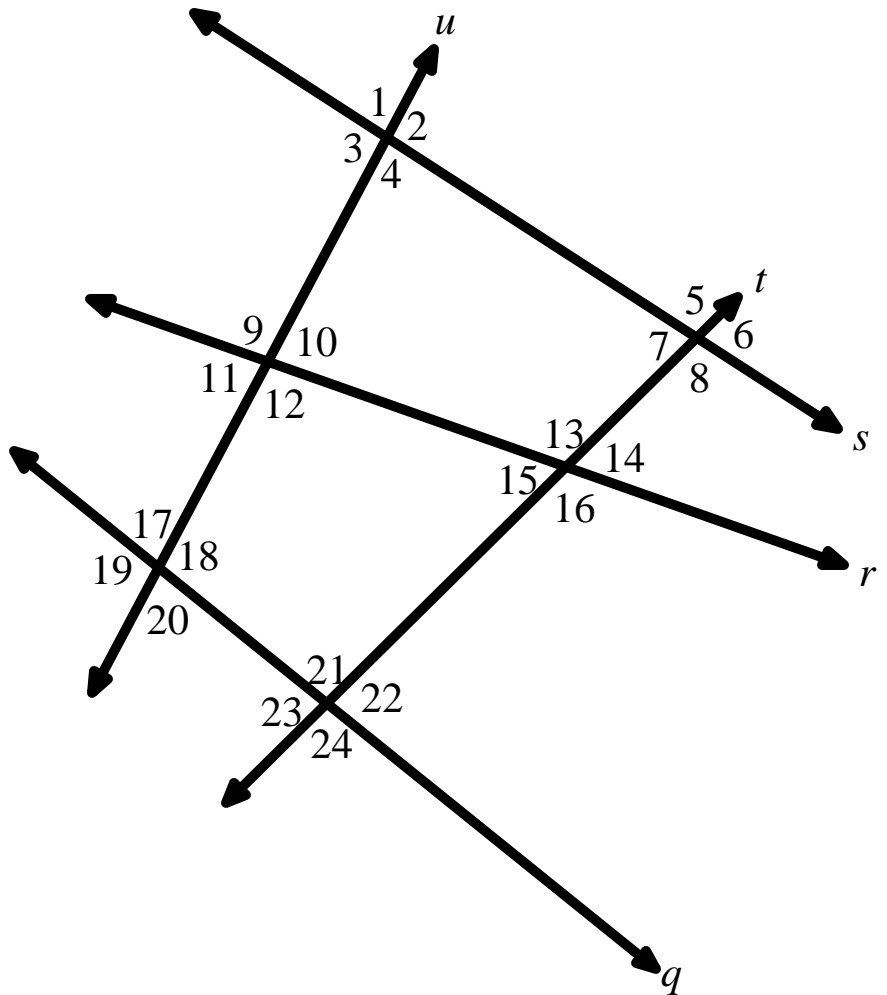
f. $\angle 9$ and $\angle 20$

g. $\angle 5$ and $\angle 7$

h. $\angle 22$ and $\angle 23$

i. $\angle 1$ and $\angle 5$

j. $\angle 21$ and $\angle 8$



10.1b Homework: Special Angles Formed by Transversals

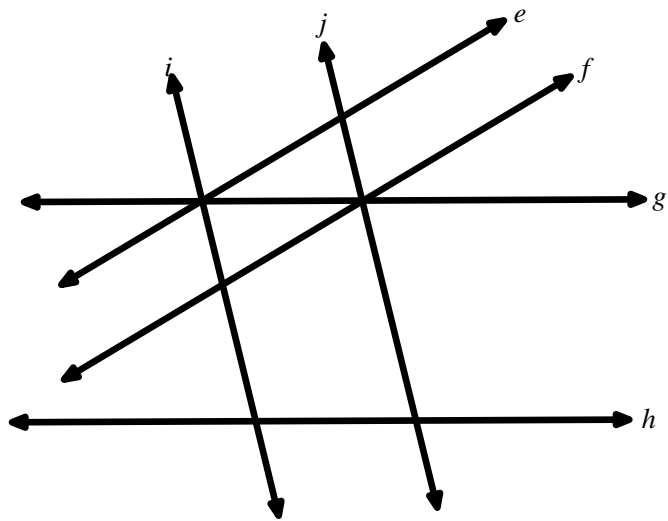
1. Identify the sets of given lines to which each line is a transversal.

a. line e

b. line g

c. line h

d. line j



2. Refer to the figures below. State if $\angle 1$ and $\angle 2$ are alternate exterior angles, alternate interior angles, corresponding angles, vertical angles, or straight angles.

<p>a.</p>	<p>b.</p>
<p>c.</p>	<p>d.</p>

3. Refer to the figure below; state if the following pairs of angles are alternate exterior angles, alternate interior angles, corresponding angles, vertical angles, or straight angles.

a. $\angle 4$ and $\angle 9$

b. $\angle 12$ and $\angle 11$

c. $\angle 1$ and $\angle 5$

d. $\angle 1$ and $\angle 8$

e. $\angle 6$ and $\angle 7$

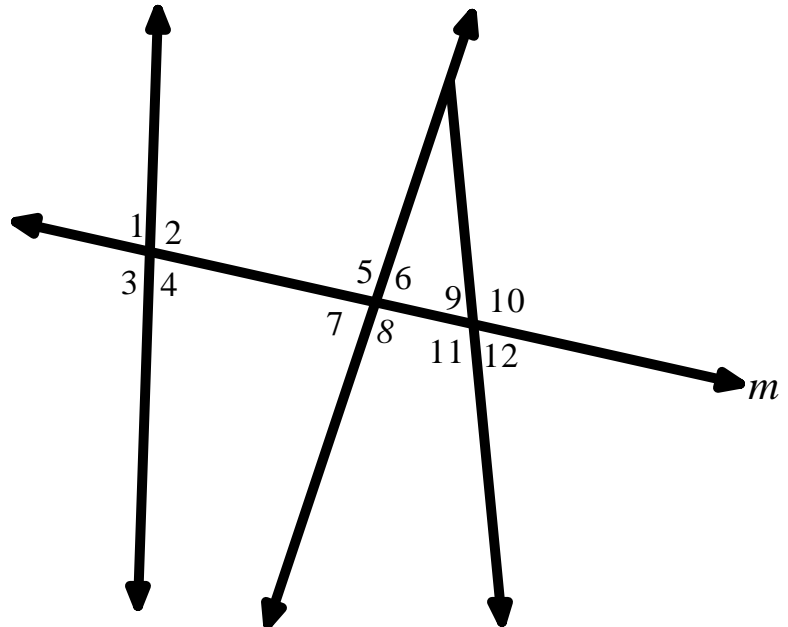
f. $\angle 1$ and $\angle 3$

g. $\angle 8$ and $\angle 9$

h. $\angle 7$ and $\angle 11$

i. $\angle 3$ and $\angle 10$

j. $\angle 10$ and $\angle 11$



Find, Fix, and Justify

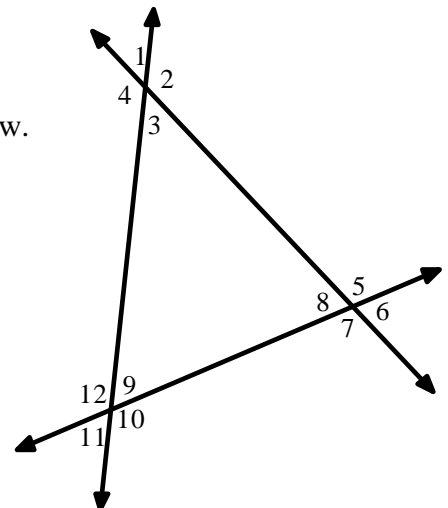
4. Patel and Ari are naming alternate interior angles for the figure below. They are listing alternate interior angle pairs for angle 3. Their work is shown below.

Patel

$\angle 3$ and $\angle 12$
 $\angle 3$ and $\angle 5$

Ari

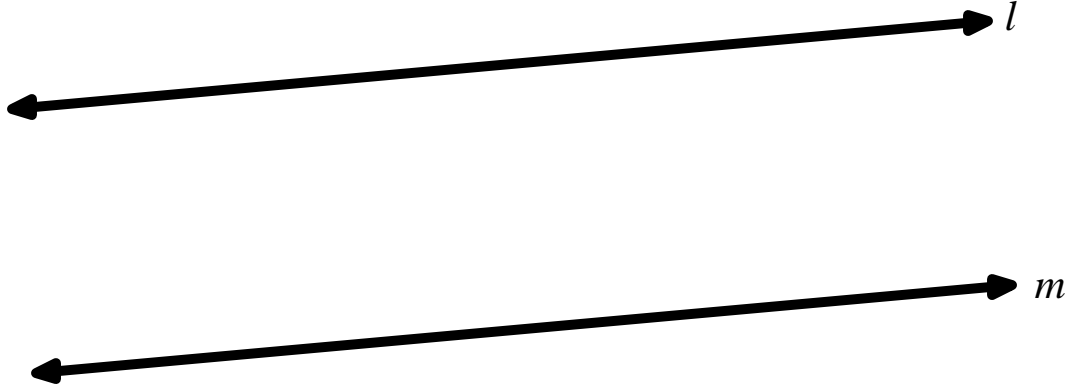
$\angle 3$ and $\angle 9$
 $\angle 3$ and $\angle 5$



Who is correct? Explain your reasoning.


10.1c Class Activity: Parallel Lines and Transversals

1. Use the picture given below to describe what parallel lines are. Use the correct notation to denote that line l is parallel to line m .



2. Draw a transversal for the two parallel lines above and label it line t . Label the angles formed by the transversal and the parallel lines with numbers 1 through 6. *Be sure to number in the same order as your teacher.

Transversals that intersect two or more parallel lines create angle pairs that have special properties. Use what you know about rigid motions to discover some of these relationships.

3. What type of angle pair is $\angle 2$ and $\angle 6$? 
4. Copy $\angle 2$ on a piece of tracing paper (or patty paper). Describe the rigid motion that will carry $\angle 2$ to $\angle 6$. Determine the relationship between $\angle 2$ and $\angle 6$.
5. Use a similar process to see if the same outcome holds true for all of the corresponding angles in the figure. Start by listing the remaining pairs of corresponding angles and then state the relationship.
6. List the pairs of angles that are vertical angles, what do you know about vertical angles?

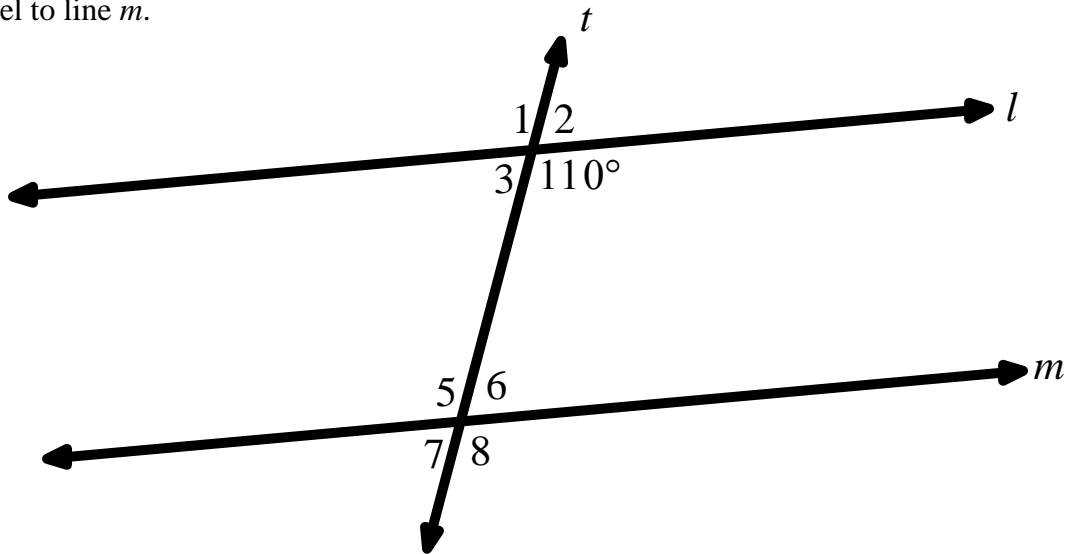
7. Continue to use rigid motions and what you know about vertical angles to discover other relationships that exist between alternate interior angles and alternate exterior angles. Be sure to provide justification for your claims.
8. Complete the following statements in the box below.

Properties of Transversals to Parallel Lines

If two parallel lines are intersected by a transversal,

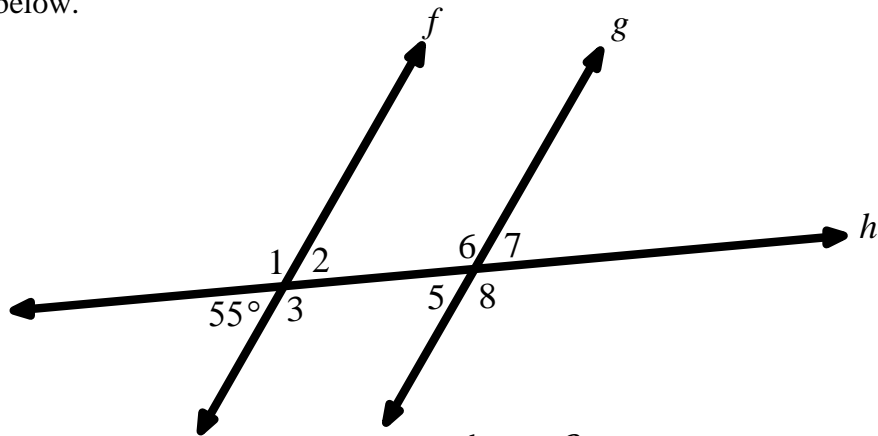
- Corresponding angles are _____.
- Alternate interior angles are _____.
- Alternate exterior angles are _____.

9. In the diagram below one angle measure is given. Find the measure of each remaining angle if line l is parallel to line m .



- a. $m\angle 1 =$ _____
- b. $m\angle 2 =$ _____
- c. $m\angle 3 =$ _____
- d. $m\angle 5 =$ _____
- e. $m\angle 6 =$ _____
- f. $m\angle 7 =$ _____
- g. $m\angle 8 =$ _____

10. Line $f \parallel$ line g and one angle measure is given in the diagram. Determine the measures of the remaining angles in the diagram below.



a. $m\angle 1 =$ _____

b. $m\angle 2 =$ _____

c. $m\angle 3 =$ _____

d. $m\angle 5 =$ _____



e. $m\angle 6 =$ _____

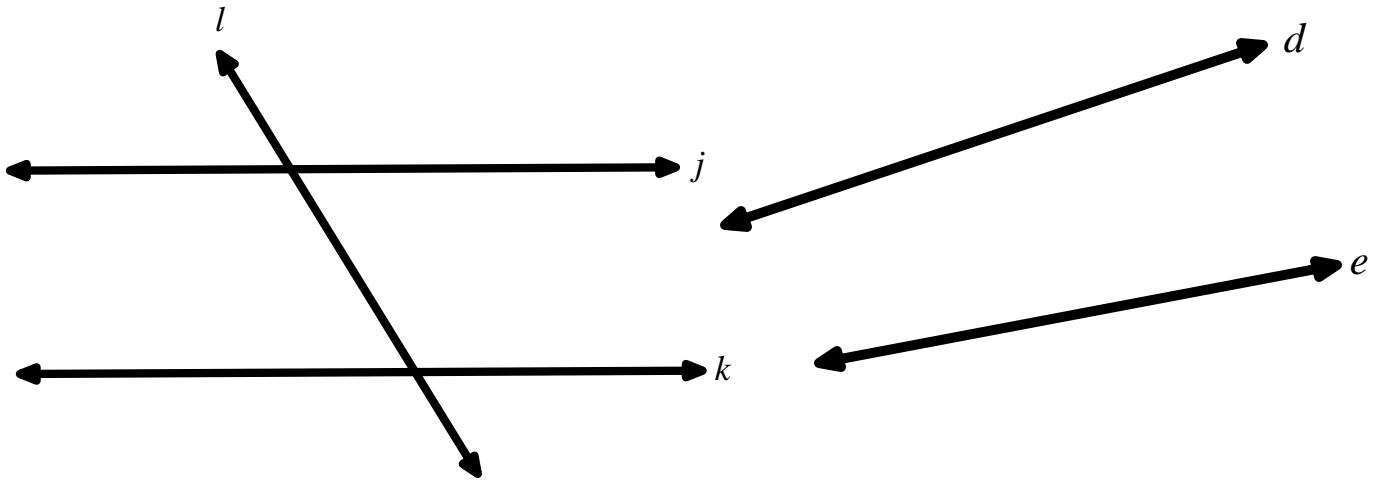
f. $m\angle 7 =$ _____

g. $m\angle 8 =$ _____

11. Given that line $l \parallel$ line m solve for x and then find the measure of all the remaining angles. Write the angle measures on the picture.

<p>a.</p> <p>$x =$</p>	<p>b.</p> <p>$x =$</p>
<p>c.</p> <p>$x =$</p>	<p>d.</p> <p>$x =$</p>

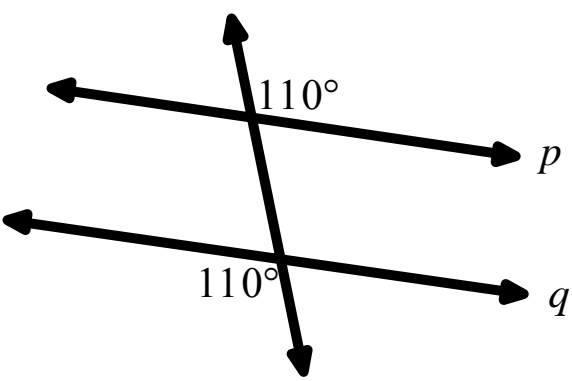
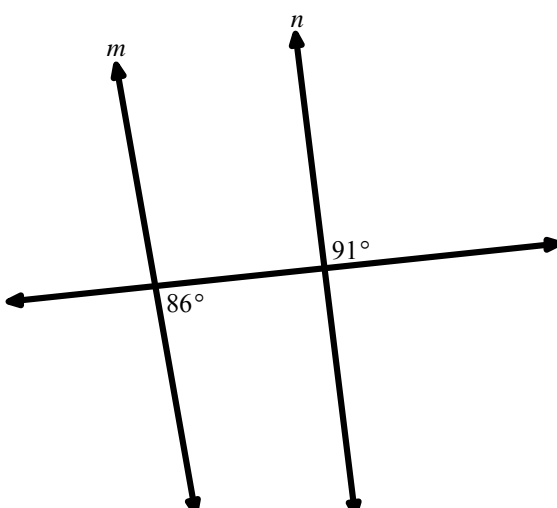
12. Given two lines j and k in a picture below with transversal l devise a strategy to determine whether the two lines are parallel using what you know about the properties of rigid motion. Also use your strategy to determine whether lines d and e are parallel. Stating that the lines do not look parallel, is not a justification.  



13. Complete the statement below.

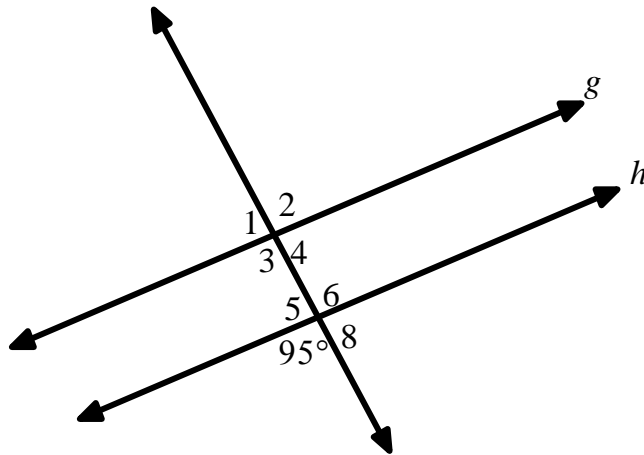
Given two lines, if a transversal cuts through both lines so that corresponding angles are _____, then the two lines are _____.

14. Determine whether the following sets of lines are parallel or not. Provide a justification for your response.

<p>a. Is p parallel to q? Why or why not?</p> 	<p>b. Is m parallel to n? Why or why not?</p> 
---	--

10.1c Homework: Parallel Lines and Transversals

Directions: Use the diagram below to answer questions #1 and 2 given that $g \parallel h$.



1. For each of the following pairs of angles, describe the relationship between the two angles (corresponding angles, alternate interior angles, alternate exterior angles, or vertical angles).

a. $\angle 3$ and $\angle 6$

b. $\angle 4$ and $\angle 8$

c. $\angle 1$ and $\angle 8$

d. $\angle 1$ and $\angle 5$

2. One angle measure is given in the diagram. Determine the measures of the remaining angles in the diagram.

a. $m\angle 1 = \underline{\hspace{2cm}}$

b. $m\angle 2 = \underline{\hspace{2cm}}$

c. $m\angle 3 = \underline{\hspace{2cm}}$

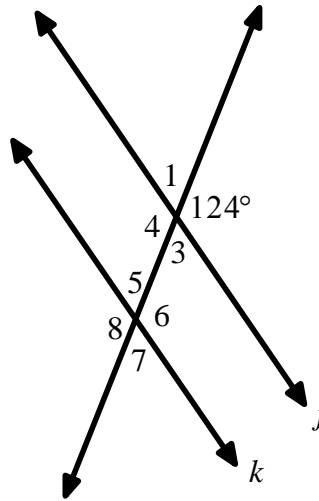
d. $m\angle 4 = \underline{\hspace{2cm}}$

e. $m\angle 5 = \underline{\hspace{2cm}}$

f. $m\angle 6 = \underline{\hspace{2cm}}$

g. $m\angle 8 = \underline{\hspace{2cm}}$

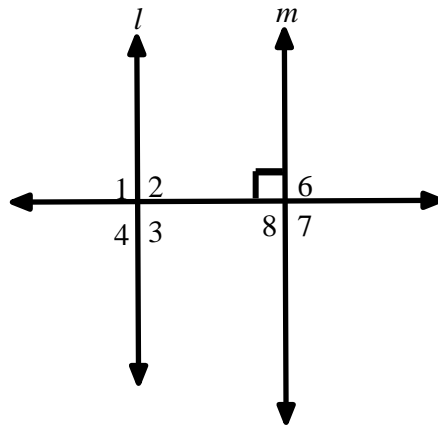
Directions: Use the diagram below to answer question #3 given that line $j \parallel$ line k .



