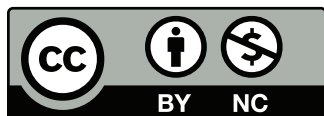


Area

Apprenticeship and Workplace Mathematics (Grade 10/Literacy Foundations Level 7)

qt
inches mL °C
pounds cm³
centimetres
Ounces LITRES
FAHRENHEIT Hectares
KILOMETRES
MILES²
yd²

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Course History

New, March 2012

Project Partners

This course was developed in partnership with the Distributed Learning Resources Branch of Alberta Education and the following organizations:

- Black Gold Regional Schools
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- Edmonton Public Schools
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Viewing Your PDF Learning Package

This PDF Learning Package is designed to be viewed in Acrobat. If you are using the optional media resources, you should be able to link directly to the resource from the pdf viewed in Acrobat Reader. The links may not work as expected with other pdf viewers.



Download Adobe Acrobat Reader:

<http://get.adobe.com/reader/>

Section Organization

This section on Area is made up of several lessons.

Lessons

Lessons have a combination of reading and hands-on activities to give you a chance to process the material while being an active learner. Each lesson is made up of the following parts:

Essential Questions

The essential questions included here are based on the main concepts in each lesson. These help you focus on what you will learn in the lesson.

Focus

This is a brief introduction to the lesson.

Get Started

This is a quick refresher of the key information and skills you will need to be successful in the lesson.

Activities

Throughout the lesson you will see three types of activities:

- Try This activities are hands-on, exploratory activities.
- Self-Check activities provide practice with the skills and concepts recently taught.
- Mastering Concepts activities extend and apply the skills you learned in the lesson.

You will mark these activities using the solutions at the end of each section.

Explore

Here you will explore new concepts, make predictions, and discover patterns.

Bringing Ideas Together

This is the main teaching part of the lesson. Here, you will build on the ideas from the Get Started and the Explore. You will expand your knowledge and practice your new skills.

Lesson Summary

This is a brief summary of the lesson content as well as some instructions on what to do next.

At the end of each section you will find:

Solutions

This contains all of the solutions to the Activities.

Appendix

Here you will find the Data Pages along with other extra resources that you need to complete the section. You will be directed to these as needed.

Glossary

This is a list of key terms and their definitions.

Throughout the section, you will see the following features:

Icons

Throughout the section you will see a few icons used on the left-hand side of the page. These icons are used to signal a change in activity or to bring your attention to important instructions.



AWM online resource (optional)

This indicates a resource available on the internet. If you do not have access, you may skip these sections.



Solutions

My Notes

The column on the outside edge of most pages is called “My Notes”. You can use this space to:

- write questions about things you don’t understand.
- note things that you want to look at again.
- draw pictures that help you understand the math.
- identify words that you don’t understand.
- connect what you are learning to what you already know.
- make your own notes or comments.

Materials and Resources

There is no textbook required for this course.

You will be expected to have certain tools and materials at your disposal while working on the lessons. When you begin a lesson, have a look at the list of items you will need. You can find this list on the first page of the lesson, right under the lesson title.

In general, you should have the following things handy while you work on your lessons:

- a scientific calculator
- a ruler
- a geometry set
- Data Pages (found in the appendix)

Area



Photo by Tischenko Irina © 2010

Honey bees are master designers. Mathematicians have proven that the hexagonal pattern on the comb is the best way of dividing up a region into cells of equal area. The amount of wax required, and the energy expended by the worker-bees, are kept to a minimum. The cells are all the exact same size. There are approximately four cells per square centimetre or 25 per square inch.

This section is all about the area measurements of shapes. You will estimate areas and use those estimates to ensure that your answers to area problems are reasonable. Some of the problem situations you will meet in this section involve areas of two-dimensional shapes such squares, rectangles, parallelograms, and circles. As well, you will explore the surface area of prisms, pyramids, cylinders, and cones. You will also investigate how changing the dimensions of geometric shapes affects the perimeters and areas of those shapes.

In this section you will:

- solve problems that involve SI and imperial area measurements of a variety of 2-D and 3-D shapes, including decimal and fractional measurements.

Lesson A

Estimating Area

To complete this lesson, you will need:

- a ruler or a tape measure that shows imperial and SI units

In this lesson, you will complete:

- 5 activities

Essential Questions

- How are referents used to estimate area measurements using SI and imperial units?
- In what situations are SI and/or imperial units for measurement used?
- How can the area of a regular or irregular shape be estimated using a grid?

