Version 01

Angles

Apprenticeship and Workplace Mathematics (Grade 10/Literacy Foundations Level 7)

OPEN SCHOOL BC

Course History

New, March 2012

Project Partners

This course was developed in partnership with the Distributed Learning Resources Branch of Alberta Education and the following organizations:

- Black Gold Regional Schools
- Calgary Board of Education
- Edmonton Public Schools
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Viewing Your PDF Learning Package

This PDF Learning Package is designed to be viewed in Acrobat. If you are using the optional media resources, you should be able to link directly to the resource from the pdf viewed in Acrobat Reader. The links may not work as expected with other pdf viewers.

Download Adobe Acrobat Reader: *http://get.adobe.com/reader/*

Section Organization

This section on Angles is made up of several lessons.

Lessons

Lessons have a combination of reading and hands-on activities to give you a chance to process the material while being an active learner. Each lesson is made up of the following parts:

Essential Questions

The essential questions included here are based on the main concepts in each lesson. These help you focus on what you will learn in the lesson.

Focus

This is a brief introduction to the lesson.

Get Started

This is a quick refresher of the key information and skills you will need to be successful in the lesson.

Activities

Throughout the lesson you will see three types of activities:

- Try This activities are hands-on, exploratory activities.
- Self-Check activities provide practice with the skills and concepts recently taught.
- Mastering Concepts activities extend and apply the skills you learned in the lesson.

You will mark these activities using the solutions at the end of each section.

Explore

Here you will explore new concepts, make predictions, and discover patterns.

Bringing Ideas Together

This is the main teaching part of the lesson. Here, you will build on the ideas from the Get Started and the Explore. You will expand your knowledge and practice your new skills.

Lesson Summary

This is a brief summary of the lesson content as well as some instructions on what to do next.

At the end of each section you will find:

Solutions
This contains all of the solutions to the Activities.

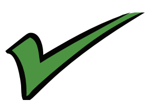
Appendix
Here you will find the Data Pages along with other extra resources that you need to complete the section. You will be directed to these as needed.

Glossary
This is a list of key terms and their definitions.

Throughout the section, you will see the following features:

Icons
Throughout the section you will see a few icons used on the left-hand side of the page. These icons are used to signal a change in activity or to bring your attention to important instructions.

AWM online resource (optional)
This indicates a resource available on the internet. If you do not have access, you may skip these sections.



Solutions

My Notes

The column on the outside edge of most pages is called "My Notes". You can use this space to:

- write questions about things you don't understand.
- note things that you want to look at again.
- draw pictures that help you understand the math.
- identify words that you don't understand.
- connect what you are learning to what you already know.
- make your own notes or comments.

Materials and Resources

There is no textbook required for this course.

You will be expected to have certain tools and materials at your disposal while working on the lessons. When you begin a lesson, have a look at the list of items you will need. You can find this list on the first page of the lesson, right under the lesson title.

In general, you should have the following things handy while you work on your lessons:

- a scientific calculator
- a ruler
- a geometry set
- Data Pages (found in the Appendix)

Angles

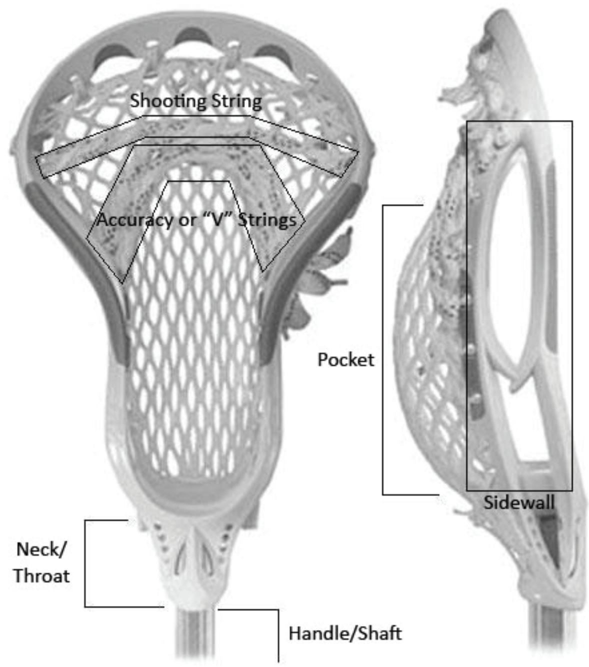


Photo by Larry St. Pierre © 2010

The game of lacrosse originated among the First Peoples of North America and has been an important part of their culture for almost a thousand years. Recognized by an act of Parliament in 1994 as Canada's national summer sport, it is a vigorous game enjoyed by young men and women across our country!

The graphic is of a contemporary lacrosse stick. The webbing geometric design changes from the pocket to the shooting string to make it easier to either cradle or shoot the ball. The parallel lines, four-sided figures, and the repeated angles in the webbing illustrate that geometry plays an important role in sports equipment.

In this section you will explore the geometry of angles. In particular, you will investigate how they are defined, measured, classified, duplicated, and bisected. As well, you will explore relationships among the angles formed when two parallel lines are cut by a third line. In each case, you will apply definitions and relationships to solve a variety of practical problems.

In this section you will:

- classify angles and pairs of angles.
- draw, bisect, replicate, and construct a variety of angles.
- solve problems that involve parallel, perpendicular and transversal lines, and the pairs of angles formed between them.

Lesson A
Sketching and Measuring Angles

To complete this lesson, you will need:

- a protractor
- a ruler
- scissors
- a compass
- several blank sheets of paper

In this lesson, you will complete:

- 5 activities

Essential Questions

- How can you sketch and describe angles of various measures?
- How are referents used to estimate the measure of a given angle?
- How is the protractor used to measure angles in a variety of orientations?

My Notes

Focus

Photo by Vitaly M © 2010

Cue sports such as snooker require a steady hand, practice, and an eye for angles. What separates the novice and the skilled player is the ability to predict how the billiard balls will behave when struck and when they carom of the cushions. Spaced at regular intervals along the table's perimeter are diamond shapes, which assist a player in assessing possible angles to line up the shot. When a cue ball strikes another, what is the largest possible angle the ball struck can be deflected from the line along which the cue ball was travelling?

Get Started

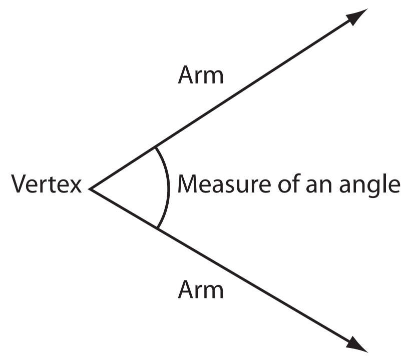
To move through this lesson and further through the section you will need to be able to measure and draw angles. In this part of the lesson you will review the parts of an angle and how to sketch and measure angles using a protractor.

My Notes

Parts of an Angle

In previous math courses you drew and measured **angles** of various sizes.

An angle is a geometric shape formed by two rays with a common endpoint. Each ray is called an *arm of the angle*. The common endpoint of the arms of the angle is the vertex of the angle.



The measure of an angle is commonly given in degrees. There are 360 degrees (360°) in one rotation.

To see all the angles possible in one 360 degree rotation, go and look at *Angle Definition* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/glossary/Division03/Angle/index.html*).

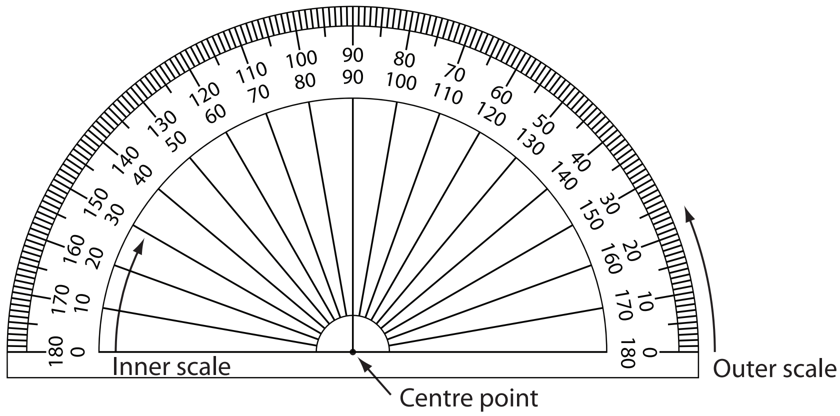
Did You Know?

In ancient times, the peoples of the Middle East and India divided the circle into 360 parts. This was partly because there are approximately 360 days in a year and the stars move in the sky in a great circle that takes a year to complete. They didn't use 365 because the number 360 is much more convenient to use. Besides 1 and 360, the number 360 can be divided evenly by 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, and 180. Arithmetic is simpler using 360 than it is using 365!

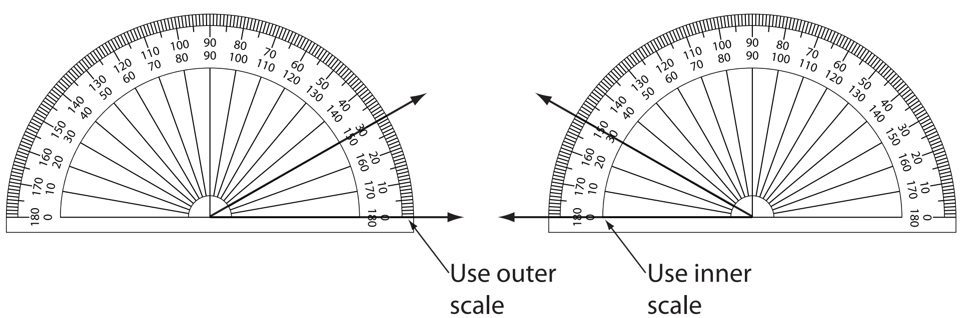
My Notes

Measuring and Drawing Angles

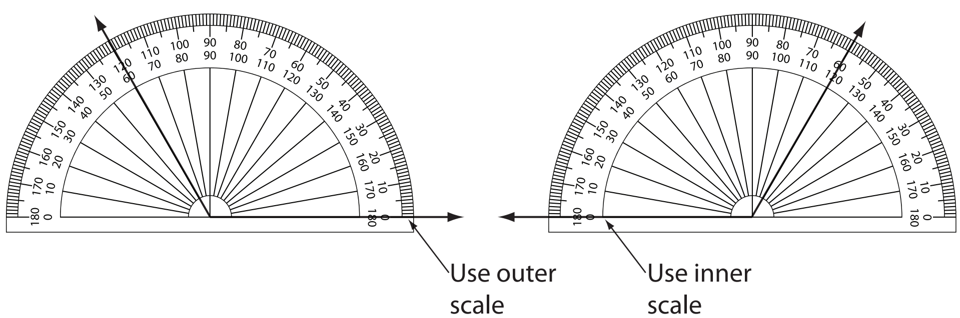
First review how to use your protractor.



The two scales on your protractor are used to measure angles drawn in different orientations. Remember, line up the centre-point of the protractor with the angle's vertex. Then start at zero on one of the protractor's scales. Both of the angles pictured below measure 30º.



Obtuse angles can be measured the same way. Both of the angles pictured below measure 120°.



Go an look at *Protractor* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/glossary/Division03/Protractor/index.html*). Use the demonstration applet to practise measuring angles. Watch the angle measurements changing as you drag the coloured dot.

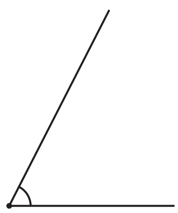
Activity 1
Self-Check

My Notes

To complete this activity, you'll need your protractor and straightedge.

1. Use a protractor to measure each angle. Record all measurements to the nearest degree.

a.



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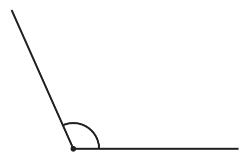
b.



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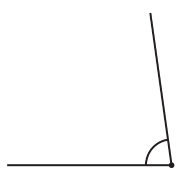
c.



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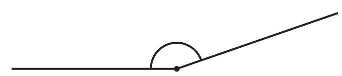
d.



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2. Draw each angle from the measurements below. Draw the angles in parts a and b opening to the right as in 1. a and b. Draw the angles in parts c and d opening to the left as in 1. c and d.

   a. 49º

My Notes

b. 149^{o}

c. 16^{o}

d. 127^{o}

Turn to the solutions at the end of the section and mark your work.

Explore

My Notes

You may have used **referents** in math to estimate length and area in SI and in imperial units. Now, we will examine referent angles.

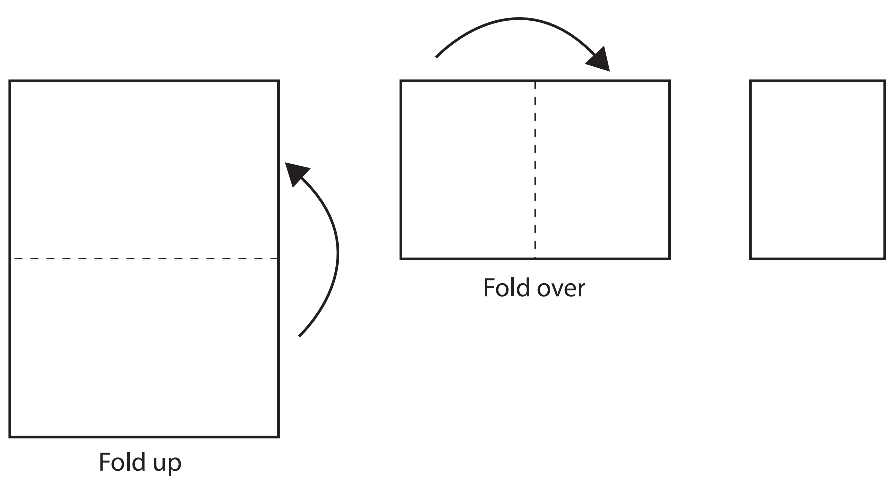
Referent angles can help you estimate the sizes of angles you encounter. If you know approximately how big a few referent angles are, you can compare their measures with the angle you must estimate.

In the next activity you will create several referent angles through a paper-folding exercise.

Activity 2
Try This

In this activity you develop referents for estimating the measure of an angle from 0º to 180º. You will need a blank sheet of paper, scissors, a protractor, and compasses to complete this activity.

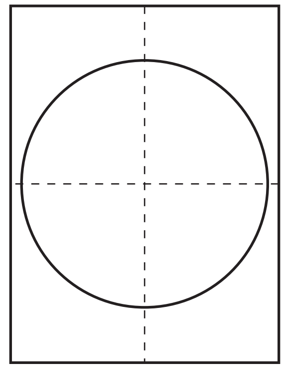
Step 1: Fold a blank sheet of paper in half from top to bottom. Then fold it in half again, this time from side to side.



Step 2: Unfold the sheet of paper and use your straightedge to draw lines along the creases.

My Notes

Step 3: Draw a circle with a radius of at least 8 cm. The centre of the circle is the point at the centre of the page where the lines cross.



1. With your protractor, measure the angle between the two lines. Why must this angle measure 90º?

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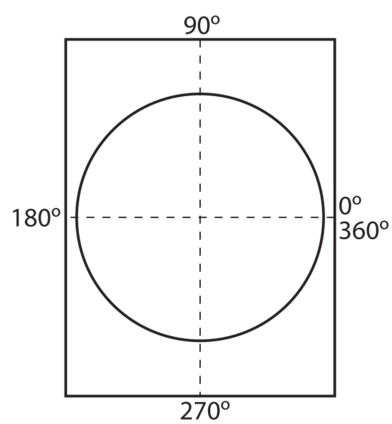
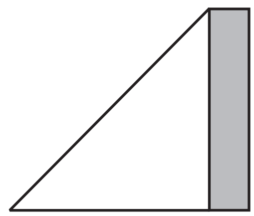
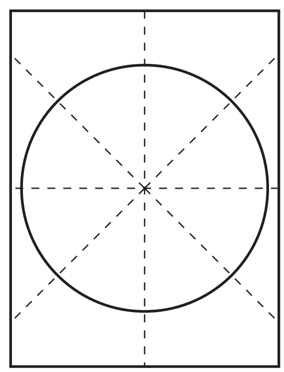
Step 4: Write the measures of the angles as you go once around the circle as shown.



My Notes

Step 5: Fold the paper along the folds you created in Step 1. Then fold it at the centre to form a triangle. The folded edges shown at the bottom of the diagram must match. Then cut off the excess paper on the right (shaded in the diagram below) so that when you unfold the paper it is a square, as shown. Use your straightedge to draw lines along the new creases.





2. Without using a protractor, how large is each of the 8 angles? Explain how you know this is true.

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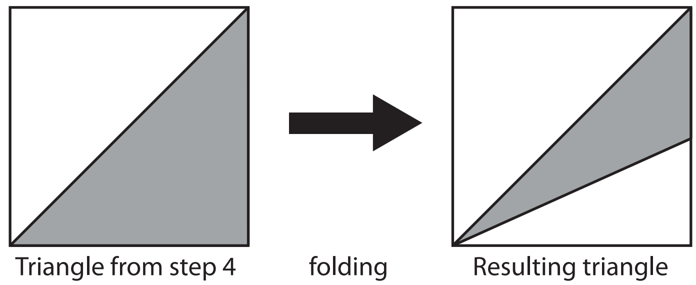
Step 6: Write the measures on each of the new angles as you did in Step 4.

My Notes

Step 7: Fold your paper back into the triangle of Step 5.

Now, fold the triangle from Step 5 in half, as shown below, to form a smaller triangle.



Now, unfold the paper. You should have 16 small angles formed around the centre of the paper.

3. How large is each of the 16 small angles? How do you know this is true?

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Step 8: Write the measures on the new angles as you did in Steps 4 and 6.

Turn to the solutions at the end of the section and mark your work.

Referent Angles

You have divided your circle into quarters, eighths, and sixteenths. You have created angles that measure 22.5°, 45°, and 90°. You have also created angles that are multiples of these measures. You can use the angles you created as benchmarks, or referents, to help you estimate the size of a given angle.

These are not the only useful referents. It is also helpful to know the sizes of 30-degree and 60-degree angles to use as referents. These two angles will help you refine your estimates. In the next activity you will create 30-degree and 60-degree angles.

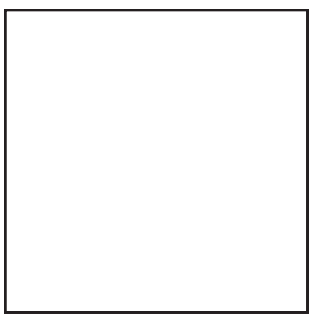
My Notes

Activity 3
Try This

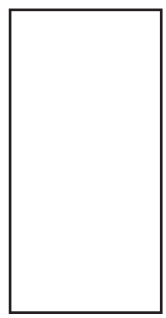
To complete this activity, you will need a square sheet of blank paper, scissors, a protractor, and compasses.

Step 1: You are going to fold your square piece of paper as shown in the diagram below.

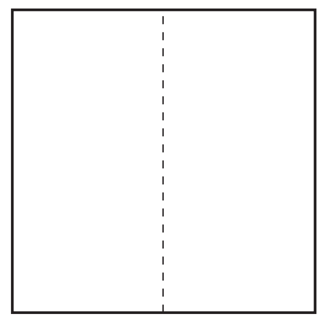
Fold the square sheet of paper in half. Unfold the sheet. Then, fold the right half in toward the centre crease. Now, unfold the sheet to reveal the two creases.



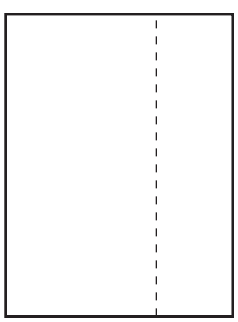
Original sheet



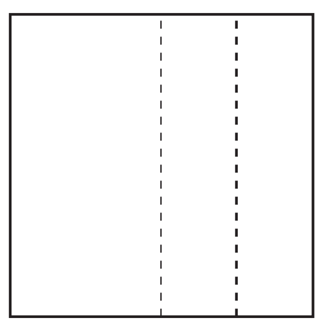
Folded in half



Unfolded showing crease



Right half folded inward

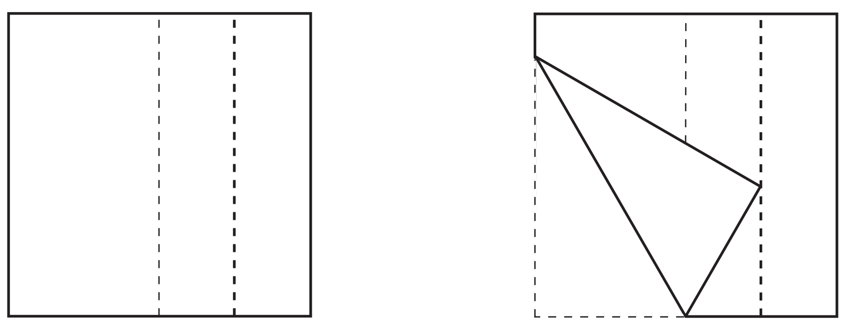


Unfolded showing both creases (dashed)

My Notes

1. What fraction of the entire sheet is each narrow vertical strip? How do you know this is true?

Step 2: Take the bottom left corner and place it on the right quarter line as shown in the diagram below. Holding the corner on that line, crease the paper so that the crease passes through the bottom end point of the centre line. Cut out the triangle (shown in bold below).



2. Using your protractor, measure each angle of the triangle. How large is each angle?

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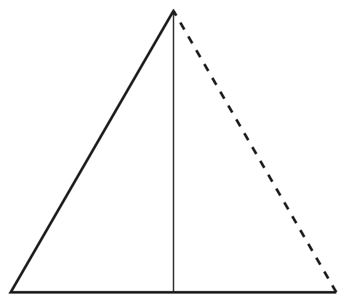
Step 3: On each angle of the triangle, write its measure. Cut out the triangle and keep it to use as a referent.

Turn to the solutions at the end of the section and mark your work.

My Notes

More Referent Angles

In Activity 3, you created two new referent angles: 30º and 60º. To explain why these angles are these measures, let's think about a special type of triangle. An *equilateral triangle* is a triangle that has sides of equal lengths. Equilateral triangles also have three angles of equal measure. When you folded the square page, you created an equilateral triangle.

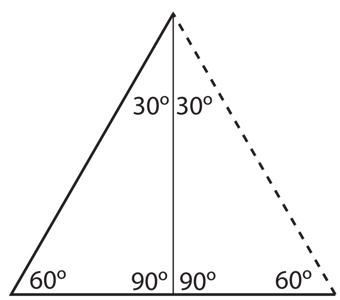


In any triangle the sum of the angles is 180°.

$$180^\circ \div 3 = 60^\circ$$

So each of the three equal angles must be 60°.

It follows that, if you split the triangle in half, the angles will be as shown below. These are the angles you measured in the activity.



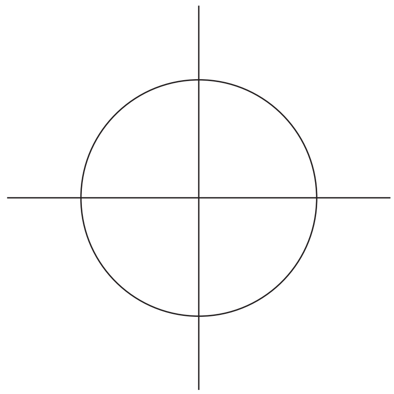
Bringing Ideas Together

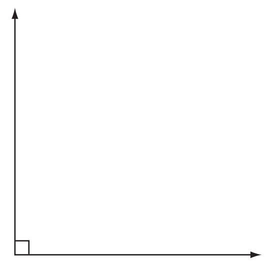
In Getting Started and Explore, you reviewed the parts of angles, how they are measured, and what referents may be used to estimate their size. Before developing your estimation skills further, you will investigate how angles are classified by their sizes.

My Notes

Right Angles

In Explore you began by folding a sheet of paper to divide one complete rotation into quarters. Each of the four angles was ¼ × 360º = 90º in measure. A 90-degree angle is called a **right angle**.





a right angle

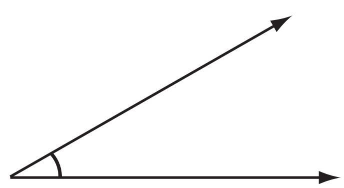
Most home designs involve countless right angles. Wall and floors meet at right angles—corners are square. A small square is often drawn between the arms of a right angle.

Acute Angles

Did You Know?

The word *acute* means *sharp*.

An acute angle is an angle that measures less than 90º.



To investigate acute angles, go and look at *Acute Angles* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/glossary/Division03/Acute%20Angle/index.html*).

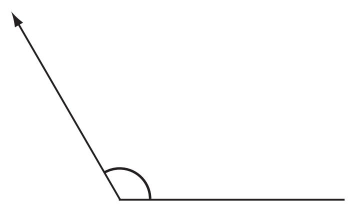
My Notes

Obtuse Angles

Did You Know?

The word *obtuse* means *dull*.

An obtuse angle is an angle with a measure greater than 90° but less than 180°.



To investigate obtuse angles, go and look at *Obtuse Angles* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/glossary/Division03/Obtuse%20Angle/index.html*).

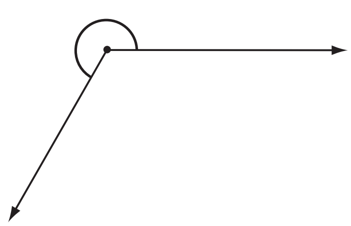
Straight Angles

A straight angle measures exactly 180°.

To investigate straight angles, go and look at *Straight Angles* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/glossary/Division03/Straight%20Angle/index.html*).

Reflex Angles

A reflex angle is an angle with a measure greater than 180° but less than 360°.



To investigate reflex angles, go and look at *Reflex Angles* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/glossary/Division03/Reflex%20Angle/index.html*).).

My Notes

Full Rotation

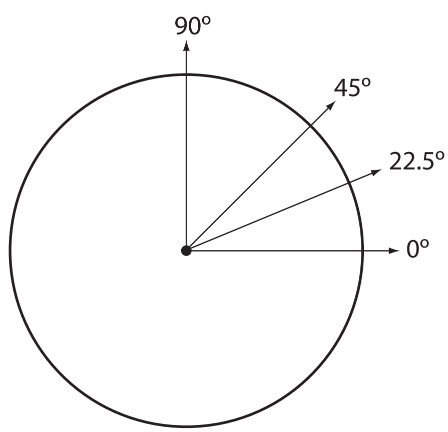
A **full rotation** is an angle having a measure of 360°.

A full rotation angle looks a lot like an angle of 0º. The difference is that, to show a full rotation, we draw a circle around the vertex.

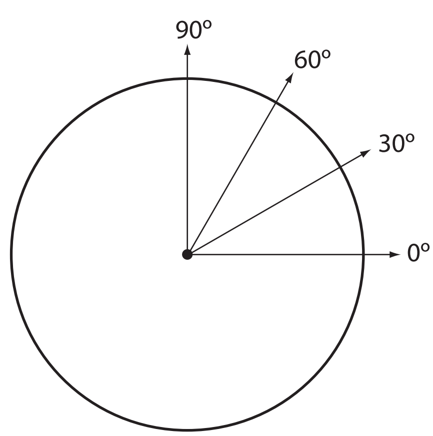
Estimating Angle Measures with Referents

Next, you will estimate angles using the referents you discovered in Explore. You will need the circle diagram you prepared through paper folding and the triangle containing the 30º and 60º angles.

Look at your circle diagram again.



If you were to place 30º and 60º on the circle, those angles would divide the right angle into three parts.

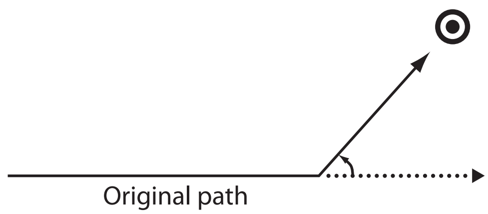


My Notes

Study the following examples to hone your estimation skills. You will be working with acute, obtuse, and reflex angles. You will need a protractor to check the accuracy of the estimates.

Example 1

A curling stone strikes a rock in the house and is deflected from its original course.



a. State whether the angle is acute, obtuse, or reflex.

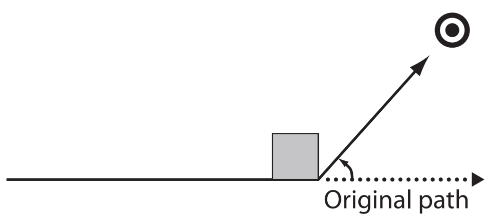
b. Estimate the angle the stone was deflected.

c. Measure the angle to check your estimate.

Solution

a. The angle is less than 90º. So the angle is acute.

b.



If a square is placed on the figure as shown, the angle appears to be just a little larger than half of a right angle.

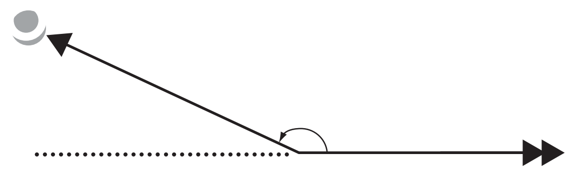
Since $\frac{1}{2} \times 90^\circ = 45^\circ$ (the referent), the angle probably measures about 50º.

c. With a protractor, you'll find that the angle measures 48º.

My Notes

Example 2

A proton is fired from the left at the nucleus of a gold atom. The ray with the double arrow head shows the original course of the electron, which it would have taken if it hadn't collided. The electron is scattered from its original course as shown by the ray with the single arrow head.



a. State whether the angle is acute, obtuse, or reflex.

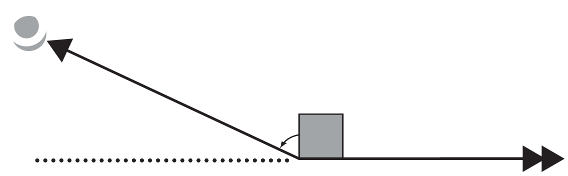
b. Estimate the scattering angle.

c. Measure the angle to check your estimate.

Solution

a. The angle is greater than 90º but less than 180º. So the angle is obtuse.

b.



If the corner of a sheet of paper is placed on the angle, it looks as though the small angle between the dashed line and the arm of the angle is about one quarter of a right angle (22.5° is the referent). So the scattering angle is about 22.5° less than 180º.

180º – 22.5° = 157.5°

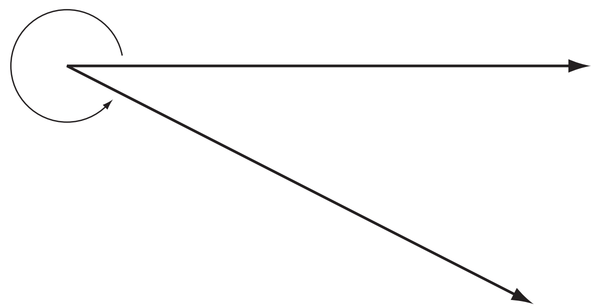
The scattering angle is approximately 158º.

c. Using a protractor, you will find that the angle measures 155º.

My Notes

Example 3

A wrench is turned counter clockwise to loosen a bolt through an angle as shown.



a. State whether the angle is acute, obtuse, or reflex.