3. One angle measure is given in the diagram. Determine the measures of the remaining angles in the diagram.

- | | |
|------------------------|------------------------|
| a. $m\angle 1 =$ _____ | b. $m\angle 3 =$ _____ |
| c. $m\angle 4 =$ _____ | d. $m\angle 5 =$ _____ |
| e. $m\angle 6 =$ _____ | f. $m\angle 7 =$ _____ |
| g. $m\angle 8 =$ _____ | |

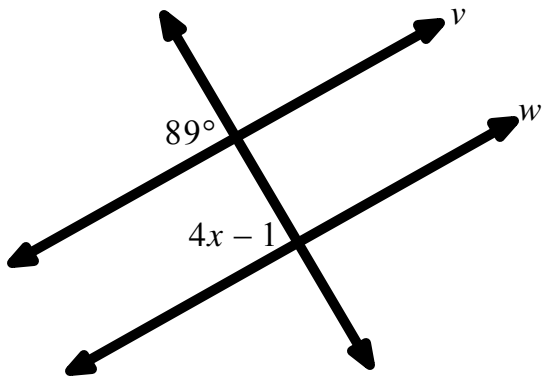
Directions: Use the diagram below to answer question #4 given that line $l \parallel$ line m .



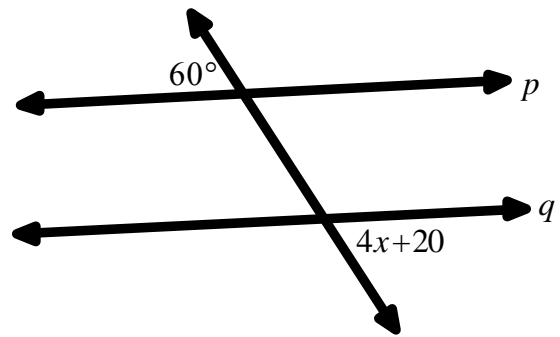
4. One angle measure is given in the diagram. Determine the measures of the remaining angles in the diagram.

- | | |
|------------------------|------------------------|
| a. $m\angle 1 =$ _____ | b. $m\angle 2 =$ _____ |
| c. $m\angle 3 =$ _____ | d. $m\angle 4 =$ _____ |
| e. $m\angle 6 =$ _____ | f. $m\angle 8 =$ _____ |
| g. $m\angle 7 =$ _____ | |

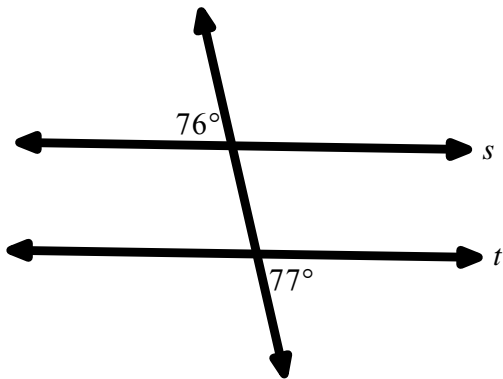
5. Given line $v \parallel$ line w , solve for x .



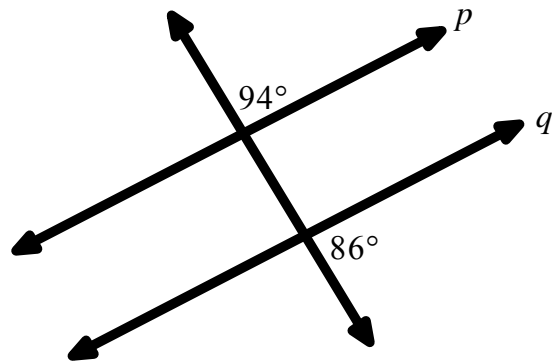
6. Given line $p \parallel$ line q , solve for x .



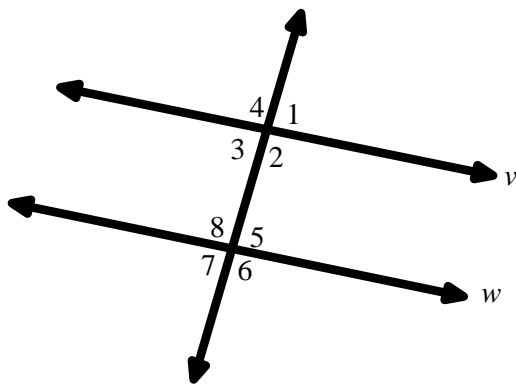
7. Determine whether lines s and t are parallel. Provide a justification for your response.



8. Determine whether lines p and q are parallel. Provide a justification for your response.

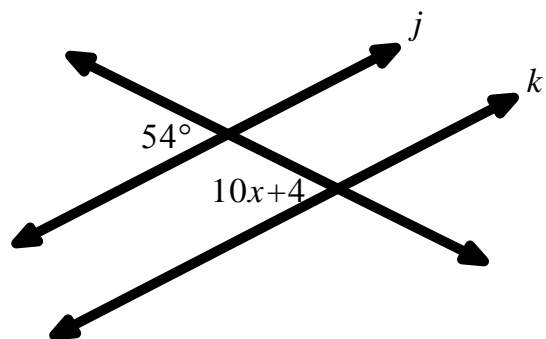


9. Given: line $v \parallel$ line w .



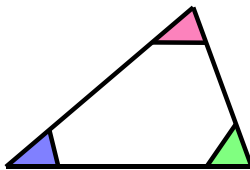
- Which angles are congruent to $\angle 1$?
- Which angles are congruent to $\angle 8$?
- Name **three** pairs of supplementary angles.

10. What value of x will make line j parallel to line k ?

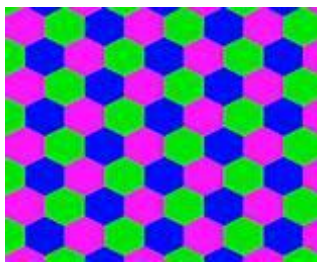


10.1d Class Activity: Tessellating Triangles

1. Take the index card that has been given to you and using a ruler draw an obtuse scalene triangle or an acute scalene triangle. Remember, in a scalene triangle, the side lengths of the triangle are all different. If the triangle has to be acute or obtuse, that means it can't have a right angle.
2. Cut out the triangle and color the angles each a different color as shown below.




3. Tessellate an 8 ½" x 11" white piece of paper with copies of your triangle. A **tessellation** is when you cover a surface with one or more geometric shapes, called tiles, with no overlaps or gaps. A tessellation by regular hexagons is shown below.



After each tessellation of your triangle, color each angle with its corresponding color.

4. What types of motion did you use to tessellate the plane with your triangle?

5. Look back at some of the facts we have studied so far in this section. How does your tessellation support these facts? 

- **Angles that lie on the same line are supplementary and have a common vertex.**
- **Vertical angles have the same measure.**
- **If two lines are parallel and they are intersected by a transversal, then corresponding angles at the points of intersection have the same measure.**
- **Given two lines, if a third line cuts through both lines so that corresponding angles are congruent, then the two lines are parallel.**

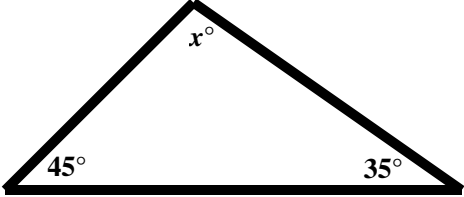
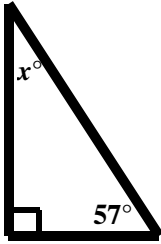
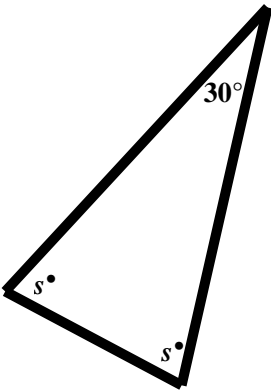
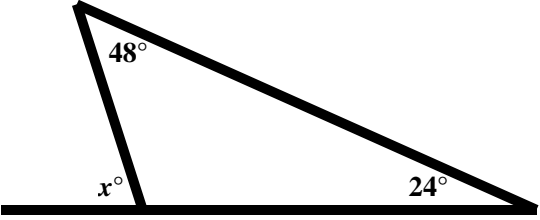
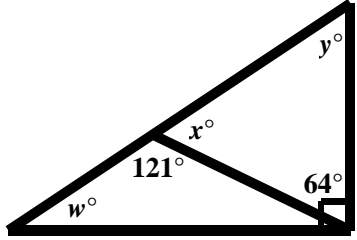
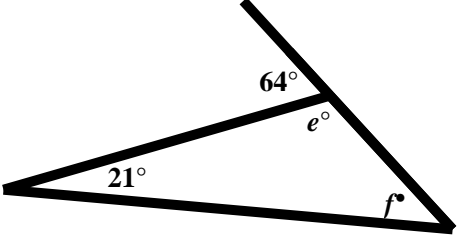
6. The following bolded bullets are additional facts we can observe in our tessellation. Use your tessellation to observe each fact and then provide a mathematical explanation as to why each fact is true.

- **The sum of the interior angles of a triangle is a straight angle (180°).**

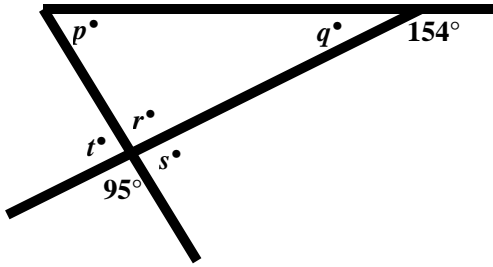
- **The sum of the interior angles of a quadrilateral is 360° .**

- **The measure of an exterior angle of a triangle is equal to the sum of the measures of the non-adjacent angles.**

Directions: In the following problems, solve for the missing angle(s).

<p>7.</p>  <p>$x =$ _____</p>	<p>8.</p>  <p>$x =$ _____</p>
<p>9.</p>  <p>$s =$ _____</p>	<p>10.</p>  <p>$x =$ _____</p>
<p>11.</p>  <p>w _____ $x =$ _____ $y =$ _____</p>	<p>12.</p>  <p>$e =$ _____ $f =$ _____</p>

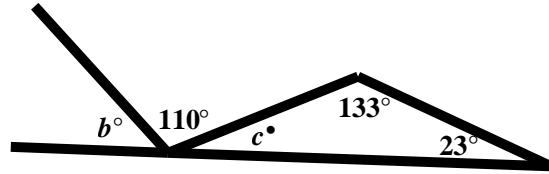
13.



$p = \underline{\hspace{2cm}}$ $q = \underline{\hspace{2cm}}$ $r = \underline{\hspace{2cm}}$

$s = \underline{\hspace{2cm}}$ $t = \underline{\hspace{2cm}}$

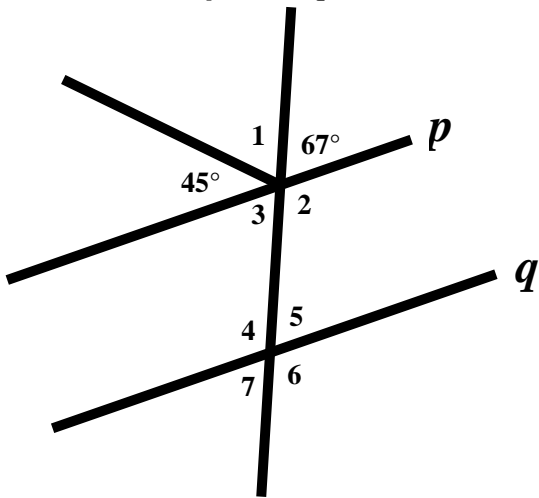
14.



$b = \underline{\hspace{2cm}}$

$c = \underline{\hspace{2cm}}$

15. Given: line $p \parallel$ line q

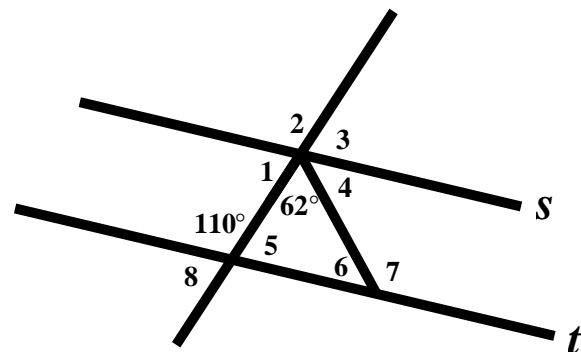


$\angle 1 = \underline{\hspace{2cm}}$ $\angle 2 = \underline{\hspace{2cm}}$ $\angle 3 = \underline{\hspace{2cm}}$

$\angle 4 = \underline{\hspace{2cm}}$ $\angle 5 = \underline{\hspace{2cm}}$ $\angle 6 = \underline{\hspace{2cm}}$

$\angle 7 = \underline{\hspace{2cm}}$

16. Given line $s \parallel$ line t



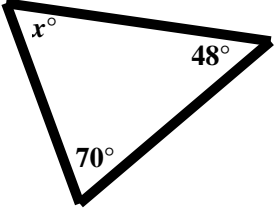
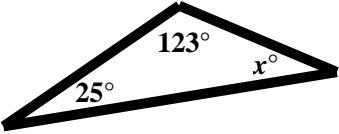
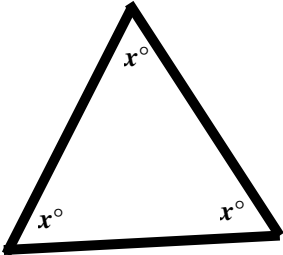
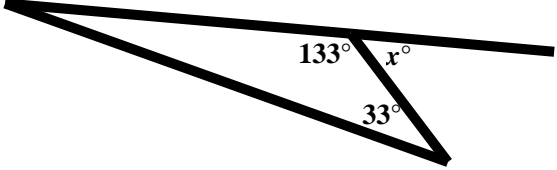
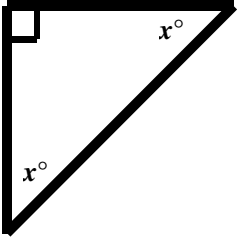
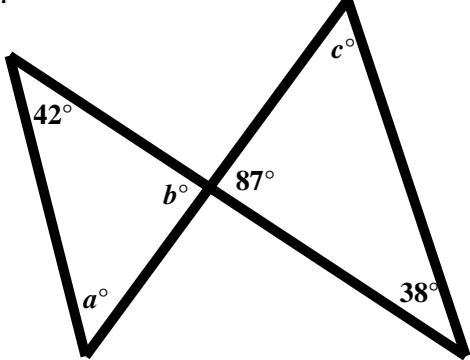
$\angle 1 = \underline{\hspace{2cm}}$ $\angle 2 = \underline{\hspace{2cm}}$ $\angle 3 = \underline{\hspace{2cm}}$

$\angle 4 = \underline{\hspace{2cm}}$ $\angle 5 = \underline{\hspace{2cm}}$ $\angle 6 = \underline{\hspace{2cm}}$

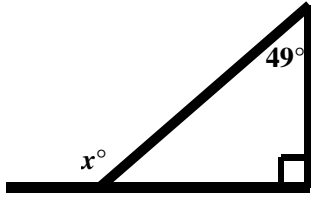
$\angle 7 = \underline{\hspace{2cm}}$ $\angle 8 = \underline{\hspace{2cm}}$

10.1d Homework: Finding Angle Measures in Triangles

Directions: In the following problems, solve for the missing angle(s).

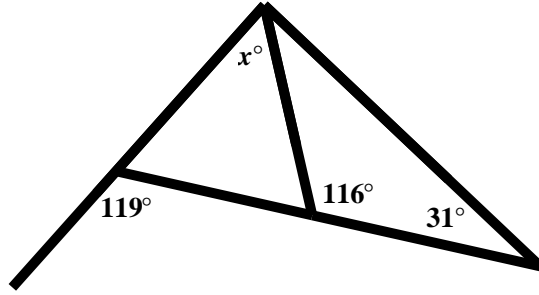
<p>1.</p>  <p>$x = \underline{\hspace{2cm}}$</p>	<p>2.</p>  <p>$x = \underline{\hspace{2cm}}$</p>
<p>3.</p>  <p>$x = \underline{\hspace{2cm}}$</p>	<p>4.</p>  <p>$x = \underline{\hspace{2cm}}$</p>
<p>5.</p>  <p>$x = \underline{\hspace{2cm}}$</p>	<p>6.</p>  <p>$a = \underline{\hspace{1cm}} \quad b = \underline{\hspace{1cm}} \quad c = \underline{\hspace{1cm}}$</p>

7.



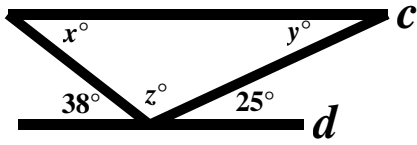
$x = \underline{\hspace{2cm}}$

8.



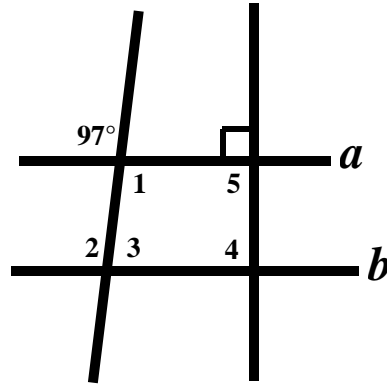
$x = \underline{\hspace{2cm}}$

9. Given: line $c \parallel$ line d



$x = \underline{\hspace{1cm}} \quad y = \underline{\hspace{1cm}} \quad z = \underline{\hspace{1cm}}$

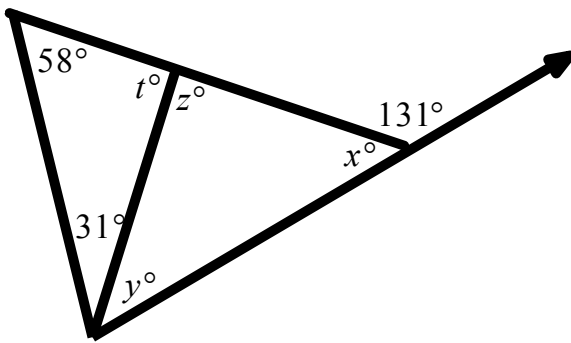
10. Given: line $a \parallel$ line b



$m\angle 1 = \underline{\hspace{1cm}} \quad m\angle 2 = \underline{\hspace{1cm}} \quad m\angle 3 = \underline{\hspace{1cm}}$

$m\angle 4 = \underline{\hspace{1cm}} \quad m\angle 5 = \underline{\hspace{1cm}}$

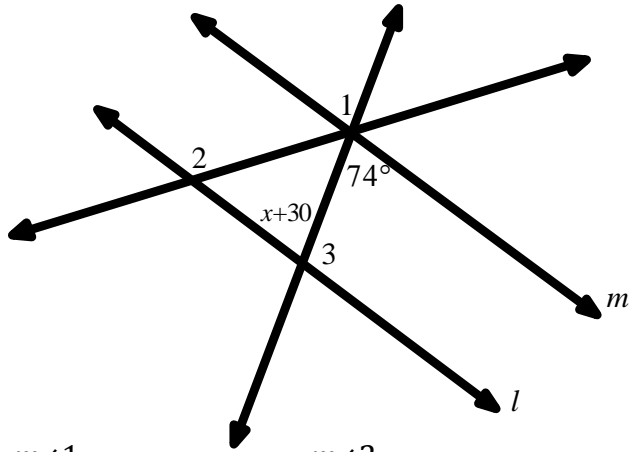
11.



$t = \underline{\hspace{1cm}} \quad y = \underline{\hspace{1cm}}$

$z = \underline{\hspace{1cm}} \quad x = \underline{\hspace{1cm}}$

12. Given: line $l \parallel$ line m



$m\angle 1 = \underline{\hspace{1cm}} \quad m\angle 2 = \underline{\hspace{1cm}}$

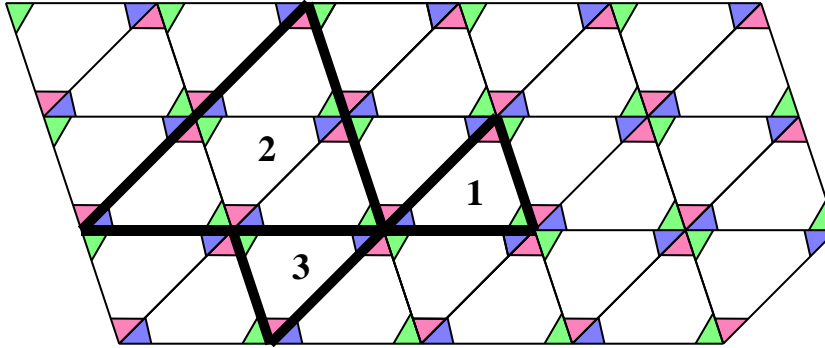
$m\angle 3 = \underline{\hspace{1cm}} \quad x = \underline{\hspace{1cm}}$

10.1e Class Activity: Similar Triangles

Revisit some of the following facts about similar triangles from Chapter 9.

- **If two triangles are similar, then the ratios of the lengths of corresponding sides are the same.**
- **If two triangles are similar, then corresponding angles have the same measure.**


Use the tessellation you made to continue your study of triangles.