My Notes

Focus

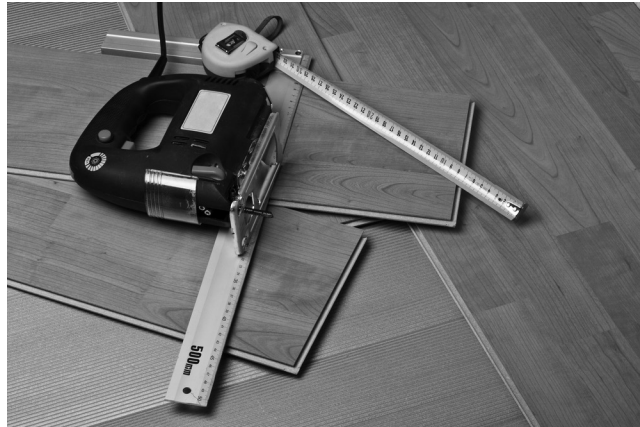


Photo by Dmitry Melnikov © 2010

Suppose you and your dad plan to renovate the family room in your home. To estimate the cost of new laminate flooring, you need to prepare an estimate of:

- the total coverage area of the floor
- any possible waste (as a result of cuts)

The flooring will have to be cut to fit:

- along the walls
- around a fireplace (if there is one)
- on and/or around stairs
- around furnace vents (if there are vents)
- any smaller areas of the room, e.g., a closet

Does your family room have any unusual features? How would you estimate the amount of flooring you should purchase?

Get Started

Try this next activity to refresh your memory about units of area in both the SI and imperial measurement systems.

Activity 1 Self-Check

My Notes

Unit Name	Symbol	SI or Imperial?
square foot	ft ²	imperial
	m ²	
square inch		
acre		
	ha	
square centimetre		
	yd ²	
square kilometre		
square mile		



Turn to the solutions at the end of the section and mark your work.

Explore

You may find it helpful to have reference objects for units of area measure. In the next activity, you'll measure some common objects in order to establish your own area referents.

My Notes

Choosing Units

Before you estimate or measure an area, consider which units would be the best ones to use for a particular object.

For example, the book cover in the picture has an area of 99 in^2 . This area is also equal to 0.076 yd^2 —not a whole number of square yards. It would be a bit strange to say that the area of the book cover is 0.000016 acres! Not only would it be strange, but it would also be hard to visualize a small fraction of an acre. So, why do you think a square inch is the best unit for measuring this area?

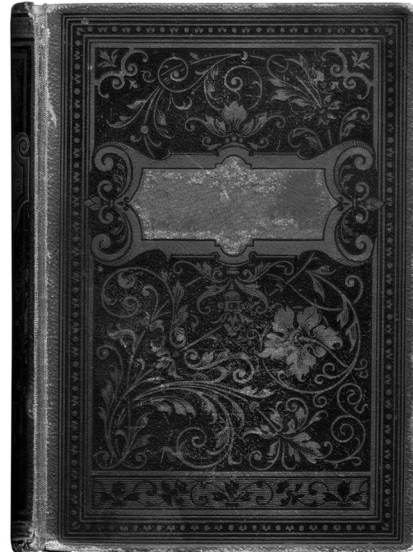


Photo by AKaiser © 2010

A good choice of units allows for small, whole numbers to be used in the measurement. By choosing square inches, you can express the area measurement as 99 in^2 .

Now think of your bedroom. Would it be appropriate to express the area of your bedroom in square inches? What would a more appropriate imperial unit be?

You would likely use the square foot (ft^2) or the square yard (yd^2) to measure the area of your bedroom. In SI units, you would probably use the square metre (m^2).

Activity 2 Self-Check

My Notes

Circle the appropriate unit of area measurement for each of the following objects.

Area of:

- | | | | |
|-----------------------------------|---------------|---------------|---------------|
| 1. a TV screen | in^2 | ft^2 | ac |
| 2. the main floor of a home | in^2 | ft^2 | ac |
| 3. a plot of land | cm^2 | m^2 | km^2 |
| 4. the screen on a digital camera | cm^2 | m^2 | km^2 |



Turn to the solutions at the end of the section and mark your work.

Measurement Systems

In your work in Module 1, you learned that in Canada many items are advertised in both imperial and SI units.

The following is a partial guide for your reference. See if you can think of one or two more items and add them to the table.

Item	Advertised In:		
	ft^2	yd^2	m^2
laminated flooring	✓		✓
carpets		✓	✓
house and floor area	✓		✓
linoleum		✓	✓
lawn area on bags of fertilizer	✓		✓

My Notes

Bringing Ideas Together

Estimation can be a useful technique in many different applications. You may use a referent to estimate the area of an object when you don't have a ruler or tape measure handy—or if you just want a quick estimate without having to perform any calculations.