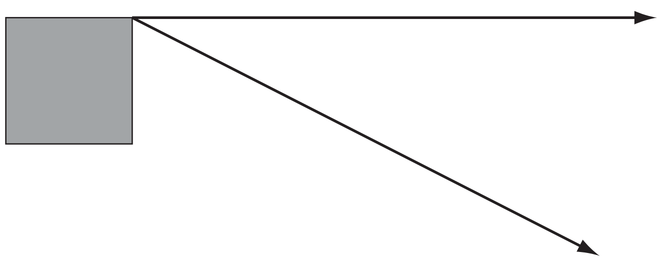
b. Estimate the angle

c. Use a protractor to check your estimate.

Solution

a. The angle is greater than 180º but less than 360º. The angle is a reflex angle.

b.



If the square corner of a sheet of paper is positioned at the vertex as shown, the small angle appears to be one third of a right angle (30º is the referent). So the reflex angle is about 30º less than one full rotation.

360º – 30º = 330º

The reflex angle is approximately 330º.

c. Using a protractor, the small angle is 27º.

360º – 27º = 333º

The reflex angle is 333º.

My Notes

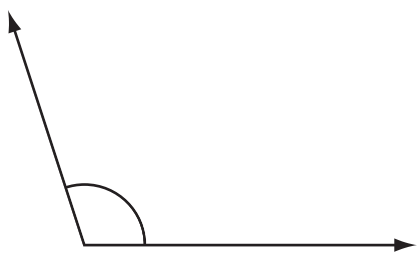
Activity 4
Self-Check

Please complete the following questions.

a. Describe the angle as acute, right, obtuse, straight, reflex, or a full rotation.

b. Use a referent to estimate the measure. State the referent you used: 22.5º, 30º, 45º, or 60º.

c. Use a protractor to check your estimate.

1. 

a. ______________________________

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b. ______________________________

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c. ______________________________

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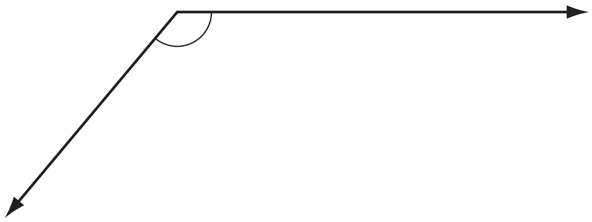
2. 

a. ______________________________

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b. ______________________________

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c. ______________________________

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My Notes

My Notes

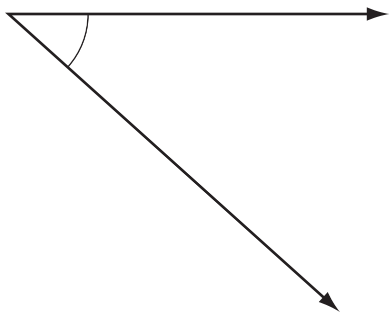
3.

a. ____

b. ____

c. ____

My Notes

4. 

a. ______________________________

____

b. ______________________________

____

c. ______________________________

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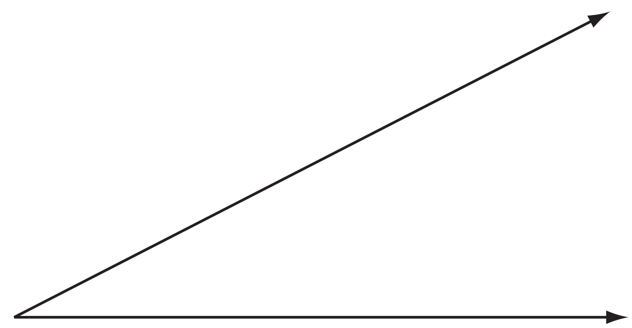
Turn to the solutions at the end of the section and mark your work.

My Notes

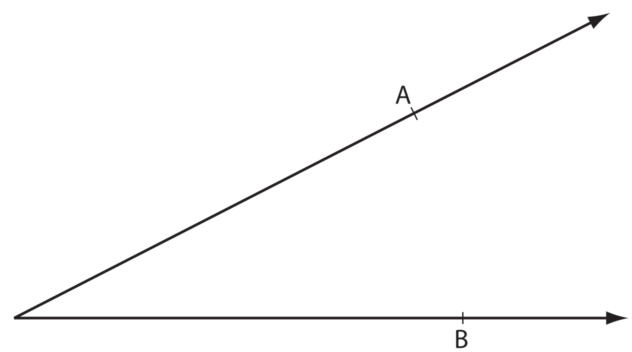
Activity 5
Mastering Concepts

You have seen that referents help in estimating the measure of an angle. But you can also estimate angles using a ruler.

Step 1: Draw any acute angle.



Step 2: From the vertex, measure out 6 cm along each arm and mark these points A and B as shown.



Step 3: Measure the distance between A and B to the nearest tenth of a centimetre.

Step 4: Multiply the distance between A and B by 10. This product will be close to the measure of the angle (within a few degrees). For example, if AB = 2.3 cm, then an estimate of the angle would be 2.3 × 10 = 23 degrees.

Step 5: Check with a protractor to see how close the estimate was.

My Notes

Try the above procedure with several acute angles. Then, answer the following questions to see why this works.

1. Points A and B are 6 cm from the vertex. If you drew a circle centred at the vertex and passing through A and B, what would its approximate circumference be?

2. What is 10 times this circumference?

3. How does the number of degrees in a circle compare to your answer in Question 2?

____

____

____

____

Turn to the solutions at the end of the section and mark your work.

My Notes

Lesson Summary

Photo by Phillip Durand © 2010

Curling is another game in which predicting the angles at which rocks will travel when struck is a key element of strategy. If you have played the game, there is nothing more satisfying than seeing a well-executed shot. For fans, the satisfaction may be in seeing the side they are cheering for make a double or triple takeout.

In this lesson, you reviewed how protractors are used to sketch and measure angles. From their measures, you described these angles as acute, right, obtuse, straight, reflex, or a full rotation. You also explored angle referents, such as 22.5°, 30°, 45°, and 60°, to estimate the measure of any given angle.

Lesson B
Constructing Congruent Angles

To complete this lesson, you will need:

- a protractor
- a compass
- a square from a geometry set
- several blank sheets of paper
- a straightedge or ruler
- grid paper from the Appendix at the end of this module

In this lesson, you will complete:

- 5 activities

Essential Questions

- How can you determine if two angles are congruent?
- How can you construct congruent angles?

My Notes

Focus

Lakota star quilts remind us that art and design played an important role in the lives of the First Nation peoples of the prairies and plains. This design represents the Morning Star and signifies a fresh start to a person's life. Blankets and quilts, such as the one in the photograph, were given as gifts to celebrate births, weddings, and other milestones.

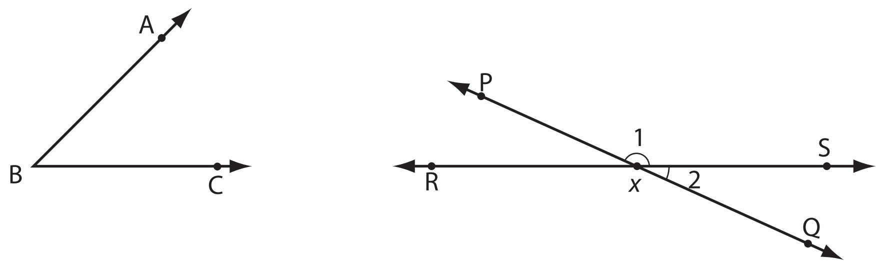
Photo by Alice Day © 2010

The pattern incorporates the repetition of angles and geometric shapes. If you examine the star more closely, you can see how carefully identical—that is, congruent—geometric figures are stitched together to create a star. In the star you can see congruent parallelograms, triangles, rectangles, and trapezoids. How many sets of congruent angles can you find?

Get Started

My Notes

Let's review how to identify angles.



The labels "∠ABC" "∠CBA" and "∠B" can all be used to identify the same angle. However, you can only use the label "∠B" if the angle stands alone as it does in the diagram above. Notice that when a three-letter name is used, the middle letter corresponds to the vertex. Likewise, the vertex is the only letter you can use for a single-letter name.

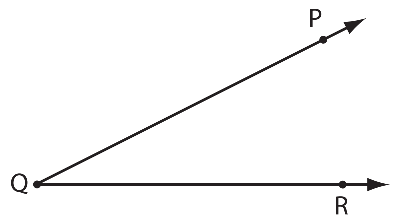
When there are two or more angles that share a vertex, you should not use a single letter label. In the diagram above, line PQ intersects line RS at point X. Referring to "∠X" is confusing in this situation—it could refer to any of four angles. To distinguish among the angles, you must use a three-letter name or you can number the angles. So at the intersection of the lines, ∠PXS is ∠1 and ∠QXS is ∠2.

My Notes

Activity 1
Self-Check

1. Write the name of each angle.

a.



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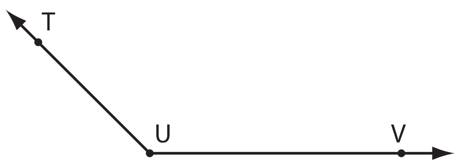
b.



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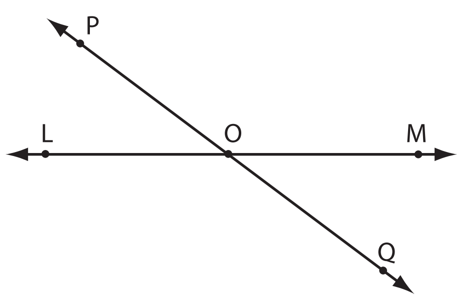
c.



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My Notes

2. Draw and label each angle.

   a. ∠ABC is 50º

   b. ∠RST is 135º

   c. ∠M is 193º

Turn to the solutions at the end of the section and mark your work.

My Notes

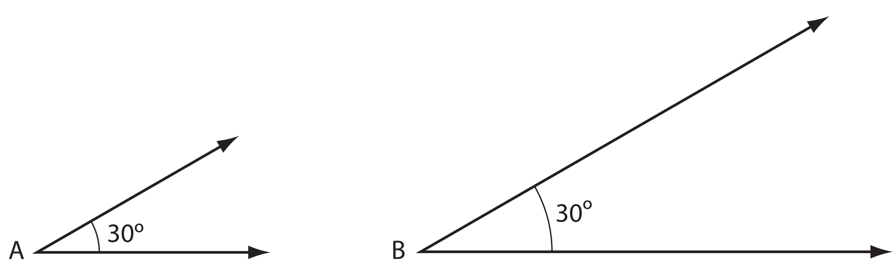
Explore

You may recall working with **congruent angles** in previous math courses. Do you remember what a congruent angle is?

Take a look at the images of the star quilt below. Some of the congruent angles are highlighted for you in the quilt. You can use your protractor to find more congruent angles in the quilt.

Photo by Alice Day © 2010

Congruent angles are angles that have the same measure. Please note: congruent angles have the same measure, but their arm-lengths may be different. For example the angles shown below are congruent: they both measure 30º. However, their arm lengths are different.



Mathematicians use the symbol ≅ which means “is congruent to.” So, $\angle A \cong \angle B$ means “angle A is congruent to angle B.”

My Notes

Now, take a few minutes to look around you. Inside or out, there are many examples of congruent angles. For instance, look at where the corner of the room meets the ceiling.

Patterns in nature often display congruent angles. Look for them on images of a snake's skin, a butterfly's wings, and a snowflake. Can you identify the congruent angles in these images?

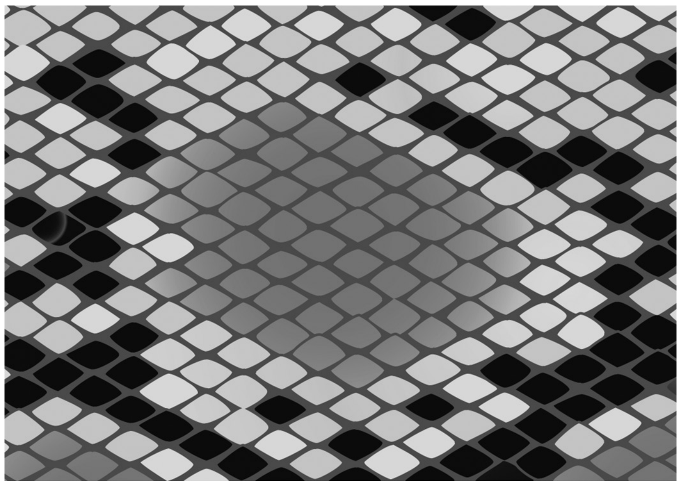


Photo by Outsider © 2010

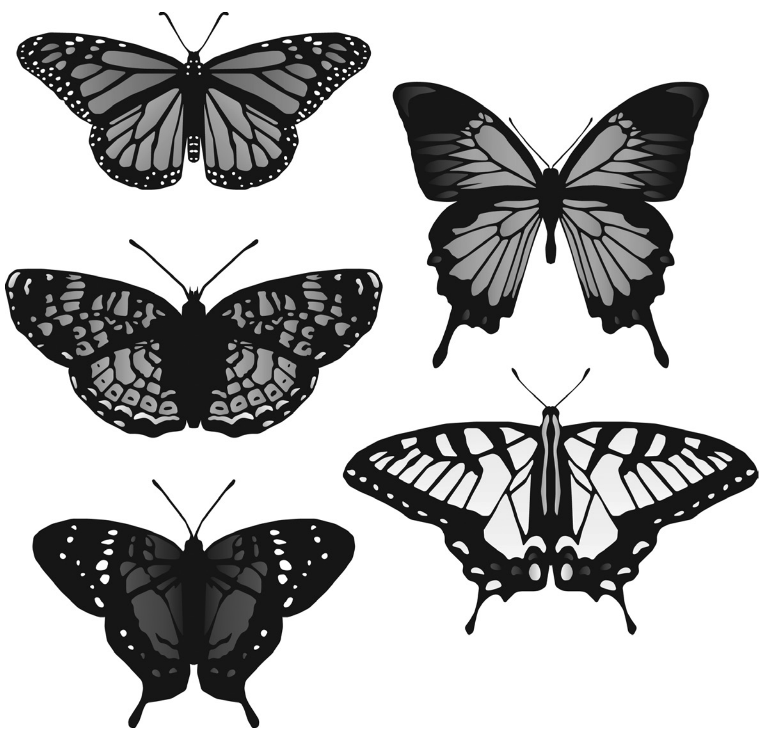


Photo by John David Bigl III © 2010

My Notes

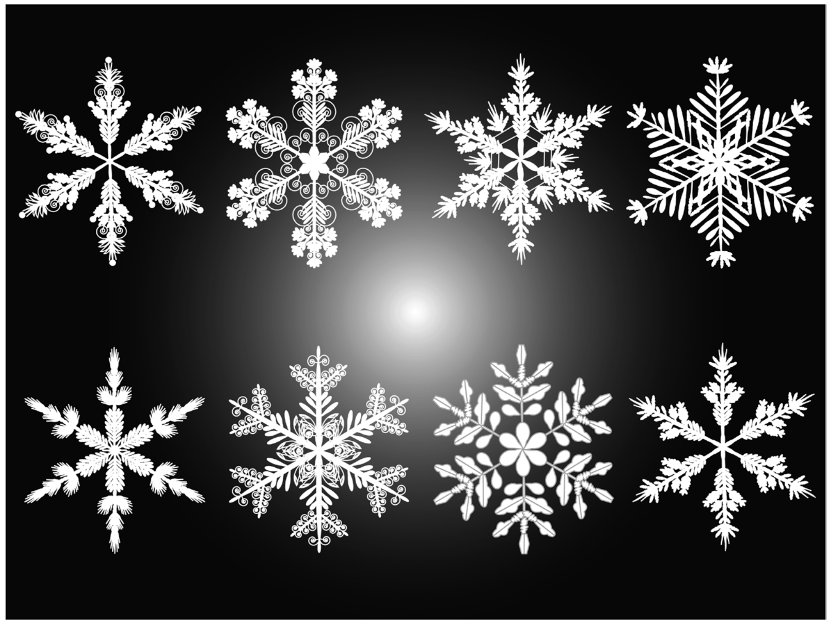
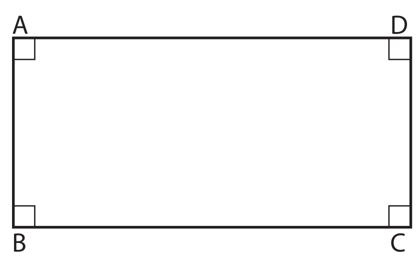
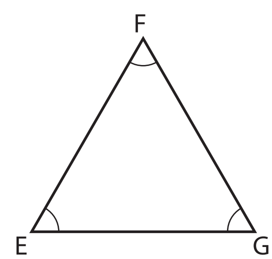


Photo by Inna Petyakina © 2010

As well as in your home or in nature, you encountered congruent angles in geometric shapes you explored in previous mathematics courses.





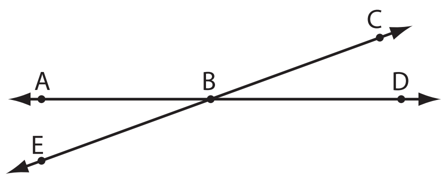
Rectangles have four right angles. Since all right angles measure 90º, the four angles in a rectangle are congruent. Equilateral triangles—triangles whose three sides are equal in length—have three congruent angles. Each of the angles in an equilateral triangle are 60º.

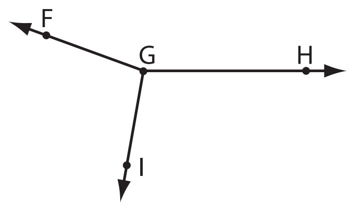
Test out your skills at finding congruent angles by trying the questions in Activity 2.

Activity 2
Self-Check

My Notes

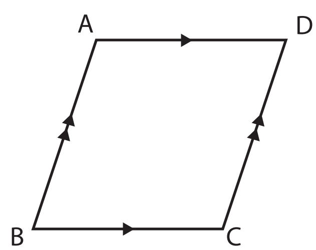
1. Look at the angles below. Use your protractor to find all the sets of congruent angles.





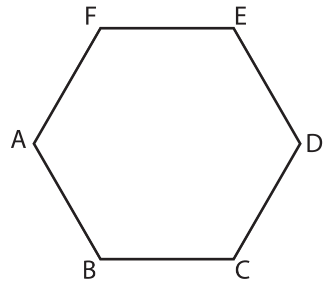
2. For each shape below, list the sets of congruent angles. You may use your protractor.

   a. ABCD is a parallelogram.

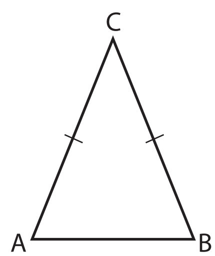


My Notes

b. ABCDEF is a regular hexagon—a polygon with six equal sides.



c. ABC is an isosceles triangle—a triangle with two equal sides.



Turn to the solutions at the end of the section and mark your work.

Bringing Ideas Together

My Notes

In Explore, you practised identifying congruent angles. Now, we'll look at a three different ways that you can draw congruent angles:

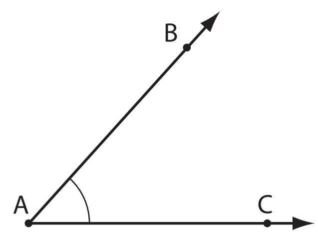
- using a protractor
- folding the page that the angle is drawn on
- using a geometric construction technique

Using a Protractor

The most straightforward method of drawing congruent angles is by using a protractor. You can simply measure the original angle and then draw a second angle of equal measure.

Example 1

Use a protractor to draw an angle that is congruent to ∠ABC.



Solution

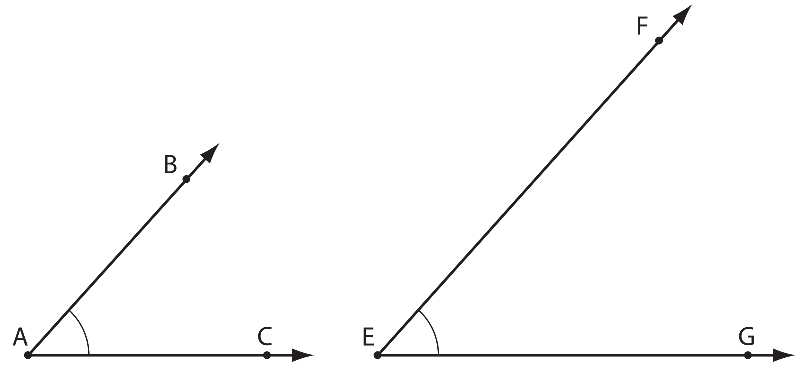
First, use your protractor to measure ∠ABC.

∠ABC = 48º

Now, use your protractor to draw a 48-degree angle.

Use the **straightedge** on the bottom of your protractor to make the first arm. Then place the centre-point of the protractor on the endpoint of the arm. Find 48º and make a mark on your page. Using the straightedge again, connect the endpoint of the arm to the mark you just made. Label the angle.

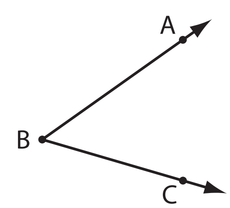
My Notes



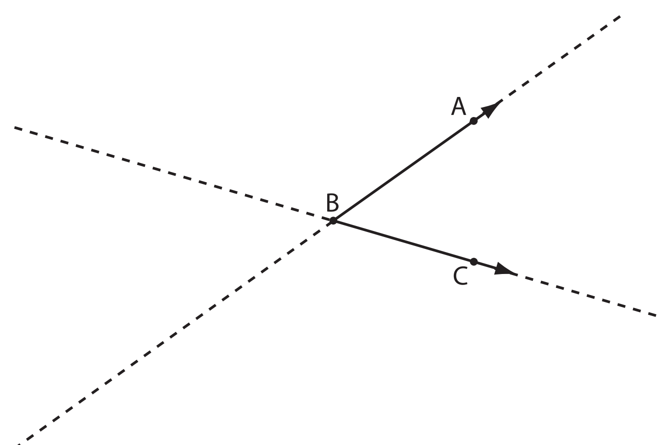
Folding the Page

Another simple technique for creating congruent angles is by folding your page in a very specific way.

Step 1: Draw the angle you want to duplicate on a piece of paper.

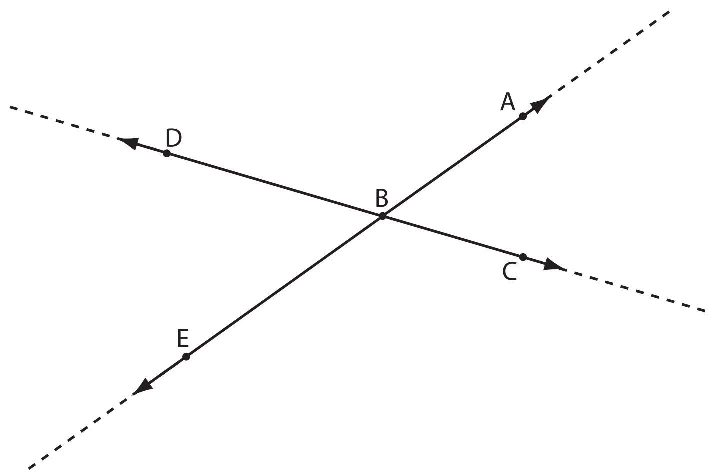


Step 2: Fold the paper along each of the arms. The fold lines are illustrated with dashed lines in the diagram.



My Notes

Step 3: Draw a second angle by tracing the fold lines.

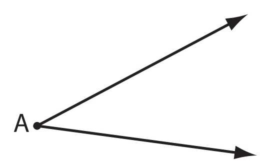


∠ABC ≅ ∠DBE. Use your protractor to check that the angles are, in fact, congruent.

Geometric Construction

This method for constructing congruent angles is one that geometers have used for thousands of years. You will use your compass and a straightedge to draw congruent angles.

Follow the steps below to create an angle that is congruent to ∠A.

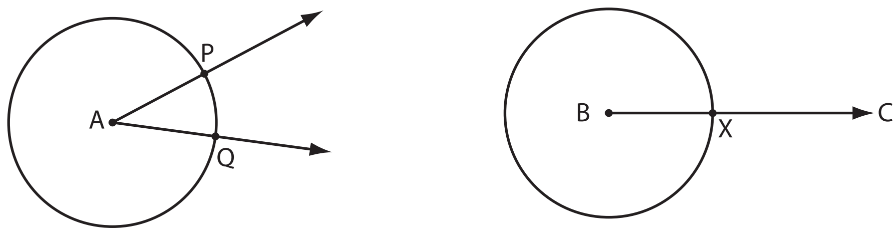


Step 1: Use your straightedge to draw a ray, BC. This will be the lower arm of the new angle. It doesn't have to point in the same direction as the lower arm of ∠A, but in this example, we'll draw it that way.

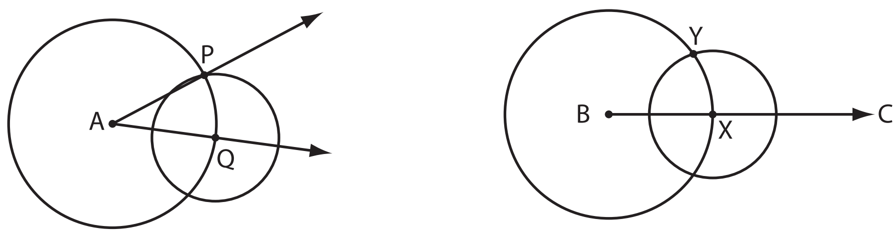
Step 2: Use your compass to draw two circles with the same radius, one centred at A and one centred at B.

My Notes

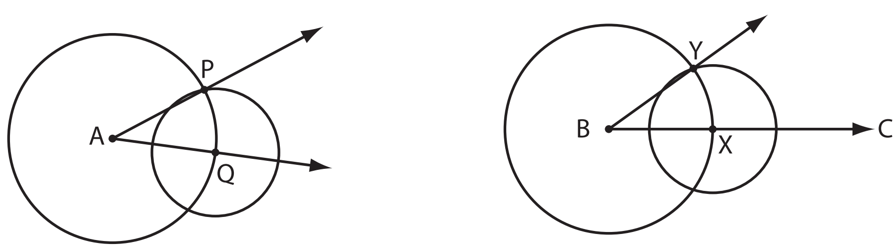
The first circle cuts through the arms of ∠A at two points: P and Q. The second circle cuts across ray BC at the point X.



Step 3: You will draw two more circles. The radii of these two circles will be the distance from P to Q. Place the point of your compass on point Q and the pencil in your compass on point P. Draw a circle. Lift the compass, and with the same radius, draw a circle centred at point X. This circle will intersect with the circle you drew in Step 2. Label this intersection point Y.



Step 4: Use your straightedge to draw ray BY. ∠A ≅ ∠B.



Now, you should use your protractor to ensure that ∠A ≅ ∠B.

To view the construction of congruent angles using a compass and straightedge, go and look at *Constructing Congruent Angles* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/congruent_angles/m10_3_m5_009.htm*).

Activity 3

Self-Check

My Notes

You will need a protractor, a compass, and a straightedge to complete this activity.

Draw any obtuse angle and label it ∠ABC. Use each of the three methods described in this lesson to construct three angles congruent to ∠ABC. Then check whether each of the angles you constructed is *actually* congruent to ∠ABC.

Original Obtuse Angle

Congruent Angle Using Method 1

My Notes

Congruent Angle Using Method 2

Congruent Angle Using Method 3

Turn to the solutions at the end of the section and mark your work.

Stairs and Roofs

In construction, carpenters use their carpenter squares to measure horizontally and vertically to obtain the angle required for projects such as stairs or roofs.

Think of a set of stairs.

Photo by mrfotos © 2010

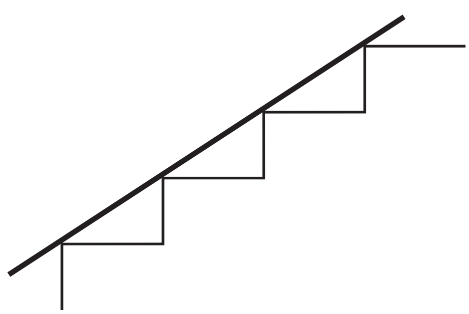
The angle of the stairs does not change. Can you suggest a reason why that is the case?

____

____

The steps are uniform. The treads (the part of the stairs you step on) are all the same size. The risers (the vertical portion of each step) are all the same size.

This uniformity guarantees that the angles are the same from step to step. You could lay a straight board on the steps to check. The board would rest on all the step edges.



My Notes

My Notes

The ratio of the riser height to the tread length affects the steepness, that is, the angle, of the stairway. In other words, the vertical and horizontal heights of a stairway affect its angle. Also for a roof, vertical and horizontal distances affect the angle of the roof.

The following is an example of how the slope of a roof can be determined from vertical and horizontal distances.

Example 2

Photo by Jim Parkin © 2010

Akiko and her father are building a house and an attached garage. The slope of the garage roof is a 4 inch rise for every 12 inch measured horizontally.

a. Draw a diagram of the roof using grid paper.

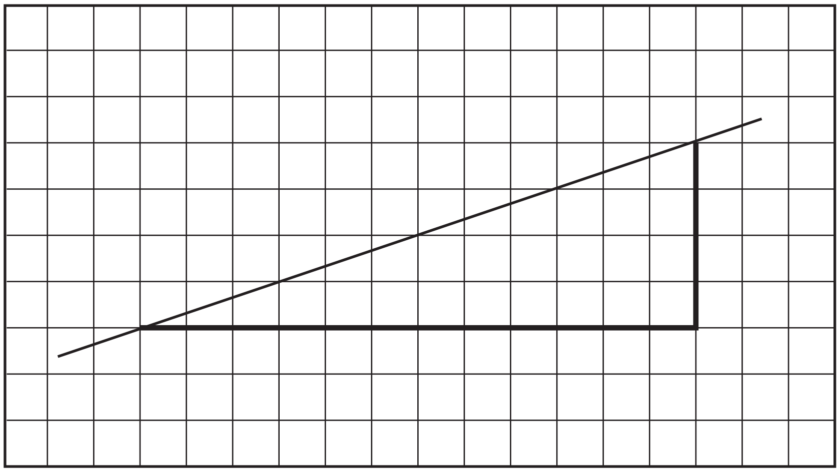
b. Measure the angle of the roof rises from your diagram.

My Notes

Solution

Use one square to represent one inch.

a.



b. Use your protractor to measure the angle.

The angle of the roof is approximately 18.5°.

My Notes

Activity 4
Self-Check

1. For a roof that slopes at 45°, what is the rise (in inches) for a horizontal run of 12 in?

2. Determine the angle at which a ladder rests against a vertical wall. The foot of the ladder is 3 ft from the wall, and the top of the ladder rests against the wall 8 ft above the ground.

Turn to the solutions at the end of the section and mark your work.

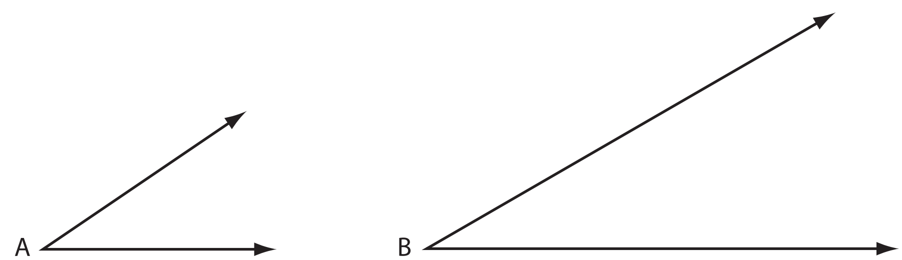
My Notes

My Notes

Activity 5
Mastering Concepts

To do this question you will need your protractor.