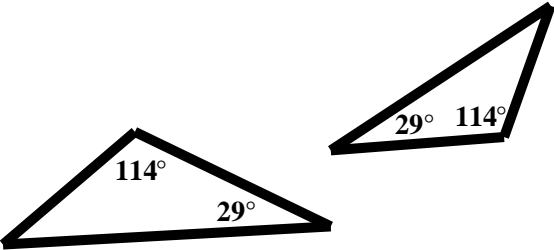
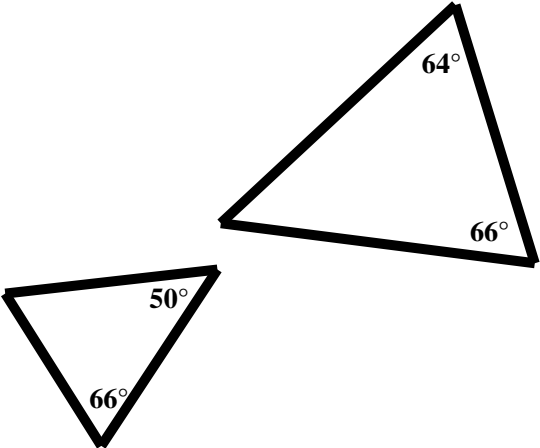
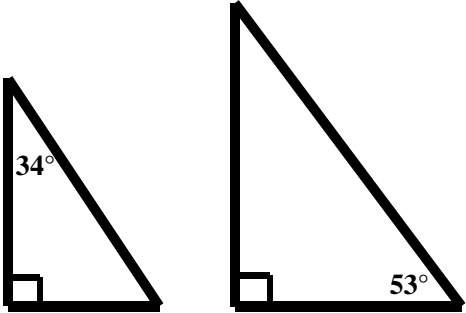
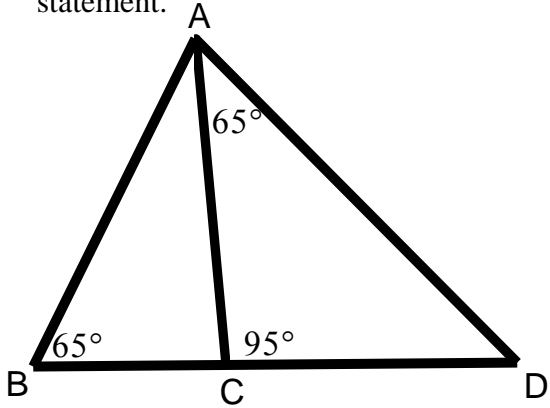
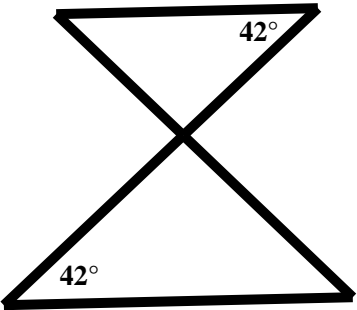
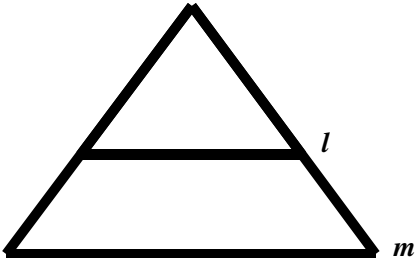
1. In Chapter 9 we learned that if one figure can be carried onto another by a series of rigid motions and dilations, then the two figures are similar.
 - a. In the picture above triangle 1 is similar to triangle 2. Describe the sequence of transformations that will carry triangle 1 onto triangle 2. What is the scale factor?
 - b. In the picture above triangle 2 is similar to triangle 3. Describe the sequence of transformations that will carry triangle 2 onto triangle 3. What is the scale factor?
 - c. What do you notice about the corresponding angles of similar triangles?
2. Can you find a triangle that is a dilation of triangle 1 with a scale factor of 3? Trace the triangle. What do you notice about the angle measures in the new triangle you created?

Below is another fact about similar triangles.

- **Given two triangles, if the corresponding angles have the same measure, then the triangles are similar.**
3. We will be using the tessellation you made to explore the proposition above. Find and highlight in black two triangles that have the same angle measures but are a different size.

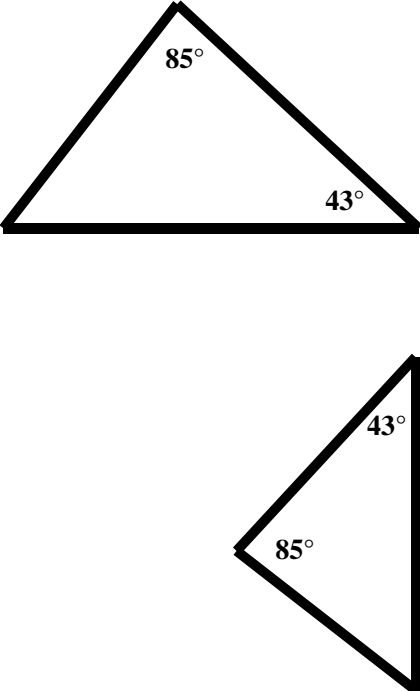
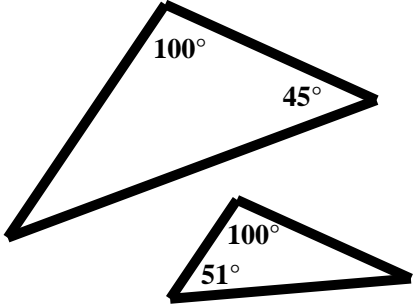
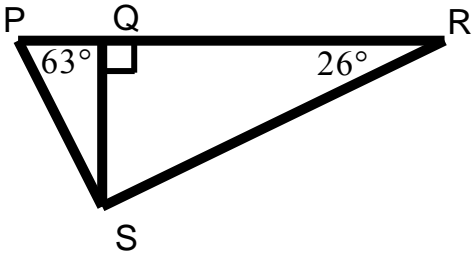
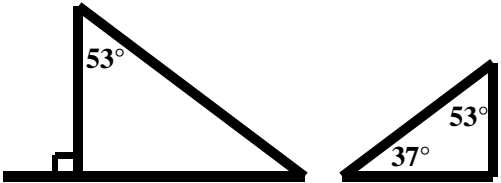
4. What is the relationship between the triangles formed by the dark lines? Justify your answer. 
5. Find a third triangle that is a different size than the other two you highlighted. Highlight the third triangle. What is the relationship of this triangle to the other triangles? Justify your answer.
6. Complete the following statement. It two triangles have corresponding angles that are the same measure, then one triangle can be mapped to the other using _____; therefore the triangles are _____.
7. Do all 3 pairs of corresponding angles have to be congruent in order to say that the two triangles are similar? What if only 2 pairs of corresponding angles are congruent? Would the triangles still be similar? Why or why not?

Directions: Are the triangles similar? If they are similar justify why.

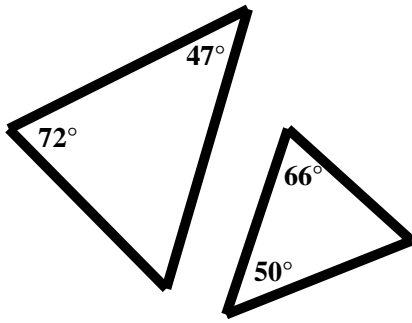
<p>8.</p> 	<p>9.</p> 
<p>10.</p> 	<p>11. In the picture below be sure to consider all three triangles shown. If any of the triangles are similar write a similarity statement.</p> 
<p>12.</p> 	<p>13. Given line $l \parallel$ line m</p> 

10.1e Homework: Similar Triangles

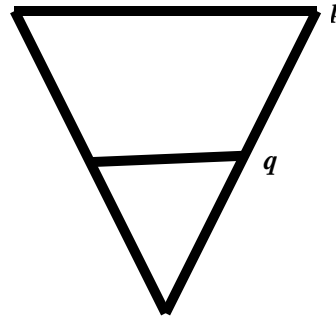
Directions: Are the triangles similar? If they are similar justify why.

<p>1.</p> 	<p>2.</p> 
<p>3. In the picture below be sure to consider all three triangles shown. If any of the triangles are similar write a similarity statement.</p> 	<p>4.</p> 

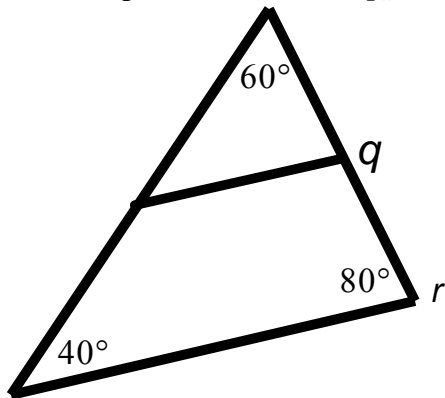
5.



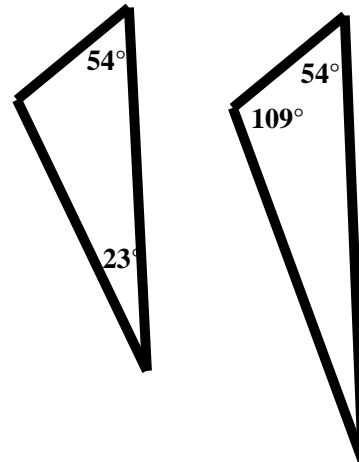
6. In the picture below p is not parallel to q .



7. In the picture below line $q \parallel$ line r .



8.



10.1f Self-Assessment: Section 10.1

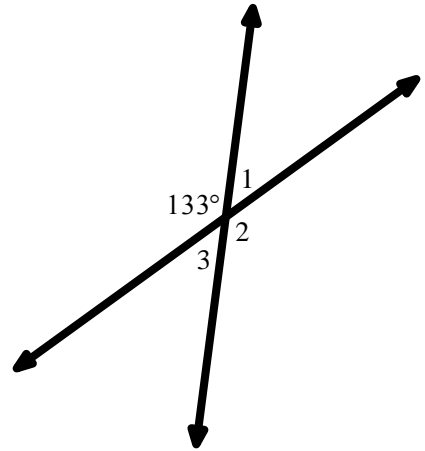
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provide on the next page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Know that straight angles sum to 180° and that vertical angles are congruent. <i>See sample problem #1</i>				
2. Know that the sum of the angles in a triangle is 180° . Understand that the measure of an exterior angle of a triangle is equal to the sum of the measures of the non-adjacent angles. Use these properties to find missing angle measures related to a triangle. <i>See sample problem #2</i>				
3. Determine the relationship between angles formed when a transversal intersects parallel lines. Use these relationships to find missing angle measures. <i>See sample problem #3</i>				
4. Determine whether two lines are parallel based on the angle measures when a transversal intersects the lines. <i>See sample problem #4</i>				
5. Understand and apply the angle-angle criterion to determine whether two triangles are similar. <i>See sample problem #5</i>				

10.1f Sample Problems: Section 10.1

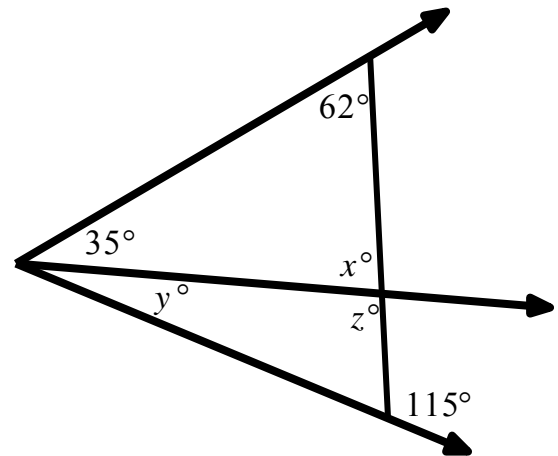
Sample Problem #1

Determine the measures of angles 1, 2, and 3. Justify your answers.



Sample Problem #2

In the figure to the right find the value for x , y , and z . Justify your answers.



Sample Problem #3

Use the figure to the right to answer each question given that line g is parallel to line h

- a. State the relationship between the following pairs of angles.

$\angle 1$ and $\angle 8$

$\angle 4$ and $\angle 8$

$\angle 3$ and $\angle 6$

- b. Find the measure of the angles given below.

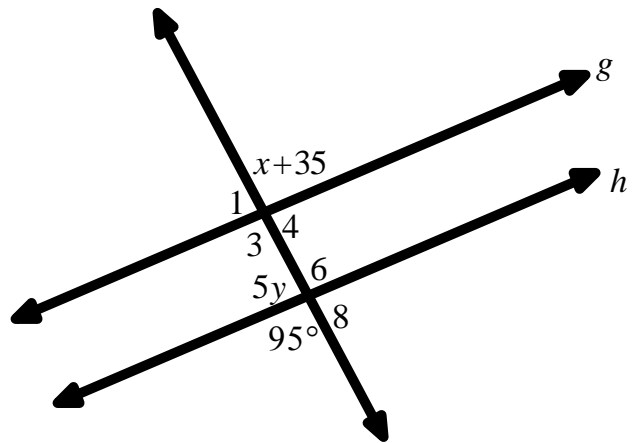
$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 3 = \underline{\hspace{2cm}}$

$m\angle 4 = \underline{\hspace{2cm}}$

$m\angle 6 = \underline{\hspace{2cm}}$

$m\angle 8 = \underline{\hspace{2cm}}$

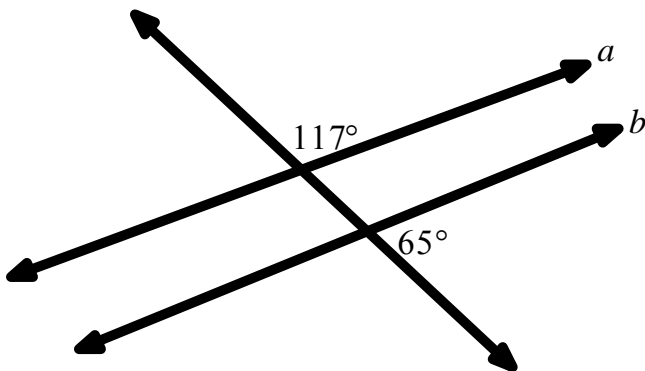


- c. Find the value of x and y .

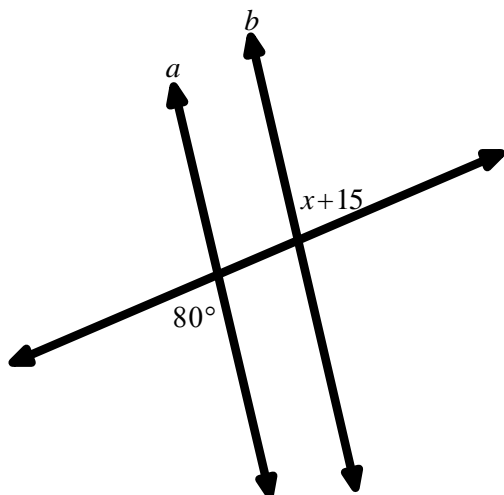
$x = \underline{\hspace{2cm}}$ $y = \underline{\hspace{2cm}}$

Sample Problem #4

a. Determine if line $a \parallel$ line b . Justify your answer.



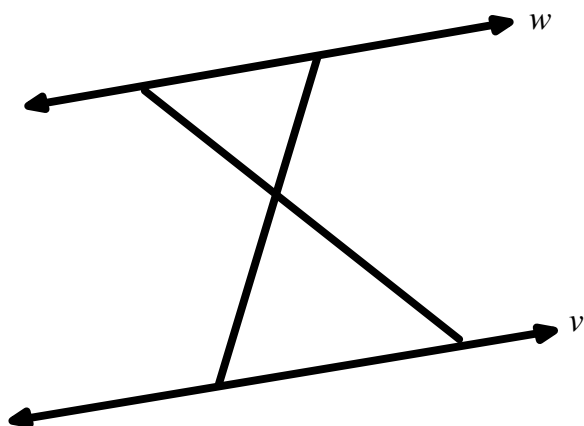
b. Find the value of x that will make line $a \parallel$ line b .



$x =$ _____

Sample Problem #5

Given that line $w \parallel$ line v , determine if the triangles formed below are similar. If they are similar justify why.



Section 10.2 The Pythagorean Theorem

Section Overview:

In this section students begin to formalize many of the ideas learned in Chapter 7. They will transition from using the area of a square to find the length of a segment to generalizing the relationship between the side lengths of a right triangle, i.e. the Pythagorean Theorem, to find the length of a segment. They begin this transition by finding the areas of the squares adjacent to a given right triangle. Using these concrete examples, students describe the relationship between the sides of a right triangle. From here, students work to explain a proof by picture and subsequently a paragraph proof of the Pythagorean Theorem, starting first with a right triangle of side lengths 3, 4, and 5. Students then use a similar process to explain a proof of the Pythagorean Theorem for any right triangle with side lengths a , b , and c where a and b are the legs of the right triangle and c is the hypotenuse. Students arrive at the Pythagorean Theorem: $a^2 + b^2 = c^2$ where a and b are the legs of the right triangle and c is the hypotenuse. Throughout the section, students are connecting the Pythagorean Theorem to work done in Chapter 7. Next, students use the Pythagorean Theorem to solve for the missing side length of a right triangle given the measurements of the other two sides, relying on skills learned in Chapters 7 and 8. This is followed by explaining a proof of the converse of the Pythagorean Theorem: For a triangle with side lengths a , b , and c if $a^2 + b^2 = c^2$, then the triangle is a right triangle. Using this theorem, students determine whether three given side lengths form a right triangle. Throughout this section emphasis is placed on creating good arguments and explanation. Students are not formally proving the Pythagorean Theorem and its converse but explaining why the theorems are true by learning how to provide sufficient explanations and arguments. In addition students are providing evidence and warrants for claims that they make. At the end of the section is an optional exploration on Pythagorean triples.

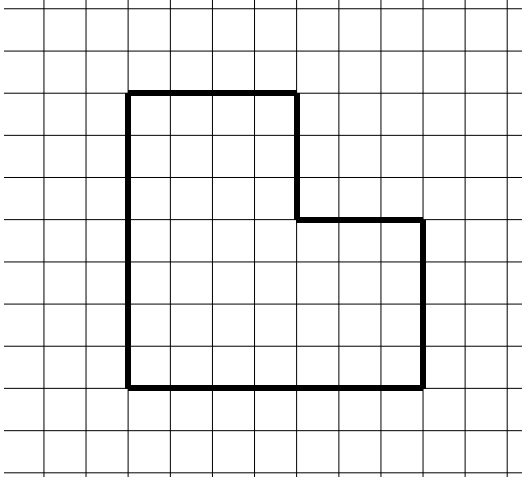
Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Know that in a right triangle $a^2 + b^2 = c^2$, where a and b are the legs of the right triangle and c is the hypotenuse.
2. Understand and explain a proof of the Pythagorean Theorem.
3. Use the Pythagorean Theorem to solve for the missing side length of a right triangle given the measurements of the other two sides.
4. Understand and explain a proof of the converse of the Pythagorean Theorem. That is, for a triangle with side lengths a , b , and c if $a^2 + b^2 = c^2$, then the triangle is a right triangle.
5. Use the converse of the Pythagorean Theorem to determine whether three given measurements are possible side lengths of a right triangle.

10.2a Class Activity: A Proof of the Pythagorean Theorem

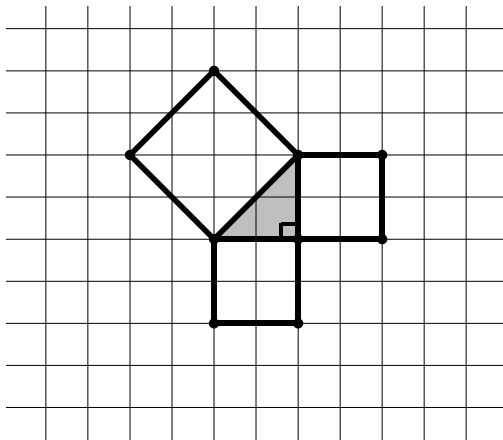
1. Find the area of the shape below. Each square on the grid has a side length of 1 unit.



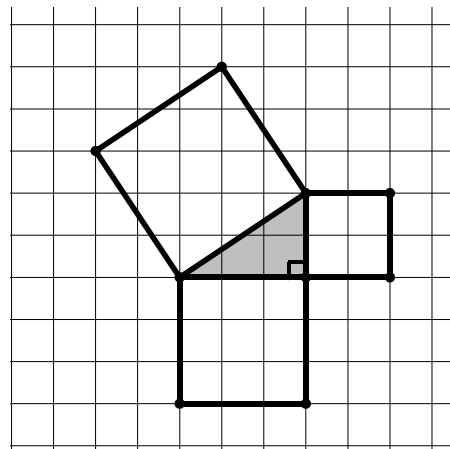
In numbers 2 and 3, a right triangle is shown in gray. The shorter sides of a right triangle are referred to as **legs**. The longer side of the right triangle (the side opposite of the right angle) is called the **hypotenuse**.


Directions: Squares have been drawn adjacent to the sides of the right triangle. Find the area of each of the squares. Assuming each square on the grid has a side length of 1 unit. Write the areas inside each of the squares.

2.