Estimating an area can also help you decide whether or not a calculated answer is reasonable. Let's look at a few examples.

Example 1

A room measures 3.05 metres by 3.96 metres.

- Estimate its area.
- Calculate its area in square metres. Round to two decimal places.

Solution

- To estimate, first round each measure to the nearest metre.
3.05 m is close to 3 m and 3.96 m is close to 4 m
So, the area of the room is about $3 \text{ m} \times 4 \text{ m} = 12 \text{ m}^2$.
- To calculate, use the formula for the area of a rectangle.

$$\begin{aligned} A &= l \times w \\ &= 3.05 \text{ m} \times 3.96 \text{ m} \\ &= 12.078 \text{ m}^2 \\ &\approx 12.08 \text{ m}^2 \end{aligned}$$

Find this formula on your Data Pages.

The area of the room is 12.08 m^2 . This answer is reasonable, since the estimate was 12 m^2 .

Example 2

Estimate the number of square yards of carpet needed for a room that measures $14\frac{1}{2}$ feet by 12 feet.



Photo by Lepas © 2010

My Notes**Solution**

Remember $1\text{ yd} = 3\text{ ft}$.

So, 14.5 ft is almost 5 yd.

And, $12\text{ ft} = 4\text{ yd}$.

Therefore, the area is about $5\text{ yd} \times 4\text{ yd} = 20\text{ yd}^2$.

About 20 square yards of carpet are needed to cover the floor of the room.

Activity 3
Self-Check

Please answer the following questions.

1. Emile's rectangular driveway is $12\frac{1}{2}$ feet wide and 59 feet long.
 - a. Estimate the area in square yards.

My Notes

b. Calculate the area to the nearest tenth of a square yard.

c. Compare your answer for question b to the estimate. What does the estimate tell you?

2. A binder cover measures 27 cm by 29 cm.

a. Estimate its area.

- b. Calculate the area. Is your answer reasonable?

My Notes

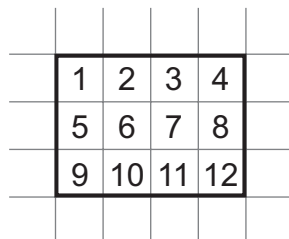


Turn to the solutions at the end of the section and mark your work.

Estimating Areas of Shapes that are not Rectangular

In your work to this point, you have been calculating the areas of rectangles and squares. You know that the areas you have found for these figures are the number of square units (cm^2 , in^2 , ft^2 , m^2 , and so on) within each figure.

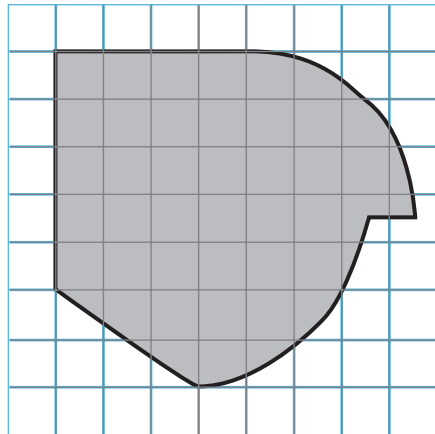
For example: in the figure below, if each square represents 1 cm^2 , then you can count the number of squares within the shape to find its area. This shape has an area of 12 cm^2 .



Of course, not all areas are square or rectangular. The area of an irregular **two-dimensional** shape can be approximated by counting unit squares.

My Notes

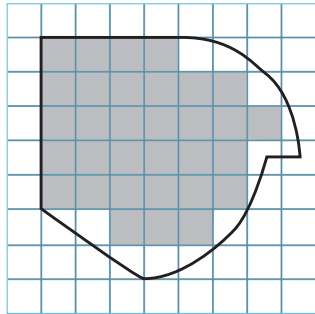
How would you find the area of the following shape?



You may have suggested that you can estimate the area of the shape by counting up all the full squares within the lines, or by counting the full squares and adding on the partial squares. Keep whatever method you suggested in mind—you'll return to it a bit later.

My Notes

Let's look at one method for estimating the area of irregular shapes.



Unit squares totally contained by the figure

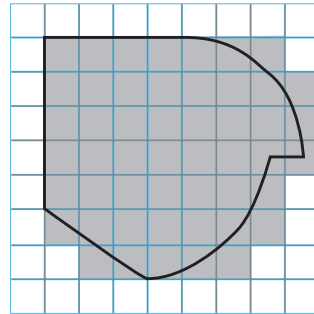
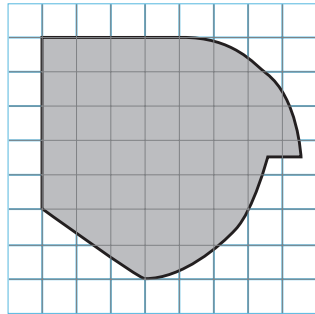
32

≤

Area of the figure

≤

50



Unit squares totally containing the figure

The area of the figure is shown in the middle. You can see that the actual area of the figure lies between the number of unit squares contained by the figure and the number of unit squares that contain the figure. There are 32 squares that lay entirely within the figure. There are 50 squares that cover the entire figure.