Compare the following angles.



Which of the following statements best describes the relationship between ∠A and ∠B?

a. ∠A is smaller than ∠B
b. ∠A is congruent to ∠B
c. ∠A is larger than ∠B

Justify your answer.

Lesson Summary

My Notes

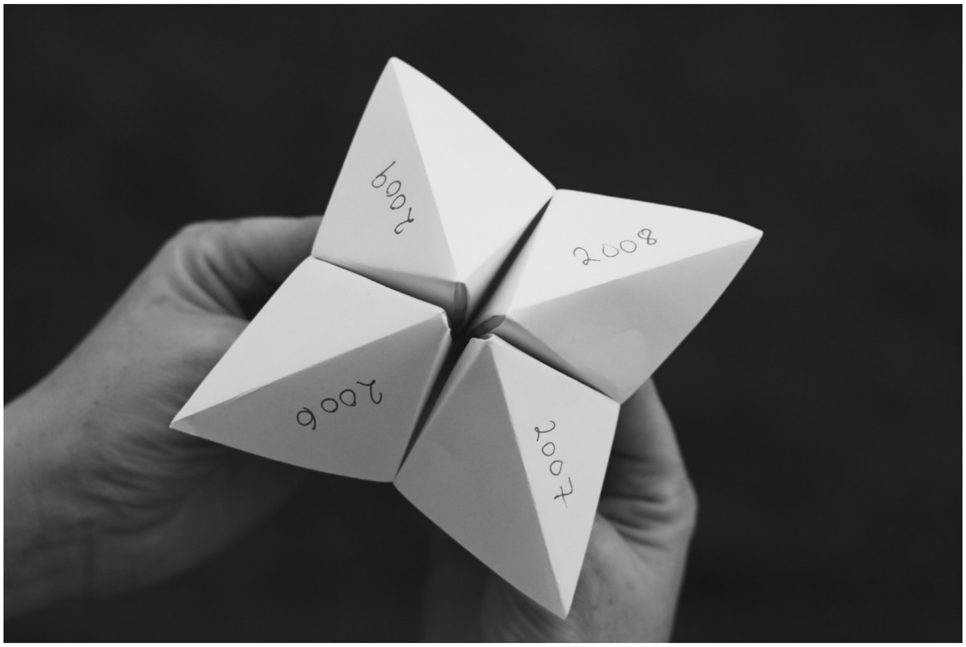


Photo by Ingvald Kaldhussater © 2010

The game being played in the photograph involves first folding a square sheet of paper into the shape shown and then opening and closing the shape as a name is spelled or a number is counted out. The process of folding a piece of paper into various shapes is a traditional Japanese folk art called *origami*. The folds create an interplay of angles and geometric shapes. Can you identify some congruent angles in the figure?

Lesson C
Bisecting Angles

To complete this lesson, you will need:

- a mirror
- a protractor
- a compass
- a square from a geometry set
- several blank sheets of paper
- a straightedge or ruler

In this lesson, you will complete:

- 5 activities

Essential Questions

- What does it mean to bisect an angle?
- How are different techniques used to bisect angles?

My Notes

Focus

Photo by James Steidl © 2010

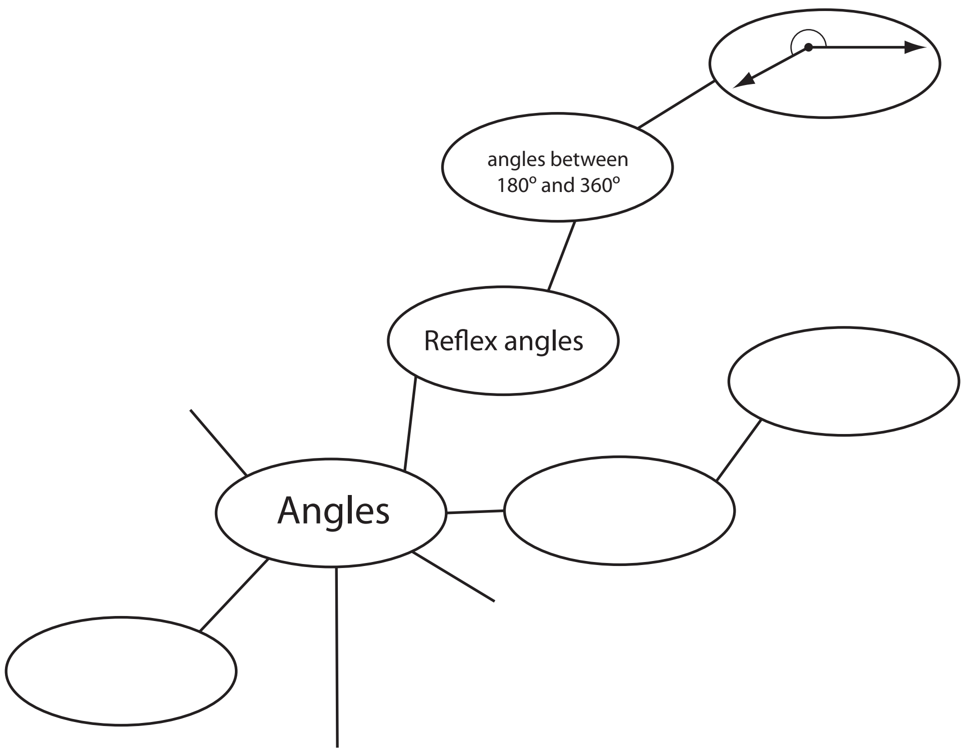
Have a look at the baseball diamond in the picture above. The first and third base lines form the angle that determines whether a ball is hit fair or foul. Second base, the pitcher's mound, and home plate split that angle into halves. What is the measure of each half of that angle?

In this lesson you will continue to explore the geometry of angles and learn about the lines that divide angles into halves.

Get Started

My Notes

To warm-up for this lesson, review what you've learned so far about angles. Complete the following mindmap using what you know about angles. You can fill in some of the empty bubbles and/or create your own. Feel free to look back at Lessons A and B to refresh your memory. You may also include things you learned in other courses about angles.



My Notes

Explore

You may recall, from your previous math experience, finding the midpoint of a line segment. The midpoint of a line divides the line segment in half—that is, into two equal parts. The term we use in geometry to describe dividing something in half is **bisect**.

So far in this section, you've worked with several types of angles. You will continue to work with angles in this lesson. In fact, you will learn to bisect angles using a variety of methods.

Can you think of some different ways to divide an acute angle into two equal parts? What about other types of angles?

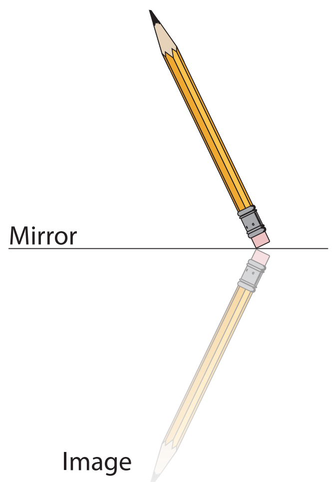
Activity 1
Try This

My Notes

You will need a pen or pencil and a mirror to complete this activity.

Step 1: Take a pencil (or pen) and place one end against a mirror.

Notice that the pencil and its image form an angle, with the vertex located at the point where the pencil touches the mirror.



Step 2: Change the size of the angle between the pencil and its image, all the while keeping the end of the pencil on the mirror.

Questions:

1. Compare the size of the angle created between the mirror and the pencil with the size of the angle created between the mirror and the pencil's image. What do you notice?

____

____

My Notes

2. Compare the size of the angle created between the mirror and the pencil with the size of the angle created between the pencil and its image. What do you notice?

____

____

Turn to the solutions at the end of the section and mark your work.

Symmetry

Bisecting an angle is like creating a line of symmetry through an angle. In Activity 1, the mirror acted as the angle **bisector** of the angle created by the pencil and its image. An angle bisector always divides an angle into two identical halves.

Bisectors are common features in design, nature, and art.

For example, look at the veins in a leaf. In a maple leaf, as in most other leaves, there is **symmetry**. One half of the leaf is the mirror image of the other.

Photo by Marta Tobolova © 2010

Can you sketch an angle and its bisector formed by the veins in the leaf?

The angles formed by the veins are bisected by the central vein running from the stem to the top of the leaf. An angle and its bisector are drawn on the leaf below.

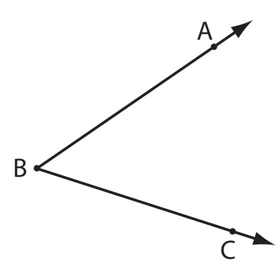
Photo by Marta Tobolova © 2010

Notice that ∠ABD measures the same as ∠CBD. Or, ∠ABD ≅ ∠CBD.

We can say that an angle bisector divides an angle into two congruent halves. Now, how do we find the bisector of an angle?

Activity 2
Try This

Step 1: On a blank sheet of paper, draw any angle and label it ∠ABC. Measure and record the size of ∠ABC



My Notes

My Notes

Step 2: Experiment with different folding methods. Your goal is to fold the sheet of paper to construct the bisector of ∠ABC.

Step 3: Label the bisector BD.

Step 4: Measure and record the size of ∠ABD and ∠CBD.

Questions:

1. Explain how you formed the bisector of ∠ABC by paper folding.

____

____

____

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____

____

2. Give the reason why the line you labelled BD must be the angle bisector.

____

____

3. State the measures of ∠ABC, ∠ABD and ∠CBD. How are these measures related?

____

____

Turn to the solutions at the end of the section and mark your work.

Bringing Ideas Together

My Notes

In Explore, you examined what it means to bisect an angle. You also learned one technique for finding the angle bisector: by folding! Now, we'll look at three more methods for finding the angle bisector using geometry tools. The three methods you'll learn are:

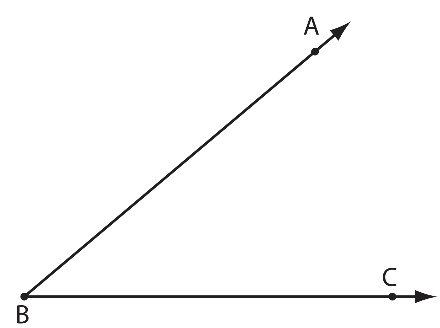
- Using a Protractor
- Geometric Construction: Compass and Straightedge
- Geometric Construction: Carpenter's Square

Using a Protractor

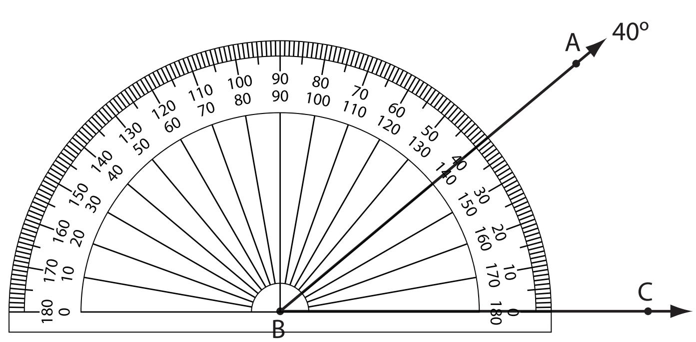
One simple method for finding the bisector of an angle is to use a protractor. Work through the example below to see how this is done.

Example 1

Bisect ∠ABC.



Step 1: Use your protractor to measure the angle you intend to bisect.

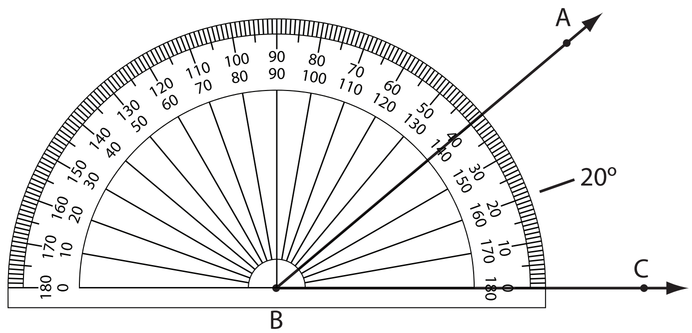


My Notes

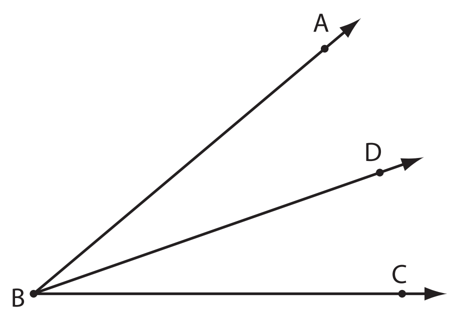
Step 2: Divide the measurement in half.

40º ÷ 2 = 20º

Step 3: Line up the protractor so that the centre-point is on the vertex and the baseline runs along one arm of the angle. Make a mark on the page at the half-measurement you determined in Step 2.



Step 4: Draw a line from the vertex of the angle to the mark you made in Step 3. This is your angle bisector.



Measure ∠ABD to make sure it is, in fact, the same size as ∠CBD.

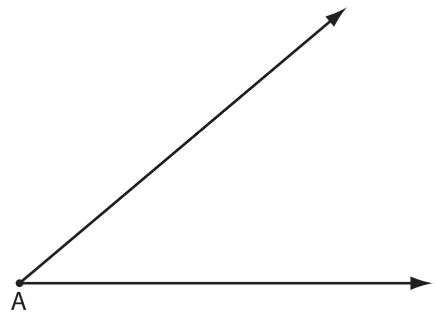
Geometric Construction: Compass and Straightedge

In Lesson B you constructed a congruent angle using only a compass and a straightedge. Now, you will construct the bisector of an angle using this classical approach—one that geometers have used for thousands of years.

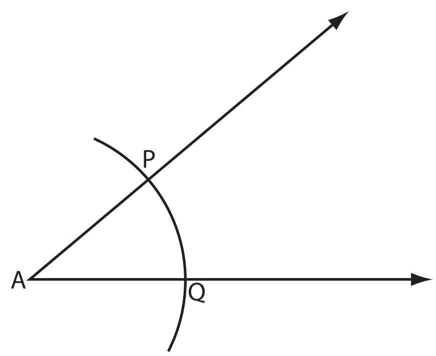
My Notes

Example 2

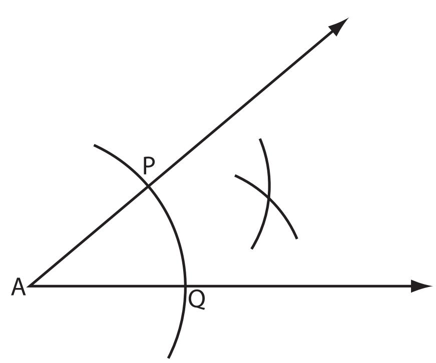
Use compasses and a straight edge to bisect ∠A.



Step 1: Set your compass to a suitable radius. With centre A, the vertex of the angle, draw an arc of a circle cutting the arms of the angle at points P and Q.

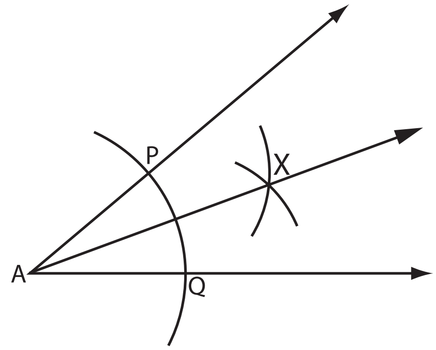


Step 2: Open your compasses to a radius that is at least greater than one-half the distance between points P and Q. With centre at P, draw an arc between the arms of ∠A. Then, with centre at Q, draw a second arc between the arms of ∠A. Name the point where these arcs intersect, X.



My Notes

Step 3: Draw ray AX.



Use your protractor to measure ∠A, ∠PAX, and ∠XAQ.

If the work is done carefully, ∠PAX ≅ ∠XAQ; that is, both angles are equal in measure.

And, ∠PAX = ∠XAQ = ½∠A.

To see a demonstration of this method, go and look at *Bisect an Angle* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/glossary/Division03/BisectAngle/index.html*).

Activity 3
Self-Check

My Notes

Do these questions. When you are finished, check your answers.

1. Draw any obtuse angle. Use your compasses and a straightedge to bisect the angle. Check the accuracy of your construction with your protractor.

My Notes

2. A student bisected a 234º angle. A protractor was used to measure each half. The student said that each half was 63º. What mistake might have been made? How large should each half have been?

Turn to the solutions at the end of the section and mark your work.

Geometric Construction: Carpenter's Square

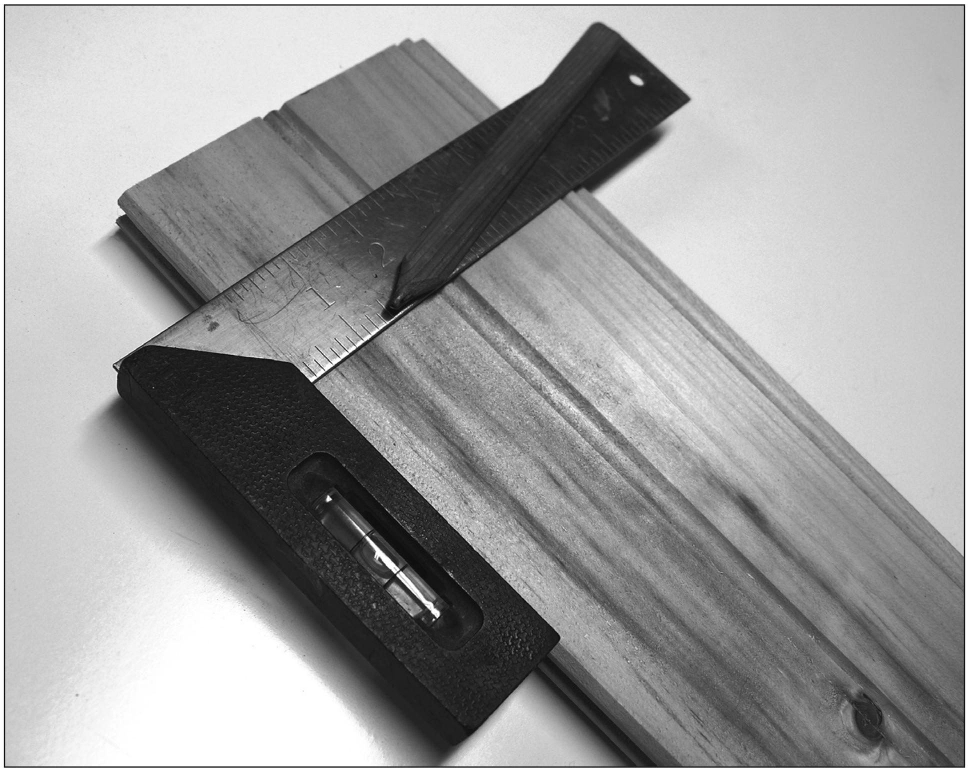


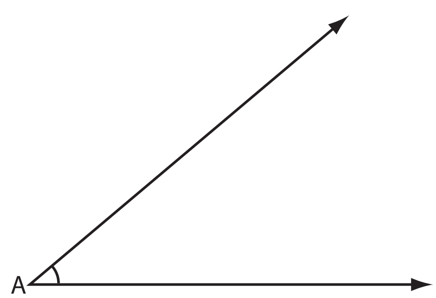
Photo by Tom Oliveira © 2010

My Notes

In construction, a carpenter would use his or her carpenter's square to bisect angles. You can use one of the squares from your geometry set to bisect an angle.

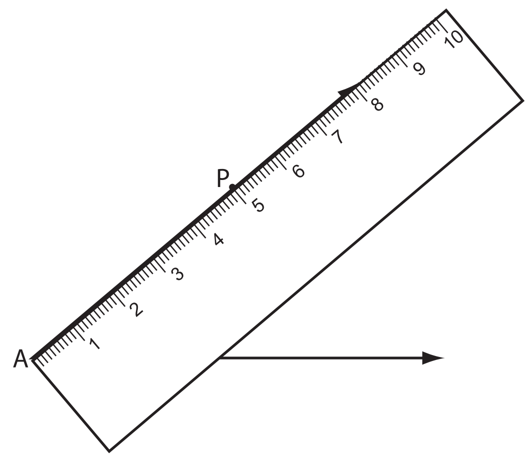
Example 3

Bisect ∠A using a carpenter's square.

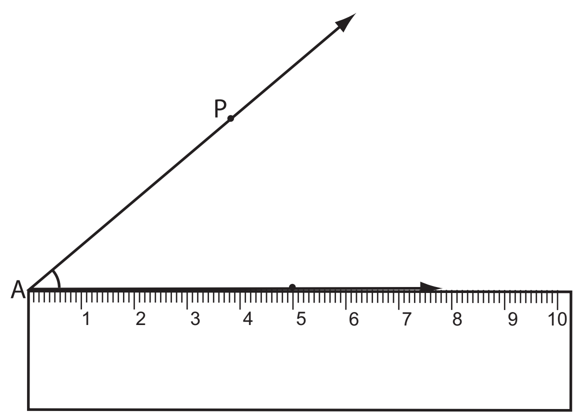


My Notes

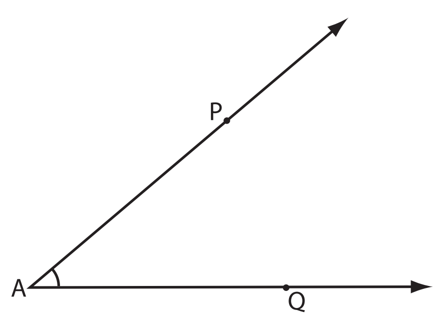
Step 1: From vertex A, measure out a specified distance along the arm. In this example we will use 5 cm. Mark and label the first point P.



Step 2: Measure out the same distance along the other arm of angle A. Mark and label this point Q.

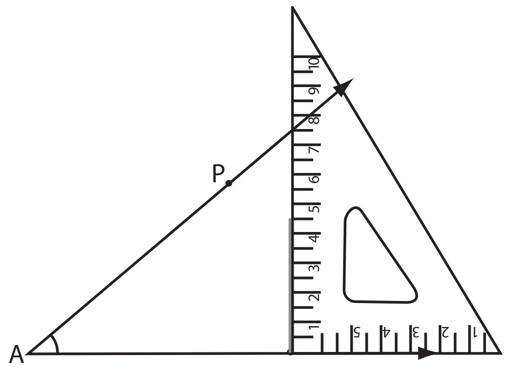


Your angle should now look like this:

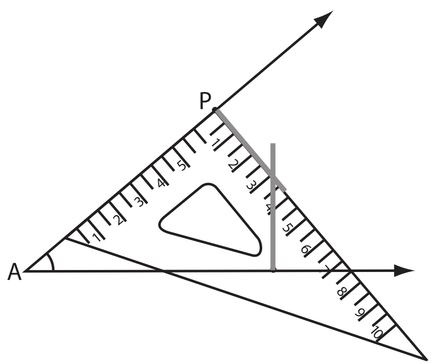


My Notes

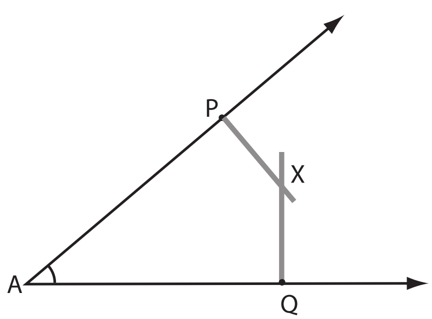
Step 3: Place the right angle of your square along the arm at point Q. Draw a line along the square towards the centre of the angle.



Step 4: Place the right angle of your square along the arm at point P. Draw a line along the square towards the centre of the angle.

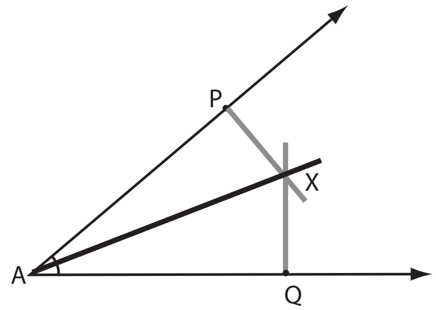


Step 5: Name the point where the lines cross, X.



My Notes

Step 6: Draw ray AX.



Ray AX bisects angle A. You can use a protractor to check that this is true.

To see the technique of bisecting an angle using a square, go and look at *Bisect an Angle Using a Square* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/m10_3_m5_013/m10_3_m5_013.htm*).

Activity 4
Self-Check

My Notes

Use the Carpenter's square method to bisect a 120º angle.

Turn to the solutions at the end of the section and mark your work.

My Notes

Activity 5
Mastering Concepts

Zander says he can bisect a 300° angle by bisecting the 60° angle that completes the full rotation and extending the bisector of the 60° angle in both directions. Is he correct?

If he is correct, explain why. If he is incorrect, explain why.

Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes

Photo by Pichugin Dmitry © 2010

Many photographs and paintings of alpine scenery involve reflections in the mirror surfaces of the mountain lakes. The angles formed by the mountains and their reflections are bisected at the shoreline. This lesson involved similar relations between the arms of angles and the bisectors of those angles. Bisectors behave like mirrors, reflecting each arm into the other.

In this lesson, you investigated a variety of techniques that can be used to construct angle bisectors. You constructed angle bisectors by paper folding, using a protractor using a compass and a straightedge, and using squares from a geometry set.

Lesson D

Relationships Among Angles

To complete this lesson, you will need:

- a protractor
- scissors
- several blank sheets of paper
- a straightedge or ruler

In this lesson, you will complete:

- 6 activities

Essential Questions

- What are some of the relationships between sets of angles?
- How can classifying the relationships between sets of angles help you solve problems?

My Notes

Focus

Photo by Tina Rencelj © 2010

Seeking patterns and relationships is a function of the human mind. Art and architecture, games and music all make use of patterns. The same is true of math, and specifically geometry. Look at the interplay between light and shadows by the goal on the soccer pitch in the photo. The photographer knew that the lines, shapes, and variety of angles would make this an interesting photograph.

The shadow of the goal post nearest you divides a right angle into two acute angles as you can see in the second image. What terms describe this angle pair?

Get Started

You may have studied triangles in previous courses. Do you remember anything about the relationship between the angles of a triangle?

____

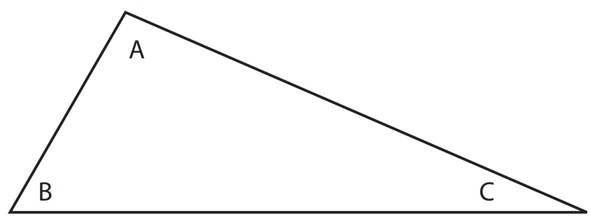
____

Activity 1
Try This

My Notes

You will need a protractor to complete this activity.

Step 1: On a blank sheet of paper, draw a triangle of any shape and size. Label the angles as shown.



Step 2: Measure the three angles with a protractor. Record these measures.

∠A = _______

∠B = _______

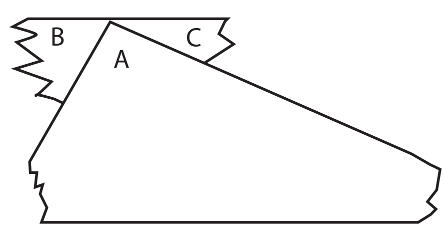
∠C = _______

Step 2: Calculate the sum of the angles.

∠A + ∠B + ∠C = _______

Step 3: Cut out the triangle, and then tear off angles B and C.

Step 4: Place angles B and C next to ∠A so that all three vertices lie at the same point.



Step 5: Repeat Steps 1–4 for a different triangle.

My Notes

Questions

1. What do you notice about the angle formed after Step 4?

____

____

____

____

2. Will the sum of the angles in a triangle be the same for any triangle you draw?

____

____

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____

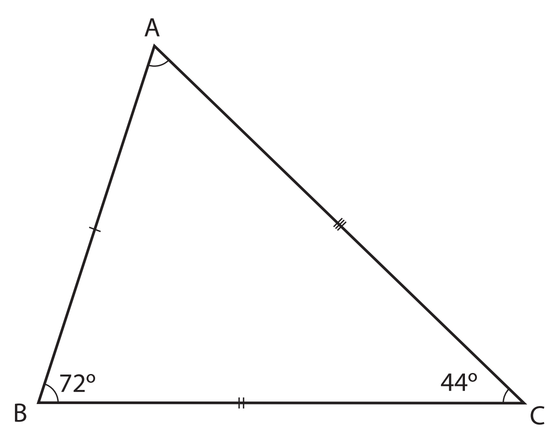
Turn to the solutions at the end of the section and mark your work.

Angles in a Triangle

The sum of the angles in any triangle is 180º. Let's apply this relationship.

Example 1

Find the missing measure of ∠A.



My Notes

Solution

In a triangle, all the angles add up to 180°, so the following must be true:

$$\angle A + \angle B + \angle C = 180^\circ$$
$$\angle A + 72^\circ + 44^\circ = 180^\circ$$
$$\angle A + 116^\circ = 180^\circ$$
$$\angle A = 180^\circ - 116^\circ$$
$$\angle A = 64^\circ$$

Subtract 116 from both sides of the equation to isolate ∠A.

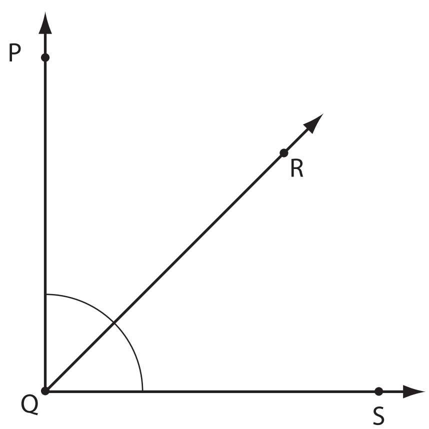
Explore

In Get Started, you reviewed the relationship between the angles in a triangle. Now, you will look at several relationships between pairs of angles.

Adjacent Angles

The word *adjacent* means *neighbouring* or *nearby*. **Adjacent angles** are angles that share a vertex and an arm. If it helps you remember, you can think of them as neighbours.

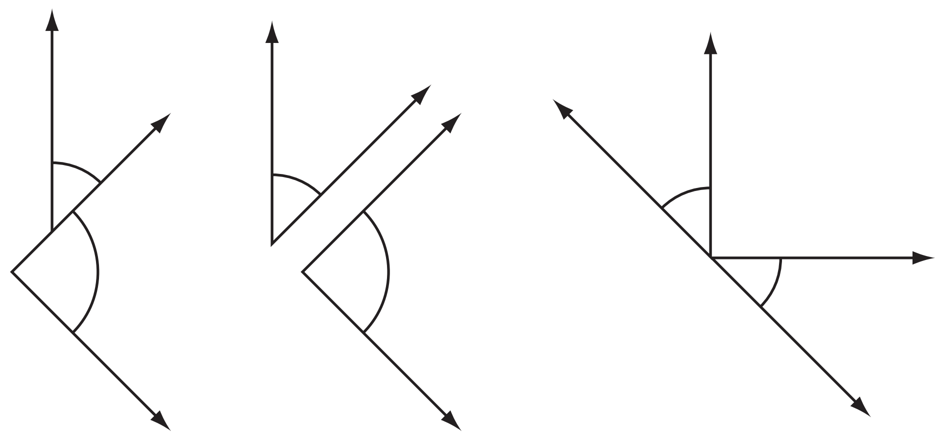
In the diagram below, ∠PQR and ∠RQS are adjacent angles.