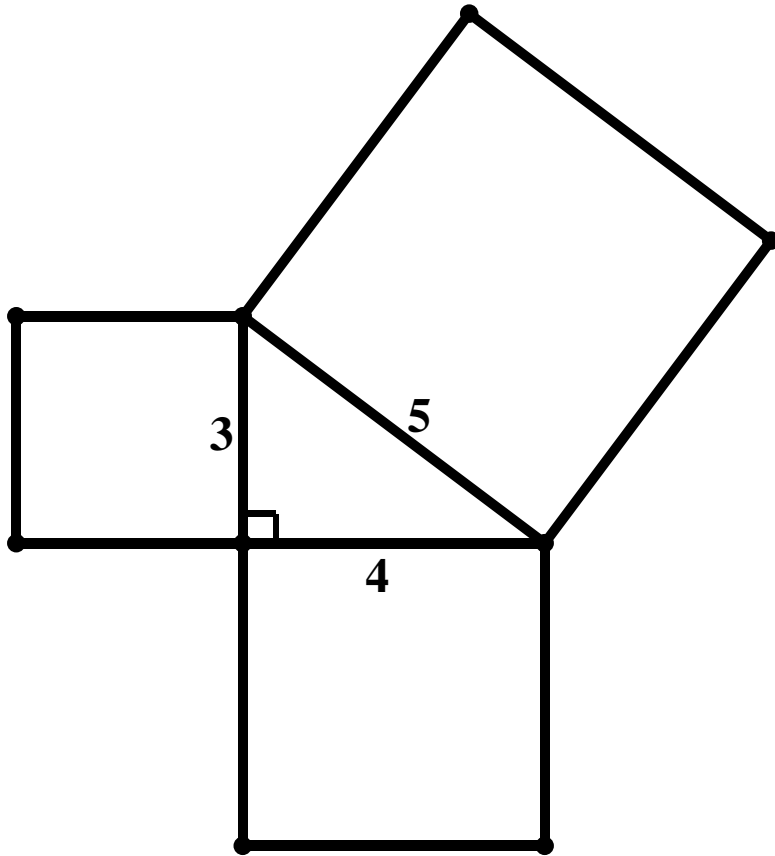


3.

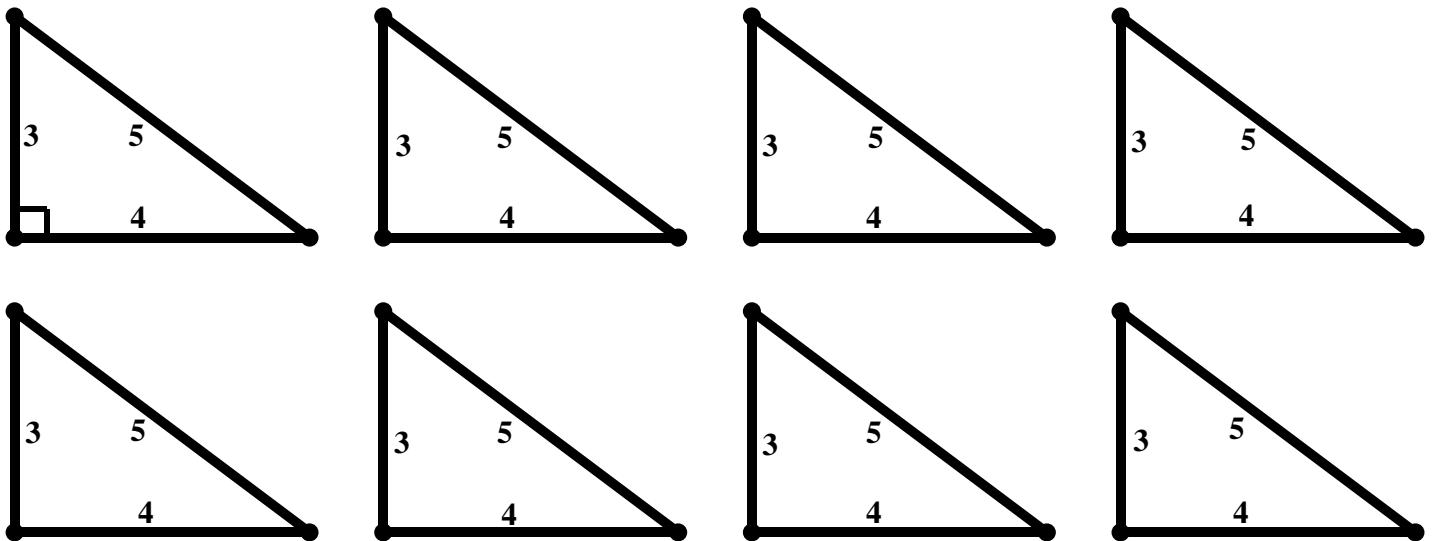


4. What do you notice about the relationship between the areas of the squares formed adjacent to the legs of a right triangle? 

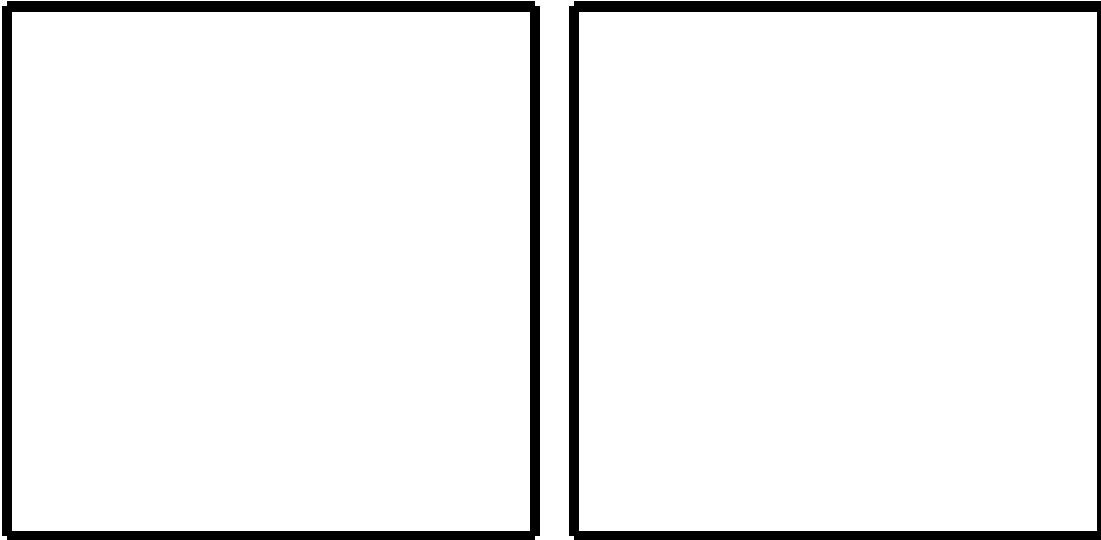
5. Below is a right triangle with side lengths 3, 4, and 5. Squares have been drawn adjacent to the sides of the right triangle.



- a. Find the area of each of the squares. Write the area inside each of the squares. Then, cut out the three squares very carefully.
- b. Below are 8 copies of the original right triangle. Cut out the 8 triangles very carefully.



- c. Below are two **congruent squares**. Since the squares are congruent, we know that their sides have the same length and subsequently they have the same area. Use your square with an area of 25 and four of the triangles from the previous page to cover one of the squares. Use your squares with areas 9 and 16 and four of the triangles from the previous page to cover the other square. Tape the pieces into place.

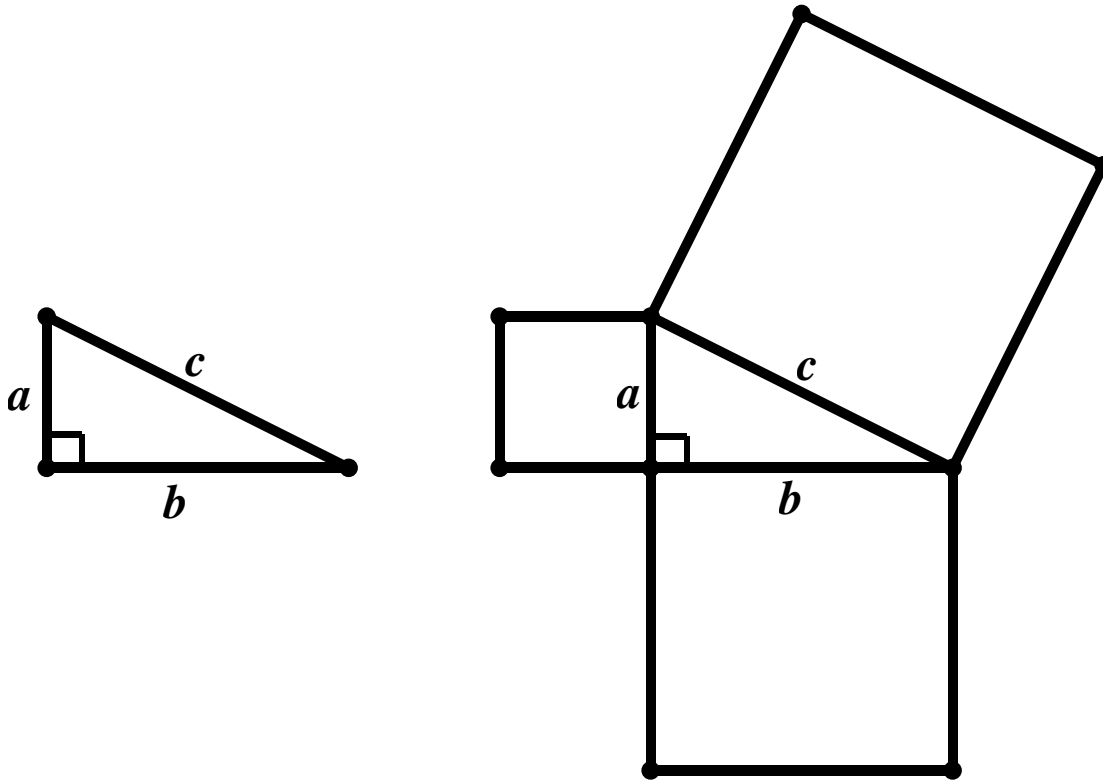


- d. Use the large squares in part c) to explain the relationship you discovered in #2 – 4 between the squares formed adjacent to the sides of a right triangle.

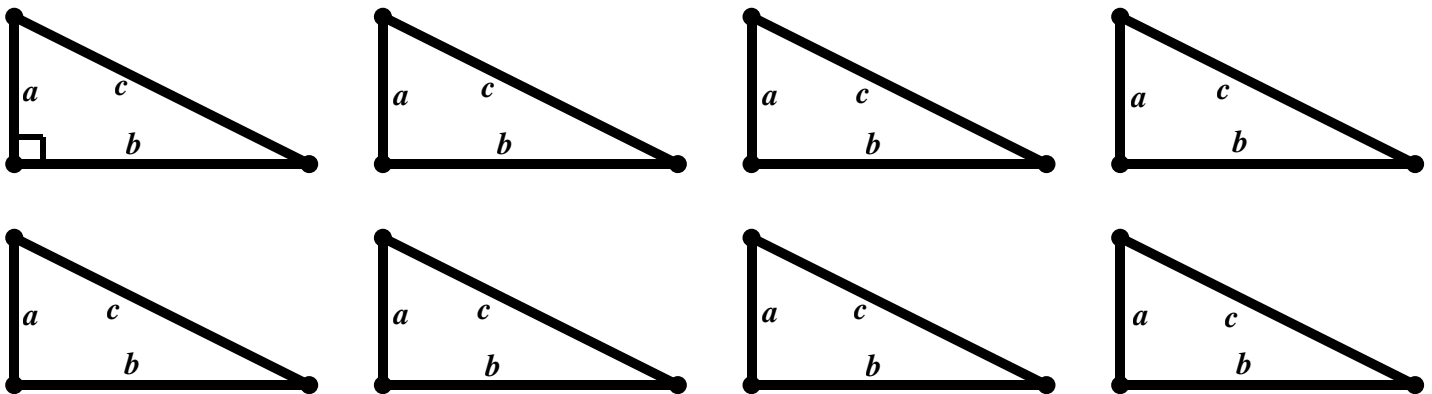


6. In the previous problems, we saw that for specific triangles **the sum of the areas of the squares along the legs of the right triangle equals the area of the square along the hypotenuse of the triangle** by looking at several examples. Now, we want to show that this relationship holds true for *any* right triangle.

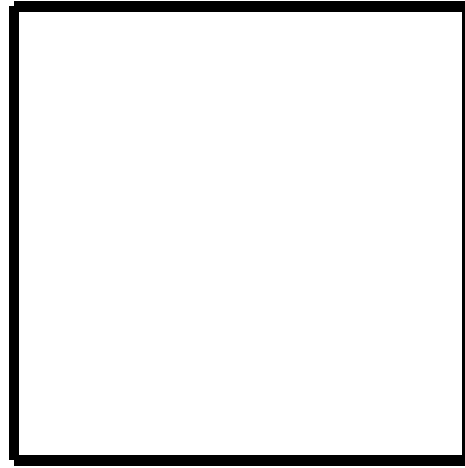
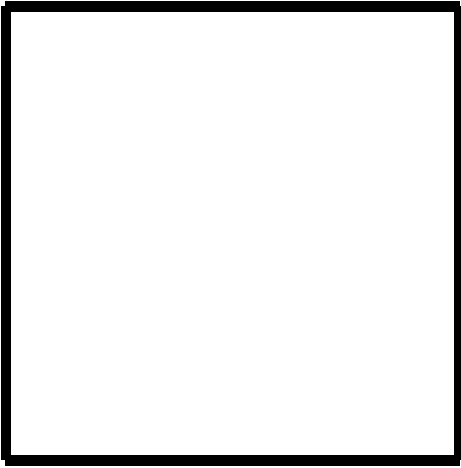
Suppose you have a right triangle with any side lengths a , b , and c where a and b are the legs of the triangle and c is the hypotenuse of the right triangle as shown below. The squares have been drawn along the sides of the right triangle. Our goal is to show that $a^2 + b^2 = c^2$ is always true.



- Find the area of each of the squares adjacent to the sides of the right triangle. Write the areas inside each square.
- Cut out the squares formed on the sides of the triangle above as well as the 8 copies of the triangle with side lengths a , b , and c below.



- c. Arrange the 3 squares and 8 triangles to cover the 2 squares shown below.



- d. Using the picture above, show that **the sum of the areas of the squares adjacent to the legs of the right triangle equals the area of the square adjacent to the hypotenuse of the triangle for any right triangle.**



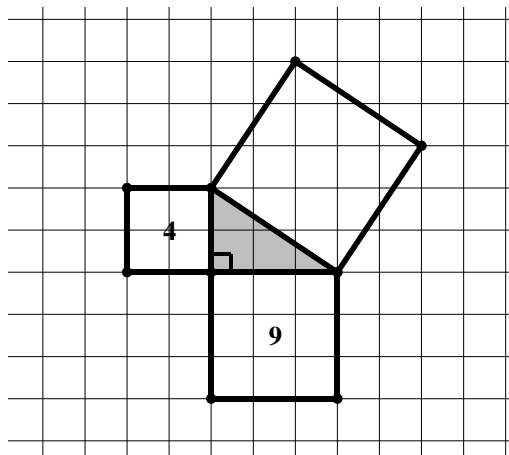
- e. Conventionally, the leg lengths of a right triangle are denoted using the variables a and b and the hypotenuse of a right triangle is denoted using the variable c . State the relationship between the side lengths of a right triangle using the words **legs** and **hypotenuse**.



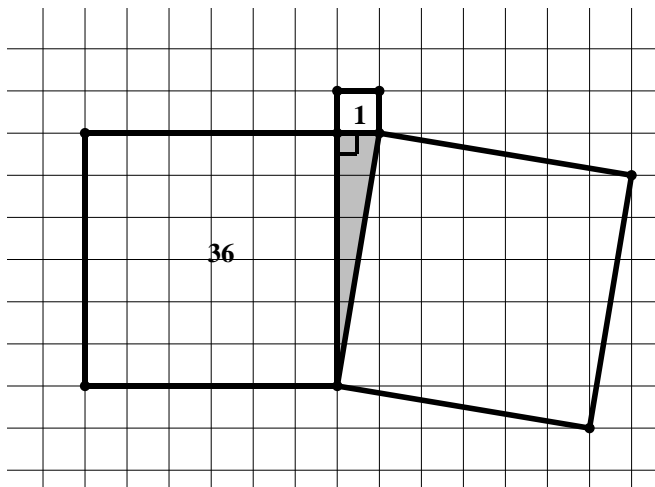
- f. Write an **equation** that shows the relationship between the side lengths of a right triangle using a and b for the lengths of the legs and c for the length of the hypotenuse.

Directions: In each of the problems below, a right triangle is shown in gray. The squares along each of the three sides of the triangles have been drawn. The area of two of the squares is given. Determine the area of the third square. Write your answer in the square. Also find the side length of each square, write the sides lengths below each picture.

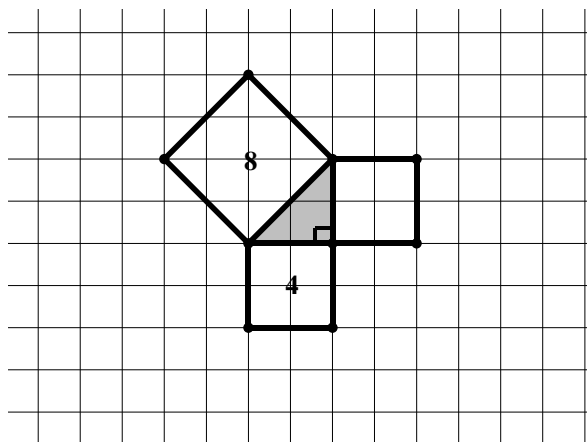
1.



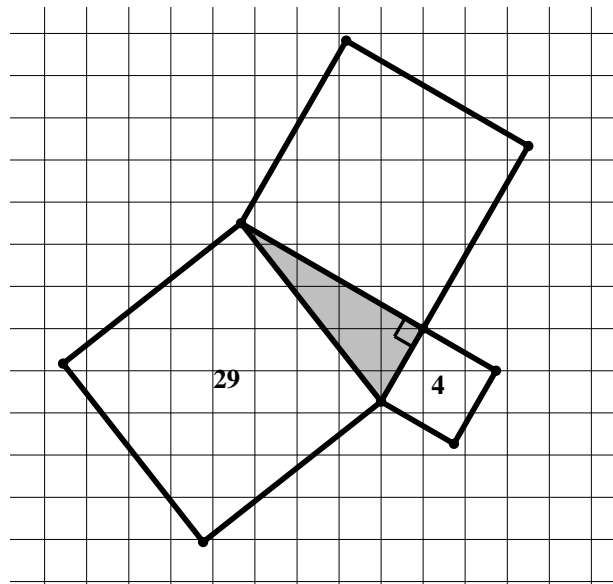
2.



3.



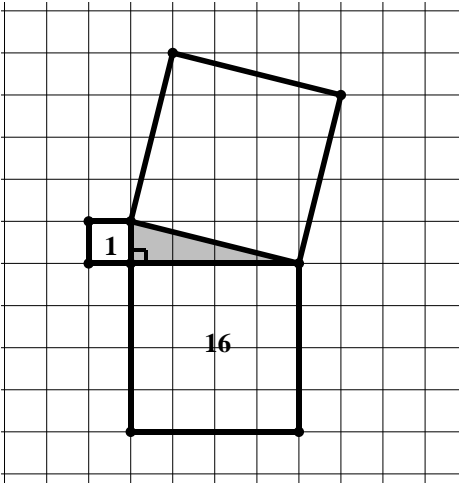
4.



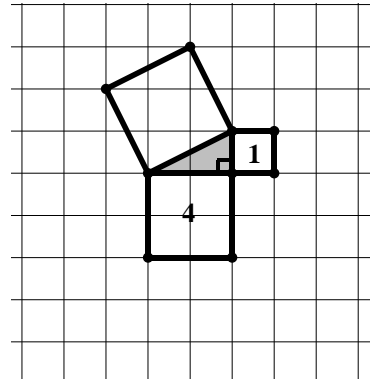
10.2a Homework: A Proof of the Pythagorean Theorem

Directions: In each of the problems below, a right triangle is shown in gray. The squares along each of the three sides of the triangles have been drawn. The area of two of the squares is given. Determine the area of the third square. Write your answer in the square.

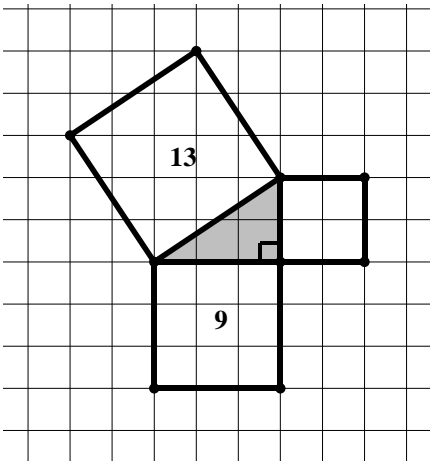
1.



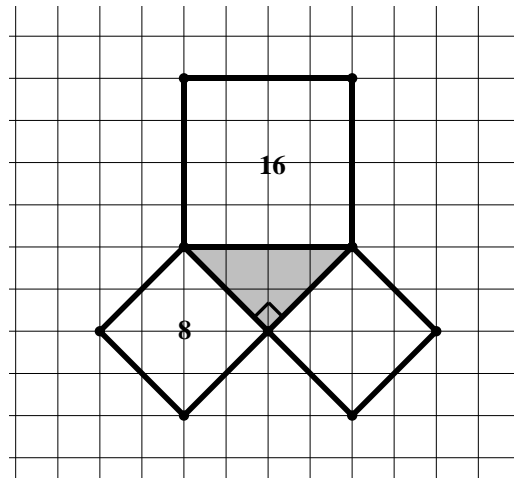
2.



3.

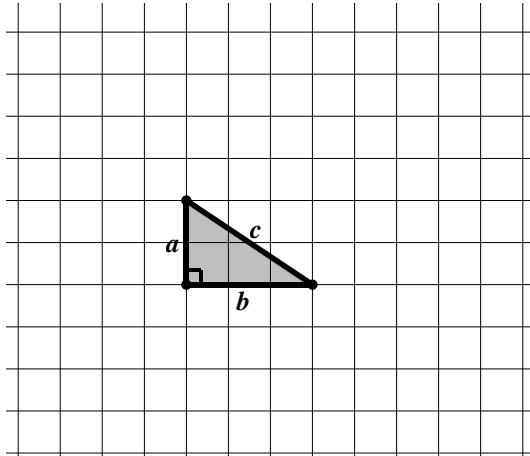


4.