A better estimate than either 32 or 50 unit squares is the *average* of these estimates.

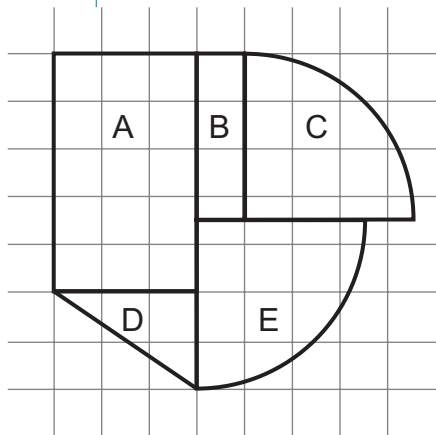
What is the average of 32 and 50? Remember, the average of two numbers is found by adding them together and dividing the result by 2.

$$\begin{aligned} \text{Average} &= \frac{32 + 50}{2} \\ &= \frac{82}{2} \\ &= 41 \end{aligned}$$

My Notes

Think back to the estimate you made earlier. How close was it to 41 square units? Which do you think is a better method: yours or the average-method? (Or is your method the same as the average-method?)

The actual area of the shape is 40.74 units². The calculations are shown below. Don't worry about the details of these calculations now. You will explore a variety of area formulas in later lessons in this section. For now, consider how close our estimate was to the actual area of the figure. You can try this technique to estimate the area of the 2-D shape in the next activity.



$$\begin{aligned} \text{Area}_A &= l_A \times w_A \\ &= 5 \times 3 \\ &= 15 \text{ units}^2 \end{aligned}$$

$$\begin{aligned} \text{Area}_B &= l_B \times w_B \\ &= 3.5 \times 1 \\ &= 3.5 \text{ units}^2 \end{aligned}$$

$$\begin{aligned} \text{Area}_C &= \frac{1}{4} \pi r_C^2 \\ &= \frac{1}{4} \pi (3.5)^2 \\ &= 9.6211 \dots \text{ units}^2 \end{aligned}$$

$$\begin{aligned} \text{Area}_D &= \frac{1}{2} b \times h \\ &= \frac{1}{2} (2)(3) \\ &= 3 \text{ units}^2 \end{aligned}$$

$$\begin{aligned} \text{Area}_E &= \frac{1}{4} \pi r_E^2 \\ &= \frac{1}{4} \pi (3.5)^2 \\ &= 9.6211 \dots \text{ units}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of Figure} &= \text{Area}_A + \text{Area}_B + \text{Area}_C + \text{Area}_D + \text{Area}_E \\ &= 15 + 3.5 + 9.6211 \dots + 3 + 9.6211 \dots \\ &= 40.7422 \dots \text{ units}^2 \end{aligned}$$

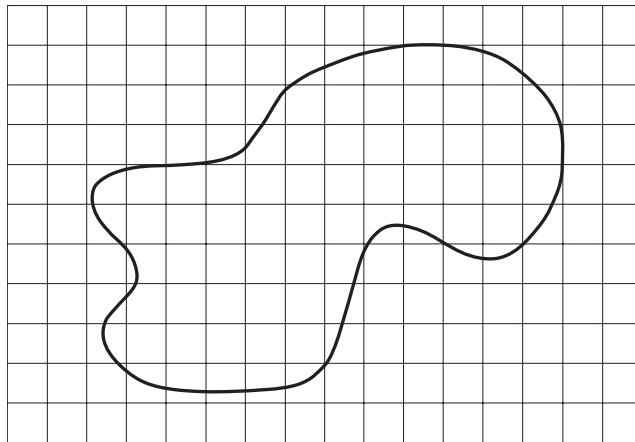
Activity 4

Self-Check

My Notes

1. Estimate the area of the irregular two-dimensional object.

Hint: Use a highlighter to highlight squares to make the count easier. Use one colour for the squares contained within the figure, and a different colour for the squares that contain the figure.



My Notes

2. If each square in the diagram is 1 cm on a side, what units would your estimate have?



Turn to the solutions at the end of the section and mark your work.