My Notes

Activity 2
Self-Check

This illustration shows three pairs of angles.



Explain why none of these pairs are adjacent angles.

____

____

____

____

____

____

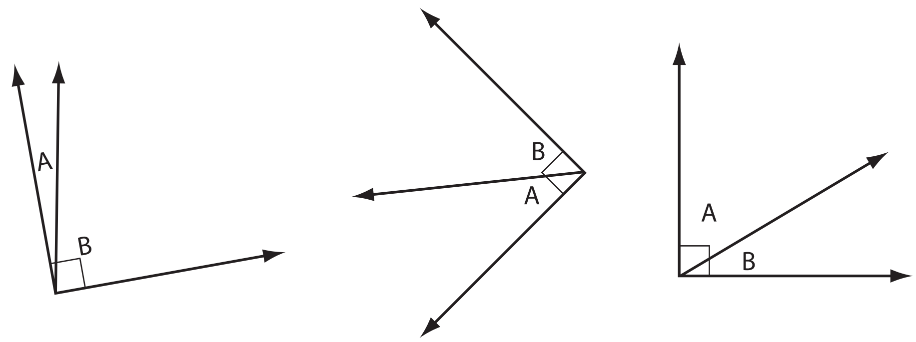
Turn to the solutions at the end of the section and mark your work.

My Notes

Complementary Angles

There is a special connection between the sizes of adjacent angles making up a right angle.

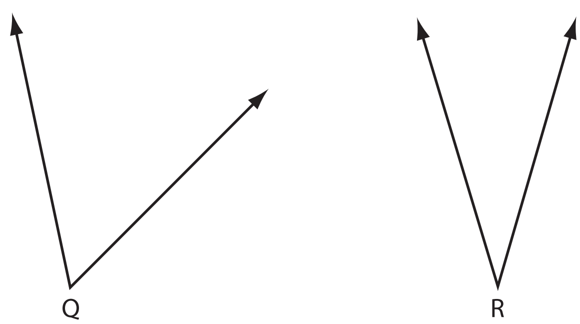
Look at angles A and B in the following frames.



∠A + ∠B = __________

From the diagrams, you should see that ∠A + ∠B = 90º. Two angles whose sum is 90º are said to be **complementary angles**. The angles in the diagrams above are both complementary and adjacent. However, not all complementary angles are adjacent.

The angles shown below are complementary, but not adjacent. Use your protractor to measure these angles and ensure they are complementary.



∠Q + ∠R = _____ + ______

= _________

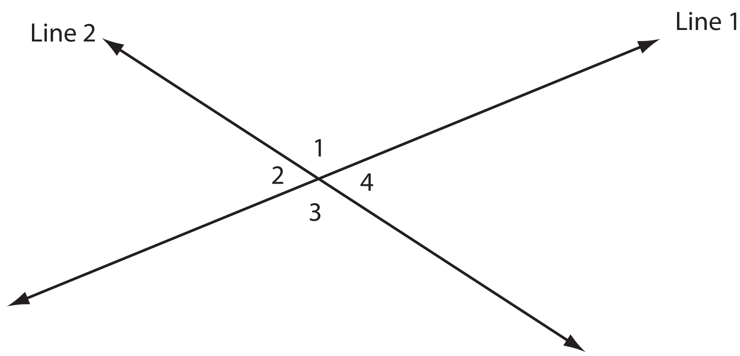
You should have seen that angle Q is 57º and angle R is 33º. The sum of the angle measures is 90º and therefore the angles are complementary.

My Notes

Intersecting Lines

The sizes of adjacent angles at the intersection of two lines are connected in a special way.

Can you identify pairs of adjacent angles at the intersection of Line 1 and Line 2?



The following are pairs of adjacent angles:

- $\angle 1$ and $\angle 2$
- $\angle 1$ and $\angle 4$
- $\angle 3$ and $\angle 4$
- $\angle 2$ and $\angle 3$

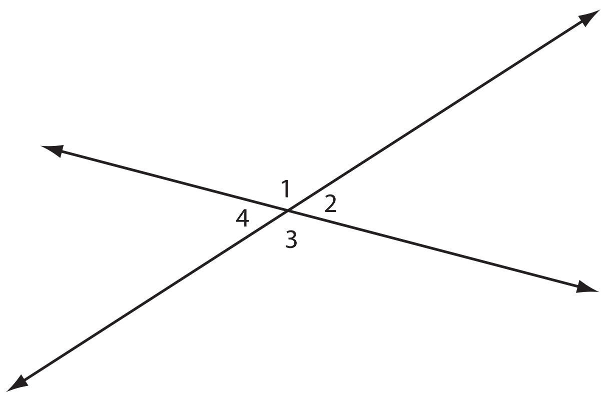
In the next activity you will explore the relationships between the angles formed at the intersection point of two lines.

My Notes

Activity 3
Try This

You will need a protractor to complete this activity.

Two intersecting lines are shown below. Use this diagram as you work through the activity.



1. Use your protractor to measure each of the four angles. Record your measurements below.

   $\angle 1$ = ______

   $\angle 2$ = ______

   $\angle 3$ = ______

   $\angle 4$ = ______

2. Find the sum of the measures of each pair of adjacent angles. Record your answers below.

   $\angle 1 + \angle 2$ = ______

   $\angle 1 + \angle 4$ = ______

   $\angle 3 + \angle 4$ = ______

   $\angle 2 + \angle 3$ = ______

My Notes

3. What do you notice about the sum of the measures of adjacent angles formed by intersecting lines?

____

____

4. a. What do you notice about ∠1 and ∠3?

____

____

b. What do you notice about ∠2 and ∠4?

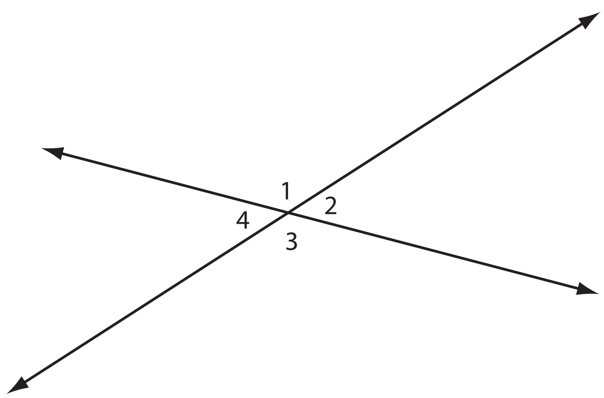
____

____

Turn to the solutions at the end of the section and mark your work.

Supplementary Angles

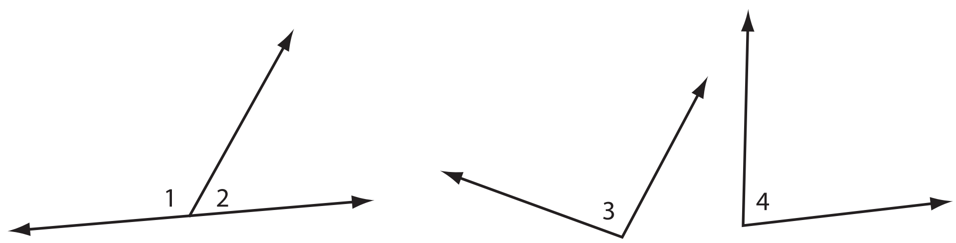
The diagram below is the same as the one you used in Activity 3.



You found that the measures of each of the pairs of adjacent angles formed by the two intersecting lines add up to 180º. This is because the angles in each pair combine to form a straight line. Angles whose measures add up to 180º are called **supplementary angles**.

My Notes

As you saw in Activity 3, some supplementary angles are adjacent. However, not all supplementary angles are adjacent. The diagram below shows two sets of supplementary angles. Use your protractor to measure the angles and ensure that each pair is, in fact, supplementary.

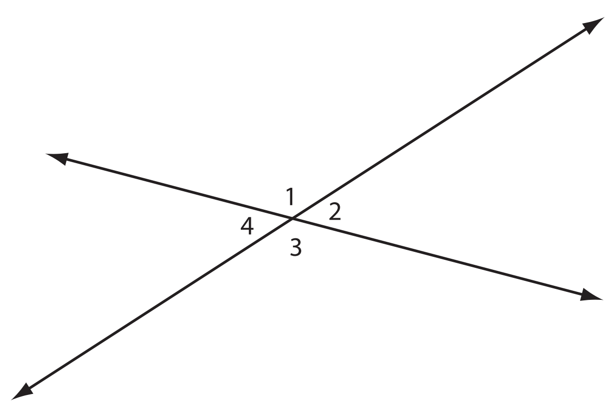


∠1 + ∠2 = _____ + _____ = _____   ∠3 + ∠4 = _____ + _____ = _____

You should have found that angle 1 is 124º and angle 2 is 56º. The sum of the angle measures is 180º and therefore the angles are supplementary. Further, angle 3 is 98º and angle 4 is 82º. The sum of the angle measures is 180º and therefore the angles are supplementary.

Opposite Angles

The diagram below is the same as the one you used in Activity 3.



You found that the measures of ∠1 and ∠3 are equal, and so are the measures of ∠2 and ∠4. We can say that ∠1 and ∠3 are congruent and ∠2 and ∠4 are congruent.

Have a look at where these pairs of angles are located in the diagram above. ∠1 and ∠3 share a vertex and are opposite each other. ∠2 and ∠4 also share a vertex and are opposite each other. Notice that these pairs of angles are not adjacent.

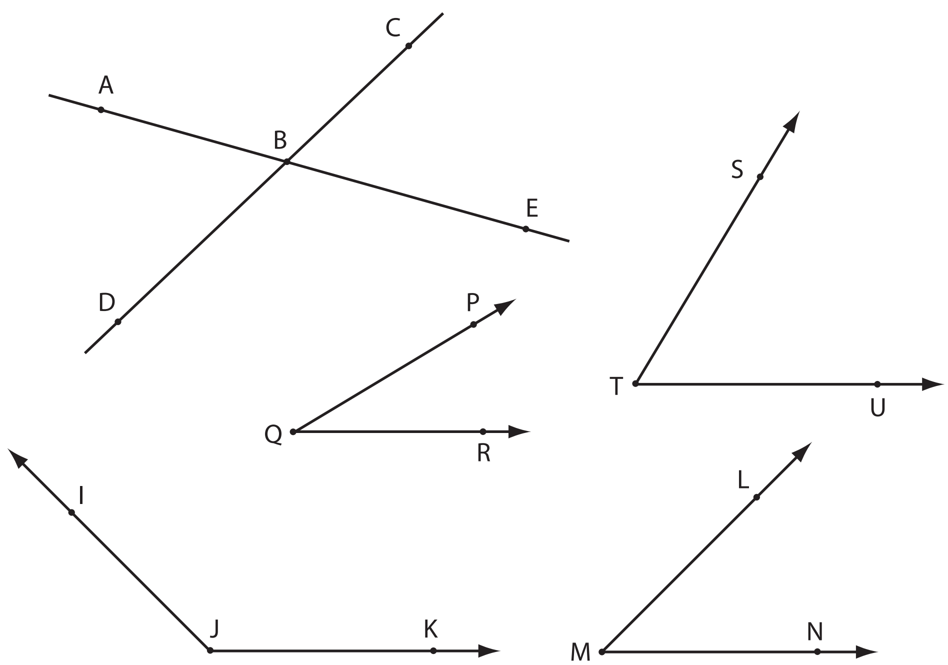
My Notes

These pairs of angles are called **vertically opposite angles** or simply *opposite angles*. Opposite angles are created when two lines intersect. In the diagram above, ∠1 and ∠3 are opposite angles. ∠2 and ∠4 are also opposite angles.

To explore opposite angles further, go and look at *Angle Relationships* and then *Opposite Angles* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/glossary/Division03/OppositeAngles/index.html*). You can move the lines to adjust the sizes of the angles.

Activity 4
Self-Check

You will need a protractor to complete this activity.



Using the diagram above, identify all the pairs of angles that are:

1. adjacent

____

____

My Notes

2. complementary

____

____

3. supplementary

____

____

4. vertically opposite

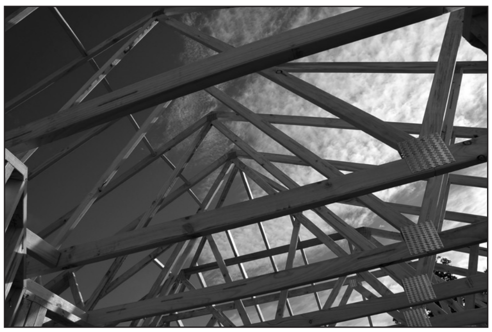
____

____

Turn to the solutions at the end of the section and mark your work.

My Notes

Bringing Ideas Together



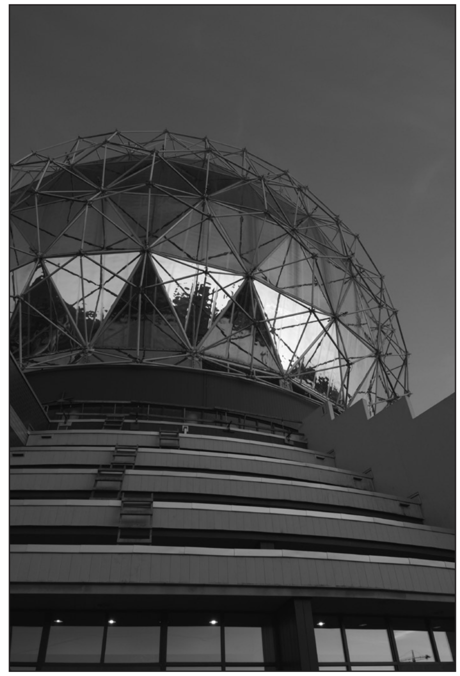


Photo of spider web by Martin Maun © 2010, Photo of rafters by John Leung © 2010, and Photo of Vancouver Science World by Lijuan Guo © 2010

There are countless examples of angles in nature, construction, art, and architecture. Have a look at the pictures above and see if you can find some adjacent, complementary, supplementary, and opposite angles.

Using Angle Relationships

In the following examples and activities you will apply the angle relationships you examined in the Explore. These angle relationships are:

- angles in a triangle
- adjacent angles
- complementary angles
- supplementary angles
- vertically opposite angles

Example 2

Photo by Mark Stout Photography © 2010

The photograph above shows an A-frame house. The roofline forms an isosceles triangle.

The angle at the top of the roof is 94º. The two angles at the base are equal. What is the measure of each?

Solution

The sum of the measures of the three angles of the triangle is 180º.

$$x + x + 94^\circ = 180^\circ$$
$$2x + 94^\circ = 180^\circ$$
$$2x = 180^\circ - 94^\circ$$
$$2x = 66^\circ$$
$$\frac{2x}{2} = \frac{66^\circ}{2}$$
$$x = 33^\circ$$

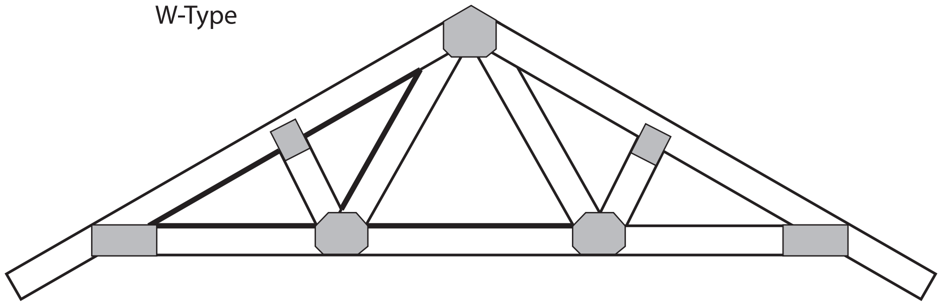
Each angle at the base of the triangle measure 33º.

Example 3

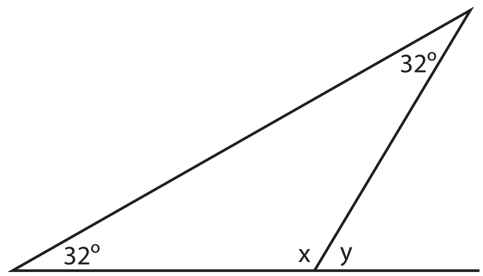
The W-type roof truss is the most common type in simple wood-frame construction. This type of roof truss is shown in the following diagram. The angles we will examine are highlighted.

My Notes

My Notes



The angles will depend on the pitch of the roof. If the pitch of the roof is 32º, calculate the values of *x* and *y*.



Solution

First find the value of *x*.

The sum of the angles in the triangle is 180º.

$$32^\circ + 32^\circ + x = 180^\circ$$
$$64^\circ + x = 180^\circ$$
$$x = 180^\circ - 64^\circ$$
$$x = 116^\circ$$

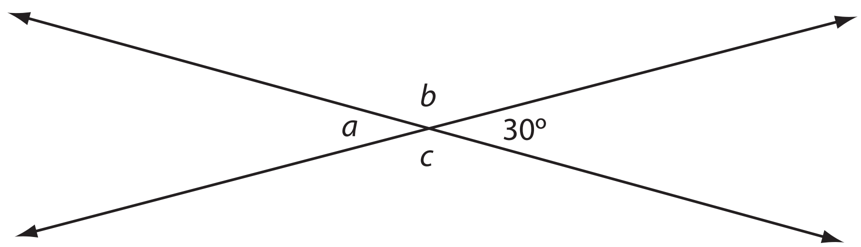
The angles with measures *x* and *y* are supplementary.

$$x + y = 180^\circ$$
$$116^\circ + y = 180^\circ$$
$$y = 180^\circ - 116^\circ$$
$$y = 64^\circ$$

My Notes

Example 4

Two straight paths cross at 30° as shown. Find the measures of the other three angles, which are represented by *a*, *b*, and *c*.



Solution

The angle with measure *a* is opposite the 30°. Therefore *a* = 30°.

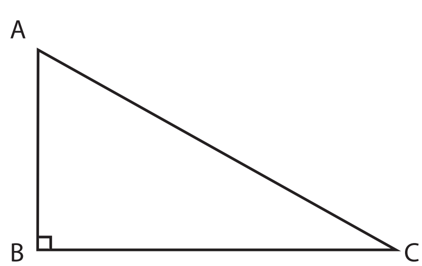
The angles with measures *a* and *b* are supplementary.

$$\begin{aligned} a + b &= 180° \\ 30° + b &= 180° \\ b &= 180° - 30° \\ b &= 150° \end{aligned}$$

The angle with measures *c* is opposite *b*. Therefore *c* = *b* = 150°

Example 5

Angles A and C are angles of a right triangle.



Show that ∠A and ∠C are complementary.

My Notes

Solution

If angles A and C are complementary, they will add up to 90º. Start with what you know and see if you can show that angles A and C add up to 90º.

$$\begin{aligned} \angle A + \angle B + \angle C &= 180^\circ \\ \angle A + 90^\circ + \angle C &= 180^\circ \\ \angle A + 90^\circ + \angle C - 90^\circ &= 180^\circ - 90^\circ \\ \angle A + \angle C &= 90^\circ \end{aligned}$$

Subtract 90º from both sides of the equation.

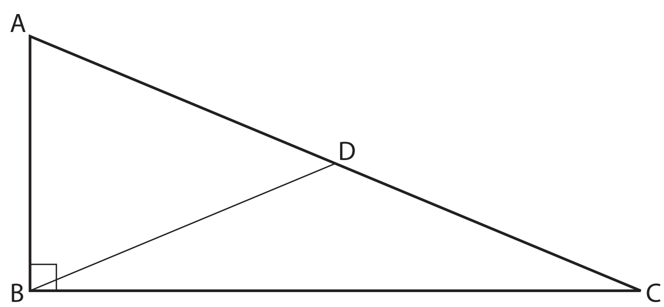
Therefore angles A and C are complementary.

My Notes

Activity 5
Self-Check

Complete the following questions. Please note: The diagrams may not be drawn to scale.

1.



a. Name two pairs of complementary angles.

____

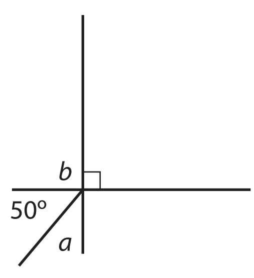
____

b. Name one pair of supplementary angles.

____

____

2. Calculate the angle measures *a* and *b*.



My Notes

3. Find the measures of *a*, *b*, *c*, and d for the rectangular envelope shown in the photograph. In the figure *b* = *c*.

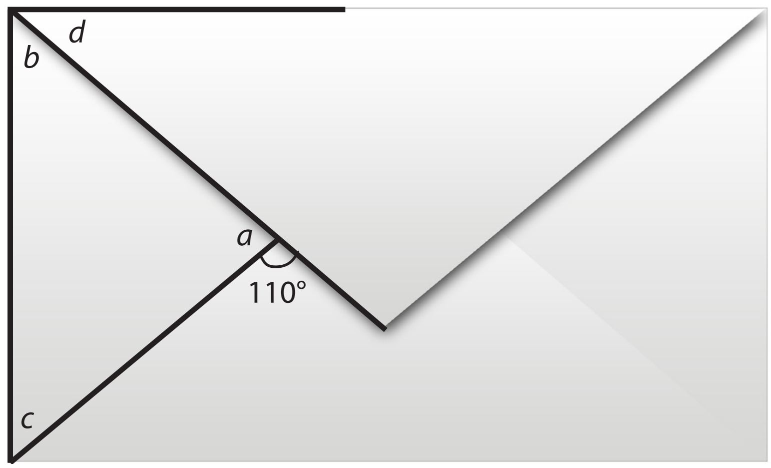
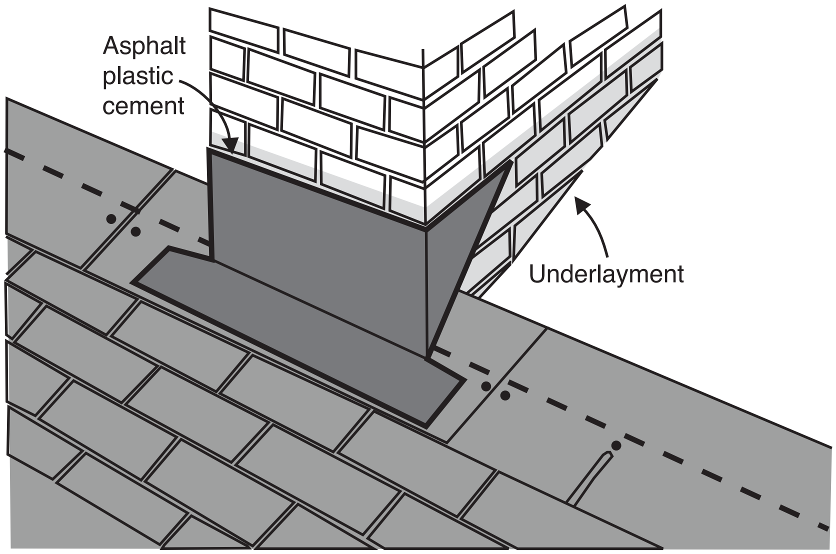
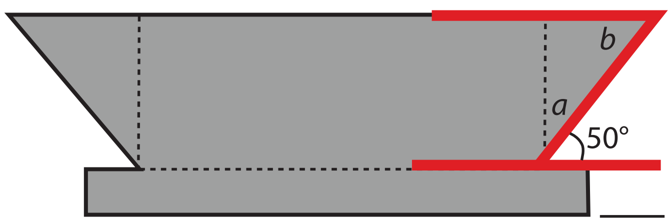


Photo by mayamaya © 2010

4. The layout for a sheet-metal front-cap flashing to be positioned at the front of a chimney is shown. Find the missing measures *a* and *b*.

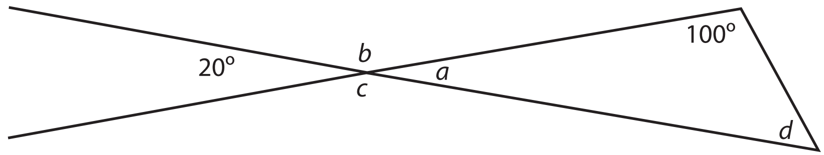




My Notes

My Notes

5. Find the missing measures indicated in the diagram.



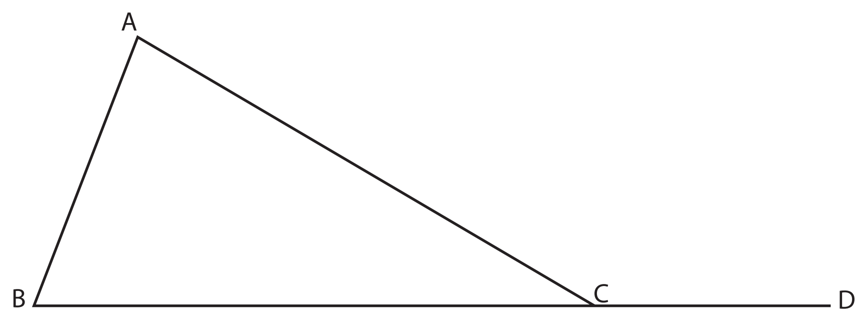
Turn to the solutions at the end of the section and mark your work.

Activity 6

Mastering Concepts

My Notes

1. The angles within a triangle are called interior angles. If a side of a triangle is extended, it forms an exterior angle.

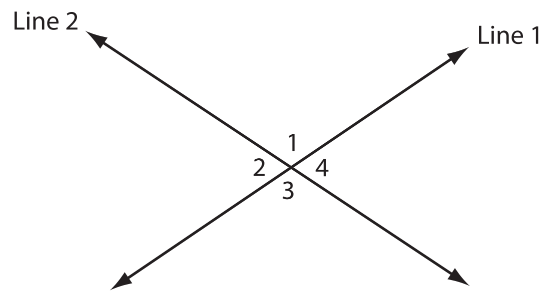


In ∠ABC, ∠A, ∠B and ∠ACB are interior angles. ∠ACD is an example of an exterior angle.

Show ∠A+ ∠B = ∠ACD

My Notes

Use the diagram below to answer Question 2.



2. a. Complete the following statement. Then justify your answer.

$\angle 1 + \angle 2 =$ ____°.

____

b. Complete the following statement. Then justify your answer.

$\angle 3 + \angle 2 =$ ____°.

____

c. Using the results from (a) and (b), show (in a mathematical way) why $\angle 1 \cong \angle 3$.

d. Use a similar approach to show $\angle 2 \cong \angle 4$.

Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes


Photo by EDHAR © 2010 and Photo by Danylchenko Iaroslav © 2010

Many forms of art utilize the visual appeal of angles. A photographer looks for the best angle to shoot his or her subject and makes use of the lines and angles created by the subject and the background. The subjects of the photos you see here are dancers. Dance is another art form in which angles are of great importance. Dancers are always aware of the angles they are creating with their bodies. See how many angles you can trace in the photos above.

In this lesson you examined some relationships between sets of angles. You looked at the relationships between the angles in a triangle, adjacent angles, complementary angles, supplementary angles, and vertically opposite angles.

Lesson E
Parallel and Perpendicular Lines

To complete this lesson, you will need:

- several blank sheets of paper
- straightedge/ruler
- scissors
- protractor

In this lesson, you will complete:

- 5 activities

Essential Questions

- What relationships exist between angles formed when a line intersects a set of two parallel lines?
- How can the relationship among angles formed when a line intersects parallel lines be used to find missing angles in geometric shapes?

My Notes

Focus


Photo by Péter Gudella © 2010 and Photo by maigi © 2010

The photographs above involve very ordinary subjects. You might think that a photo of railroad tracks or a stack of two-by-fours would be boring, but these photos are certainly not that! So, what makes these images interesting?

In both photographs, the photographers have captured patterns formed by parallel and perpendicular lines.

In this lesson you'll investigate the angles formed when a line intersects a set of parallel lines. You'll also explore the relationships between those angles and apply the relationships to practical situations.

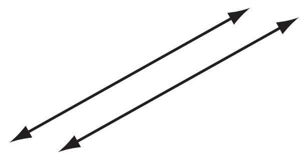
Get Started

You have probably studied **parallel** and **perpendicular** lines in previous courses. It is important that you understand the characteristics of parallel and perpendicular lines before you continue through this lesson.

My Notes

Parallel Lines

The diagram below shows two parallel lines.



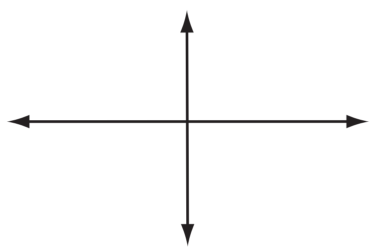
What do you notice about the lines? What happens if you extend the lines in all directions? Do the lines intersect?

____

____

Perpendicular Lines

The diagram below shows two perpendicular lines.



What do you notice about the lines? Use your protractor to measure the angles formed by the intersection of the two lines.

____

____

To investigate parallel lines, go and look at *Parallel Lines* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/glossary/Division03/Parallel/index.html*). To investigate perpendicular lines, go and look at *Perpendicular Lines* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/glossary/Division03/Perpendicular/index.html*). Drag the lines around and read the resulting text. Rotate the lines by clicking on one of the dots and dragging and read the resulting text.

My Notes

Activity 1
Self-Check

1. a. In your own words, explain what it means for two lines to be parallel.

   ___

   ___

   b. In your own words, explain what it means for two lines to be perpendicular.

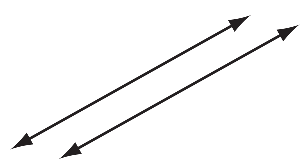
   ___

   ___

2. For each pair of lines below:
   i. state whether they are parallel, perpendicular, or neither parallel nor perpendicular.
   ii. explain why your classification from part (i).

   You may use a protractor and/or a straightedge.

   a.

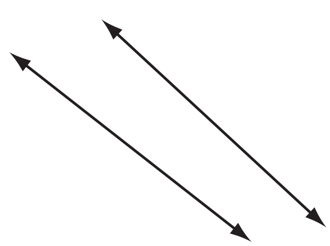


   ___

   ___

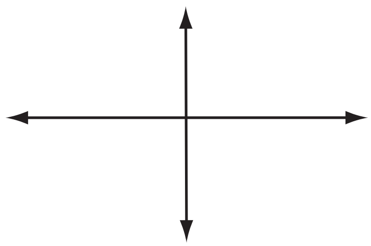
   ___

   b.

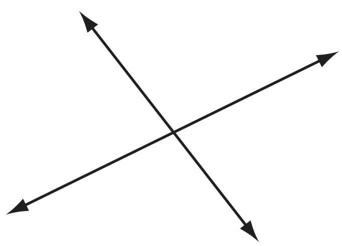


My Notes

c.