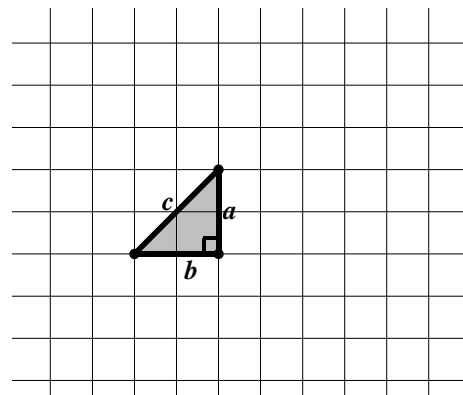
Directions: For each of the following problems, the gray triangle is a right triangle. Draw the squares adjacent to each of the three sides of the triangles. Find the area of each square and write the area in each square. Then, find the side lengths a , b , c of each triangle.

5.



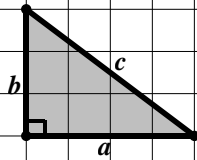
$a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$

6.



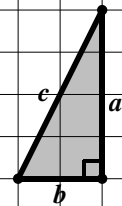
$a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$

7.



$a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$

8.

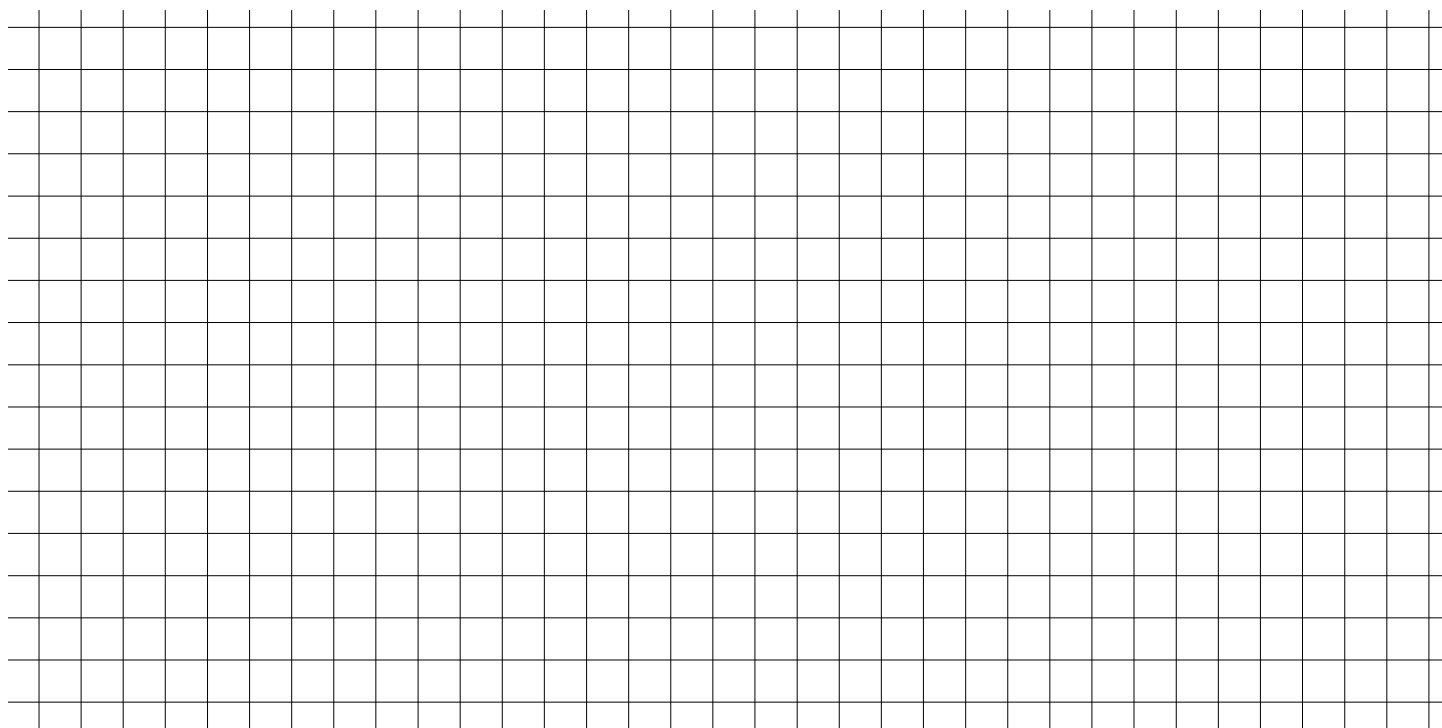
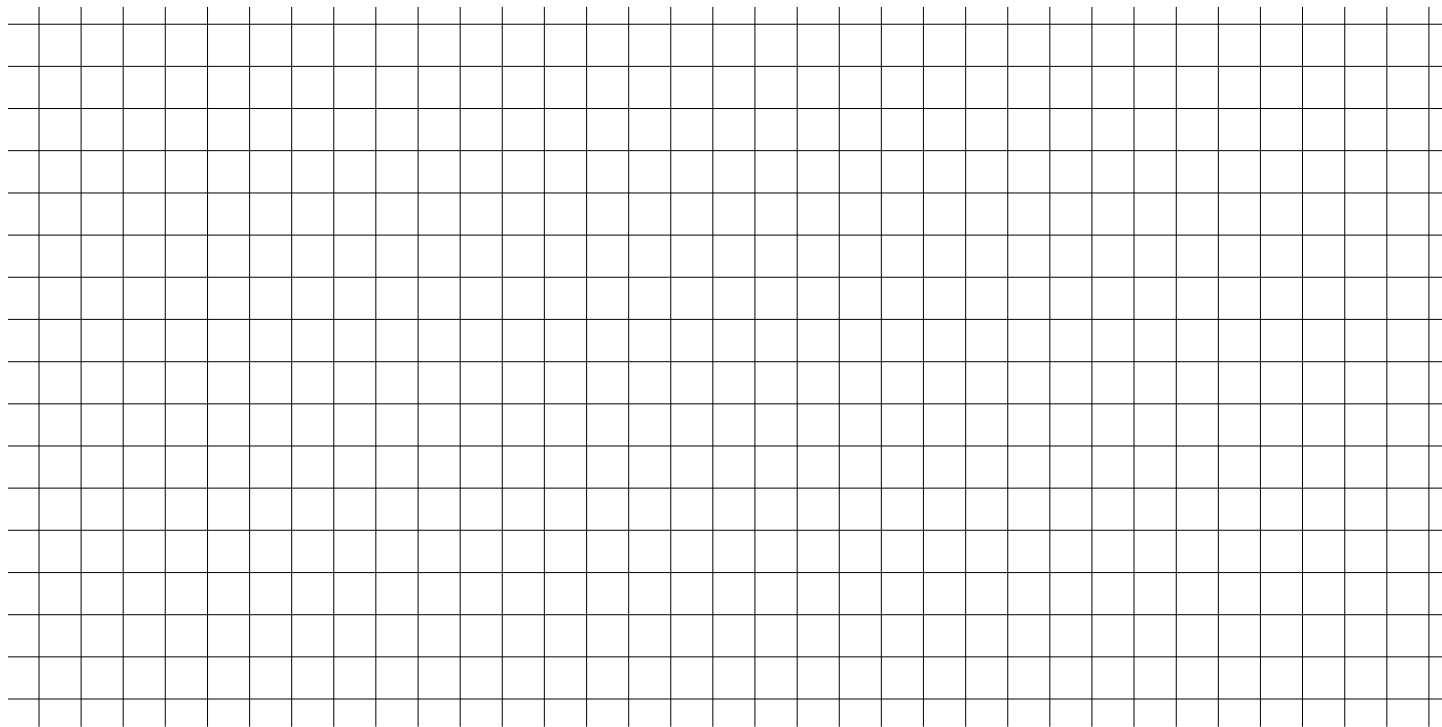


$a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$

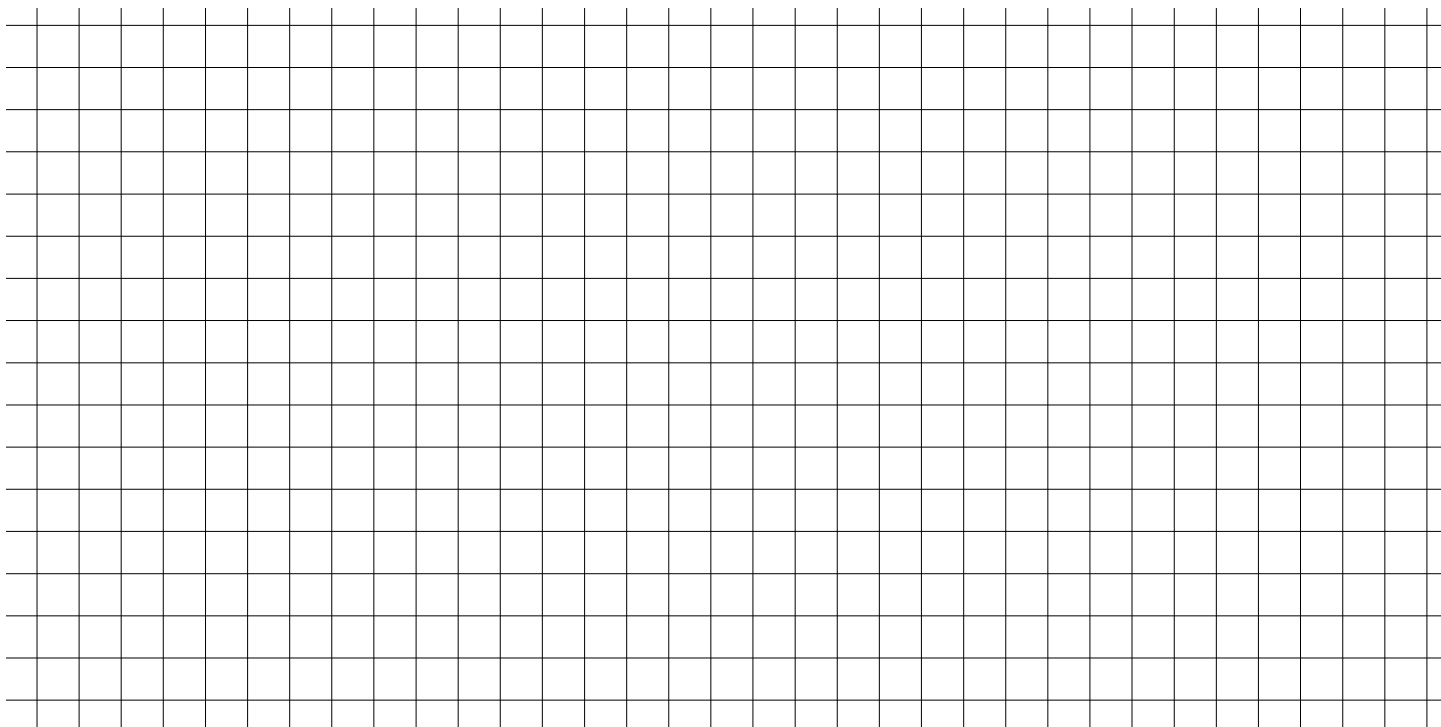
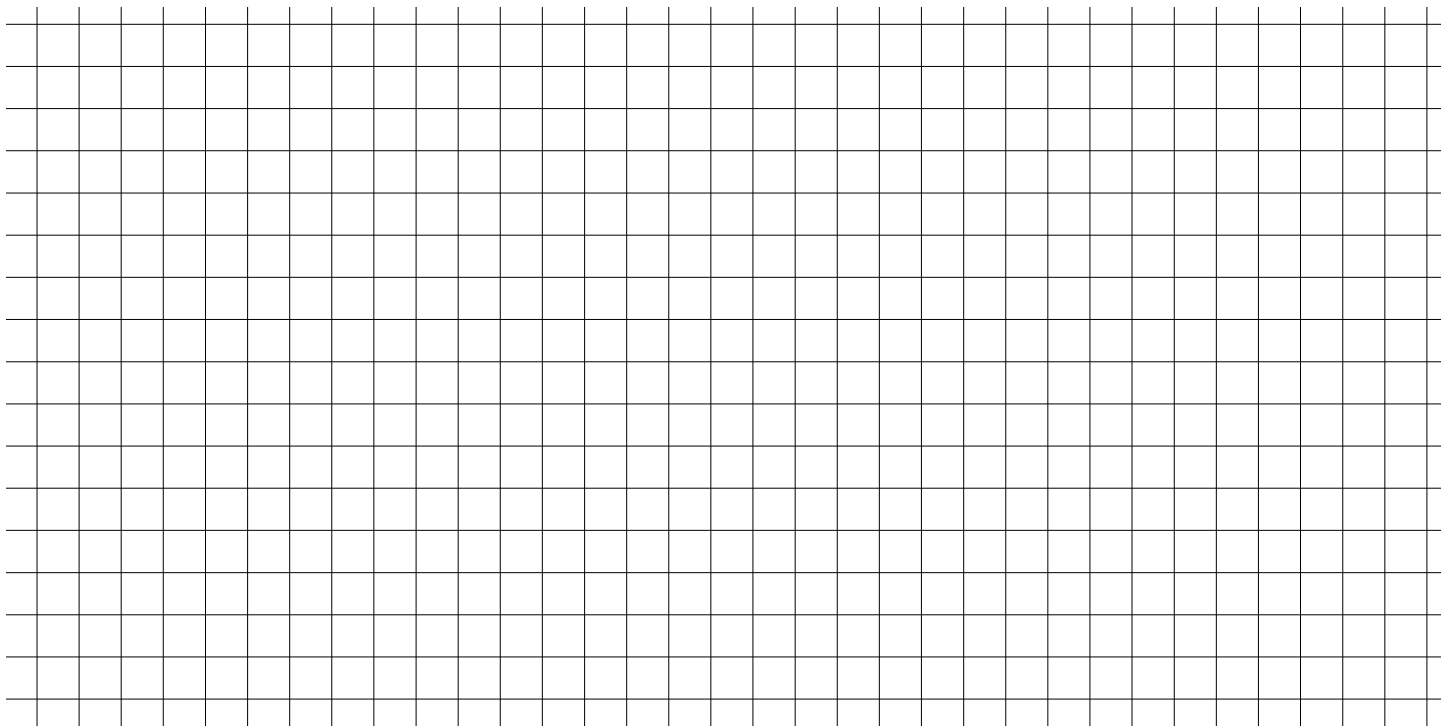
10.2b Class Activity: The Pythagorean Theorem and Tilted Squares



1. On the grids below, construct the following and clearly label each object:
 - a. Square $ABCD$ that has an area of 40 square units
 - b. Square $PQRS$ that has an area of 10 square units
 - c. \overline{EF} that has a length of $\sqrt{8}$ units
 - d. \overline{LM} that has a length of $\sqrt{17}$ units

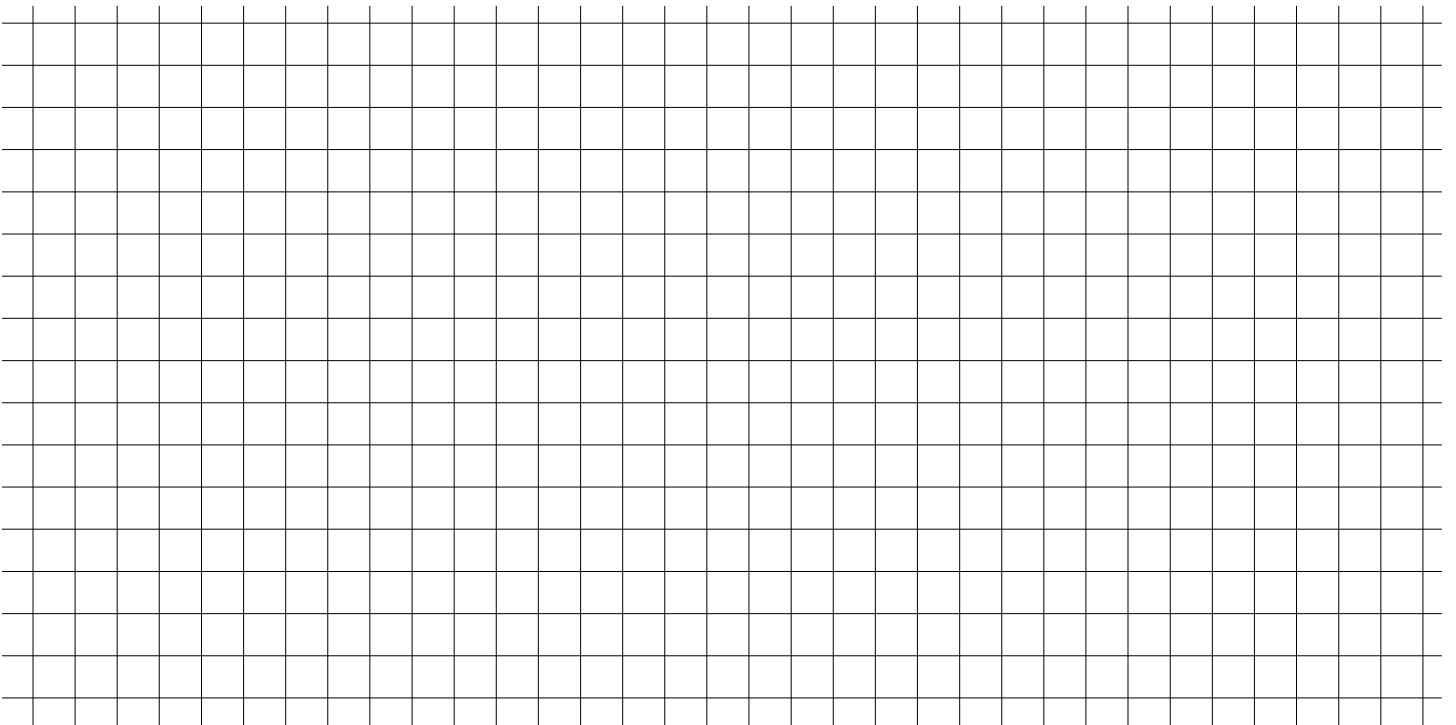
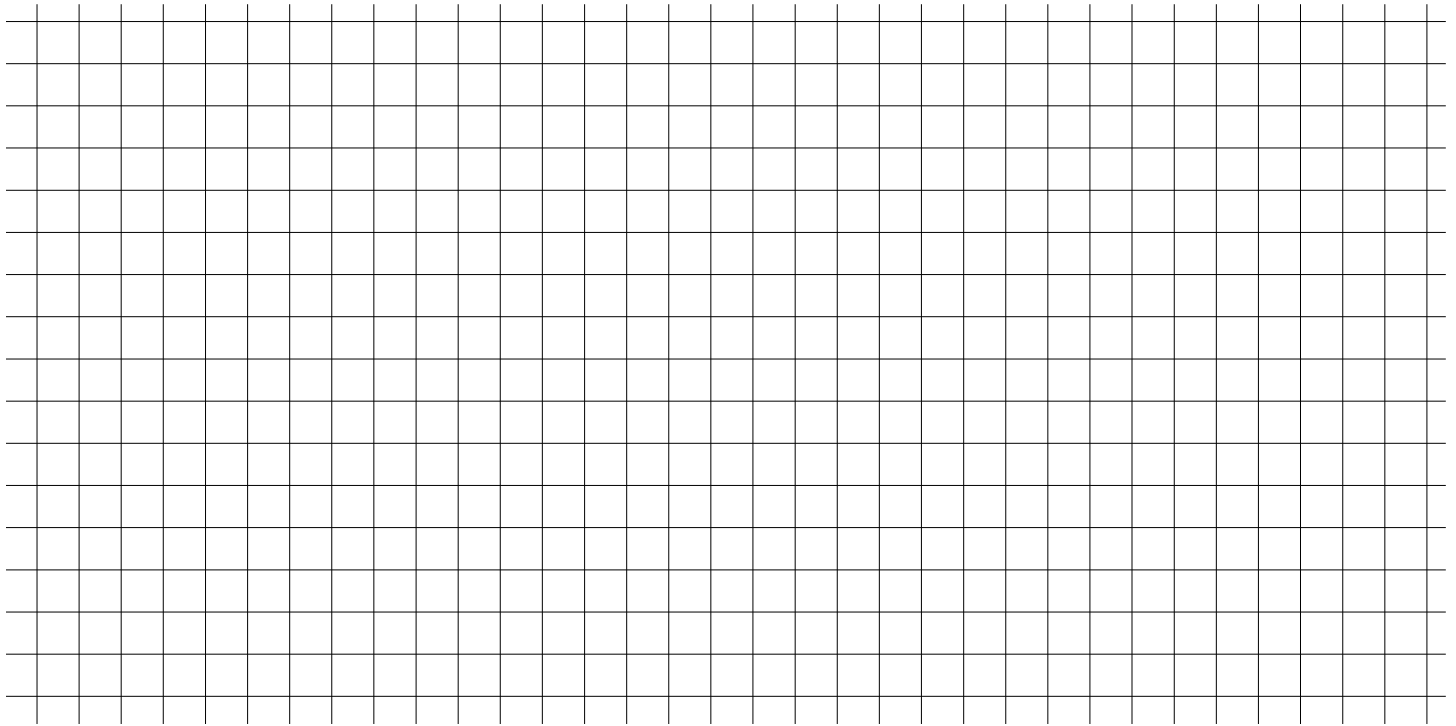


2. Draw as many different squares as you can with an area of 25 square units on the grids below. In this problem, different means that the squares are not tilted the same way.