Activity 5
Mastering Concepts

Have you watched workers shingle a roof or maybe even helped with the job? Asphalt shingles are sold by the square or by the bundle. A *square* is the number of shingles needed to cover 100 ft^2 —an area 10 ft by 10 ft. A *bundle* is one-third of a square. The weight of a bundle can usually be handled by one person.



Photo by L. Barnwell © 2010

1. How many square feet of shingles are in a bundle?

2. Contractors estimate that a bundle will normally cover 32 ft^2 , as there is always waste at the ends of the roof and by chimneys. If a bundle is $\frac{1}{3}$ of a square, how many square feet of shingles per bundle is accounted for as waste?

My Notes

3. What percent of a square is accounted for as waste?

My Notes

4. A roof is 1000 ft^2 in area. How many bundles of shingles would a contractor estimate are needed to shingle that roof?



Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes



Photo by GWImages © 2010

If you have looked at home advertisements in flyers or the newspaper, the area of houses for sale is quoted in either square feet or square metres. These areas, derived from sales person's measurements, are estimates only.

This lesson dealt with area estimates too.

You discovered that your skill in estimating area depends on associating linear units in both the SI and imperial systems with common items, such as a thumbprint for the square inch. These items are called *referents*, and your referents are likely different from another person's referents.

You practised estimating areas in problem situations to confirm whether or not your calculated answers were reasonable. As well, you used grids to estimate irregular areas.

Lesson B

Area Formulas 1

To complete this lesson, you will need:

- a ruler or tape measure that shows imperial and SI units
- “Parallelogram Template” from the templates section of the appendix
- “Triangle Template” from the templates section of the appendix
- scissors
- tape
- a calculator

In this lesson, you will complete:

- 6 activities

Essential Questions

- How are the areas of parallelograms and triangles calculated?
- How can you find the areas of composite shapes involving rectangles, squares, parallelograms, and triangles?

My Notes

Focus

Drivers know from the shape of a certain traffic sign what the sign represents. Yield signs are triangular. Stop signs are octagonal. Information signs are rectangular. Most countries follow the same standards for shape, graphics, and colour.¹ These standards make it easier for international travellers who may not speak the local language to know what the sign represents.



Photo by J.D.S. © 2010

Traffic signs are covered with a reflective coating, so they can be seen more easily at night and in low-lighting conditions. How much coating used depends on the area of the sign. How would you determine the quantity of coating needed for the yield sign in this photograph?

Get Started

In your work in previous lessons, you found areas of rectangles and squares. Rectangles and squares are only two of many shapes. Try the following activity to test how much you remember about geometric shapes.

¹In the United States and Canada, for instance, orange signs are used at construction sites.

Activity 1 Self-Check

My Notes

Match the shapes on the right to the names on the left.

1. ___ Circle

2. ___ Hexagon

3. ___ Octagon

4. ___ Parallelogram

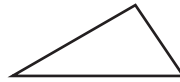
5. ___ Rectangle

6. ___ Square

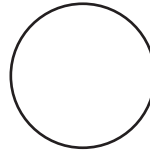
7. ___ Trapezoid

8. ___ Triangle

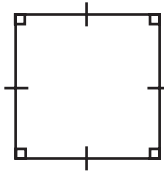
A.



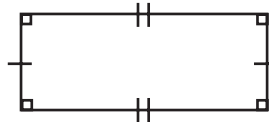
B.



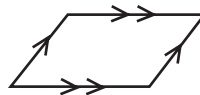
C.



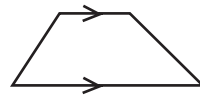
D.



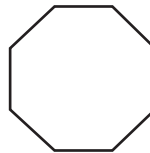
E.



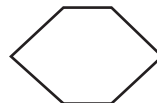
F.



G.



H.



Turn to the solutions at the end of the section and mark your work.

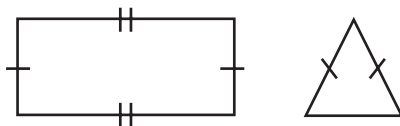
My Notes

Sketching Geometric Figures

When you sketch geometric figures, certain symbols are used to communicate information. A few of these are described below.

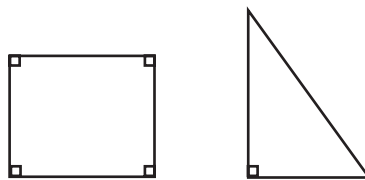
Side-lengths

Hatch marks are used to indicate sides that have equal lengths.



Right angles

The small squares shown in the diagram of a rectangle and a square tell you that these corners are right angles.



Parallel sides

The pair of sides that are parallel are marked by arrows.

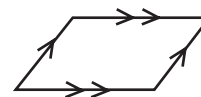


Explore

We have already worked with the area formulas for rectangles and squares. Now, let's explore the areas of two new shapes: the **parallelogram** and the triangle.