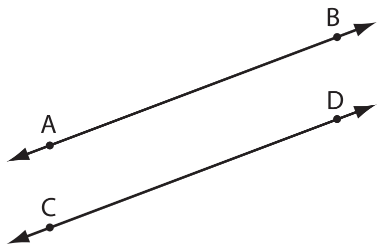
d.

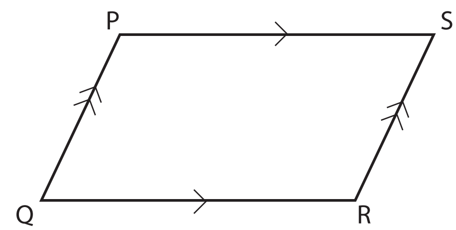


Turn to the solutions at the end of the section and mark your work.

My Notes

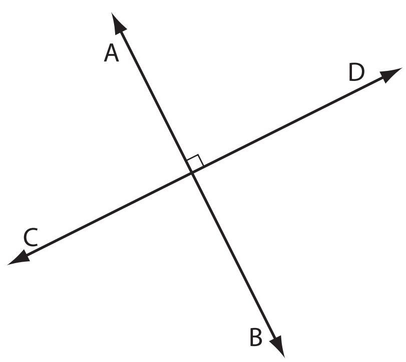
Parallel and Perpendicular Line Notation

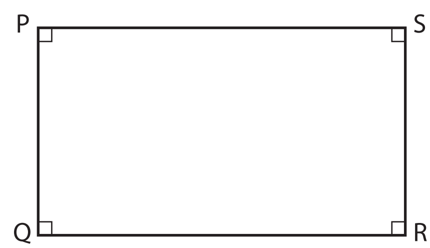




In the diagram above, line AB is parallel to line CD. The symbol || means "is parallel to." You may write $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. Note the symbol $\longleftrightarrow$ means "line."

Also in the diagram above, the parallelogram PQRS, has some parallel sides. The small arrows on the sides of the parallelogram indicate this. The two sides that have a single arrow are parallel and the two sides that have double arrows are parallel. A side with a single arrow is not parallel to a side with double arrows. You can write: $\overline{PS} \parallel \overline{PQ}$ and $\overline{PQ} \parallel \overline{RS}$. The symbol $\overline{\quad}$ means "line segment"—a part of a line with definite endpoints.





In the diagram above, line AB is perpendicular to line CD. The symbol $\perp$ means "is perpendicular to." You could write $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$.

In the rectangle PQRS, $\overline{PS} \perp \overline{PQ}$, $\overline{PS} \perp \overline{RS}$, $\overline{PQ} \perp \overline{QR}$, and $\overline{SR} \perp \overline{QR}$.

Parallel Planes

My Notes

In the same way that lines can be parallel or perpendicular, planes can also be parallel or perpendicular. A plane is a flat, two-dimensional surface. You can think of it as a stiff sheet of paper or like a thin wall.

Consider the place where a wall meets a floor. We can think of the surfaces of the wall and the floor as two planes. What angle is created at the intersection of these two planes?

____

Photo by Sergey Shlyaev © 2010

A wall should meet the floor at a 90° angle. You can use a carpenter's square (or a square from your geometry set) to check this using a wall in your house. Of course, buildings settle over time, and in many cases the walls won't be perfectly perpendicular to the floor. Can you think of other examples of parallel or perpendicular planes?

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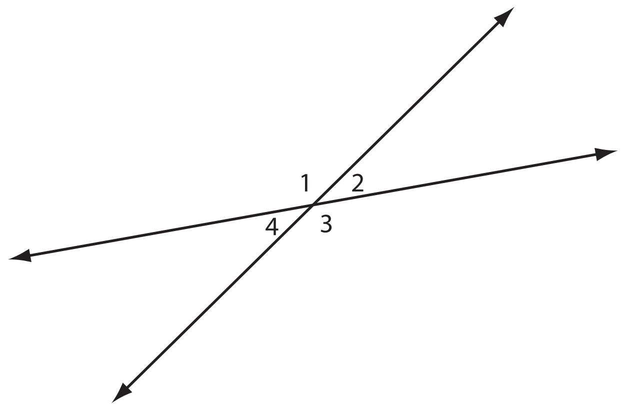
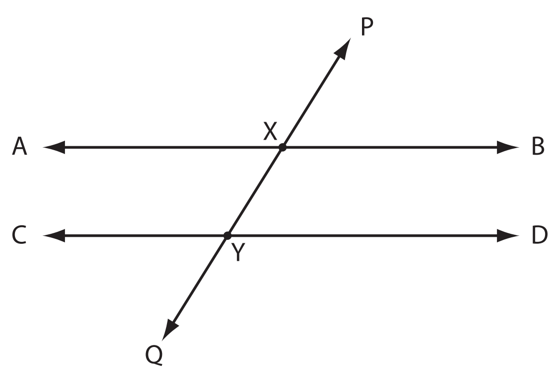
My Notes

Explore

In Lesson D you explored the relationships between pairs of angles. You looked at the angle relationships that result from the intersection of two lines, as shown below.



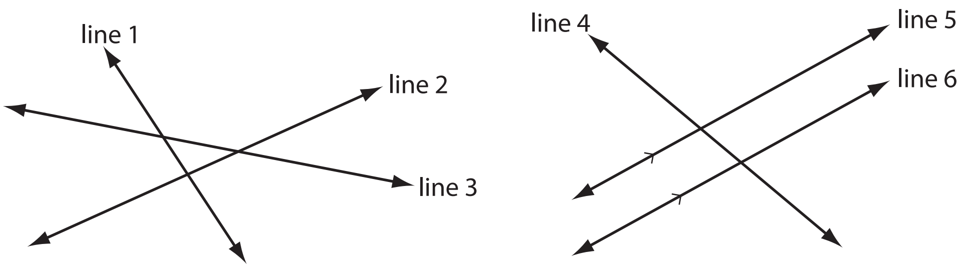
In the next activity lesson, you will explore the relationships among angles formed when two parallel lines are intersected by a third line.



In the diagram above, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, and $\overleftrightarrow{PQ}$ intersects the parallel lines at points X and Y. The intersecting line, $\overleftrightarrow{PQ}$, is called a **transversal.**

My Notes

By definition, a transversal is a line that crosses at least two other lines. Have a look at the diagram below. Can you identify which lines are transversals?

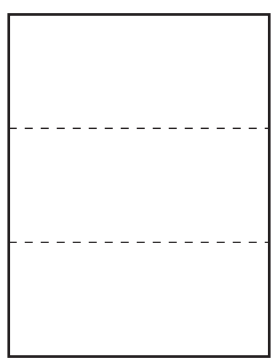


Lines 1, 2, 3 and 4 are transversals because each of them intersects two other lines. Lines 5 and 6 are not transversals because they each only intersect one other line.

Activity 2
Try This

In this activity, you will explore angles formed by a transversal. You will need a blank sheet of paper, a ruler or straightedge, and a pair of scissors.

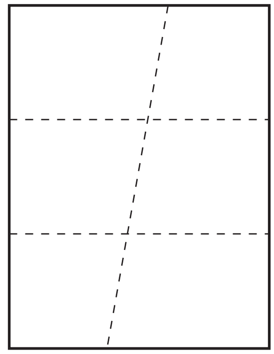
Step 1: Take a blank sheet of paper and fold it twice to form two parallel creases when unfolded.



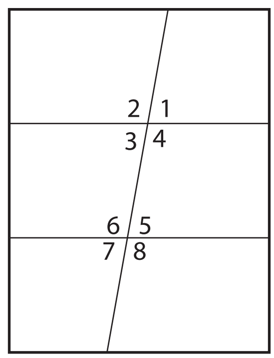
Step 2: Answer Question 1.

My Notes

Step 3: Fold the paper again. When unfolded the crease must cross both parallel creases.



Step 4: Using a ruler or other straightedge, draw lines on all the creases. Number the angles as shown below.



Step 5: Using scissors, cut along each of the lines. You should end up with six pieces that have numbers on them.

Step 6: Pick up ∠1 and compare it to each of the other angles by placing it on top of each one. Are any of the angles congruent to (the same measure as) ∠1? If there are, list those angles in the "Congruent to..." column in the following table.

Step 7: Repeat Step 5 for each of the other angles until you have compared them all. Please note: ∠3 and ∠6 are on the same piece of paper. ∠4 and ∠5 are also on the same piece of paper. Think about how you can compare ∠3 with ∠6 and ∠4 with ∠5. Question 2 will ask you to explain your strategy.

Angle	Congruent to...
$\angle 1$	
$\angle 2$	
$\angle 3$	
$\angle 4$	
$\angle 5$	
$\angle 6$	
$\angle 7$	
$\angle 8$	

My Notes

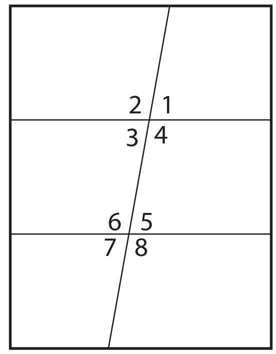
Questions

1. Explain how you folded the paper and why, when unfolded, the creases must be parallel.

2. $\angle 3$ and $\angle 6$ are on the same piece of paper. $\angle 4$ and $\angle 5$ are also on the same piece of paper. How did you compare $\angle 3$ with $\angle 6$ and $\angle 4$ with $\angle 5$?

My Notes

3. If you were to shade all congruent angles in the same colour, how many different colours would you need for this diagram. Explain your answer.

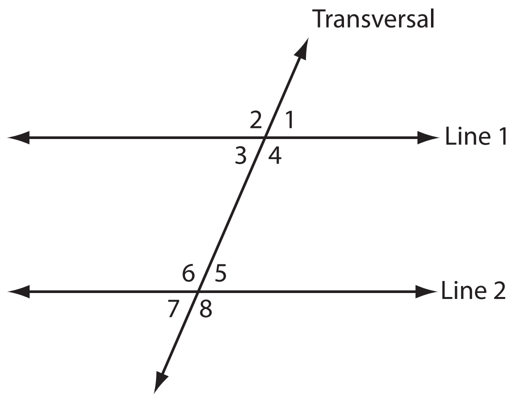


Turn to the solutions at the end of the section and mark your work.

Bringing Ideas Together

My Notes

In Explore, you compared the angles formed when two parallel lines are cut by a transversal.

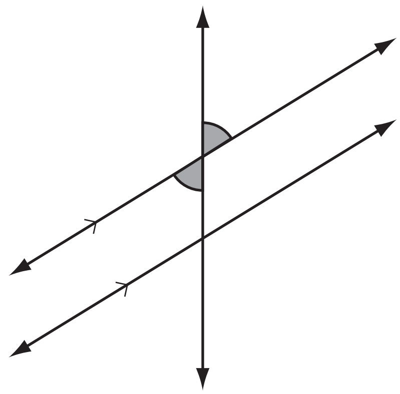


Now, we'll expand on what you found in Explore. We'll define several relationships between the angles formed when two parallel lines are cut by a transversal. We'll look at:

- vertically opposite angles
- corresponding angles
- alternate interior angles
- alternate exterior angles
- co-interior angles
- co-exterior angles

My Notes

Vertically Opposite Angles



You should remember from Lesson D that **vertically opposite angles** are congruent. In Lesson D we only looked at the angles around a single intersection point. In Activity 2 in this lesson, you saw that the same rule applies when there is more than one intersection point.

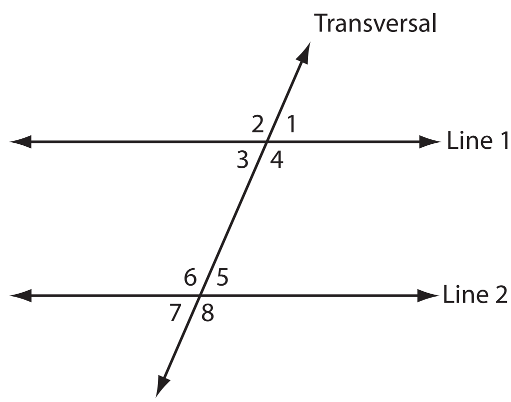
In the diagram shown, there are four pairs of vertically opposite angles:

∠1 and ∠3

∠2 and ∠4

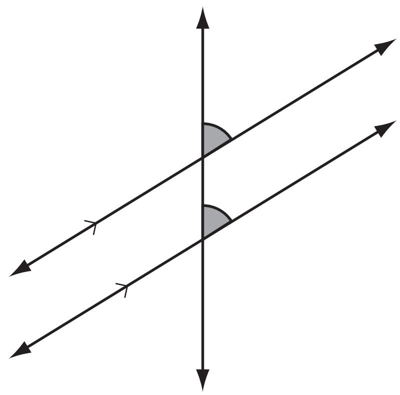
∠5 and ∠7

∠6 and ∠8



Corresponding Angles

My Notes



In Activity 2, you discovered that angles in the same relative positions at the two points of intersection are congruent. These sets of congruent angles are called **corresponding angles**.

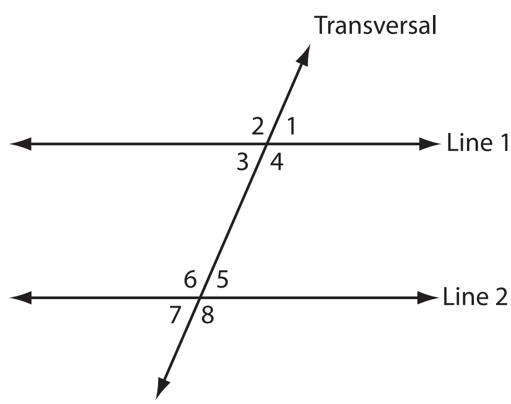
In the diagram shown, there are four pairs of corresponding angles:

$\angle 1$ and $\angle 5$

$\angle 2$ and $\angle 6$

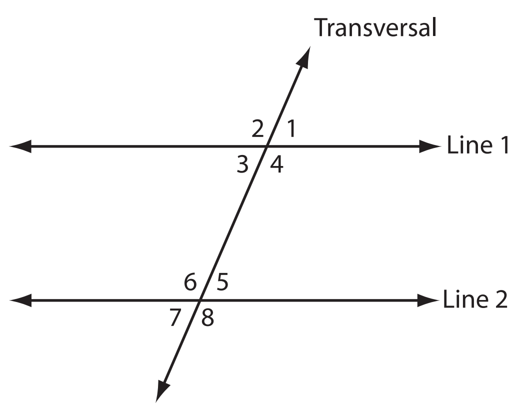
$\angle 3$ and $\angle 7$

$\angle 4$ and $\angle 8$



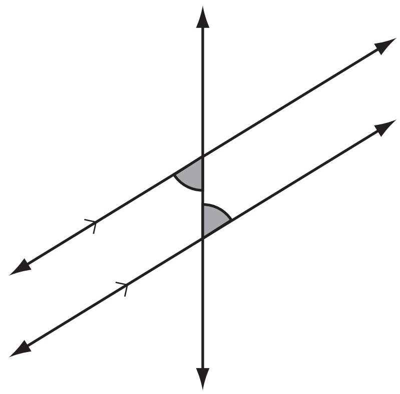
Interior and Exterior Angles

Look at the four angles between the parallel lines. $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ are termed **interior angles**. The other four angles ($\angle 1$, $\angle 2$, $\angle 7$, and $\angle 8$) are **exterior angles**.



My Notes

Alternate Interior Angles



Do you remember from Activity 2, which interior angles are congruent?

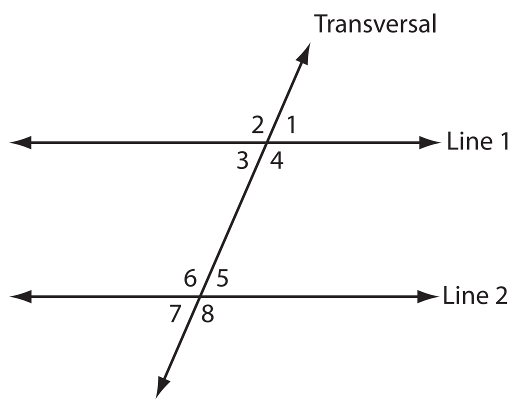
____

The congruent pairs of interior angles are called alternate interior angles. The two angles in each pair of **alternate interior angles** lie on opposite sides of the transversal.

In the diagram shown, there are two pairs of alternate interior angles:

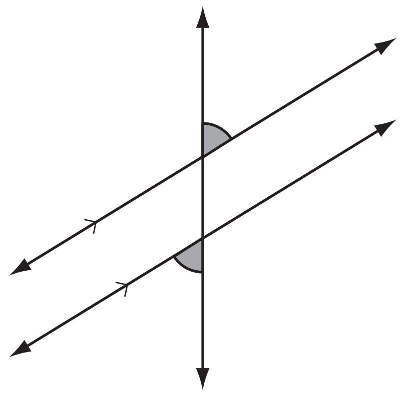
$\angle 3$ and $\angle 5$

$\angle 4$ and $\angle 6$



Alternate Exterior Angles

My Notes



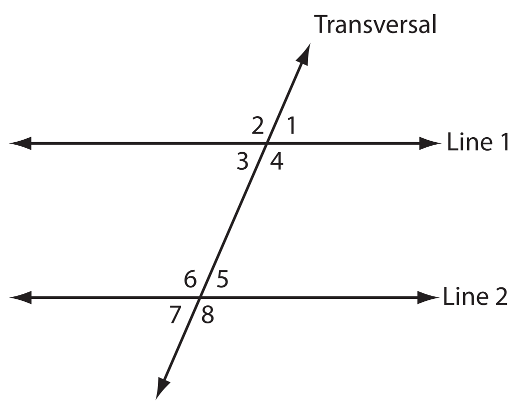
Do you remember from Activity 2, which exterior angles are congruent?

The congruent pairs of exterior angles are called **alternate exterior angles**. The two angles in each pair of alternate exterior angles lie on opposite sides of the transversal.

In the diagram shown, there are two pairs of alternate exterior angles:

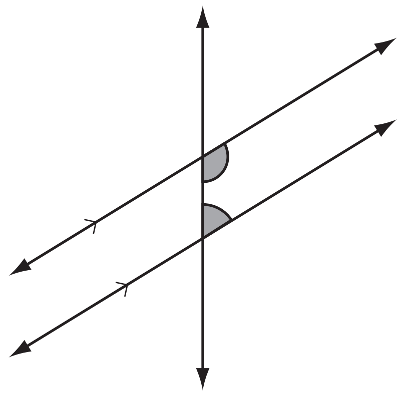
$\angle 1$ and $\angle 7$

$\angle 2$ and $\angle 8$



My Notes

Co-interior Angles



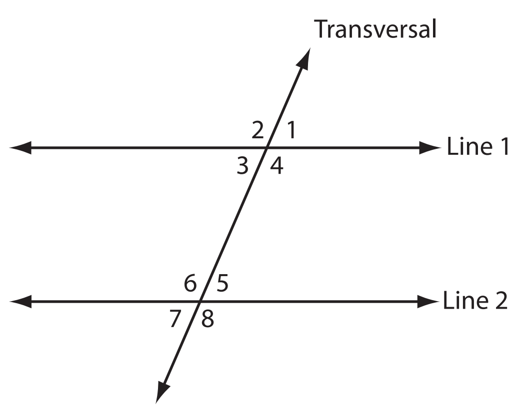
Co-interior angles are interior angles (angles that lie between the parallel lines) that are located on the same side of the transversal. Use your protractor to measure the co-interior angles in the diagram above. What do you notice?

The co-interior angles are supplementary—that is, their measures add up to 180º.

In the diagram shown, there are two pairs of co-interior angles:

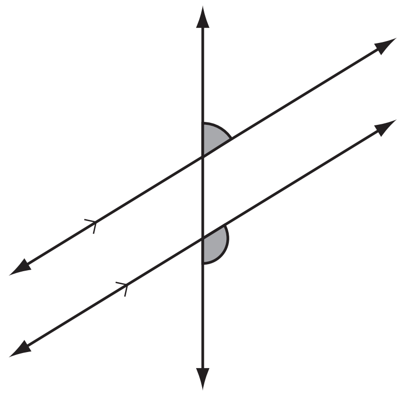
$\angle 3$ and $\angle 6$

$\angle 4$ and $\angle 5$



Co-exterior Angles

My Notes



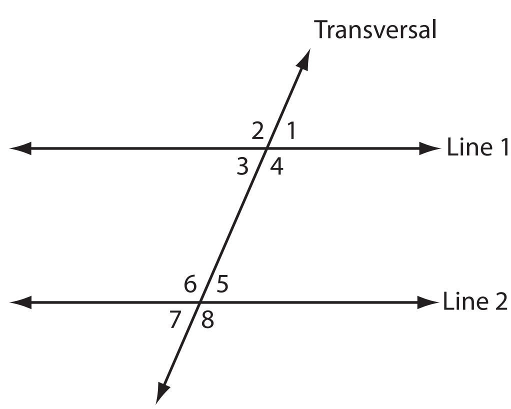
Co-exterior angles are exterior angles (angles that lie outside the parallel lines) that are located on the same side of the transversal. Use your protractor to measure the co-exterior angles in the diagram above. What do you notice?

Like co-interior angles, co-exterior angles are supplementary—that is, their measures add up to 180º.

In the diagram shown, there are two pairs of co-exterior angles:

∠1 and ∠8

∠2 and ∠7



My Notes

Summarizing Angle Relationships

You have learned about several angle relationships in this lesson:

- vertically opposite angles
- corresponding angles
- alternate interior angles
- alternate exterior angles
- co-interior angles
- co-exterior angles

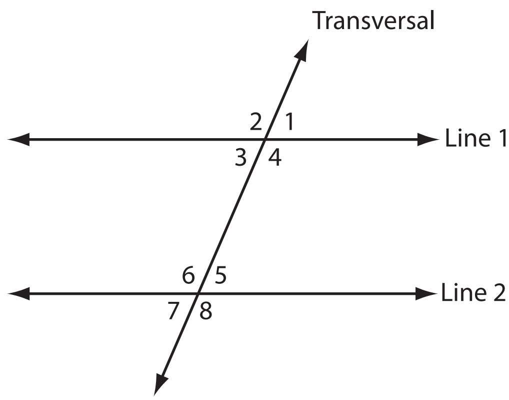
To summarize what you learned, and solidify your understanding, explore the media *Angle Relationships* and then work through Activity 3.

If you have Internet access and want further review before trying Activity 3, go and look at *Angle Relationships* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/glossary/Division03/Correspondingangles/index.html*) and select the angle relationship you'd like to review. Be sure to try out the applets! Move LINE 1 (the purple one) to change the angles formed at the intersection points. As you move the line, notice the relationships between the angles.

Activity 3
Self-Check

My Notes

1. Have a look at the diagram below.



Classify each angle pair below based on the angle relationships discussed in this lesson. Also, state whether the pair is congruent or supplementary.

Angle Pair	Angle Relationship	Congruent or Supplementary?
∠1 and ∠3		
∠4 and ∠8		
∠4 and ∠5		
∠3 and ∠5		
∠2 and ∠8		
∠2 and ∠7		

My Notes

2. a. Determine the missing angle measures in the diagram below.

∠1 = ______

∠2 = ______

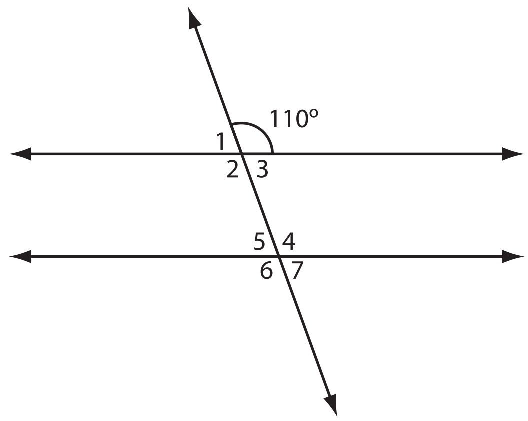
∠3 = ______

∠4 = ______

∠5 = ______

∠6 = ______

∠7 = ______



b. Explain how you determined the measures of ∠1, ∠4, and ∠7.

____

____

____

____

____

____

____

____

Turn to the solutions at the end of the section and mark your work.

Using Angle Relationships

Now that you're familiar with the relationships between the angles formed when two parallel lines are crossed by a transversal, let's look at how we can use these relationships to find missing angle measures.

My Notes

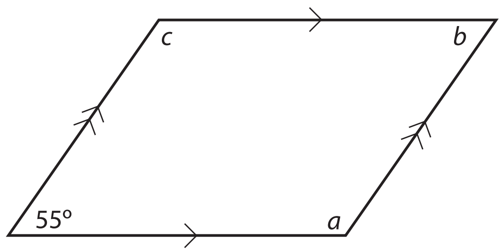
Before we you look at the examples, you should note that all of the angle relationships work in the reverse direction. For example:

- If the corresponding angles are congruent, the lines are parallel.
- If the alternate interior or alternate exterior angles are equal, the lines are parallel.
- If the co-interior or co-exterior angles are supplementary the lines are parallel.

Now, work carefully through the examples and then complete Activity 4.

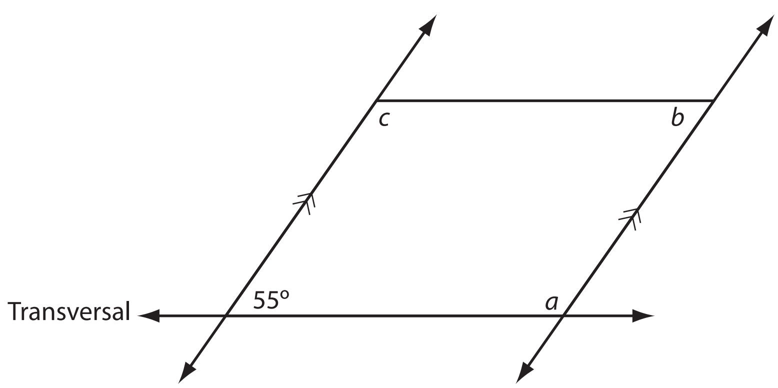
Example 1

Find the missing angle measures in the parallelogram below.



Solution

We can extend the lines of the parallelogram to make the angle relationships more apparent. To find angle measure a, the bottom of the parallelogram will be the transversal.



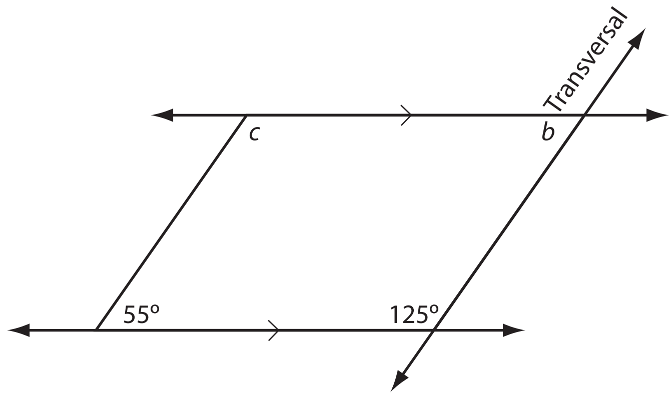
$a + 55° = 180°$ (co-interior angles)

$a = 180° - 55°$

$a = 125°$

My Notes

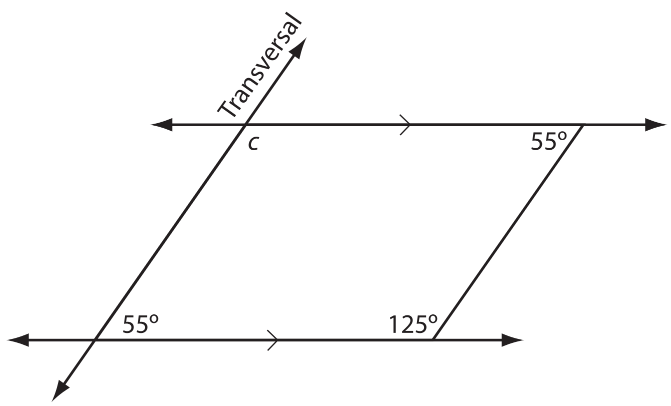
Add angle measure *a* to your diagram. To find angle measure *b*, the right side of the parallelogram becomes the transversal.



$125° + b = 180°$
$b = 180° - 125°$
$b = 55°$

co-interior angles

Add angle measure *b* to your diagram. To find angle measure *c*, the left side of the parallelogram becomes the transversal.



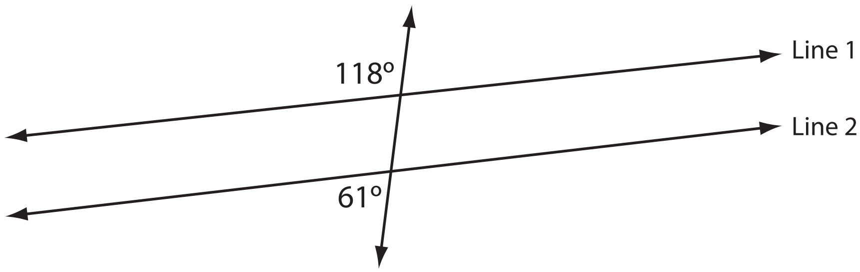
$c + 55° = 180°$
$c = 180° - 55°$
$c = 125°$

co-interior angles

My Notes

Example 2

Are lines 1 and 2 parallel? Why or why not?



Solution

The given angles are co-exterior angles.

If the lines were parallel, the co-exterior would be supplementary (add up to 180º).

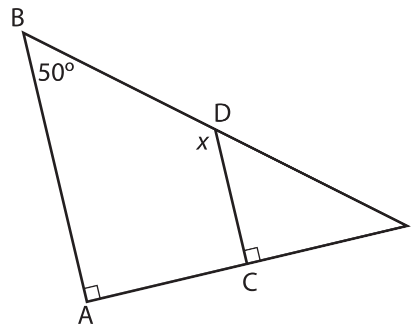
But, 118º + 61º = 179º.

So, Line 1 and Line 2 are not parallel.

My Notes

Activity 4
Self-Check

Please complete the following questions.

1. 

Based on the figure above, answer the following:

a. Is $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$? Justify your answer.

____

____

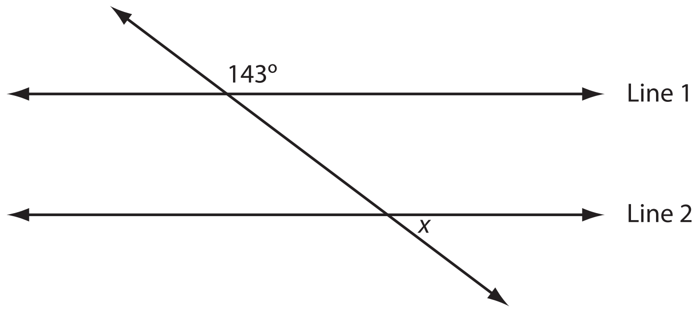
____

____

b. Calculate the value of x. State the angle property you used to justify your answer.

My Notes

2. Line 1 is parallel to Line 2.



Find the value of *x*.