10.2b Homework: The Pythagorean Theorem and Tilted Squares

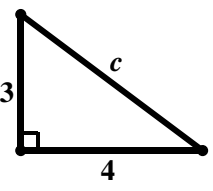
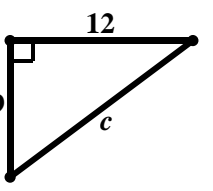
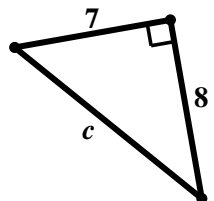
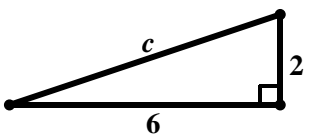
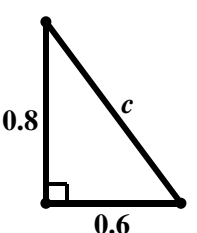
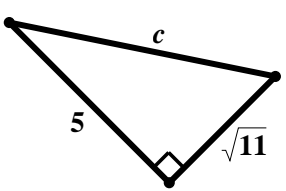
1. On the grids below, construct the following and clearly label each object:
 - a. Square $ABCD$ that has an area of 5 square units
 - b. Square $PQRS$ that has an area of 29 square units
 - c. \overline{EF} that has a length of $\sqrt{18}$ units
 - d. \overline{LM} that has a length of $\sqrt{13}$ units



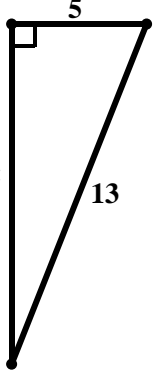
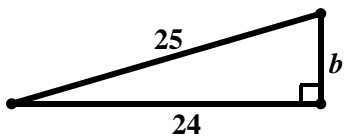
10.2c Class Activity: The Pythagorean Theorem and Unknown Side Lengths

Directions: Find the length of the hypotenuse of each right triangle shown below using the Pythagorean Theorem. Leave your answer in simplest radical form.

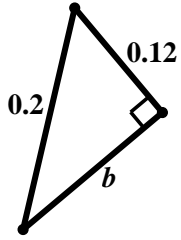


<p>1.</p>  <p>$c = \underline{\hspace{2cm}}$</p>	<p>2.</p>  <p>$c = \underline{\hspace{2cm}}$</p>
<p>3.</p>  <p>$c = \underline{\hspace{2cm}}$</p>	<p>4.</p>  <p>$c = \underline{\hspace{2cm}}$</p>
<p>5.</p>  <p>$c = \underline{\hspace{2cm}}$</p>	<p>6.</p>  <p>$c = \underline{\hspace{2cm}}$</p>

Directions: Find the length of the leg of each right triangle shown below using the Pythagorean Theorem. Leave your answer in simplest radical form.

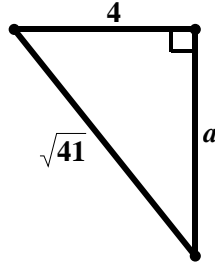
<p>7.</p>  <p>$a = \underline{\hspace{2cm}}$</p>	<p>8.</p>  <p>$b = \underline{\hspace{2cm}}$</p>
--	---

9.



$b = \underline{\hspace{2cm}}$

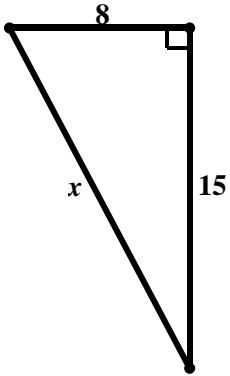
10.



$a = \underline{\hspace{2cm}}$

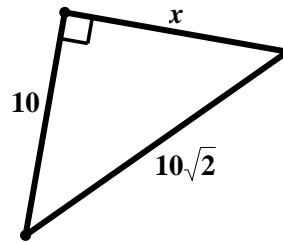
Directions: Find the value of x using the Pythagorean Theorem. Leave your answer in simplest radical form.

11.



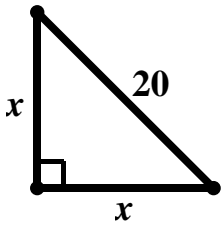
$x = \underline{\hspace{2cm}}$

12.



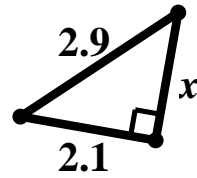
$x = \underline{\hspace{2cm}}$

13.



$x = \underline{\hspace{2cm}}$

14.



$x = \underline{\hspace{2cm}}$

10.2c Homework: The Pythagorean Theorem and Unknown Side Lengths

Directions: Two side lengths of a right triangle have been given. Solve for the missing side length if a and b are leg lengths and c is the length of the hypotenuse. Leave your answer in simplest radical form.

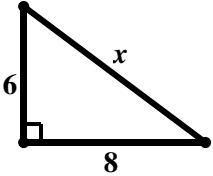
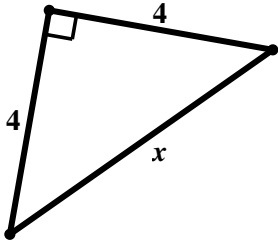
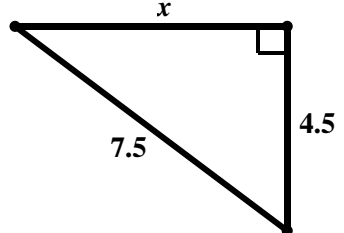
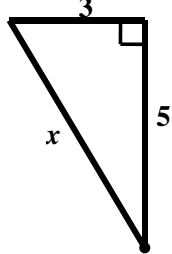
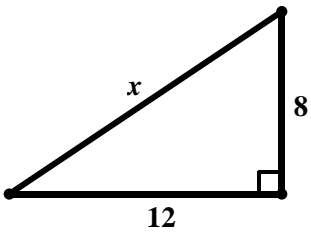
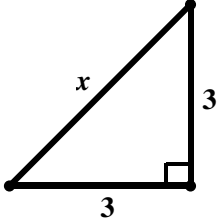
1. $a = 16, b = 30, c = ?$

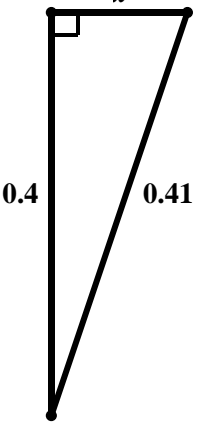
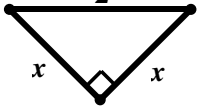
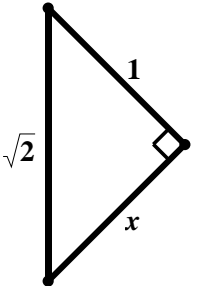
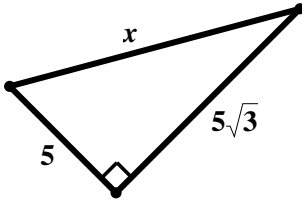
3. $a = 40, b = ?, c = 50$

2. $a = 2, b = 2, c = ?$

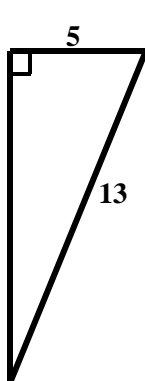
4. $a = ?, b = 4\sqrt{3}, c = 8$

Directions: Find the value of x using the Pythagorean Theorem. Leave your answer in simplest radical form.

<p>5.</p>  <p>$x =$ _____</p>	<p>6.</p>  <p>$x =$ _____</p>
<p>7.</p>  <p>$x =$ _____</p>	<p>8.</p>  <p>$x =$ _____</p>
<p>9.</p>  <p>$x =$ _____</p>	<p>10.</p>  <p>$x =$ _____</p>

<p>11.</p>  <p>$x = \underline{\hspace{2cm}}$</p>	<p>12.</p>  <p>$x = \underline{\hspace{2cm}}$</p>
<p>13.</p>  <p>$x = \underline{\hspace{2cm}}$</p>	<p>14.</p>  <p>$x = \underline{\hspace{2cm}}$</p>

15. **Find, Fix, and Justify:** Megan was asked to solve for the unknown side length in the triangle below. Her work is shown below. She made a mistake when solving. Explain the mistake she made and then solve the problem correctly.



Megan's Solution:

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 5^2 + 13^2 &= c^2 \\
 25 + 169 &= c^2 \\
 194 &= c^2 \\
 \sqrt{194} &= c
 \end{aligned}$$

Correct Solution:

Explain Mistake:

16. **Find, Fix, and Justify:** Raphael was asked to solve for the length of the hypotenuse in a right triangle with legs that have side lengths of 4 and 5. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.

Raphael's Solution:

$$a^2 + b^2 = c^2$$

$$4^2 + 5^2 = c^2$$

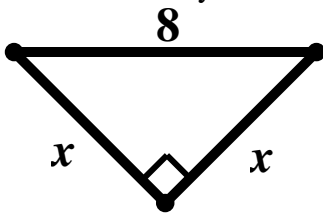
$$16 + 25 = c^2$$

$$41 = c$$

Correct Solution:

Explain Mistake:

17. **Find, Fix, and Justify:** Nataani was asked to solve for the unknown side length in the triangle below. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.



Nataani's Solution:

$$a^2 + b^2 = c^2$$

$$x^2 + x^2 = 8$$

$$2x^2 = 8$$

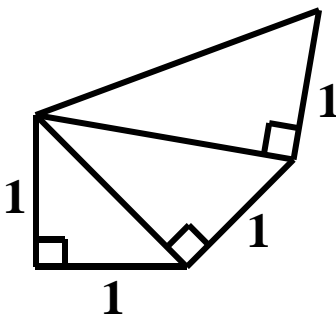
$$x^2 = 4$$

$$x = 2$$

Correct Solution:


Explain Mistake:

Extra for Experts: Use the picture below to answer questions a) and b).



- Find all the missing side lengths and label the picture with the answers.
- Using the picture above, devise a strategy for constructing a segment with a length of $\sqrt{5}$. Explain your strategy below.

10.2d Class Activity: The Converse of the Pythagorean Theorem

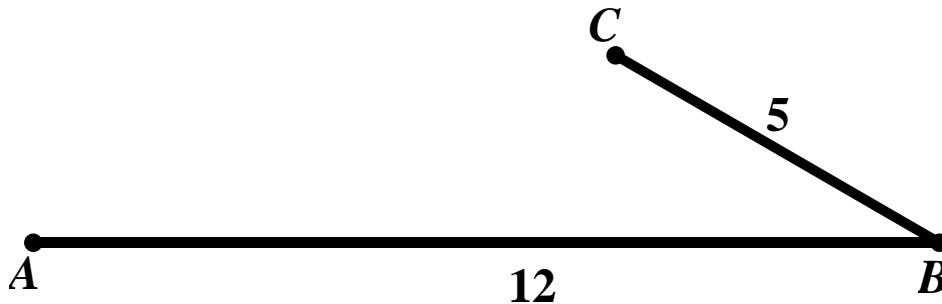
1. Mr. Riley's 8th grade class has been studying the Pythagorean Theorem. One day, he asked his class to find numbers a , b , and c where $a^2 + b^2 = c^2$, and draw triangles with those side lengths. 

Oscar determined that the numbers 5, 12, and 13 satisfy the Pythagorean Theorem as shown below:

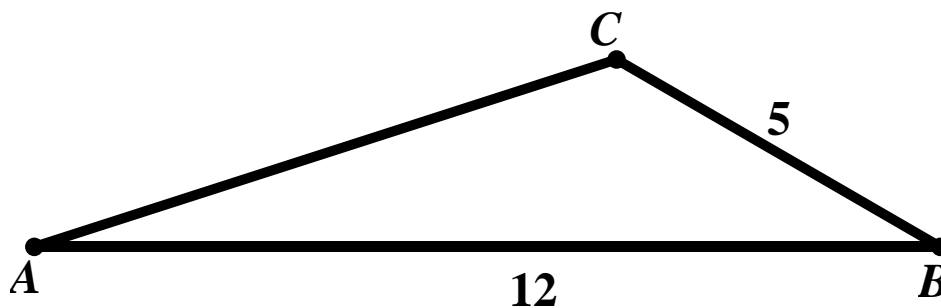
$$\begin{aligned}a^2 + b^2 &= c^2 \\5^2 + 12^2 &= 13^2 \\25 + 144 &= 169 \\169 &= 169\end{aligned}$$

Mr. Riley then said, "OK, so you have found three numbers that satisfy the Pythagorean Theorem. Now, show me that the triangle formed with these side lengths is a right triangle."

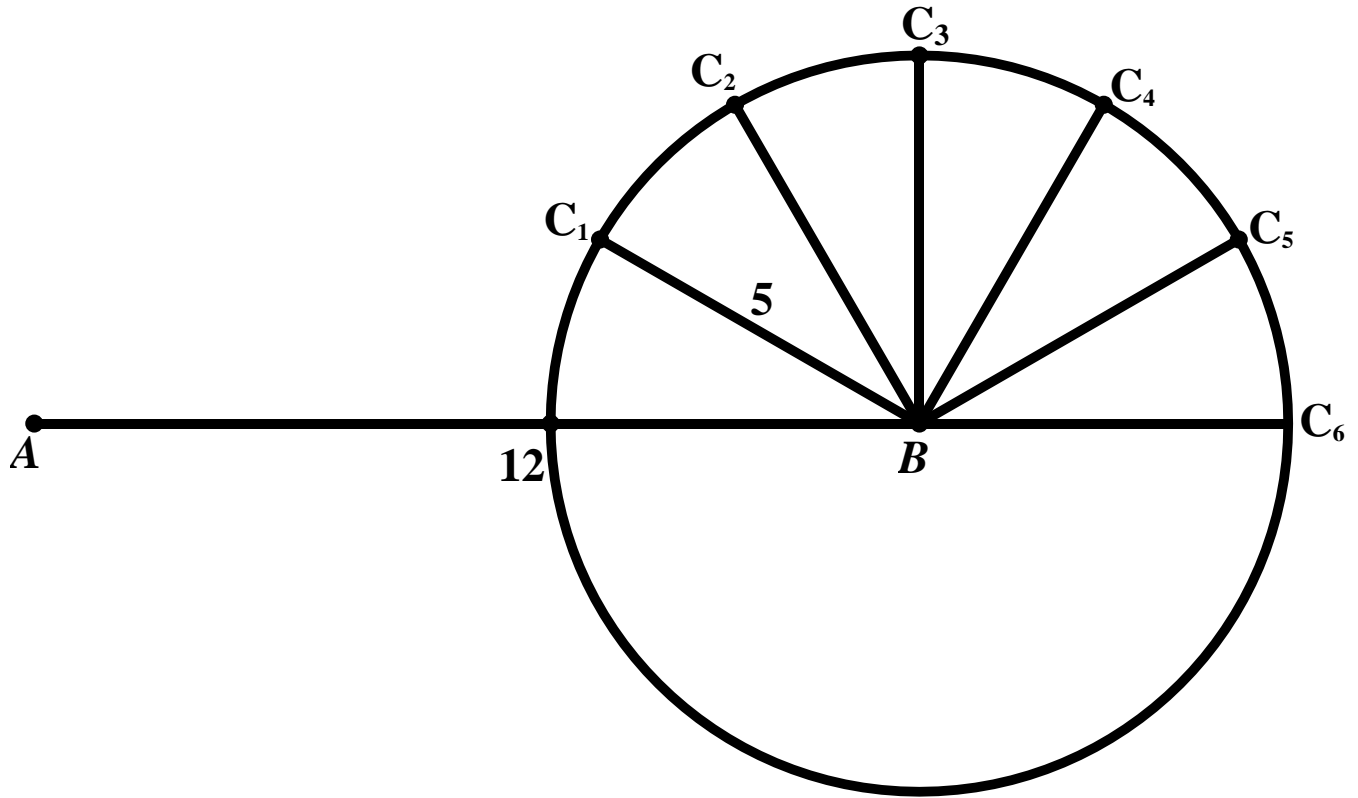
- a. Oscar continued working on the problem. He constructed a segment with a length of 12 cm and labeled the segment AB . From the endpoint B , he constructed a segment with a length of 5 cm and labeled the segment BC as shown in the picture below. Using a ruler, verify the lengths of the segments below.



- b. Then, he thought to himself, "I need to make the third side length AC equal to 13 because I know the triple 5, 12, 13 satisfies the Pythagorean Theorem." He connected A and C as shown below. He measured the length of AC and determined it did not measure 13 cm. Using a ruler, verify that AC does not measure 13 cm.



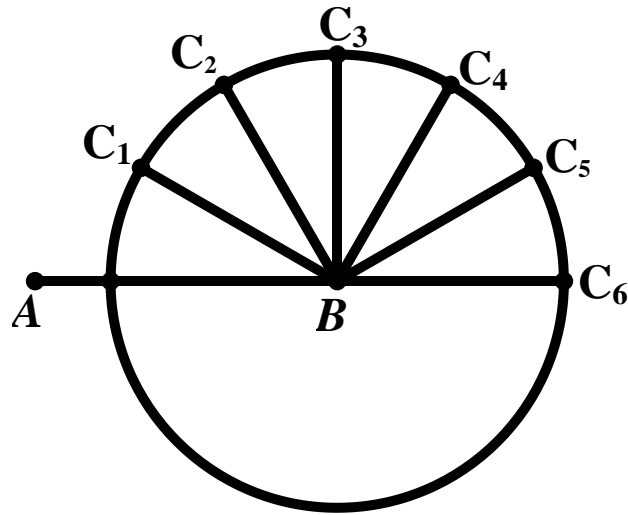
- c. Then, he thought to himself, “What if I rotate \overline{BC} around point B until AC measures 13 cm?” He began to rotate \overline{BC} clockwise about B in increments as shown below. Help Oscar to find the location of C on the circle below that will give him a triangle with side lengths 5, 12, and 13.




- d. What type of triangle is formed when AC equals 13 cm?

2. Lucy also found a set of numbers that satisfy the Pythagorean Theorem: 3, 4 and 5. Verify in the space below that Lucy’s numbers satisfy the Pythagorean Theorem.

3. Using a process similar to Oscar's, Lucy set out to prove that a triangle with side lengths 3, 4 and 5 is in fact a right triangle. In the picture below $AB = 4$ cm and $CB = 3$ cm. Help Lucy determine the location of C that will create a triangle with side lengths 3 cm, 4 cm, and 5 cm.



4. What type of triangle is formed when AC equals 5 cm?
5. Based on the problems above, what type of triangle is formed with side lengths that satisfy the Pythagorean Theorem? Write down the Converse of the Pythagorean Theorem. 

6. Do the side lengths given below satisfy the Pythagorean Theorem? Remember to distinguish between legs (shorter sides) and the hypotenuse (longest side) and enter them into the equation correctly.

a. 11, 60, 61	b. 2, 4, 6
c. 14, 50, 48	d. 1, 3, $\sqrt{10}$;
e. 2, 4, and $2\sqrt{5}$.	f. 5, 6, 8

7. Mr. Garcia then asks the class, “What if the tick marks in Lucy’s picture are each 2 cm instead of 1 cm? What are the measures of the side lengths that form the right triangle? Do they satisfy the Pythagorean Theorem?”



8. What if the tick marks in Lucy’s picture are each 3 cm? 0.1 cm? 10 cm? What are the measures of the side lengths that form the right triangles given these different scales and do they satisfy the Pythagorean Theorem?

10.2d Homework: The Converse of the Pythagorean Theorem

Directions: Determine whether the three side lengths form a right triangle. Write yes or no on the line provided.

1. 9, 12, 15 _____

2. 18, 36, 45 _____

3. 12, 37, 35 _____

4. 8, 15, 16 _____

5. $\sqrt{6}$, $\sqrt{10}$, 4 _____

6. 6.4, 12, 12.2 _____

7. 8.6, 14.7, 11.9 _____

8. 8, $8\sqrt{3}$, 16 _____

9. 8, 8, $8\sqrt{2}$ _____

10. 7, 9, 11.4 _____

10.2e Class Activity: Exploration with Pythagorean Triples Extension



While we have seen several different sets of numbers that form a right triangle, there are special sets of numbers that form right triangles called Pythagorean triples. A **Pythagorean triple** is a set of nonzero **whole numbers** a , b , and c that can be put together to form the side lengths of a right triangle. 3, 4, 5 and 5, 12, 13 are examples of Pythagorean triples. We have seen many other sets of numbers that form a right triangle such as 0.09, 0.4, 0.41 that are not Pythagorean triples because their side lengths are not whole numbers.

- a. The chart below shows some sets of numbers a , b , and c that are Pythagorean triples. Verify that the sets satisfy the equation $a^2 + b^2 = c^2$.

- b. Can you find additional Pythagorean triples? Explain the method you used.

a	b	c	a^2	b^2	c^2
3	4	5			
5	12	13			
7	24	25			

- c. The chart above starts with values for a that are odd numbers. Why didn't the chart start with a value of 1 for a .
- d. Can you find Pythagorean triples where a is even? What is the smallest Pythagorean triple you can find with a being an even number?
- e. Design a method to confirm that these numbers actually form right triangles. Write a short paragraph describing the method you used, and the results you obtained.