A parallelogram is a shape with the following characteristics:

- it has four sides
- the opposite sides are equal in length
- the opposite sides are parallel



A triangle is a shape with three sides. There are different types of triangles, which you will learn about in Module 3. For now, we'll simply look at the areas of triangles in general.



My Notes

Activity 2

Try This

In this activity, you'll explore the area of a parallelogram. You will need the "Parallelogram Template" from the templates section of the appendix, a pair of scissors, and some tape.

Follow the instructions below and then answer the questions that follow.

Step 1: Get the "Parallelogram Template" from the templates section of the appendix. Cut out the parallelogram (cutting along the solid lines).

Step 2: Cut along the dashed line segment representing the height.

Step 3: Rearrange the two pieces to make a rectangle.

Questions:

1. a. What was the length of the base of the original parallelogram?

- b. What was its height?

2. a. What is the length of the rectangle?

- b. What is its width?

My Notes

- c. How do these values compare to the base and height of the original parallelogram?
-

3. a. What is the area of this rectangle?

- b. How does this area compare to the area of the original parallelogram?
-
-



Turn to the solutions at the end of the section and mark your work.

Area of a Parallelogram

In the previous activity, you saw that the area of a parallelogram is equal to the area of a rectangle with the same base and height.

Area of a parallelogram = base \times height

$$A = bh$$

You discovered a way to find the area of a parallelogram by rearranging its area to make another figure. Let's try another rearranging technique to explore the area of a triangle.

Activity 3

Try This

My Notes

In this activity, you'll explore the area of a triangle. You will need the "Triangle Template" from the templates section of the appendix, a pair of scissors, and some tape.

Follow the instructions below and then answer the questions that follow.

Step 1: Get the "Triangle Template" from the templates section of the appendix. Cut out the triangle (cutting along the solid lines).

Step 2: Arrange the triangles to form a parallelogram. You may flip, rotate and move the triangles in order to do this.

Questions:

1. a. What is the length of the base of each triangle?

- b. What is the height of each triangle?

2. a. What is the length of the base of the parallelogram you constructed?

- b. What is the height of the parallelogram?

3. a. What is the area of the parallelogram?

My Notes

- b. What do you think the area of each triangle is? (Hint: how does the area of one triangle relate to the area of the parallelogram?)



Turn to the solutions at the end of the section and mark your work.

Area of a Triangle

In Activity 3, you may have noticed that the area of a triangle is half the area of a parallelogram with the same base and height. We can say that the formula for finding the area of a triangle is:

Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

$$A = \frac{1}{2} bh$$

Bringing Ideas Together

In the Explore you discovered how to use formulas to determine the area of a parallelogram and the area of a triangle.

- Area of a parallelogram = base \times height

$$A = bh$$

- Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

$$A = \frac{1}{2} bh$$

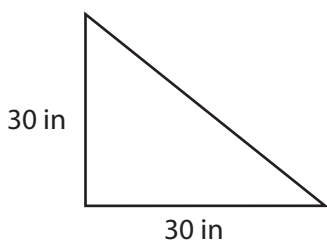
Let's work through some examples where you'll use these formulas.

Example 1

A 30 inch square of cloth is cut along the diagonal to make a triangular bandanna. What is the area of the triangular part?



Photo by vectorob © 2010

Solution

From the question, we know that:

$$b = 30 \text{ in}$$

$$h = 30 \text{ in}$$

Substitute the known values into the formula.

$$\begin{aligned} A &= \frac{1}{2} bh \\ &= \frac{1}{2} \times 30 \text{ in} \times 30 \text{ in} \\ &= 450 \text{ in}^2 \end{aligned}$$

The area of each triangle is 450 in^2 .

My Notes

My Notes

Example 2

A snake's skin is covered with small parallelograms.

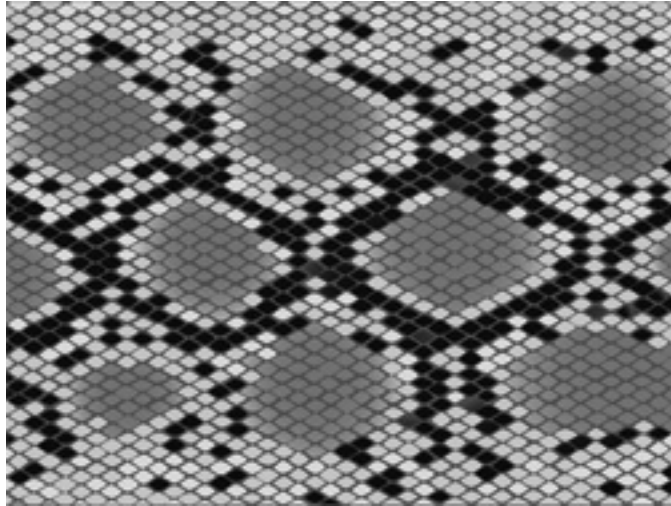
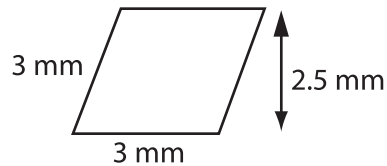


Photo by SRNR © 2010

One of the small parallelograms is pictured as follows.