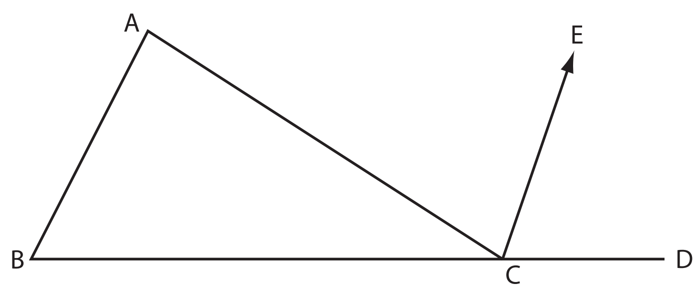
Turn to the solutions at the end of the section and mark your work.

My Notes

Activity 5
Mastering Concepts

The angles within a triangle are called **interior angles**. If a side of a triangle is extended, it forms an **exterior angle**.

In ΔABC below, ∠A and ∠B are interior angles. ∠ACD is an example of an exterior angle.



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CE}$. Show that ∠A + ∠B = ∠ACD.

Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes

Optical illusions can be created by the careful placement and sizing of lines and colours on the page. Are the lines in the image below parallel to each other? How do you know?

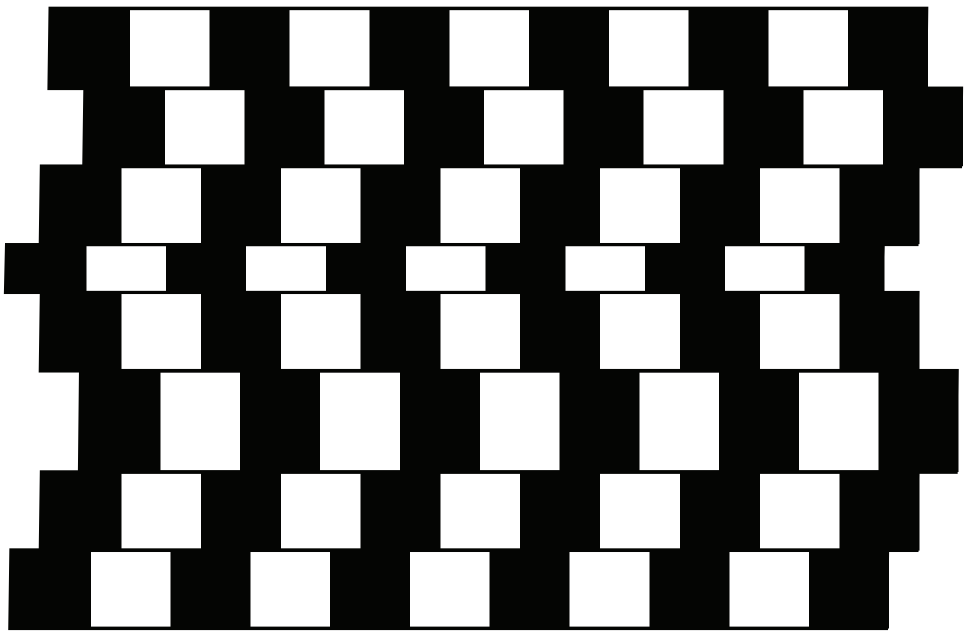


Photo by photazz © 2010

In this lesson you discovered that when two parallel lines are intersected by a transversal, corresponding angles, alternate interior, and alternate exterior angles are congruent. You also discovered that co-interior and co-exterior angles are supplementary. And if these relationships in a specific instance do not hold, then the lines simply are not parallel!

Lesson F
Solving Problems Using Angle Relationships

In this lesson, you will complete:

- 3 activities

Essential Questions

- How are the measures of angles determined in problem situations involving parallel lines and transversals?

My Notes

Focus

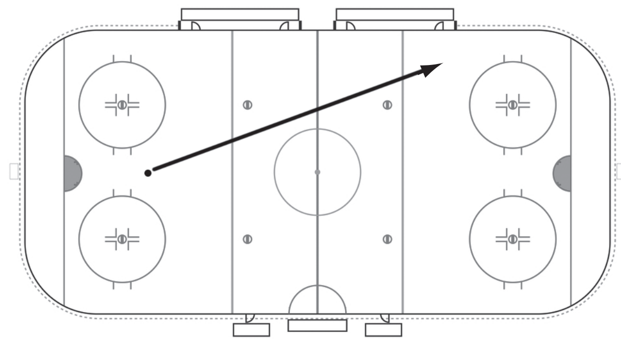


Photo by Jecowa © 2010

Whether or not you're a fan of hockey, you're probably familiar with the sight of an ice-hockey rink. Have you looked carefully at the markings on the ice? The markings consist of a series of circles, parallel, and perpendicular lines.

In the illustration above, a puck has travelled from the left side crossing both blue lines. The linesman's whistle blows and the signal for "offside" is given. If the puck travelled in a straight line, what can you say about the angles created at the two lines where the puck crossed?

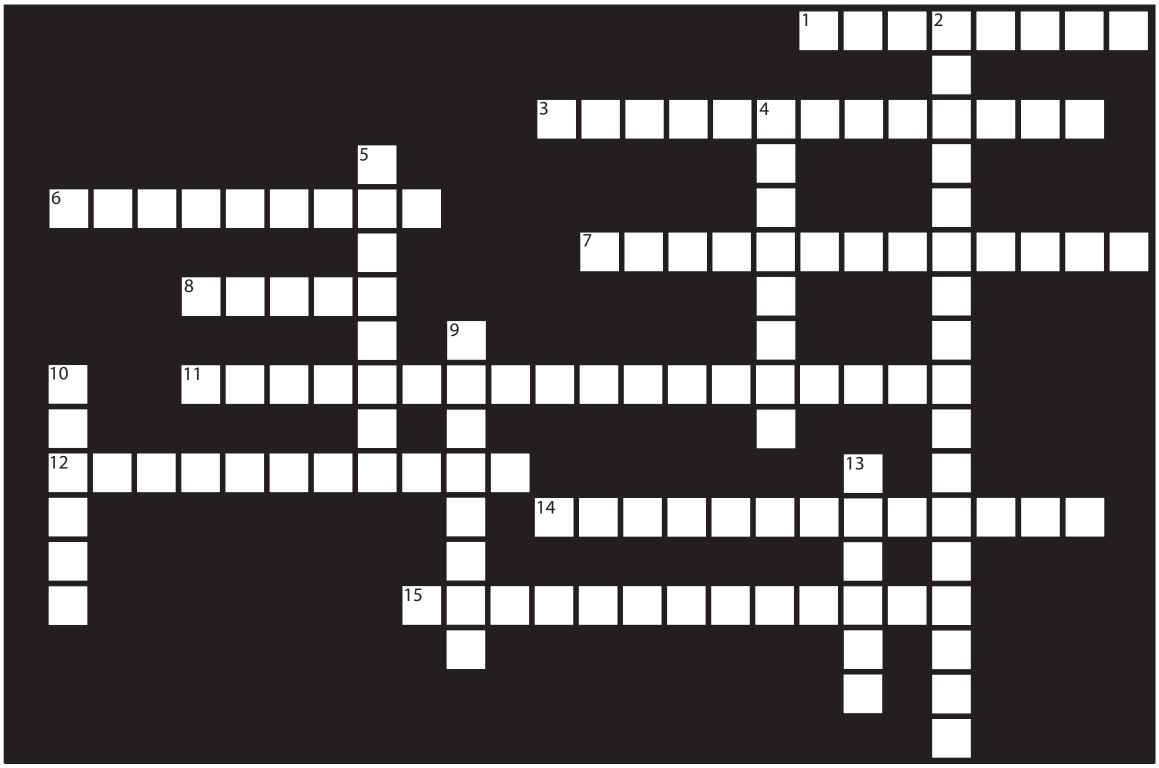
Get Started

This lesson will be structured differently than usual. We will be solving problems involving angles formed when parallel lines are crossed by a transversal. Since you've already covered these relationships, we will omit the Explore portion of this lesson.

To start the lesson, complete Activity 1. You will review what you've learned in this section about angles and angle relationships.

When you've completed the activity, you will move directly to Bringing Ideas Together. There, we will work through several examples and you'll have a chance to apply the angle relationships you've learned.

Activity 1
Self-Check



Across

1. Angles which share a common vertex and lie on opposite sides of a common arm
3. Two angles with measures that add up to 90º
6. Angles with the same measure
7. Congruent angles in the same relative positions when two lines are intersected a transversal
8. An angle having a measure greater than 0º but less than 90º
11. Angles lying across from each other at the point where two lines intersect (2 words)
12. A line that cuts across two or more lines
14. Two angles with measures that add up to 180º
15. Lines that intersect at a 90º angle

Down

2. Interior angles lying on opposite sides of the transversal. One is on the left and the other is on the right. (2 words)
4. Angles outside two lines cut by a transversal
5. Angles lying between two lines cut by a transversal
9. Lines that never meet
10. An angle having a measure greater than 90º but less than 180º
13. An angle having a measure greater than 180º but less than 360º

Turn to the solutions at the end of the section and mark your work.

My Notes

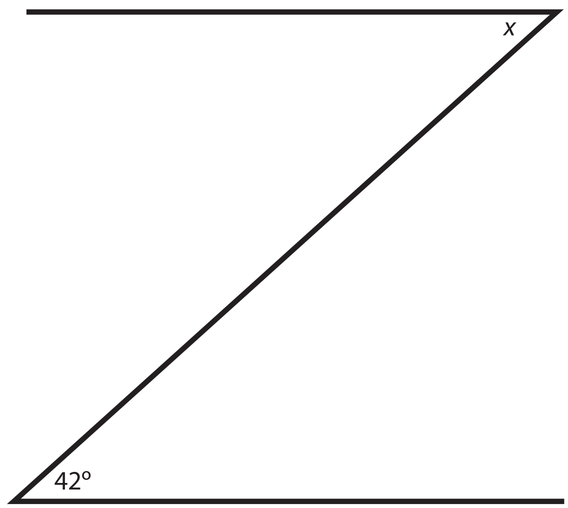
Bringing Ideas Together

In the remaining portion of this lesson, we'll solve several problems that involve the angle relationships you've learned in this section.

As you work through the examples, think about what strategies you can use to help you solve problems that involve angle relationships. Also, make sure that you understand all of the angle relationships before you move on to Activity 2.

Example 1

Photo by Paul J. West © 2010



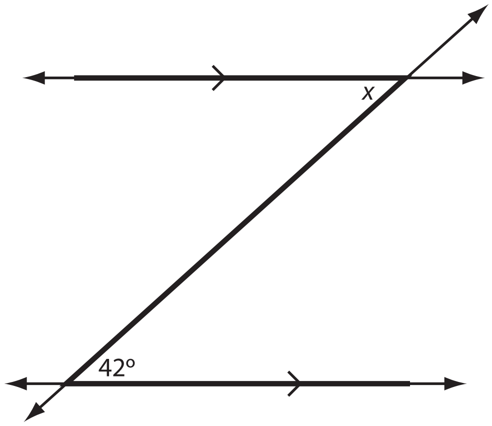
A wooden gate is cross-braced as shown. If the horizontal boards are parallel, determine the measure of angle between one of the cross braces and the top horizontal board.

Solution

The two angles in the "letter zed" are alternate interior angles. The diagonal cross brace is a "transversal" cutting across the "parallel lines" of the horizontal boards.

To make it easier to see these relationships, try extending the lines from the picture.

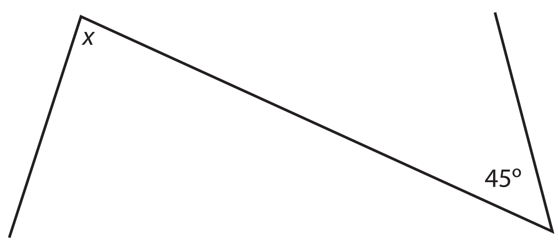
My Notes



Alternate interior angles are congruent, so $x = 42°$.

Example 2

Photo by Gulei Ivan © 2010



Classify the two angles in the diagram formed by the sides of the transmission tower and one of the diagonal braces. Do you have enough information to determine the value of x? Why or why not?

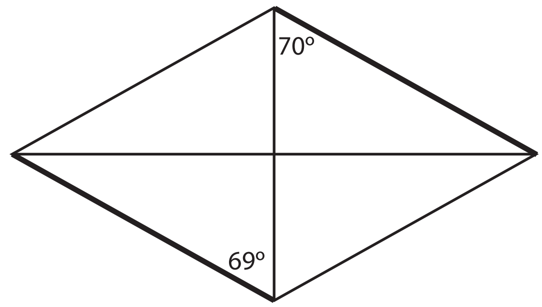
Solution

The angles in the "letter zed" are alternate interior angles. However, the sides of the tower are not parallel, so the angle relationships we have studied so not apply. There is not enough information to find the value of x.

My Notes

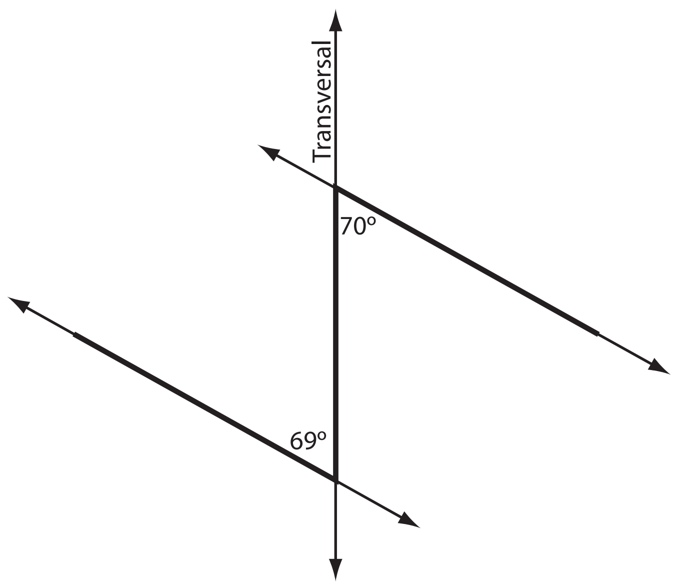
Example 3

The outline of a kite is shown below. Are the two bolded edges parallel? Justify your answer.



Solution

The 69º and 70º angles are alternate interior angles formed by the vertical transversal. Because these angles are not congruent, the two bold edges are not parallel.



If you'd like to view one more example, go and look at *Ladder Solution* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/m10_3_m5_022/m10_3_m5_022.htm*).

Problem-Solving Strategies

As you worked through the examples, what problem-solving strategies did you come up with?

____

____

____

You may have noticed that sometimes it can be difficult to see which angle relationship you can use to solve the problem. Drawing a diagram from the information given in the problem, and extending the lines in the diagram, can be very helpful strategies. In addition, you may also have your own strategies that help you remember the angle relationships.

Whatever strategies you find helpful, it's time to get them ready—you'll need them for the next activity!

My Notes

My Notes

Activity 2
Self-Check

1. This illustration shows a pair of vertical bridge supports and a cross-beam running from the top of one support to the base of the other. The angle between the second support and the beam is labelled *x*.

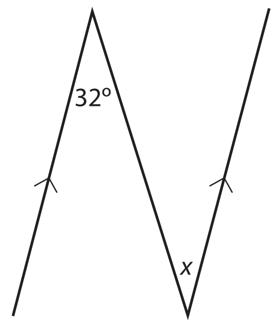
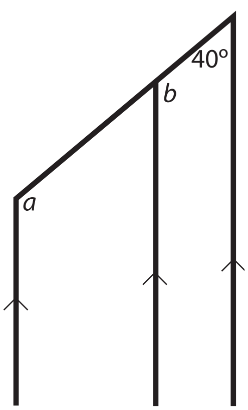


Photo by basel101658 © 2010

Calculate the value of *x*. Justify your answer. Assume the vertical bridge supports are parallel.

2. Find the missing angle measures for the windows of the chalet.

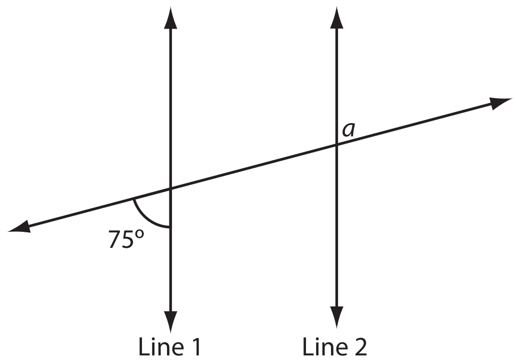
Photo by Mark Stout Photography © 2010



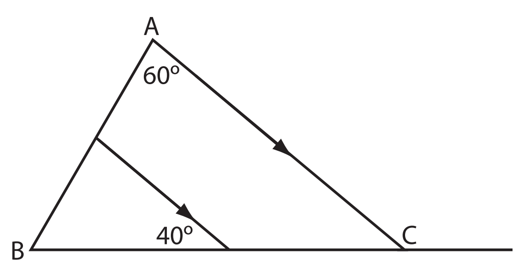
My Notes

My Notes

3. If Line 1 is parallel to Line 2, find a. What angle relationship did you use?

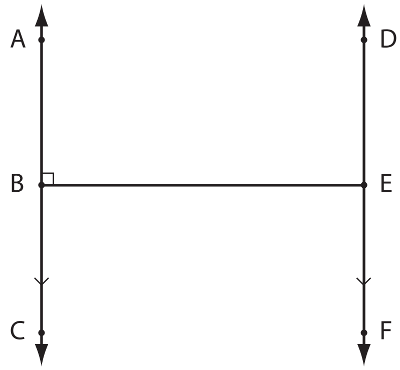


4. Find the measure of ∠B.



My Notes

5. Have a look at the diagram below. Explain why $\overrightarrow{DF} \perp \overline{BE}$.

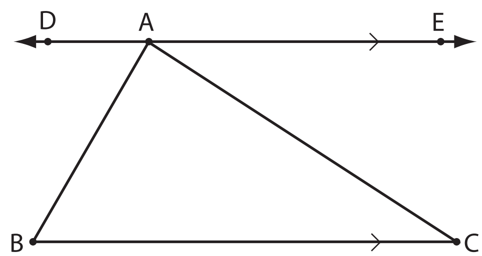


Turn to the solutions at the end of the section and mark your work.

My Notes

Activity 3
Mastering Concepts

A line, $\overleftrightarrow{DE}$ is drawn through point A, parallel to the base of ΔABC This is shown in the diagram below. Without using a protractor, prove that the three angles of ΔABC add up to 180º.



Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes

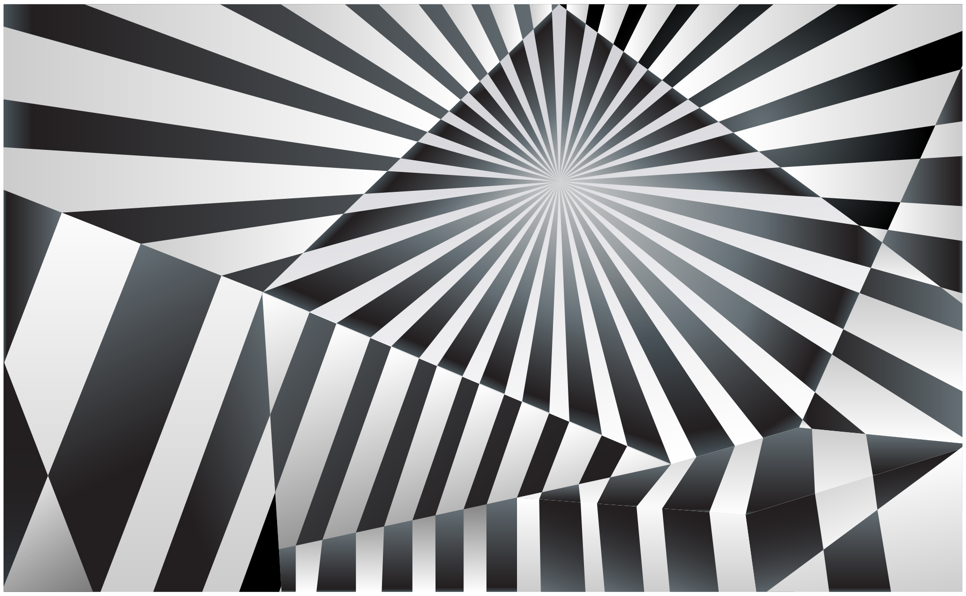


Photo by Amid © 2010

In this image, the abstract background of light and dark incorporates parallel and convergent lines. Can you identify lines and angles formed by transversals?

My Notes

Did You Know?

At the turn of the Twentieth Century, there were movements away from painting pictures which, some argued, could just as well be taken with a camera. One of those movements was cubism. Cubist paintings are not from one perspective. They show objects broken up and re-configured in an abstract form. Often, the surfaces and parts of objects intersect at a variety of angles. You can find out more about cubism through an Internet search.

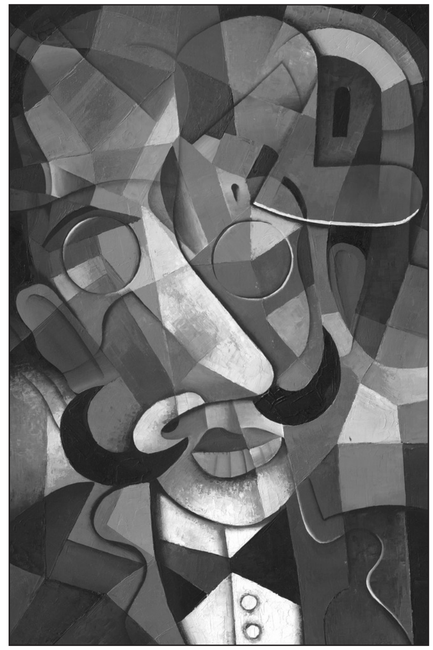


Photo by Eugene Ivanov © 2010

Try typing *cubism*, *modern art*, or *Picasso* into an Internet search engine like Google or Yahoo.

In this lesson you practised your skills at solving problems that involve angles created by the intersection of transversals with parallel lines. These problems involved vertically opposite angles, adjacent and supplementary angles, corresponding angles, alternate angles, and co-interior and co-exterior angles.

Angles—Appendix

TABLE OF CONVERSIONS

1 inch	≈	2.54 centimetres
1 foot	≈	30.5 centimetres
1 foot	≈	0.305 metres
1 foot	=	12 inches
1 yard	=	3 feet
1 yard	≈	0.915 metres
1 mile	=	1760 yards
1 mile	≈	1.6 kilometres
1 kilogram	≈	2.2 pounds
1 litre	≈	1.06 US quarts
1 litre	≈	0.26 US gallons
1 gallon	≈	4 quarts
1 British gallon	≈	$\frac{6}{5}$ US gallon

FORMULAE

Temperature

$$C = \frac{5}{9}(F - 32)$$

Trigonometry

(Put your calculator in Degree Mode)

- Right triangles

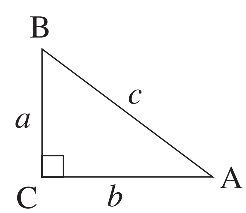
Pythagorean Theorem

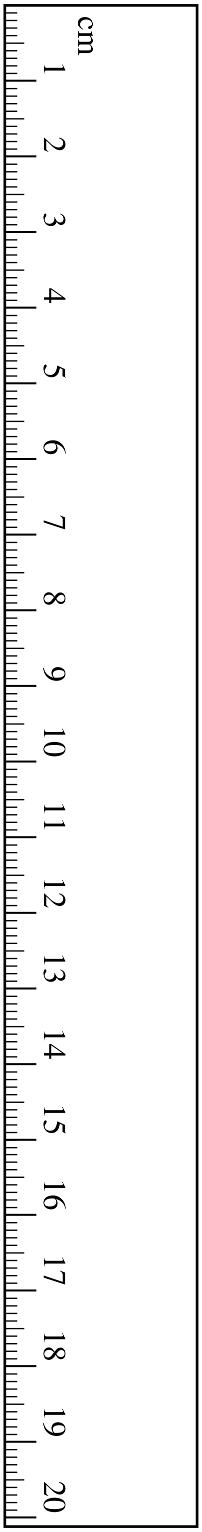
$$a^2 + b^2 = c^2$$

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

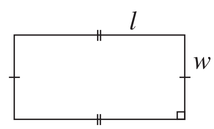
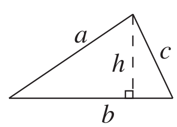
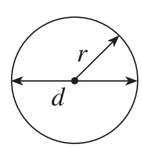
$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$



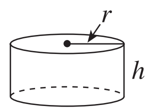
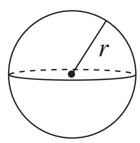
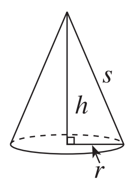
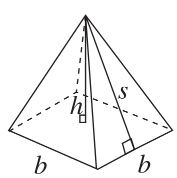
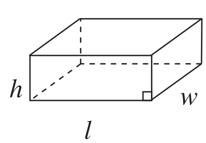


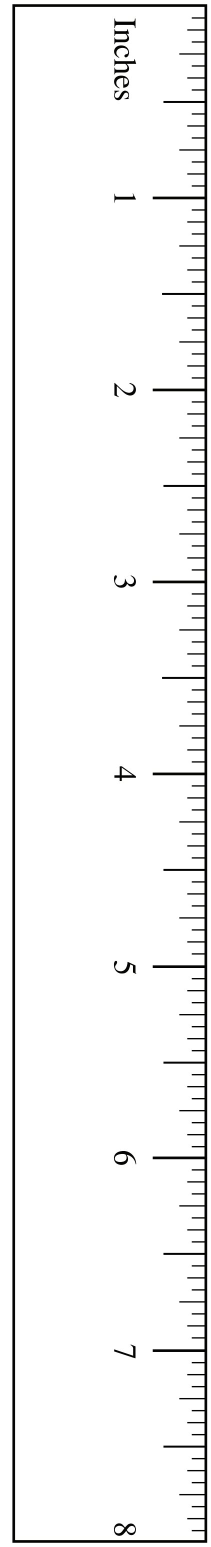
GEOMETRIC FORMULAE

Key Legend	
l = length	P = perimeter
w = width	C = circumference
b = base	A = area
h = height	SA = surface area
s = slant height	V = volume
r = radius	
d = diameter	

Geometric Figure	Perimeter	Area
Rectangle 	$P = 2l + 2w$ **or** $P = 2(l + w)$	$A = lw$
Triangle 	$P = a + b + c$	$A = \frac{bh}{2}$
Circle 	$C = \pi d$ **or** $C = 2\pi r$	$A = \pi r^2$

Note: Use the value of **π** programmed in your calculator rather than the approximation of 3.14.

Geometric Figure	Surface Area
Cylinder 	$A_{top} = \pi r^2$ $A_{base} = \pi r^2$ $A_{side} = 2\pi rh$ $SA = 2\pi r^2 + 2\pi rh$
Sphere 	$SA = 4\pi r^2$ **or** $SA = \pi d^2$
Cone 	$A_{side} = \pi rs$ $A_{base} = \pi r^2$ $SA = \pi r^2 + \pi rs$
Square-Based Pyramid 	$A_{triangle} = \frac{1}{2}bs$ (for each triangle) $A_{base} = b^2$ $SA = 2bs + b^2$
Rectangular Prism 	$SA = wh + wh + lw + lw + lh + lh$ **or** $SA = 2(wh + lw + lh)$
General Right Prism	$SA =$ the sum of the areas of all the faces
General Pyramid	$SA =$ the sum of the areas of all the faces



Note: Use the value of π programmed in your calculator rather than the approximation of 3.14.