10.2f Self-Assessment: Section 10.2

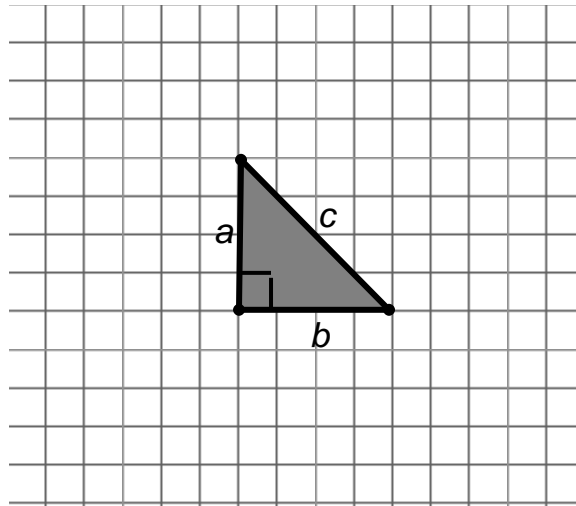
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provided on the next page that match each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Know that in a right triangle $a^2 + b^2 = c^2$, where a and b are the legs of the right triangle and c is the hypotenuse. <i>See sample problem #1</i>				
2. Understand and explain a proof of the Pythagorean Theorem. <i>See sample problem #2</i>				
3. Use the Pythagorean Theorem to solve for the missing side length of a right triangle given the measurements of the other two sides. <i>See sample problem #3</i>				
4. Understand and explain a proof of the converse of the Pythagorean Theorem. That is, for a triangle with side lengths a , b , and c if $a^2 + b^2 = c^2$, then the triangle is a right triangle. <i>See sample problem #4</i>				
5. Use the converse of the Pythagorean Theorem to determine whether three given measurements are possible side lengths of a right triangle. <i>See sample problem #5</i>				

10.2f Sample Problems: Section 10.2

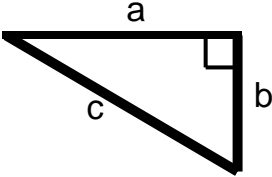
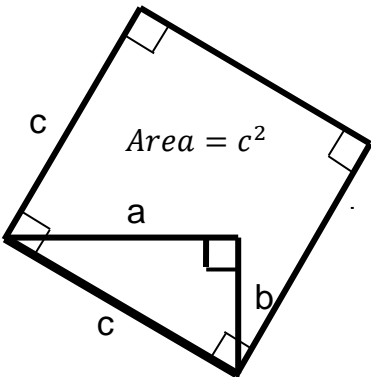
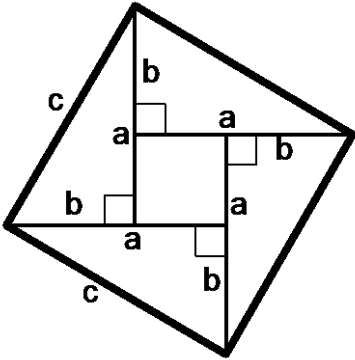
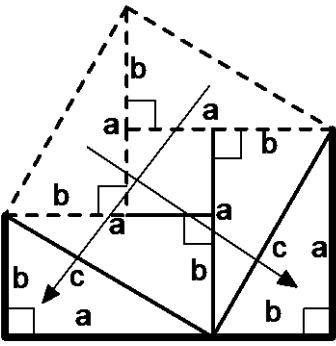
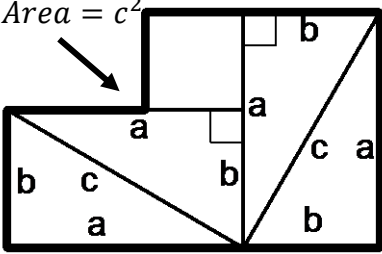
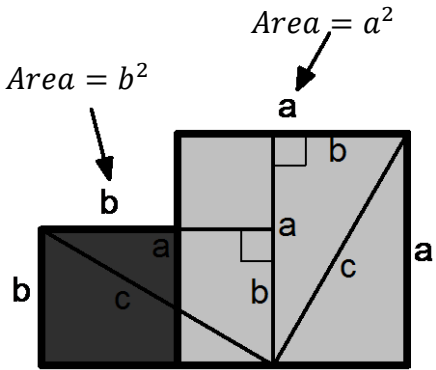
Sample Problem #1

In the picture below the gray triangle is a right triangle. Draw the squares along each of the three sides of the triangles. Find the area of each square and write the area in each square. Then, find the side lengths a , b , c of the triangle.



Sample Problem #2

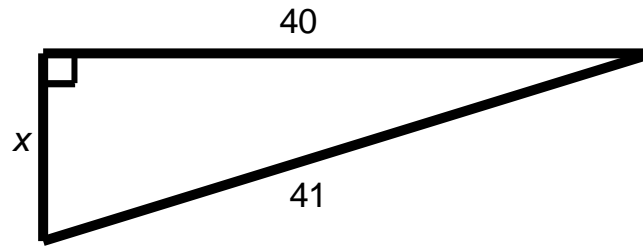
Below is a geometric explanation for a proof of the Pythagorean Theorem: Given a right triangle with side lengths a and b and a hypotenuse of c , then $a^2 + b^2 = c^2$. The figures for the proof are given in order. Choose the explanation that provides a sound argument accompanied with reasoning and warrants to support the claims given for each figure. Write the letter that matches each explanation in the space provided.

 <p>Figure 1 _____</p>	 <p>Figure 2 _____</p>	 <p>Figure 3 _____</p>
 <p>Figure 4 _____</p>	 <p>Figure 5 _____</p>	 <p>Figure 6 _____</p>

- Inside of the square draw 4 congruent right triangles with side length a and b and a hypotenuse of c .
- Draw a square with off of this triangle with a side length of c . The area of square this square is c^2 . This is because the area of a square is the side length squared
- You can view the area of this figure as the composition of two squares with sides length a and b . The area of the darker square is b^2 and the area of the lighter shaded square is a^2 . Thus the area of the whole figure is $a^2 + b^2$. As stated above this is the same as the area of the original square with side length c . Thus $a^2 + b^2 = c^2$.
- Rearrange the square by translating the top two triangles to the bottom of the figure.
- The area of this figure is the same as the area of the original square because we have not added or removed any of the pieces.
- Begin with right triangle with a horizontal side length of a and a vertical side length of b and a hypotenuse of c .

Sample Problem #3

Find the value of x using the Pythagorean Theorem. Leave your answer in simplest radical form.



Sample Problem #4

The Converse of the Pythagorean Theorem states that given a triangle with side lengths a , b , and c , if $a^2 + b^2 = c^2$, then the triangle is a right triangle. Explain the proof of the Converse of the Pythagorean Theorem that your teacher provides for you.

Sample Problem #5

Determine whether the three side lengths form a right triangle. Show your work to verify your answer.

5.5, 12.5, 13.5

Section 10.3 Applications of the Pythagorean Theorem

Section Overview:

In this section, students apply the Pythagorean Theorem to solve real-world problems in two- and three-dimensions. Then, students use the Pythagorean Theorem to find the distance between two points. After the students gain an understanding of the process being used to find the distance between two points in a coordinate system, students have the opportunity to derive the distance formula from the Pythagorean Theorem and the process being used. Rather than memorizing the distance formula, the emphasis is placed on the process used to find the distance between two points in a coordinate system and the connection between the Pythagorean Theorem and the distance formula.

Concepts and Skills to Master:

By the end of this section, students should be able to:

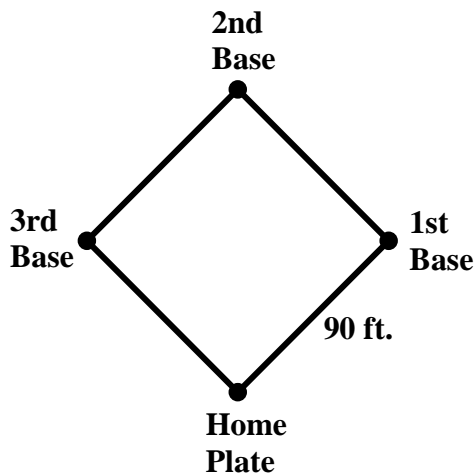
1. Use the Pythagorean Theorem to solve problems in real-world contexts, including three-dimensional contexts.
2. Find the distance between two points in a coordinate system.

10.3a Class Activity: Applications of the Pythagorean Theorem

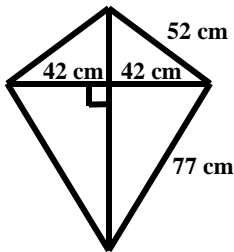
Directions: For each problem, first draw a picture if one is not provided and then solve the problem.



1. What is the length of the diagonal of a rectangle of side lengths 1 inch and 4 inches?
2. A square has a diagonal with a length of $2\sqrt{2}$ inches. What is the side length of the square?
3. Two ships leave a dock. The first ship travels 6 miles east and then 8 miles north and anchors for the night. The second ship travels 5 miles west and then 12 miles south and anchors for the night. How far are each of the ships from the dock when they anchor for the night?
4. A baseball diamond is in the shape of a square. The distance between each of the consecutive bases is 90 feet. What is the distance from Home Plate to 2nd Base?

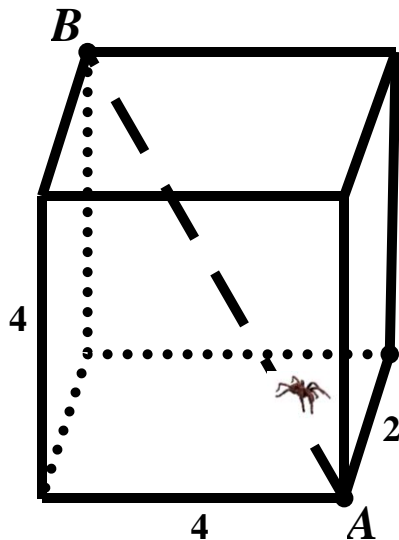


5. Ray is a contractor that needs to access his client's roof in order to assess whether the roof needs to be replaced. He sees that he can access a portion of the roof that is 15 feet from the ground. He has a ladder that is 20 feet long.
- How far from the base of the house should Ray place the ladder so that it just hits the top of the roof? Round your answer to the nearest tenth of a foot.
 - How far should he place the ladder from the base of the house if he wants it to sit 3 feet higher than the top of the roof? Round your answer to the nearest tenth of a foot.
6. The dimensions of a kite sail are shown below. The support rod that runs from the top of the kite to the bottom of the kite has been broken and needs to be replaced. What length of rod is needed to replace the broken piece? Round your answer to the nearest tenth.

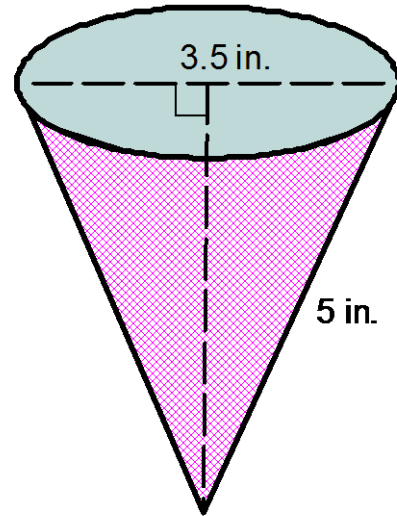


7. A new restaurant is putting in a wheelchair ramp. The landing that people enter the restaurant from is 2 feet higher than street level. Safety standards require that for every 1 foot of rise on a wheelchair ramp there must be a run of 12 feet. How long will the ramp have to be to meet safety standards? Round your answer to the nearest tenth.

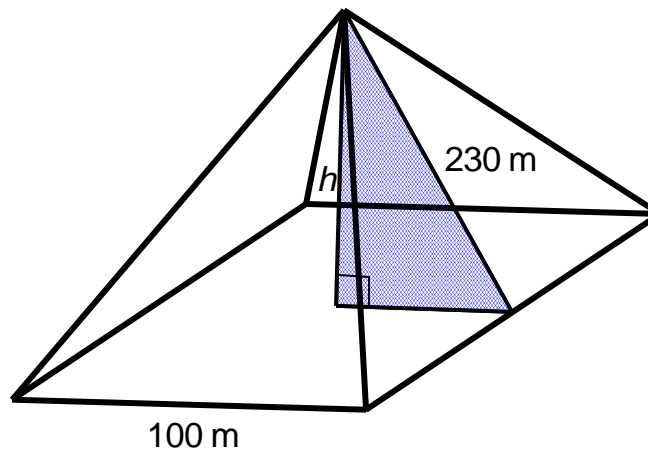
8. Melanie is having a rectangular-shaped patio built in her backyard. It is very important to Melanie that the corners of the patio are right angles. The contractor built a patio with a width of 10 feet and a length of 15 feet. The diagonal measures 20 feet. Does the patio have the right angles that Melanie requested?
9. Fred is safety conscious. He knows that to be safe, the distance between the foot of the ladder and the wall should be $\frac{1}{4}$ the height of the wall. Fred needs to get on the roof of the school building which is 20 ft. tall. How long should the ladder be if he wants it to rest on the edge of the roof and meet safety standards? Round your answer to the nearest tenth.
10. A spider has taken up residence in a small cardboard box which measures 2 inches by 4 inches by 4 inches. What is the length, in inches, of a straight spider web that will carry the spider from the lower right front corner of the box to the upper left back corner of the box?



11. Sunny made a paper cone to hold candy for favors for a baby shower. After making the cones she measures the slant height of the cone and the diameter of the base of the cone. Her measurements are shown in the picture below. Find the volume of the cone.



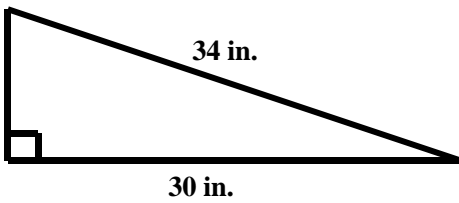
12. In the movie *Despicable Me*, an inflatable model of The Great Pyramid of Giza in Egypt was created by Vector to trick people into thinking that the actual pyramid had not been stolen. When inflated, the false Great Pyramid had a square base of side length 100 m. and the height of one of the side triangles was 230 meters. This is also called the slant height of the pyramid. What is the volume of gas that was used to fully inflate the fake Pyramid? (Hint: Recall the formula for the volume of a pyramid is $\frac{1}{3}Bh$ where B is the area of the base and h is the height of the pyramid (the distance from the base to the apex).



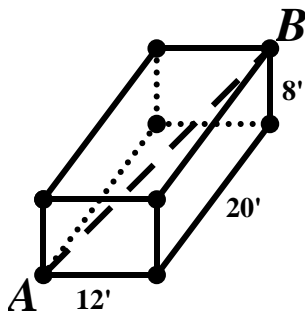
10.3a Homework: Applications of the Pythagorean Theorem

1. What is the length of the diagonal of a square with a side length of 4 cm?
2. One side length of a rectangle is 2 inches. The diagonal of the rectangle has a length of $2\sqrt{5}$ inches. What is the length of the other side of the rectangle?
3. A football field is 360 feet long and 160 feet wide. What is the length of the diagonal of a football field assuming the field is in the shape of a rectangle?
4. The length of an Olympic-size swimming pool is 55 meters. The width of the pool is 25 meters. What is the length of the diagonal of the pool assuming the pool is in the shape of a rectangle?
5. You are locked out of your house. You can see that there is a window on the second floor that is open so you plan to go and ask your neighbor for a ladder long enough to reach the window. The window is 20 feet off the ground. There is a vegetable garden on the ground below the window that extends 10 ft. from the side of the house that you can't put the ladder in. What size ladder should you ask your neighbor for?

6. Kanye just purchased a skateboarding ramp. The ramp is 34 inches long and the length of the base of the ramp is 30 inches as shown below. What is the height of the ramp?

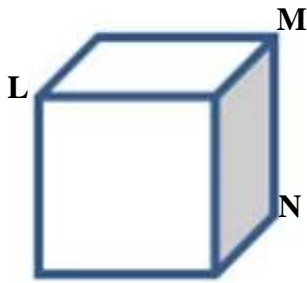


7. A rectangular-shaped room has a width of 12 feet, a length of 20 feet, and a height of 8 feet. What is the approximate distance from one corner on the floor (Point A in the figure) to the opposite corner on the ceiling (Point B in the figure)?



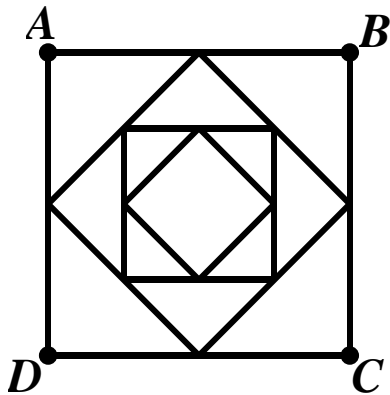
8. A large pile of sand has been dumped into a conical pile in a warehouse. The slant height of the pile is 20 feet. The diameter of the base of the sand pile is 32 ft. Find the volume of the pile of sand.

9. The cube below is a unit cube. A unit cube is a cube of side length 1.



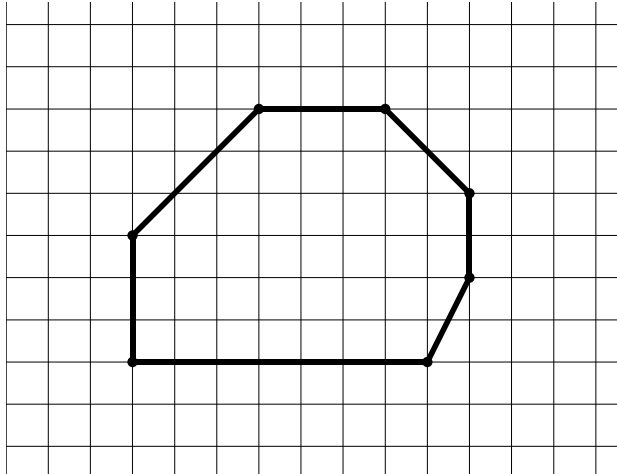
- a. What is the length of \overline{LM} ? Leave your answer in simplest radical form.
- b. What is the length of \overline{LN} ? Leave your answer in simplest radical form.

Extra for Experts: Square $ABCD$ has side lengths equal to 4 inches. Connecting the midpoints of each side forms the next square inside $ABCD$. This pattern of connecting the midpoints to form a new square is repeated.



- a. What is the side length of the inner-most square?
- b. What is the area of the inner-most square?
- c. What is the ratio of the area of each square to the area of the next square created?

Extra for Experts: The following is a scale drawing of a patio that Mr. Davis plans to build in his backyard. Each box in the scale drawing represents 1 unit.



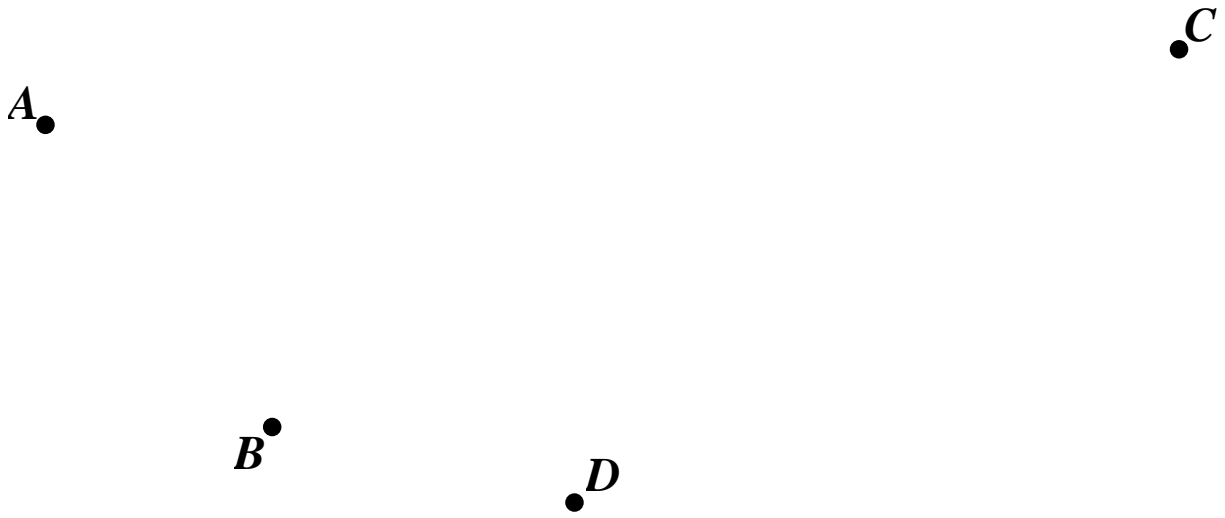
- Find the exact value of the perimeter of the scale drawing of the patio. Show all work and thinking.
- Find the area of the scale drawing of the patio. Show all work and thinking.
- If the scale on the drawing above is 1 unit = 3 feet, what is the actual measure of the perimeter of the patio? The area? Show all work.