What is its area of this parallelogram?

Solution

From the question, we know that:

$$b = 3 \text{ mm}$$

$$h = 2.5 \text{ mm}$$

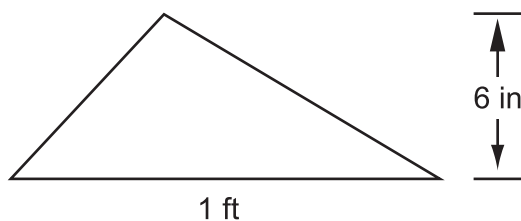
Substitute the known values into the formula.

$$\begin{aligned} A &= bh \\ &= 3 \text{ mm} \times 2.5 \text{ mm} \\ &= 7.5 \text{ mm}^2 \end{aligned}$$

The area of the parallelogram is 7.5 mm^2 .

Example 3

Find the area of the following triangle.

**Solution**

Each dimension must be expressed in the same units.

Remember that 1 ft = 12 in.

$$b = 1 \text{ ft} = 12 \text{ in}$$

$$h = 6 \text{ in}$$

$$\begin{aligned} A &= \frac{1}{2} bh \\ &= \frac{1}{2} \times 12 \text{ in} \times 6 \text{ in} \\ &= 36 \text{ in}^2 \end{aligned}$$

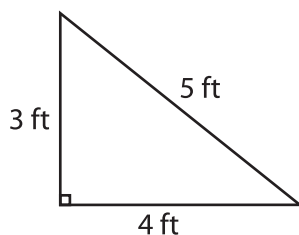
The area of the triangle is 36 in².

Now it's your turn!

Activity 4 Self-Check

Please answer the following questions.

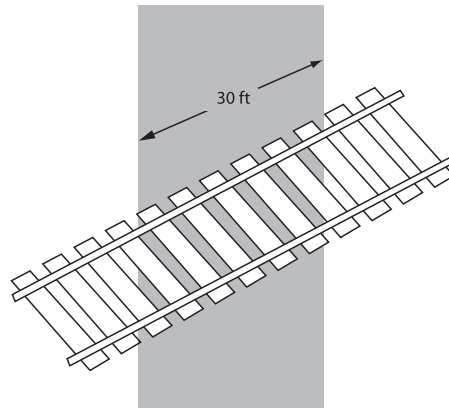
1. What is the area of the triangle shown below?



My Notes

My Notes

2. A railroad crosses a highway as shown.



The distance between the parallel rails is 4 ft 8 ½ in.

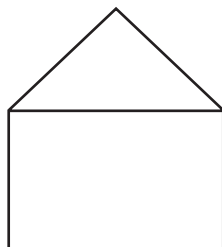
What is the area of the road covered by the tracks?



Turn to the solutions at the end of the section and mark your work.

Composite Shapes

Often more complicated figures are formed by combining simpler geometric shapes.



For instance, the end of a garage looks like a triangle on top of a rectangle. This more complicated shape can be considered to be a **composite figure**.

Combining simpler shapes such as the triangle, rectangle, parallelogram, and circle results in a composite figure.

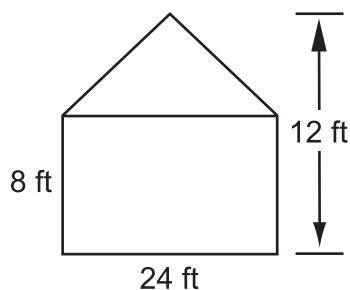


To investigate how simple shapes can be combined to form composite figures, go to *Exploring Composite Figures* (http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/html/composite_figures/index.html). (You may ignore circles for now; they will be covered in the next lesson.)

Exploring Composite Figures showed you that, to find the area of a composite figure you can add the areas of the simple shapes that make up the composite figure. In the next example, and in the activity that follows, you will examine this process more closely.

Example 4

As part of a community historical project, Anita is planning to paint a mural on the back of her garage.