Canada Pension Plan Contributions | **Cotisations au Régime de pensions du Canada**

Weekly (52 pay periods a year) | Hebdomadaire (52 périodes de paie par année)

Pay Rémunération From - De	To - À	CPP RPC	Pay Rémunération From - De	To - À	CPP RPC	Pay Rémunération From - De	To - À	CPP RPC	Pay Rémunération From - De	To - À	CPP RPC
358.11 -	358.31	14.40	372.66 -	372.85	15.12	387.20 -	387.40	15.84	401.75 -	401.94	16.56
358.32 -	358.51	14.41	372.86 -	373.05	15.13	387.41 -	387.60	15.85	401.95 -	402.14	16.57
358.52 -	358.71	14.42	373.06 -	373.25	15.14	387.61 -	387.80	15.86	402.15 -	402.35	16.58
358.72 -	358.91	14.43	373.26 -	373.46	15.15	387.81 -	388.00	15.87	402.36 -	402.55	16.59
358.92 -	359.11	14.44	373.47 -	373.66	15.16	388.01 -	388.20	15.88	402.56 -	402.75	16.60
359.12 -	359.32	14.45	373.67 -	373.86	15.17	388.21 -	388.41	15.89	402.76 -	402.95	16.61
359.33 -	359.52	14.46	373.87 -	374.06	15.18	388.42 -	388.61	15.90	402.96 -	403.15	16.62
359.53 -	359.72	14.47	374.07 -	374.26	15.19	388.62 -	388.81	15.91	403.16 -	403.36	16.63
359.73 -	359.92	14.48	374.27 -	374.47	15.20	388.82 -	389.01	15.92	403.37 -	403.56	16.64
359.93 -	360.12	14.49	374.48 -	374.67	15.21	389.02 -	389.21	15.93	403.57 -	403.76	16.65
360.13 -	360.33	14.50	374.68 -	374.87	15.22	389.22 -	389.42	15.94	403.77 -	403.96	16.66
360.34 -	360.53	14.51	374.88 -	375.07	15.23	389.43 -	389.62	15.95	403.97 -	404.16	16.67
360.54 -	360.73	14.52	375.08 -	375.27	15.24	389.63 -	389.82	15.96	404.17 -	404.37	16.68
360.74 -	360.93	14.53	375.28 -	375.48	15.25	389.83 -	390.02	15.97	404.38 -	404.57	16.69
360.94 -	361.13	14.54	375.49 -	375.68	15.26	390.03 -	390.22	15.98	404.58 -	404.77	16.70
361.14 -	361.34	14.55	375.69 -	375.88	15.27	390.23 -	390.43	15.99	404.78 -	404.97	16.71
361.35 -	361.54	14.56	375.89 -	376.08	15.28	390.44 -	390.63	16.00	404.98 -	405.17	16.72
361.55 -	361.74	14.57	376.09 -	376.28	15.29	390.64 -	390.83	16.01	405.18 -	405.38	16.73
361.75 -	361.94	14.58	376.29 -	376.49	15.30	390.84 -	391.03	16.02	405.39 -	405.58	16.74
361.95 -	362.14	14.59	376.50 -	376.69	15.31	391.04 -	391.23	16.03	405.59 -	405.78	16.75
362.15 -	362.35	14.60	376.70 -	376.89	15.32	391.24 -	391.44	16.04	405.79 -	405.98	16.76
362.36 -	362.55	14.61	376.90 -	377.09	15.33	391.45 -	391.64	16.05	405.99 -	406.18	16.77
362.56 -	362.75	14.62	377.10 -	377.29	15.34	391.65 -	391.84	16.06	406.19 -	406.39	16.78
362.76 -	362.95	14.63	377.30 -	377.50	15.35	391.85 -	392.04	16.07	406.40 -	406.59	16.79
362.96 -	363.15	14.64	377.51 -	377.70	15.36	392.05 -	392.24	16.08	406.60 -	406.79	16.80
363.16 -	363.36	14.65	377.71 -	377.90	15.37	392.25 -	392.45	16.09	406.80 -	406.99	16.81
363.37 -	363.56	14.66	377.91 -	378.10	15.38	392.46 -	392.65	16.10	407.00 -	407.19	16.82
363.57 -	363.76	14.67	378.11 -	378.31	15.39	392.66 -	392.85	16.11	407.20 -	407.40	16.83
363.77 -	363.96	14.68	378.32 -	378.51	15.40	392.86 -	393.05	16.12	407.41 -	407.60	16.84
363.97 -	364.16	14.69	378.52 -	378.71	15.41	393.06 -	393.25	16.13	407.61 -	407.80	16.85
364.17 -	364.37	14.70	378.72 -	378.91	15.42	393.26 -	393.46	16.14	407.81 -	408.00	16.86
364.38 -	364.57	14.71	378.92 -	379.11	15.43	393.47 -	393.66	16.15	408.01 -	408.20	16.87
364.58 -	364.77	14.72	379.12 -	379.32	15.44	393.67 -	393.86	16.16	408.21 -	408.41	16.88
364.78 -	364.97	14.73	379.33 -	379.52	15.45	393.87 -	394.06	16.17	408.42 -	408.61	16.89
364.98 -	365.17	14.74	379.53 -	379.72	15.46	394.07 -	394.26	16.18	408.62 -	408.81	16.90
365.18 -	365.38	14.75	379.73 -	379.92	15.47	394.27 -	394.47	16.19	408.82 -	409.01	16.91
365.39 -	365.58	14.76	379.93 -	380.12	15.48	394.48 -	394.67	16.20	409.02 -	409.21	16.92
365.59 -	365.78	14.77	380.13 -	380.33	15.49	394.68 -	394.87	16.21	409.22 -	409.42	16.93
365.79 -	365.98	14.78	380.34 -	380.53	15.50	394.88 -	395.07	16.22	409.43 -	409.62	16.94
365.99 -	366.18	14.79	380.54 -	380.73	15.51	395.08 -	395.27	16.23	409.63 -	409.82	16.95
366.19 -	366.39	14.80	380.74 -	380.93	15.52	395.28 -	395.48	16.24	409.83 -	410.02	16.96
366.40 -	366.59	14.81	380.94 -	381.13	15.53	395.49 -	395.68	16.25	410.03 -	410.22	16.97
366.60 -	366.79	14.82	381.14 -	381.34	15.54	395.69 -	395.88	16.26	410.23 -	410.43	16.98
366.80 -	366.99	14.83	381.35 -	381.54	15.55	395.89 -	396.08	16.27	410.44 -	410.63	16.99
367.00 -	367.19	14.84	381.55 -	381.74	15.56	396.09 -	396.28	16.28	410.64 -	410.83	17.00
367.20 -	367.40	14.85	381.75 -	381.94	15.57	396.29 -	396.49	16.29	410.84 -	411.03	17.01
367.41 -	367.60	14.86	381.95 -	382.14	15.58	396.50 -	396.69	16.30	411.04 -	411.23	17.02
367.61 -	367.80	14.87	382.15 -	382.35	15.59	396.70 -	396.89	16.31	411.24 -	411.44	17.03
367.81 -	368.00	14.88	382.36 -	382.55	15.60	396.90 -	397.09	16.32	411.45 -	411.64	17.04
368.01 -	368.20	14.89	382.56 -	382.75	15.61	397.10 -	397.29	16.33	411.65 -	411.84	17.05
368.21 -	368.41	14.90	382.76 -	382.95	15.62	397.30 -	397.50	16.34	411.85 -	412.04	17.06
368.42 -	368.61	14.91	382.96 -	383.15	15.63	397.51 -	397.70	16.35	412.05 -	412.24	17.07
368.62 -	368.81	14.92	383.16 -	383.36	15.64	397.71 -	397.90	16.36	412.25 -	412.45	17.08
368.82 -	369.01	14.93	383.37 -	383.56	15.65	397.91 -	398.10	16.37	412.46 -	412.65	17.09
369.02 -	369.21	14.94	383.57 -	383.76	15.66	398.11 -	398.31	16.38	412.66 -	412.85	17.10
369.22 -	369.42	14.95	383.77 -	383.96	15.67	398.32 -	398.51	16.39	412.86 -	413.05	17.11
369.43 -	369.62	14.96	383.97 -	384.16	15.68	398.52 -	398.71	16.40	413.06 -	413.25	17.12
369.63 -	369.82	14.97	384.17 -	384.37	15.69	398.72 -	398.91	16.41	413.26 -	413.46	17.13
369.83 -	370.02	14.98	384.38 -	384.57	15.70	398.92 -	399.11	16.42	413.47 -	413.66	17.14
370.03 -	370.22	14.99	384.58 -	384.77	15.71	399.12 -	399.32	16.43	413.67 -	413.86	17.15
370.23 -	370.43	15.00	384.78 -	384.97	15.72	399.33 -	399.52	16.44	413.87 -	414.06	17.16
370.44 -	370.63	15.01	384.98 -	385.17	15.73	399.53 -	399.72	16.45	414.07 -	414.26	17.17
370.64 -	370.83	15.02	385.18 -	385.38	15.74	399.73 -	399.92	16.46	414.27 -	414.47	17.18
370.84 -	371.03	15.03	385.39 -	385.58	15.75	399.93 -	400.12	16.47	414.48 -	414.67	17.19
371.04 -	371.23	15.04	385.59 -	385.78	15.76	400.13 -	400.33	16.48	414.68 -	414.87	17.20
371.24 -	371.44	15.05	385.79 -	385.98	15.77	400.34 -	400.53	16.49	414.88 -	415.07	17.21
371.45 -	371.64	15.06	385.99 -	386.18	15.78	400.54 -	400.73	16.50	415.08 -	415.27	17.22
371.65 -	371.84	15.07	386.19 -	386.39	15.79	400.74 -	400.93	16.51	415.28 -	415.48	17.23
371.85 -	372.04	15.08	386.40 -	386.59	15.80	400.94 -	401.13	16.52	415.49 -	415.68	17.24
372.05 -	372.24	15.09	386.60 -	386.79	15.81	401.14 -	401.34	16.53	415.69 -	415.88	17.25
372.25 -	372.45	15.10	386.80 -	386.99	15.82	401.35 -	401.54	16.54	415.89 -	416.08	17.26
372.46 -	372.65	15.11	387.00 -	387.19	15.83	401.55 -	401.74	16.55	416.09 -	416.28	17.27

Employee's maximum CPP contribution for the year 2009 is $2,118.60 B-6 La cotisation maximale de l'employé au RPC pour l'année 2009 est de 2 118,60 $

Employment Insurance Premiums — Cotisations à l'assurance-emploi

Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE
333.24	333.81	5.77	374.86	375.43	6.49	416.48	417.05	7.21	458.10	458.67	7.93
333.82	334.39	5.78	375.44	376.01	6.50	417.06	417.63	7.22	458.68	459.24	7.94
334.40	334.97	5.79	376.02	376.58	6.51	417.64	418.20	7.23	459.25	459.82	7.95
334.98	335.54	5.80	376.59	377.16	6.52	418.21	418.78	7.24	459.83	460.40	7.96
335.55	336.12	5.81	377.17	377.74	6.53	418.79	419.36	7.25	460.41	460.98	7.97
336.13	336.70	5.82	377.75	378.32	6.54	419.37	419.94	7.26	460.99	461.56	7.98
336.71	337.28	5.83	378.33	378.90	6.55	419.95	420.52	7.27	461.57	462.13	7.99
337.29	337.86	5.84	378.91	379.47	6.56	420.53	421.09	7.28	462.14	462.71	8.00
337.87	338.43	5.85	379.48	380.05	6.57	421.10	421.67	7.29	462.72	463.29	8.01
338.44	339.01	5.86	380.06	380.63	6.58	421.68	422.25	7.30	463.30	463.87	8.02
339.02	339.59	5.87	380.64	381.21	6.59	422.26	422.83	7.31	463.88	464.45	8.03
339.60	340.17	5.88	381.22	381.79	6.60	422.84	423.41	7.32	464.46	465.02	8.04
340.18	340.75	5.89	381.80	382.36	6.61	423.42	423.98	7.33	465.03	465.60	8.05
340.76	341.32	5.90	382.37	382.94	6.62	423.99	424.56	7.34	465.61	466.18	8.06
341.33	341.90	5.91	382.95	383.52	6.63	424.57	425.14	7.35	466.19	466.76	8.07
341.91	342.48	5.92	383.53	384.10	6.64	425.15	425.72	7.36	466.77	467.34	8.08
342.49	343.06	5.93	384.11	384.68	6.65	425.73	426.30	7.37	467.35	467.91	8.09
343.07	343.64	5.94	384.69	385.26	6.66	426.31	426.87	7.38	467.92	468.49	8.10
343.65	344.21	5.95	385.27	385.83	6.67	426.88	427.45	7.39	468.50	469.07	8.11
344.22	344.79	5.96	385.84	386.41	6.68	427.46	428.03	7.40	469.08	469.65	8.12
344.80	345.37	5.97	386.42	386.99	6.69	428.04	428.61	7.41	469.66	470.23	8.13
345.38	345.95	5.98	387.00	387.57	6.70	428.62	429.19	7.42	470.24	470.80	8.14
345.96	346.53	5.99	387.58	388.15	6.71	429.20	429.76	7.43	470.81	471.38	8.15
346.54	347.10	6.00	388.16	388.72	6.72	429.77	430.34	7.44	471.39	471.96	8.16
347.11	347.68	6.01	388.73	389.30	6.73	430.35	430.92	7.45	471.97	472.54	8.17
347.69	348.26	6.02	389.31	389.88	6.74	430.93	431.50	7.46	472.55	473.12	8.18
348.27	348.84	6.03	389.89	390.46	6.75	431.51	432.08	7.47	473.13	473.69	8.19
348.85	349.42	6.04	390.47	391.04	6.76	432.09	432.65	7.48	473.70	474.27	8.20
349.43	349.99	6.05	391.05	391.61	6.77	432.66	433.23	7.49	474.28	474.85	8.21
350.00	350.57	6.06	391.62	392.19	6.78	433.24	433.81	7.50	474.86	475.43	8.22
350.58	351.15	6.07	392.20	392.77	6.79	433.82	434.39	7.51	475.44	476.01	8.23
351.16	351.73	6.08	392.78	393.35	6.80	434.40	434.97	7.52	476.02	476.58	8.24
351.74	352.31	6.09	393.36	393.93	6.81	434.98	435.54	7.53	476.59	477.16	8.25
352.32	352.89	6.10	393.94	394.50	6.82	435.55	436.12	7.54	477.17	477.74	8.26
352.90	353.46	6.11	394.51	395.08	6.83	436.13	436.70	7.55	477.75	478.32	8.27
353.47	354.04	6.12	395.09	395.66	6.84	436.71	437.28	7.56	478.33	478.90	8.28
354.05	354.62	6.13	395.67	396.24	6.85	437.29	437.86	7.57	478.91	479.47	8.29
354.63	355.20	6.14	396.25	396.82	6.86	437.87	438.43	7.58	479.48	480.05	8.30
355.21	355.78	6.15	396.83	397.39	6.87	438.44	439.01	7.59	480.06	480.63	8.31
355.79	356.35	6.16	397.40	397.97	6.88	439.02	439.59	7.60	480.64	481.21	8.32
356.36	356.93	6.17	397.98	398.55	6.89	439.60	440.17	7.61	481.22	481.79	8.33
356.94	357.51	6.18	398.56	399.13	6.90	440.18	440.75	7.62	481.80	482.36	8.34
357.52	358.09	6.19	399.14	399.71	6.91	440.76	441.32	7.63	482.37	482.94	8.35
358.10	358.67	6.20	399.72	400.28	6.92	441.33	441.90	7.64	482.95	483.52	8.36
358.68	359.24	6.21	400.29	400.86	6.93	441.91	442.48	7.65	483.53	484.10	8.37
359.25	359.82	6.22	400.87	401.44	6.94	442.49	443.06	7.66	484.11	484.68	8.38
359.83	360.40	6.23	401.45	402.02	6.95	443.07	443.64	7.67	484.69	485.26	8.39
360.41	360.98	6.24	402.03	402.60	6.96	443.65	444.21	7.68	485.27	485.83	8.40
360.99	361.56	6.25	402.61	403.17	6.97	444.22	444.79	7.69	485.84	486.41	8.41
361.57	362.13	6.26	403.18	403.75	6.98	444.80	445.37	7.70	486.42	486.99	8.42
362.14	362.71	6.27	403.76	404.33	6.99	445.38	445.95	7.71	487.00	487.57	8.43
362.72	363.29	6.28	404.34	404.91	7.00	445.96	446.53	7.72	487.58	488.15	8.44
363.30	363.87	6.29	404.92	405.49	7.01	446.54	447.10	7.73	488.16	488.72	8.45
363.88	364.45	6.30	405.50	406.06	7.02	447.11	447.68	7.74	488.73	489.30	8.46
364.46	365.02	6.31	406.07	406.64	7.03	447.69	448.26	7.75	489.31	489.88	8.47
365.03	365.60	6.32	406.65	407.22	7.04	448.27	448.84	7.76	489.89	490.46	8.48
365.61	366.18	6.33	407.23	407.80	7.05	448.85	449.42	7.77	490.47	491.04	8.49
366.19	366.76	6.34	407.81	408.38	7.06	449.43	449.99	7.78	491.05	491.61	8.50
366.77	367.34	6.35	408.39	408.95	7.07	450.00	450.57	7.79	491.62	492.19	8.51
367.35	367.91	6.36	408.96	409.53	7.08	450.58	451.15	7.80	492.20	492.77	8.52
367.92	368.49	6.37	409.54	410.11	7.09	451.16	451.73	7.81	492.78	493.35	8.53
368.50	369.07	6.38	410.12	410.69	7.10	451.74	452.31	7.82	493.36	493.93	8.54
369.08	369.65	6.39	410.70	411.27	7.11	452.32	452.89	7.83	493.94	494.50	8.55
369.66	370.23	6.40	411.28	411.84	7.12	452.90	453.46	7.84	494.51	495.08	8.56
370.24	370.80	6.41	411.85	412.42	7.13	453.47	454.04	7.85	495.09	495.66	8.57
370.81	371.38	6.42	412.43	413.00	7.14	454.05	454.62	7.86	495.67	496.24	8.58
371.39	371.96	6.43	413.01	413.58	7.15	454.63	455.20	7.87	496.25	496.82	8.59
371.97	372.54	6.44	413.59	414.16	7.16	455.21	455.78	7.88	496.83	497.39	8.60
372.55	373.12	6.45	414.17	414.73	7.17	455.79	456.35	7.89	497.40	497.97	8.61
373.13	373.69	6.46	414.74	415.31	7.18	456.36	456.93	7.90	497.98	498.55	8.62
373.70	374.27	6.47	415.32	415.89	7.19	456.94	457.51	7.91	498.56	499.13	8.63
374.28	374.85	6.48	415.90	416.47	7.20	457.52	458.09	7.92	499.14	499.71	8.64

Yearly maximum insurable earnings are $42,300
Yearly maximum employee premiums are $731.79
The premium rate for 2009 is 1.73 %

Le maximum annuel de la rémunération assurable est de 42 300 $
La cotisation maximale annuelle de l'employé est de 731,79 $
Le taux de cotisation pour 2009 est de 1,73 %

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the provincial table

Retenues d'impôt fédéral
En vigueur le 1er janvier 2009
Hebdomadaire (52 périodes de paie par année)
Cherchez aussi les retenues d'impôt dans la table provinciale

Pay / Rémunération		Federal claim codes/Codes de demande fédéraux										
		0	1	2	3	4	5	6	7	8	9	10
From / De	Less than / Moins de	Deduct from each pay / Retenez sur chaque paie										
335	339	44.65	15.55	12.70	7.00	1.30						
339	343	45.20	16.10	13.25	7.55	1.85						
343	347	45.80	16.65	13.80	8.10	2.45						
347	351	46.35	17.20	14.35	8.65	3.00						
351	355	46.90	17.75	14.90	9.25	3.55						
355	359	47.45	18.35	15.50	9.80	4.10						
359	363	48.00	18.90	16.05	10.35	4.65						
363	367	48.60	19.45	16.60	10.90	5.25						
367	371	49.15	20.00	17.15	11.45	5.80	.10					
371	375	49.70	20.55	17.70	12.05	6.35	.65					
375	379	50.25	21.15	18.30	12.60	6.90	1.20					
379	383	50.80	21.70	18.85	13.15	7.45	1.80					
383	387	51.40	22.25	19.40	13.70	8.00	2.35					
387	391	51.95	22.80	19.95	14.25	8.60	2.90					
391	395	52.50	23.35	20.50	14.85	9.15	3.45					
395	399	53.05	23.95	21.10	15.40	9.70	4.00					
399	403	53.60	24.50	21.65	15.95	10.25	4.60					
403	407	54.20	25.05	22.20	16.50	10.80	5.15					
407	411	54.75	25.60	22.75	17.05	11.40	5.70					
411	415	55.30	26.15	23.30	17.65	11.95	6.25	.55				
415	419	55.85	26.75	23.90	18.20	12.50	6.80	1.15				
419	423	56.40	27.30	24.45	18.75	13.05	7.40	1.70				
423	427	57.00	27.85	25.00	19.30	13.60	7.95	2.25				
427	431	57.55	28.40	25.55	19.85	14.20	8.50	2.80				
431	435	58.10	28.95	26.10	20.45	14.75	9.05	3.35				
435	439	58.65	29.50	26.70	21.00	15.30	9.60	3.95				
439	443	59.20	30.10	27.25	21.55	15.85	10.20	4.50				
443	447	59.80	30.65	27.80	22.10	16.40	10.75	5.05				
447	451	60.35	31.20	28.35	22.65	17.00	11.30	5.60				
451	455	60.90	31.75	28.90	23.25	17.55	11.85	6.15	.50			
455	459	61.45	32.30	29.50	23.80	18.10	12.40	6.75	1.05			
459	463	62.00	32.90	30.05	24.35	18.65	12.95	7.30	1.60			
463	467	62.60	33.45	30.60	24.90	19.20	13.55	7.85	2.15			
467	471	63.15	34.00	31.15	25.45	19.80	14.10	8.40	2.70			
471	475	63.70	34.55	31.70	26.05	20.35	14.65	8.95	3.30			
475	479	64.25	35.10	32.30	26.60	20.90	15.20	9.55	3.85			
479	483	64.80	35.70	32.85	27.15	21.45	15.75	10.10	4.40			
483	487	65.40	36.25	33.40	27.70	22.00	16.35	10.65	4.95			
487	491	65.95	36.80	33.95	28.25	22.60	16.90	11.20	5.50			
491	495	66.50	37.35	34.50	28.85	23.15	17.45	11.75	6.10	.40		
495	499	67.05	37.90	35.10	29.40	23.70	18.00	12.35	6.65	.95		
499	503	67.60	38.50	35.65	29.95	24.25	18.55	12.90	7.20	1.50		
503	507	68.20	39.05	36.20	30.50	24.80	19.15	13.45	7.75	2.05		
507	511	68.75	39.60	36.75	31.05	25.40	19.70	14.00	8.30	2.65		
511	515	69.30	40.15	37.30	31.65	25.95	20.25	14.55	8.90	3.20		
515	519	69.85	40.70	37.90	32.20	26.50	20.80	15.15	9.45	3.75		
519	523	70.40	41.30	38.45	32.75	27.05	21.35	15.70	10.00	4.30		
523	527	71.00	41.85	39.00	33.30	27.60	21.95	16.25	10.55	4.85		
527	531	71.55	42.40	39.55	33.85	28.20	22.50	16.80	11.10	5.45		
531	535	72.10	42.95	40.10	34.45	28.75	23.05	17.35	11.70	6.00	.30	
535	539	72.65	43.50	40.70	35.00	29.30	23.60	17.90	12.25	6.55	.85	
539	543	73.20	44.10	41.25	35.55	29.85	24.15	18.50	12.80	7.10	1.40	
543	547	73.80	44.65	41.80	36.10	30.40	24.75	19.05	13.35	7.65	2.00	
547	551	74.35	45.20	42.35	36.65	31.00	25.30	19.60	13.90	8.25	2.55	
551	555	74.90	45.75	42.90	37.25	31.55	25.85	20.15	14.50	8.80	3.10	

British Columbia provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the federal table

Retenues d'impôt provincial de la Colombie-Britannique
En vigueur le 1er janvier 2009
Hebdomadaire (52 périodes de paie par année)
Cherchez aussi les retenues d'impôt dans la table fédérale

Pay / Rémunération		Provincial claim codes/Codes de demande provinciaux										
		0	1	2	3	4	5	6	7	8	9	10
From / De	Less than / Moins de	Deduct from each pay / Retenez sur chaque paie										
	343	*	.00									
343 -	345	9.30	.20									
345 -	347	9.45	.35									
347 -	349	9.60	.50									
349 -	351	9.80	.65									
351 -	353	9.95	.80									
353 -	355	10.10	.95									
355 -	357	10.25	1.15	.10								
357 -	359	10.40	1.30	.25								
359 -	361	10.55	1.45	.40								
361 -	363	10.75	1.60	.60								
363 -	365	10.90	1.75	.75								
365 -	367	11.05	1.90	.90								
367 -	369	11.20	2.10	1.05								
369 -	371	11.35	2.25	1.20								
371 -	373	11.50	2.40	1.35								
373 -	375	11.70	2.55	1.55								
375 -	377	11.85	2.70	1.70								
377 -	379	12.00	2.90	1.85								
379 -	381	12.15	3.05	2.00								
381 -	383	12.30	3.20	2.15	.10							
383 -	385	12.45	3.35	2.30	.25							
385 -	387	12.65	3.50	2.50	.45							
387 -	389	12.80	3.65	2.65	.60							
389 -	391	12.95	3.85	2.80	.75							
391 -	393	13.10	4.00	2.95	.90							
393 -	395	13.25	4.15	3.10	1.05							
395 -	397	13.40	4.30	3.30	1.20							
397 -	399	13.60	4.45	3.45	1.40							
399 -	401	13.75	4.60	3.60	1.55							
401 -	403	13.90	4.80	3.75	1.70							
403 -	405	14.05	4.95	3.90	1.85							
405 -	407	14.20	5.10	4.05	2.00							
407 -	409	14.35	5.25	4.25	2.15	.10						
409 -	411	14.55	5.40	4.40	2.35	.30						
411 -	413	14.70	5.55	4.55	2.50	.45						
413 -	415	14.85	5.75	4.70	2.65	.60						
415 -	417	15.00	5.90	4.85	2.80	.75						
417 -	419	15.15	6.05	5.00	2.95	.90						
419 -	421	15.30	6.20	5.20	3.10	1.05						
421 -	423	15.50	6.35	5.35	3.30	1.25						
423 -	425	15.65	6.50	5.50	3.45	1.40						
425 -	427	15.80	6.70	5.65	3.60	1.55						
427 -	429	15.95	6.85	5.80	3.75	1.70						
429 -	431	16.10	7.00	5.95	3.90	1.85						
431 -	433	16.25	7.15	6.15	4.10	2.00						
433 -	435	16.45	7.30	6.30	4.25	2.20	.15					
435 -	437	16.60	7.45	6.45	4.40	2.35	.30					
437 -	439	16.75	7.65	6.60	4.55	2.50	.45					
439 -	441	16.90	7.80	6.75	4.70	2.65	.60					
441 -	443	17.05	7.95	6.90	4.85	2.80	.75					
443 -	445	17.20	8.10	7.10	5.05	2.95	.90					
445 -	447	17.40	8.25	7.25	5.20	3.15	1.10					
447 -	449	17.55	8.40	7.40	5.35	3.30	1.25					
449 -	451	17.70	8.60	7.55	5.50	3.45	1.40					

*You normally use claim code "0" only for non-resident employees. However, if you have non-resident employees who earn less than the minimum amount shown in the "Pay" column, you may not be able to use these tables. Instead, refer to the "Step-by-step calculation of tax deductions" in Section "A" of this publication.

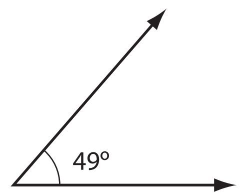
*Le code de demande «0» est normalement utilisé seulement pour les non-résidents. Cependant, si la rémunération de votre employé non résidant est inférieure au montant minimum indiqué dans la colonne «Rémunération», vous ne pourrez peut-être pas utiliser ces tables. Reportez-vous alors au «Calcul des retenues d'impôt, étape par étape» dans la section «A» de cette publication.

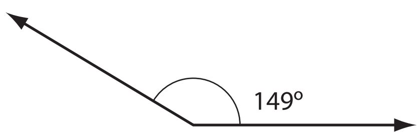
Solutions

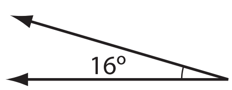
Lesson A: Sketching and Measuring Angles

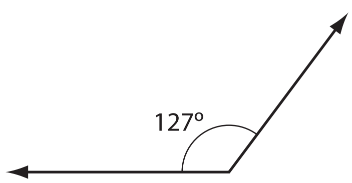
Lesson A: Activity 1: Self-Check

1. a. 63°
   b. 114°
   c. 82°
   d. 161°

2. a.


   b.


   c.


   d.


Lesson A: Activity 2: Try This

1. Each angle is one quarter of one full rotation (one full circle). There are 360º in one full rotation.

   ¼ × 360° = 90°

   One quarter of a full rotation is 90º.

2. By folding the paper again, you have divided the circle into eighths.

   $$\text{Each angle} = \frac{1}{8} \times 360^\circ$$
   $$= 45^\circ$$

   You could also think of this in the following way: You know that a quarter of a full rotation is 90°. By folding the quarter in half, you have divided the 90° angle in half. Half of 90° is 45°.

3. By folding the paper again, you have divided the circle into sixteenths.

   $$\text{Each angle} = \frac{1}{16} \times 360^\circ$$
   $$= 22.5^\circ$$

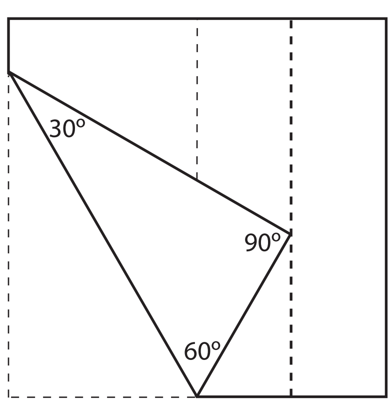
   You could also think of this in the following way: You know that an eighth of a full rotation is 45°. By folding the eighth in half, you have divided the 45° angle in half. Half of 45° is 22.5°.

Lesson A: Activity 3: Try This

1. Each narrow strip is ¼ of the entire sheet since these strips were obtained by folding in half twice.

   $$\begin{aligned} 1 \text{ page} \div 2 \div 2 &= 1 \div 2 \div 2 \\ &= 1 \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4} \end{aligned}$$

2. The angles are 90°, 60°, and 30° as shown in the picture below.



Lesson A: Activity 4: Self-Check

1. a. obtuse
   b. Estimates will vary. The angle is bigger than 90° by an amount close to the reference angle 22.5°, so the angle is close to 112.5°.
   c. 108°

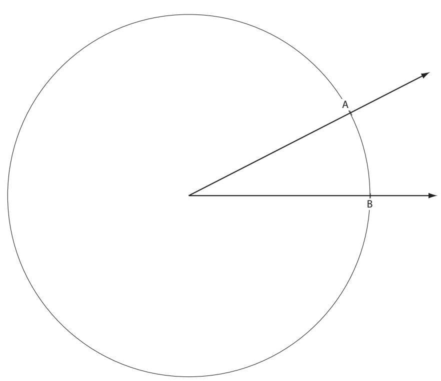
2. a. obtuse
   b. Estimates will vary. The angle is bigger than 90° by an amount close to the reference angle 45°. The angle is close to 135°.
   c. 130°

3. a. reflex
   b. Estimates will vary. The angle is bigger than 180° by at least 60°. Using the reference angles 60° and 22.5° you can see that the angle is close to 262.5°.
   c. 260°

4. a. acute
   b. Estimates will vary. The angle is close to 45°.
   c. 42°

Lesson A: Activity 5: Mastering Concepts

1.



Circumference ≈ 6 × radius
≈ 6 x 6 cm
≈ 36 cm

2. 36 × 10 = 360

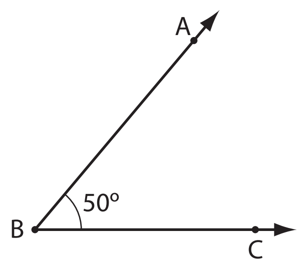
3. 360 is also the number of degrees in a circle. So, a portion of the circumference of a circle 6 cm in radius when multiplied by 10 is approximately the number of degrees in the angle at the centre of that circle.

Lesson B: Constructing Congruent Angles

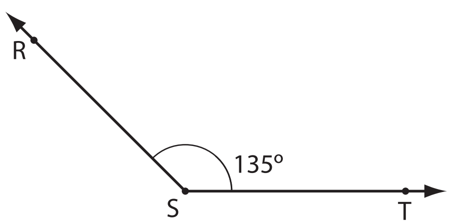
Lesson B: Activity 1: Self-Check

1. a. ∠Q or ∠PQR
   b. ∠U or ∠TUV
   c. There are four angles: ∠POL, ∠POM, ∠MOQ, and ∠QOL.

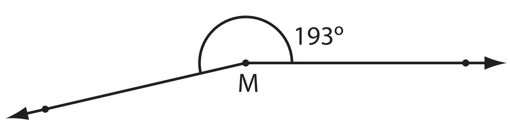
2. a.



   b.



   c.



Lesson B: Activity 2: Self-Check

1. ∠ABC ≅ ∠EBD ≅ ∠FGH = 160°
   ∠ABE ≅ ∠CBD = 120°
   ∠FGI ≅ ∠HGI = 100°

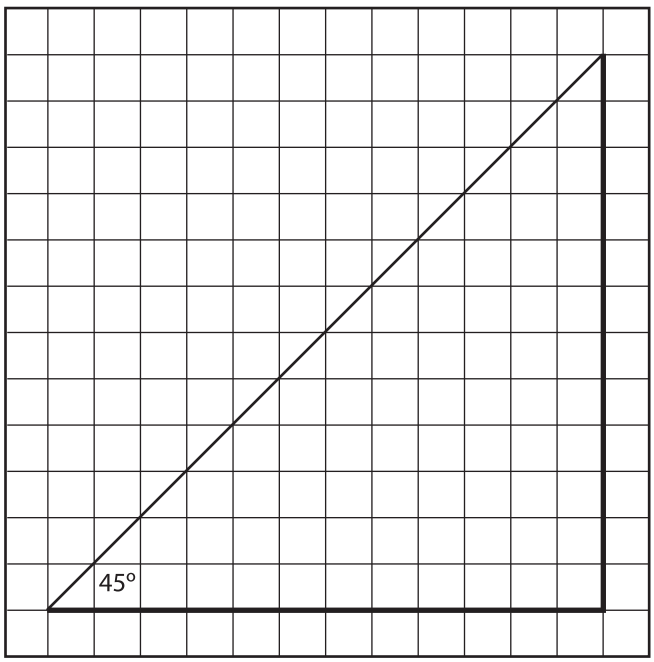
2. a. ∠ABC ≅ ∠ADC
      ∠BAD ≅ ∠BCD
   b. ∠ABC ≅ ∠BCD ≅ ∠CDE ≅ ∠DEF ≅ ∠EFA ≅ ∠FAB
   c. ∠CAB ≅ ∠CBA

Lesson B: Activity 3: Self-Check

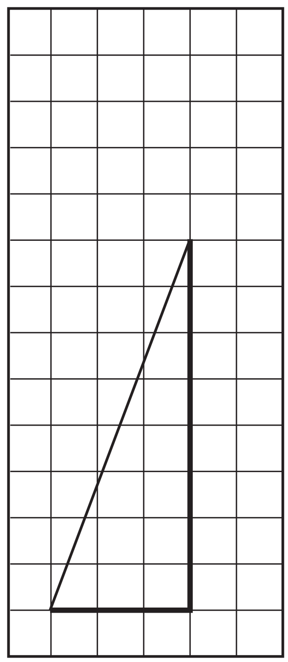
Answers will vary. Each angle you constructed should have the same measure as ∠ABC that you drew.