10.3b Class Activity: Finding Distance Between Two Points

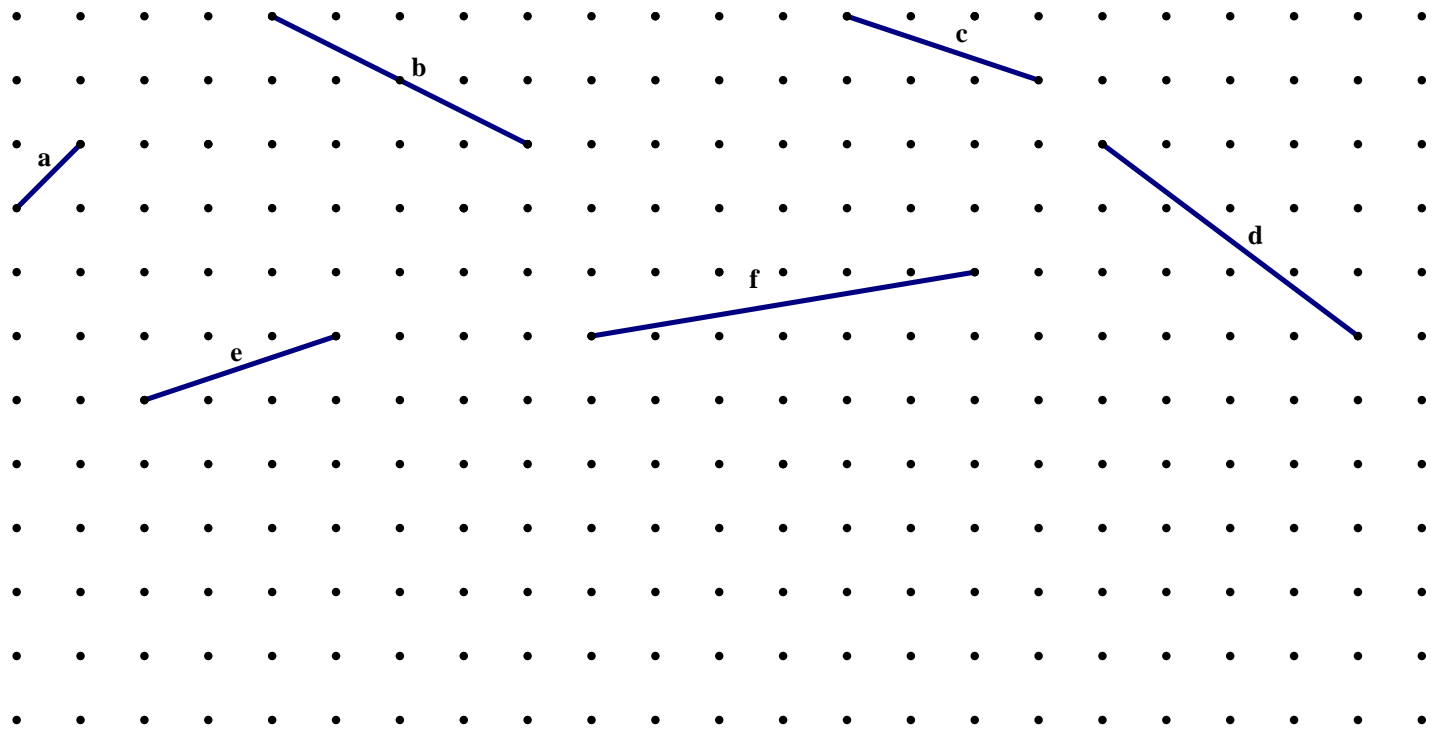
1. Using a centimeter ruler, find the distance between the following sets of points shown below. Then draw the slope triangle of each segment, measure the lengths of the rise and run, and verify that the Pythagorean Theorem holds true.



- a. A to B
- b. B to C
- c. C to D

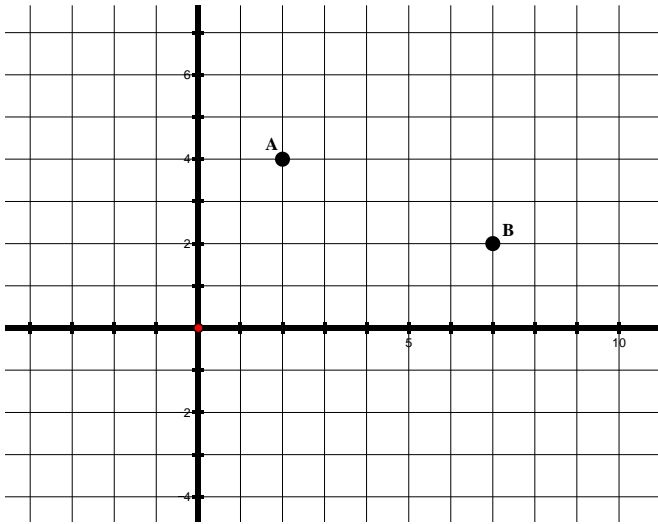


2. Find the lengths of the segments below. Assume that each horizontal and vertical segment connecting the dots has a length of 1 unit.

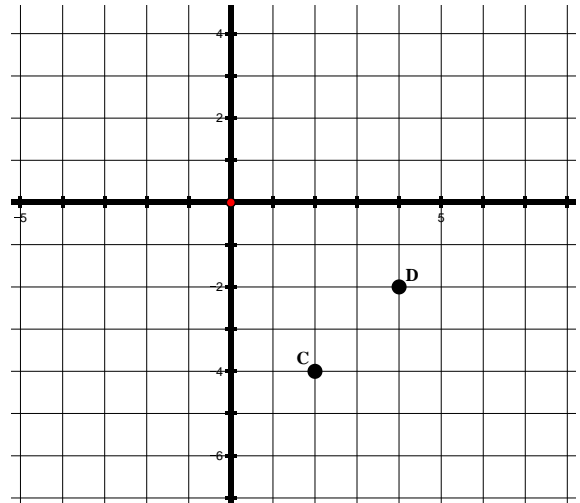


Directions: Label the coordinates of each point. Then, find the distance between the two points shown on each grid below.

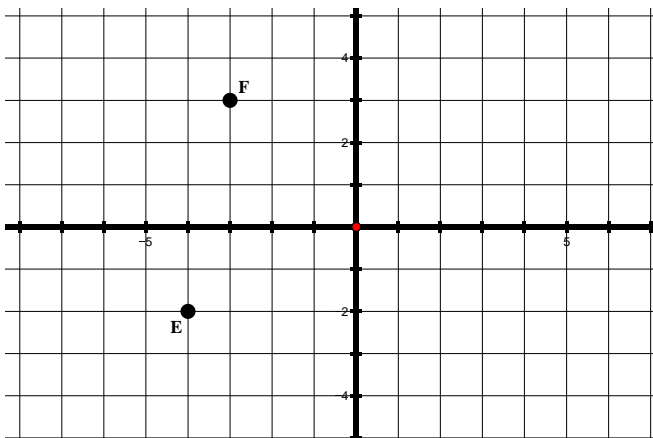
3.



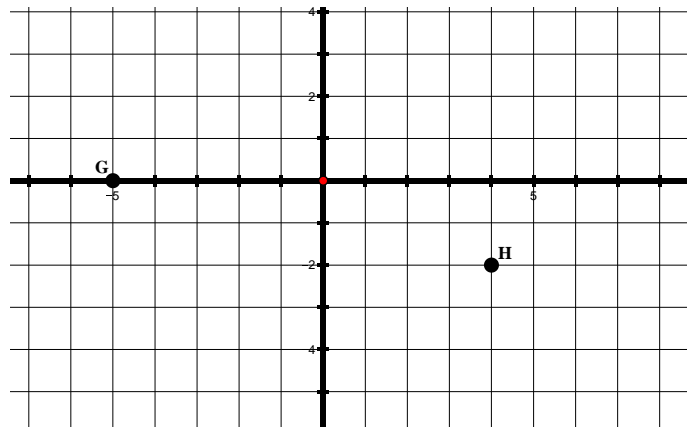
4.



5.



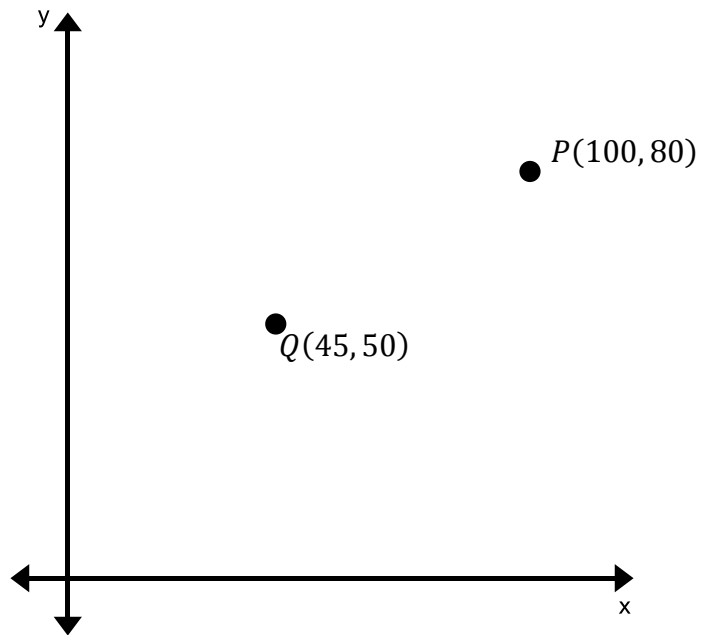
6.



The Coordinate Distance Formula



7. Find the distance between the two points given on the graph below.



8. Find the distance between the two points given below. Leave your answers in simplest radical form.

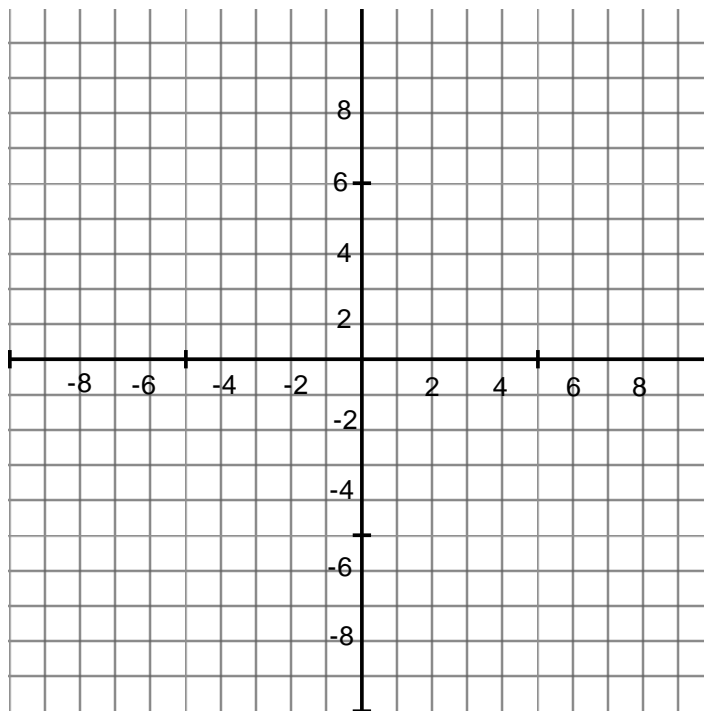
a. $A: (3, 5)$ $B: (6, 9)$

b. $R: (-1, 4)$ $S: (3, 8)$

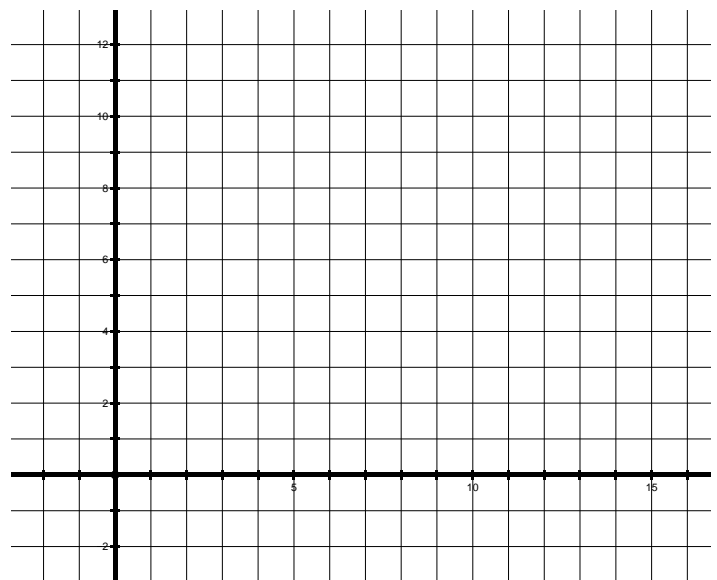
c. $C: (0, 5)$ $D: (2, -3)$

d. $S: (-3, -5)$ $T: (2, -7)$

9. A triangle has vertices at the points (2,3) and (4,8), and (6,3) on the coordinate plane.
- Find the perimeter of the triangle. Use the grid below if needed.
 - Find the area of the triangle.
 - If the triangle is dilated by a scale factor of 3 what will the new perimeter be?
 - If the triangle is dilated by a scale factor of 3 what will the new area be?
 - Plot the original triangle, label it triangle A. Then reflect the triangle over the y-axis, label the new triangle A'. Does this transformation change the perimeter of the triangle? Explain your answer.



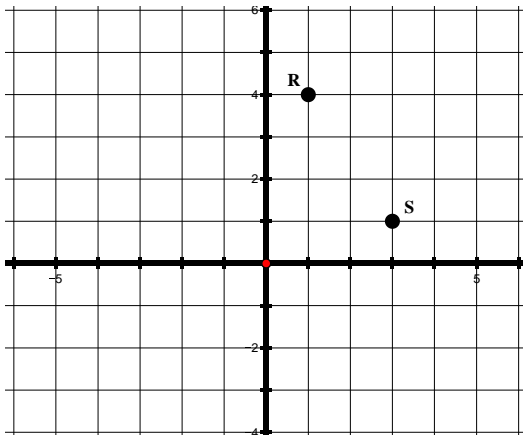
10. List **three** coordinate pairs that are 5 units away from the origin in the first quadrant. Describe how to find the points and justify your reasoning. The grid has been provided to help you. (Note: Points on the axes are not in the first quadrant).



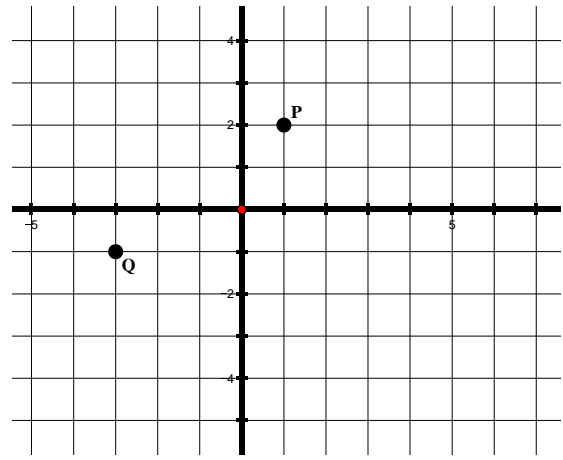
10.3b Homework: Finding Distance Between Two Points

Directions: Find the distance between the two points shown on each grid below. Leave your answers in simplest radical form.

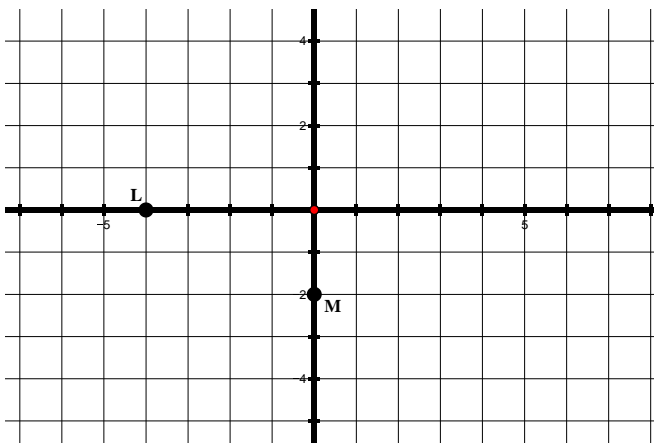
1.



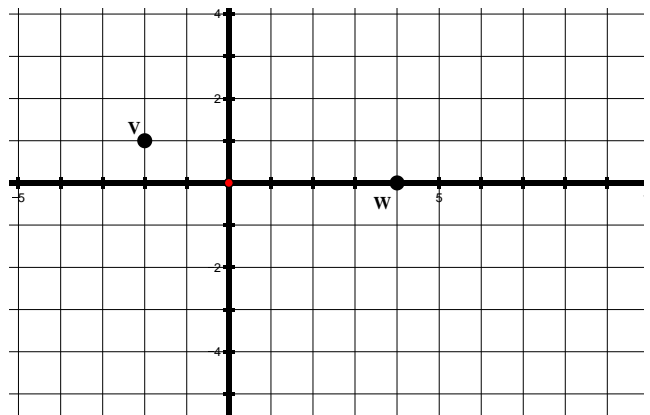
2.



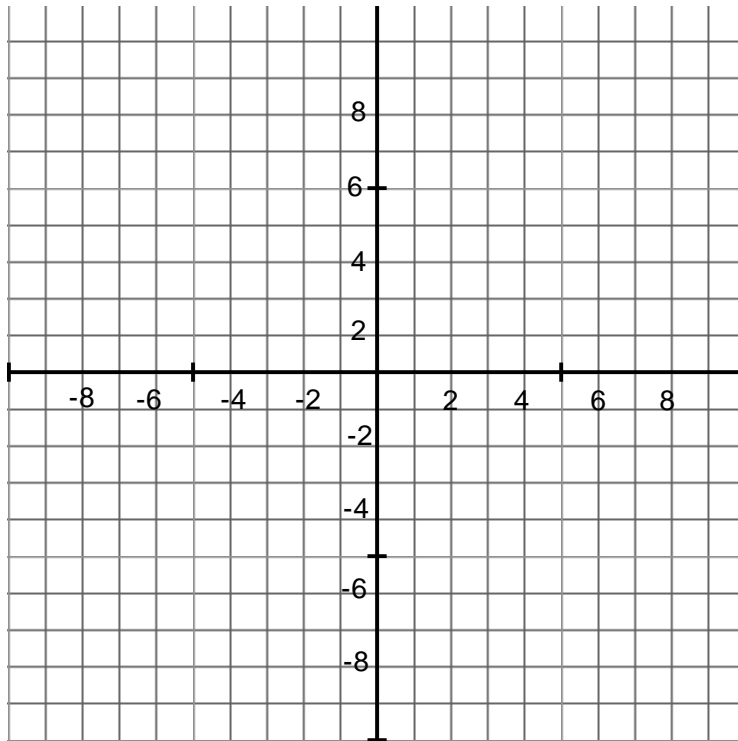
3.



4.



5. Find the distance between the two points given below. Leave your answers in simplest radical form.
- $A: (2, 1)$ $B: (4, 7)$
 - $R: (2, -1)$ $S: (8, -7)$
 - $C: (1, 0)$ $D: (2, -3)$
 - $S: (-2, -4)$ $T: (2, -5)$
6. Plot any letter of the alphabet that is made up of segments that are straight lines on the coordinate plane given below. For example you can plot the letter A, E, F, etc. but not the letter B,C, D, etc.



- Find the total distance for the segments that make up this letter.
- If you dilated this letter by a scale factor of 4 what is the total distance of the segments that make up your letter?
- If you dilated this letter by a scale factor of $\frac{1}{5}$ what is the total distance of the segments that make up your letter?
- Rotate your letter 180 degrees about the origin. Does this transformation change the size or shape of the letter? Explain your answer.

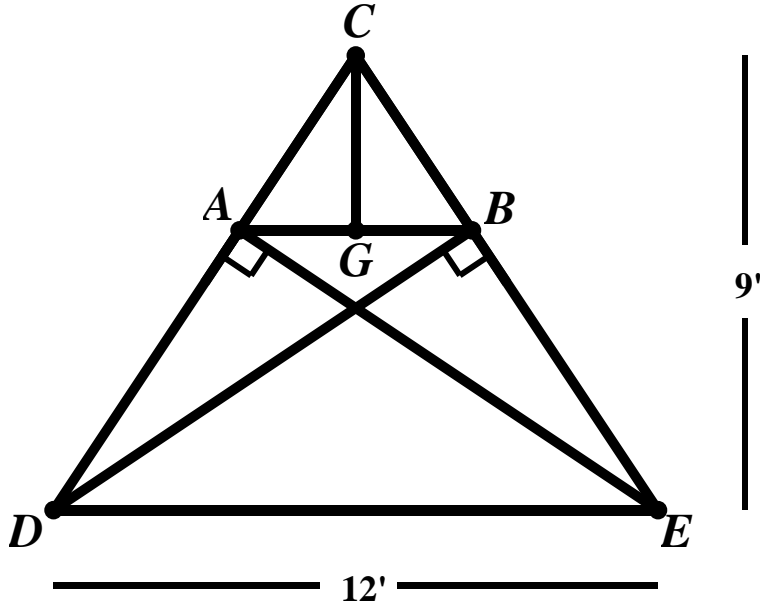
10.3c Extension: Construction

Mario is designing an A-frame for the lodge of a ski resort. Below is a scale drawing of his design.

Given: C lies over the center of the building

$$\overline{AB} \parallel \overline{DE}$$

$\angle DAE$ and $\angle EBD$ are right angles.



What are the lengths of all segments in the diagram?

10.3d Self-Assessment: Section 10.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provided on the next page that match each skill/concept.

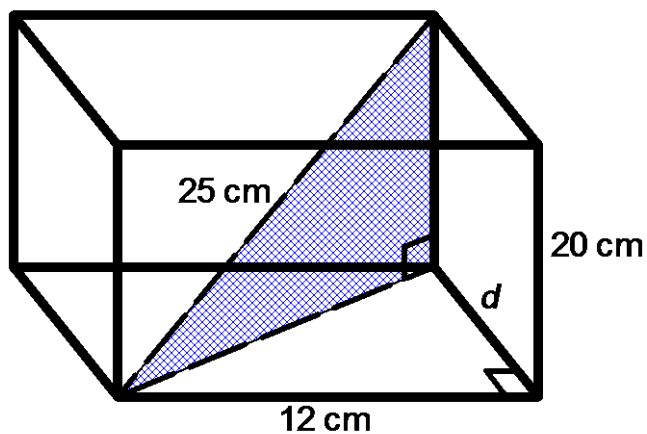
Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Use the Pythagorean Theorem to solve problems in real-world contexts, including three-dimensional contexts. <i>See sample problem #1</i>				
2. Find the distance between two points in a coordinate system. <i>See sample problem #2</i>				

10.3d Sample Problems: Section 10.3

Sample Problem #1

- a. A park is 6 miles east of your home. The bakery is 4 miles north of the park. How far is your home from the bakery as the crow flies?

- b. Find the volume of the rectangular prism given below.



Sample Problem #2

Find the distance between each set of points.

- a. $A(-10,2)$ and $B(-7,6)$
- b. $C(-2,-6)$ and $D(6,9)$
- c. $E(3,5)$ and $F(7,9)$
- d. $G(3,4)$ and $H(-2,-2)$