Lesson B: Activity 4: Self-Check

1. From the grid, the rise is 12 in for a run of 12 in. This combination yields a 45º pitch.



2. Answers will vary. A sample answer is given. Use 1 unit for every foot.



The measure of the angle is about 69.5º.

Lesson B: Activity 5: Mastering Concepts

c. ∠A is larger than ∠B

∠A = 34°

∠B = 30°

Note that the size of an angle does not depend on how long its arms are drawn. The size of an angle depends on how far one arm is rotated away from the other.

Lesson C: Bisecting Angles

Lesson C: Activity 1: Self-Check

1. The two angles are the same size.

2. The angle between the pencil and the mirror is half the size of the angle between the pencil and it’s image.

Lesson C: Activity 2: Try This

1. **Step 1**: Fold the sheet of paper so that the fold goes through the vertex of ∠ABC and ray BA falls on BC.

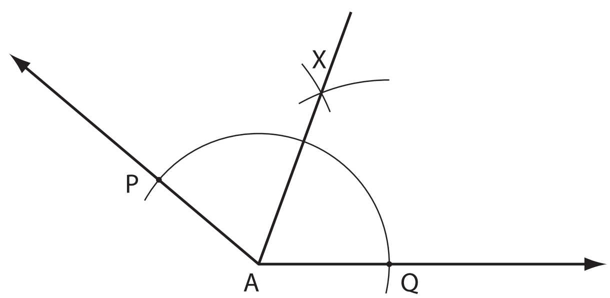
   Step 2: Unfold the sheet. Draw ray BD along the crease between the arms of the angle and. Ray BD is the bisector.

2. BD is the bisector of ∠ABC since BD divides ∠ABC into two congruent parts.

3. The measures of the angles will vary, but ∠ABD = ∠CBD = ½ ∠ABC.

Lesson C: Activity 3: Self-Check

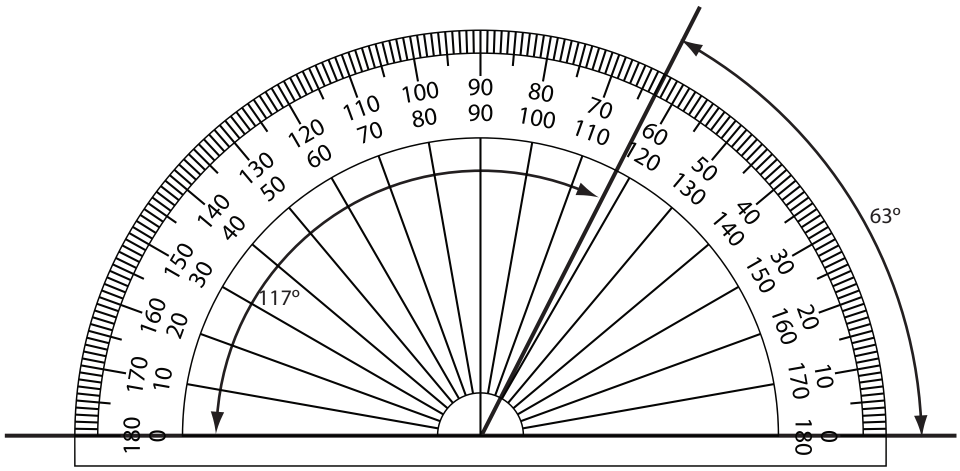
1. Answers will vary depending on the obtuse angle you drew. Refer back to the multimedia segment demonstrating the steps for bisecting an angle using a compass.



   If you have done your work carefully, ∠PAX ≅ ∠QAX; that is, both angles are equal in measure.

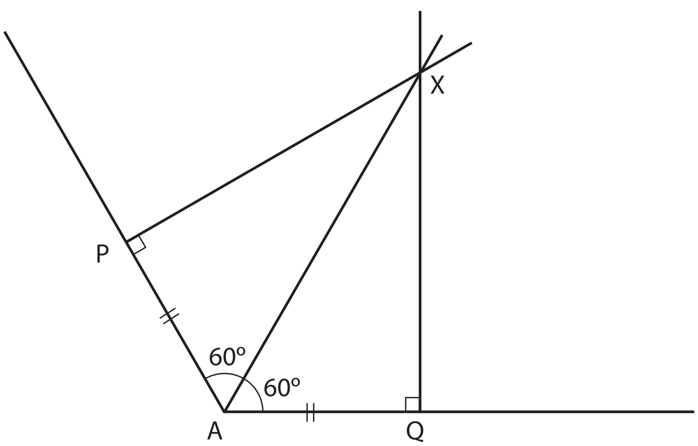
   And, ∠PAX = ∠QAX = ½ ∠PAQ.

2. The student likely used the wrong scale on the protractor when measuring half of the angle. The measure of each half of the angle should be 117º, (using the lower scale on the protractor pictured below), not 63º (the upper scale). ½ × 234º = 117º.



Lesson C: Activity 4: Self-Check

The construction is shown below.

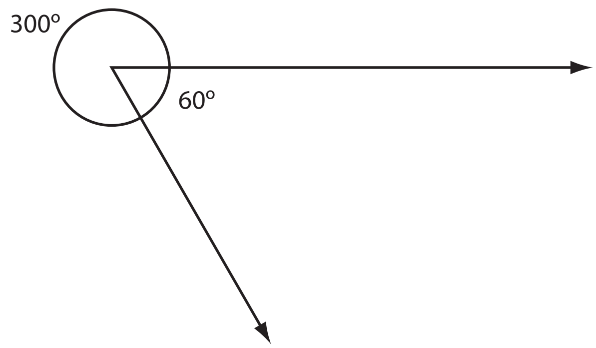


Each half of the original angle must measure 60°.

Lesson C: Activity 5: Mastering Concepts

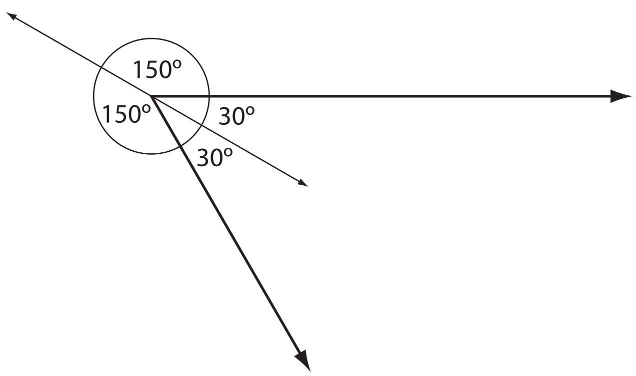
Zander is right!

Suppose Zander was asked to bisect the following 300° angle.



The 300° angle lies on the left and the 60° angle, which completes the full rotation, lies on the right. Remember, 1 full rotation = 360°, and here, 300° + 60° = 360°.

The bisector of the 60° angle divides it into two 30° angles. If you extend that bisector to the left, you will divide the 300° angle into two 150° angles. This is because 30° + 150° = 180°, and the bisector, when extended, forms straight angles.



Lesson D: Relationships Among Angles

Lesson D: Activity 1: Try This

1. A straight angle was formed. A straight angle is 180º. This is the value we got when we found the sum of the angles of the triangle.

2. The sum should be the same for any angle you draw. It was the same for the two that you drew, and you could have drawn a triangle of ANY shape and size!

Lesson D: Activity 2: Self-Check

The first pair is not adjacent because they do not share a vertex.

The second pair is not adjacent because they do not share a vertex or arm.

The third pair is not adjacent because they do not lie on opposite sides of a common arm.

Lesson D: Activity 3: Try This

1. $\angle 1 = 124^\circ$
   $\angle 2 = 56^\circ$
   $\angle 3 = 124^\circ$
   $\angle 4 = 56^\circ$
2. $\angle 1 + \angle 2 = 180^\circ$
   $\angle 1 + \angle 4 = 180^\circ$
   $\angle 3 + \angle 4 = 180^\circ$
   $\angle 2 + \angle 3 = 180^\circ$
3. The sums are all 180º.
4. a. They are the same size, but they are not adjacent.
   b. They are the same size, but they are not adjacent.

Lesson D: Activity 4: Self-Check

1. adjacent:
   ∠ABC & ∠CBE
   ∠CBE & ∠EBD
   ∠EBD & ∠DBA
   ∠DBA & ∠ABC
2. complementary:
   ∠PQR & ∠STU
3. supplementary:
   ∠ABC & ∠CBE
   ∠CBE & ∠EBD
   ∠EBD & ∠DBA
   ∠DBA & ∠ABC
   ∠IJK & ∠LMN
4. vertically opposite:
   ∠ABD & ∠CBE
   ∠ABC & ∠DBE

Lesson D: Activity 5: Self-Check

1. a. Since ∠ABC is a right angle, ∠ABD and ∠DBC are complementary.

   Also, ∠A and ∠C are complementary because if one angle in a triangle is 90°, the other two angles must add up to 90°.

   b. ∠BDA and ∠BDC are supplementary because they adjacent angles which form a straight angle.

2. $b + 90° = 180°$

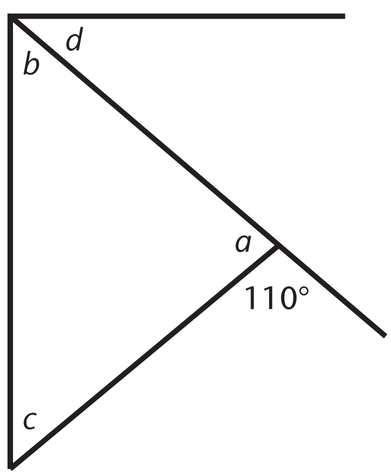
   $b = 180° – 90°$

   $b = 90°$

   $a + 50° = 90°$

   $a = 90° – 50°$

   $a = 40°$

3. 

   $a + 110° = 180°$

   $a = 180° – 110°$

   $a = 70°$

   $a + b + c = 180°$

   $70° + b + b = 180°$ ← $a = 70°$ and $c = b$

   $70° + 2b = 180°$

   $2b = 180° – 70°$

   $2b = 110°$

   $$\frac{2b}{2} = \frac{110°}{2}$$

   $b = 55°$

   Since $b = c$, $c = 55°$.

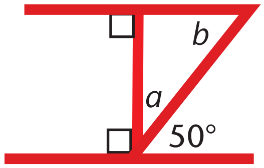
   $b + d = 90°$

   $55° + d = 90°$

   $d = 90° – 55°$

   $d = 35°$

4.



$a + 50° = 90°$

$a = 90° - 50°$

$a = 40°$

In the triangle.

$a + b + 90° = 180°$

$40° + b + 90° = 180°$

$b + 130° = 180°$

$b = 180° - 130°$

$b = 50°$

5. $a = 20°$ — Opposite angles

$20° + b = 180°$ — Supplementary angles

$b = 180° - 20°$

$b = 160°$

$c = b = 160°$ — Opposite angles

$a + d + 100° = 180°$ — Triangle sum

$20° + d + 100° = 180°$ — $a = 20°$

$d + 120° = 180°$

$d = 180° - 120°$

$d = 60°$

Lesson D: Activity 6: Mastering Concepts

1. $\angle A + \angle B + \angle ACB = 180°$
   So, $\angle A + \angle B = 180° - \angle ACB$

   $\angle ACD + \angle ACB = 180°$
   So, $\angle ACD = 180° - \angle ACB$

   Therefore, $\angle A + \angle B = \angle ACD$

   For example, if $\angle A = 40°$ and $\angle B = 60°$, then $\angle ACD = 40° + 60° = 100°$.

2. a. $\angle 1 + \angle 2 = 180°$, because they form a straight angle.

   b. $\angle 3 + \angle 2 = 180°$, because they form a straight angle.

c. Since $\angle 1 + \angle 2 = \angle 3 + \angle 2$,
$\angle 1 + \angle 2 - \angle 2 = \angle 3 + \angle 2 - \angle 2$
$\angle 1 = \angle 3$
Therefore, $\angle 1 \cong \angle 3$.

d. Since $\angle 2 + \angle 1 = \angle 4 + \angle 1$,
$\angle 2 + \angle 1 - \angle 1 = \angle 4 + \angle 1 - \angle 1$
$\angle 2 = \angle 4$
Therefore, $\angle 2 \cong \angle 4$.

Lesson E: Parallel and Perpendicular Lines

Lesson E: Activity 1: Self-Check

1. a. Parallel lines are lines that never meet. They are the same distance apart everywhere.
   b. Perpendicular lines are lines that intersect at a 90º angle.

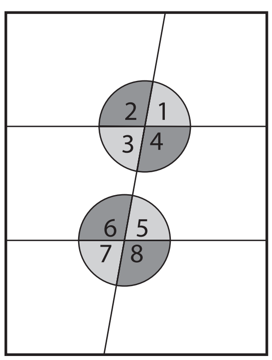
2. a. i. parallel
      ii. These lines are parallel because if you extend the lines, they will never intersect.
   b. i. neither parallel nor perpendicular
      ii. These lines are not parallel because if you extend the lines, they will intersect. These lines are not perpendicular because, when they intersect, they do not meet at a 90º angle.
   c. i. perpendicular
      ii. These lines are perpendicular because they intersect at a 90º angle.
   d. i. neither parallel nor perpendicular
      ii. These lines are not parallel because they intersect. These lines are not perpendicular because they do not meet at a 90º angle.

Lesson E: Activity 2: Try This

Angle	Congruent to...
$\angle 1$	$\angle 3$, $\angle 5$, $\angle 7$
$\angle 2$	$\angle 4$, $\angle 6$, $\angle 8$
$\angle 3$	$\angle 1$, $\angle 5$, $\angle 7$
$\angle 4$	$\angle 2$, $\angle 6$, $\angle 8$
$\angle 5$	$\angle 1$, $\angle 3$, $\angle 7$
$\angle 6$	$\angle 2$, $\angle 4$, $\angle 8$
$\angle 7$	$\angle 1$, $\angle 3$, $\angle 5$
$\angle 8$	$\angle 2$, $\angle 4$, $\angle 6$

1. If the paper is folded so that the creases are perpendicular to the edges of the paper, then the creases will be parallel.

2. Answers will vary. You may have cut each of the pieces in half so that each piece contained one numbered angle. Alternately, you could have found an angle that was congruent to $\angle 3$. Then, you could use that congruent angle to compare with $\angle 6$. (A similar strategy could be used to compare $\angle 4$ with $\angle 5$.)

3. 

You would only need two colours.

$\angle 1$, $\angle 3$, $\angle 5$, and $\angle 7$ would all be the same colour because $\angle 1 \cong \angle 3 \cong \angle 5 \cong \angle 7$.

$\angle 2$, $\angle 4$, $\angle 6$, and $\angle 8$ would all be the same colour because $\angle 2 \cong \angle 4 \cong \angle 6 \cong \angle 8$.

Lesson E: Activity 3: Self-Check

1.

Angle Pair	Angle Relationship	Congruent or Supplementary?
∠1 and ∠3	vertically opposite	congruent
∠4 and ∠8	corresponding	congruent
∠4 and ∠5	co-interior	supplementary
∠3 and ∠5	alternate interior	congruent
∠2 and ∠8	alternate exterior	congruent
∠2 and ∠7	co-exterior	supplementary

2. a. ∠1 = 70°
∠2 = 110°
∠3 = 70°
∠4 = 110°
∠5 = 70°
∠6 = 110°
∠7 = 70°

b. The measure ∠1 is found by subtracting 110º from 180º. This is because the angle marked with 110º and ∠1 are supplementary.

The measure of ∠4 makes a corresponding angle pair with the angle marked 110º. Therefore, ∠4 is also 110º.

The measure of ∠7 can be found through a number of relationships. Two such relationships are given below. You may have used one of these relationships to figure out ∠7 or you may have used a different one.

- ∠7 and the 110º angle are co-exterior angles; thus they are supplementary and ∠7 measures 70º.
- ∠7 and ∠1 are alternate exterior angles; thus they are congruent and ∠7 measures 70º.

Lesson E: Activity 4: Self-Check

1. a. $\overline{AB} \parallel \overline{CD}$ because both segments are perpendicular to the base of the triangle. The right angles marked are corresponding angles. If corresponding angles are congruent, the segments are parallel.

   b. $x + 50° = 180°$ — Co-interior angles are supplementary.

   $x = 180° – 50°$

   $x = 130°$

2. $x + 143° = 180°$ — Co-exterior angles are supplementary.

   $x = 180° – 143°$

   $x = 37°$

Lesson E: Activity 5: Mastering Concepts

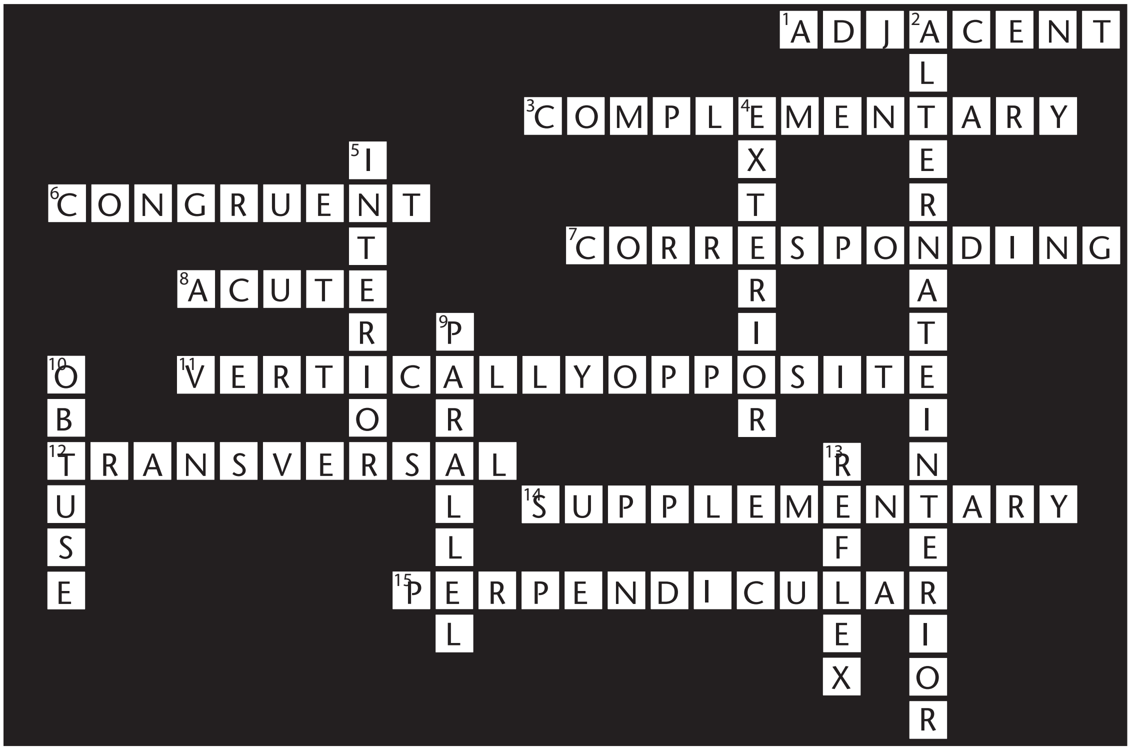
$\angle A \cong \angle ACE$ — Alternate interior angles are congruent.

$\angle B \cong \angle DCE$ — Corresponding angles are congruent.

Therefore, $\angle A + \angle B = \angle ACE + \angle DCE$
But $\angle ACE + \angle DCE = \angle ACD$.
Therefore, $\angle A + \angle B = \angle ACD$

Lesson F: Solving Problems Using Angle Relationships

Lesson F: Activity 1: Self-Check



Lesson F: Activity 2: Self-Check

1. Since the vertical bridge supports are parallel,

   $x = 32°$ — Alternate interior angles are congruent.

2. $b + 40° = 180°$ — co-interior angles

   $b = 180° - 40°$

   $b = 140°$

   $a = b$ — corresponding angles

   $a = 140°$

3. $a = 75°$ — Alternate exterior angles are congruent.

4. $\angle BCA = 40°$ — Corresponding angles are congruent.

   $\angle B + \angle BCA + \angle A = 180°$ — triangle angle sum

   $\angle B + 40° + 60° = 180°$

   $\angle B + 100° = 180°$

   $\angle B = 180° - 100°$

   $\angle B = 80°$

5. Since $\overleftrightarrow{AC}$ and $\overleftrightarrow{DF}$ are parallel, the co-interior angles formed by $\overline{BE}$ are supplementary.

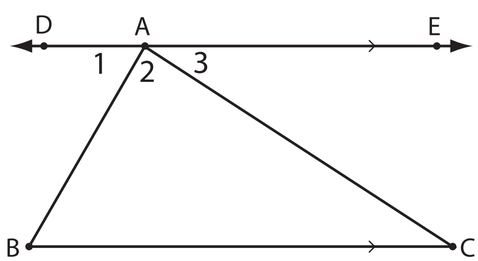
   $\angle ABE + \angle BED = 180°$

   $90° + \angle BED = 180°$

   $\angle BED = 180° - 90°$

   $\angle BED = 90°$

Lesson F: Activity 3: Mastering Concepts



$\overleftrightarrow{DE}$ is parallel to $\overline{BC}$.

$\overline{AB}$ is a transversal, so $\angle B$ and $\angle 1$ are alternate interior angles and are congruent.

$\overline{AC}$ is a transversal, so $\angle C$ and $\angle 3$ are alternate interior angles and are congruent.

Since $\angle DAE$ is a straight angle, $\angle DAE = 180°$. It follows that:
$\angle 1 + \angle 2 + \angle 3 = 180°$

Since
$\angle 1 \cong \angle B$ and $\angle 3 \cong \angle C$,

$\angle B + \angle 2 + \angle C = 180°$

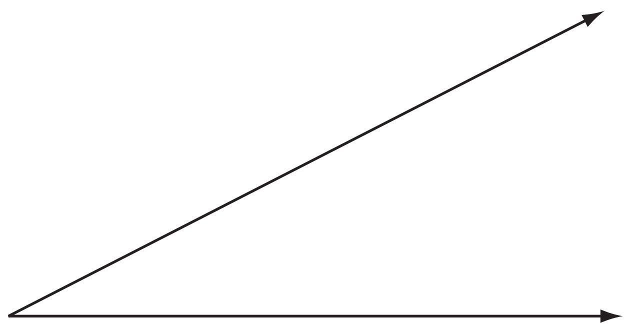
This proves that the three angles of ΔABC, $\angle B$, $\angle 2$, and $\angle C$, add up to 180°.

Glossary

acute angle

an angle greater than 0° but less than 90°

For example, this is an acute angle.



adjacent angles

angles which share a common vertex and lie on opposite sides of a common arm

adjacent side

the side next to the reference angle in a right triangle. (The adjacent side cannot be the hypotenuse.)

alternate exterior angles

exterior angles lying on opposite sides of the transversal

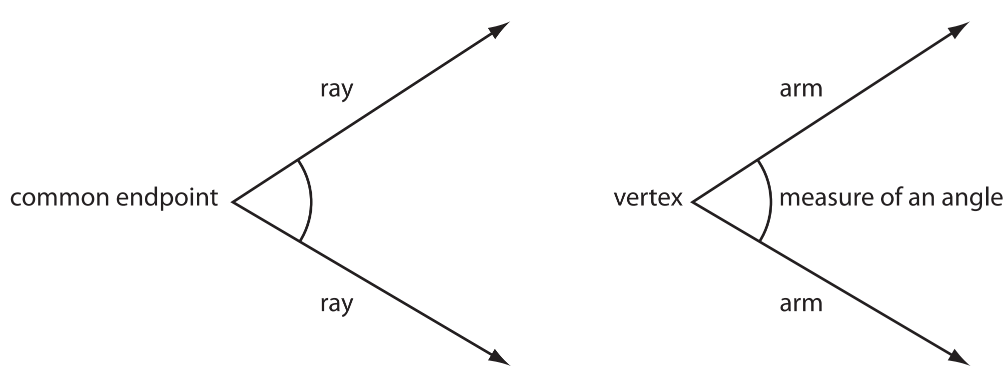
alternate interior angles

interior angles lying on opposite sides of the transversal

angle

a geometric shape formed by two rays with a common endpoint

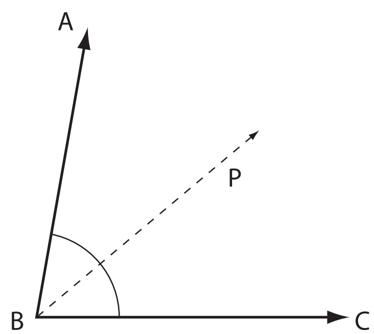
Each ray is called an *arm of the angle*. The common endpoint of the arms of the angle is the vertex of the angle.



angle of depression
an angle below the horizontal that an observer must look down to see an object that is below the observer

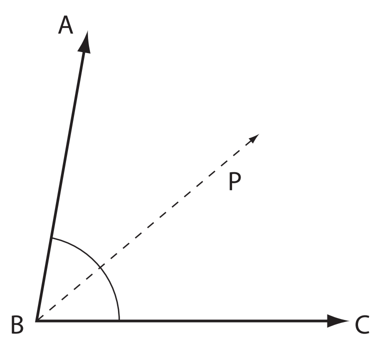
angle of elevation
the angle above the horizontal that an observer must look to see an object that is higher than the observer

bisect
divide into two congruent (equal in measure) halves



bisector
a line or ray which divides a geometric shape into congruent halves

Ray BP is a bisector of ∠ABC, since it bisects ∠ABC into two congruent halves.



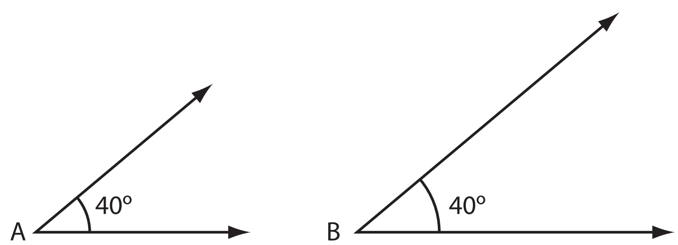
$\angle ABP \cong \angle PBC$

clinometer
a device for measuring angles to distant objects that are higher or lower than your position

complementary angles
two angles with measures that add up to 90°

One angle is called the *complement* to the other.

congruent angles
angles with the same measure



In the diagram ∠A = 40° and ∠B = 40°. So, ∠A and ∠B are congruent.

There is a special symbol for "is congruent to." The congruence symbol is ≅.

So, you can write ∠A ≅ ∠B.

corresponding angles
angles in the same relative positions when two lines are intersected by a transversal

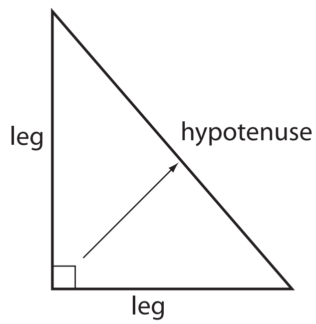
cosine ratio
the ratio of the length of the side adjacent to the reference angle, to the length of the hypotenuse of the right triangle

exterior angles
angles lying outside two lines cut by a transversal

full rotation
an angle having a measure of 360°

This is a full rotation angle.

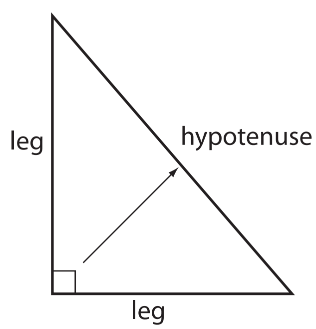
hypotenuse
in a right triangle, the side opposite the right angle; the longest side in a right triangle



indirect measurement
taking one measurement in order to calculate another measurement

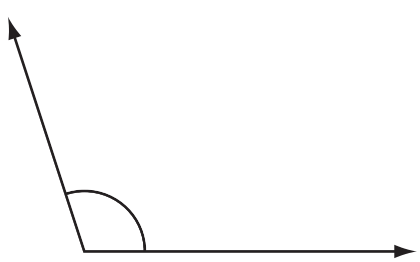
interior angles
angles lying between two lines cut by a transversal

leg
one of the two sides of a right triangle that forms the right angle



obtuse angle
an angle greater than 90° but less than 180°

For example, this is an obtuse angle.



opposite side
the side across from the reference angle in a right triangle

parallel
lines that are the same distance apart everywhere: they never meet

perpendicular
lines that meet at right angles

polygon
a many-sided figure

A triangle is a polygon with three sides, a quadrilateral is a polygon with four sides, and so on.

proportion
the statement showing two ratios are equal

Pythagorean Theorem
for any right triangle, the square of the hypotenuse is equal to the sum of the squares of the two legs

Pythagorean triple
three whole numbers, which represent the lengths of the sides of a right triangle

There are an infinite number of such triples.

reference angle
an acute angle that is specified (example, shaded) in a right triangle

referent
an object or part of the human body you can refer to when estimating length or distance

reflex angle
an angle having a measure greater than 180° but less than 360°

This is an example of a reflex angle.

regular polygon
a polygon with all its angles equal in measure and all its sides equal in measure

right angle
one quarter of a complete rotation. It is 90° in measure.

scale factor
the number by which the length and the width of a figure is multiplied to form a larger or smaller similar figure

similar figures
figures with the same shape but not necessarily the same size

A figure similar to another may be larger or smaller

sine ratio
the ratio of the length of the side opposite to the reference angle, over the hypotenuse of the right triangle

solve a right triangle
to find all the missing sides and angles in a right triangle

straight angle
one half a rotation; an angle 180°

This is a straight angle.

straightedge
a rigid strip of wood, metal, or plastic having a straightedge used for drawing lines

When a ruler is used without reference to its measuring scale, it is considered to be a straightedge.

supplementary angles
two angles, which add up to 180°

In a pair of supplementary angles, one angle is the supplement to the other.

symmetry
the property of being the same in size and shape on both sides of a central dividing line

tangent ratio
the ratio of the length of the side opposite to the selected acute angle, to the length of the side adjacent to the selected acute angle in a right triangle

transversal
a line that cuts across two or more lines

trigonometry
the branch of mathematics based originally on determining sides and angles of triangles, particularly right triangles

vertically opposite angles
angles lying across from each other at the point where two lines intersect

Vertically opposite angles are also referred to as *opposite angles*.

Grid Paper