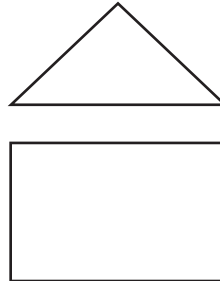
My Notes

My Notes

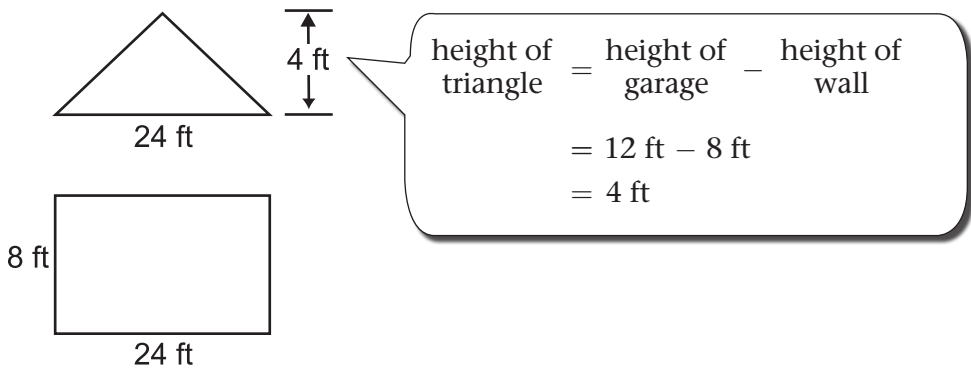
What is the total area available for her to paint?

Solution

Step 1: Separate the composite figure into simpler shapes.



Step 2: Transfer the dimensions of the composite figure to these simpler shapes.



Step 3: Calculate the area of each simple shape.

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 24 \text{ ft} \times 4 \text{ ft} \\ &= 48 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of rectangle} &= lw \\ &= 24 \text{ ft} \times 8 \text{ ft} \\ &= 192 \text{ ft}^2 \end{aligned}$$

Step 4: Combine the areas of the simpler shapes.

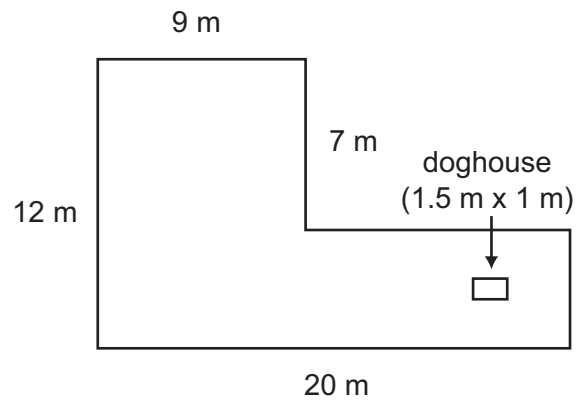
$$\begin{aligned} \text{Area of the end of the garage} &= \text{Area of triangle} + \text{Area of rectangle} \\ &= 48 \text{ ft}^2 + 192 \text{ ft}^2 \\ &= 240 \text{ ft}^2 \end{aligned}$$

Anita could paint a mural 240 ft² in area.

Activity 5
Self-Check

My Notes

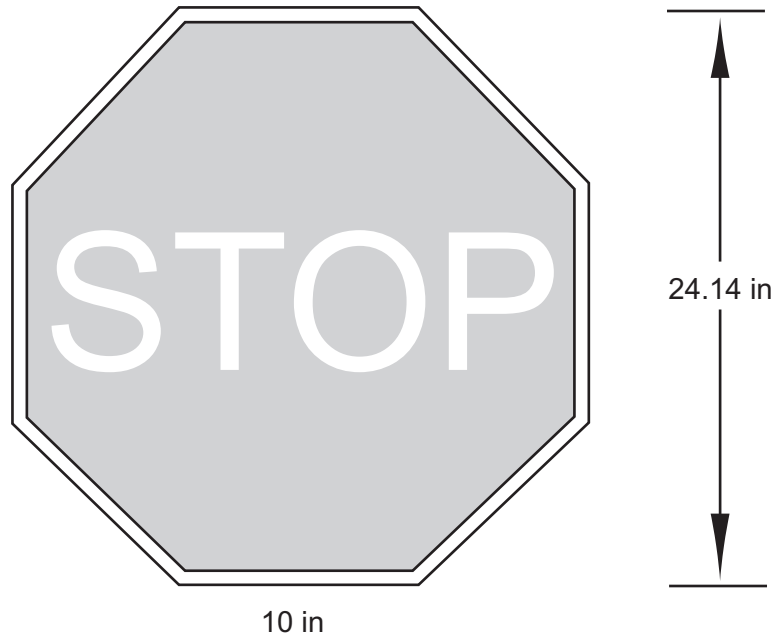
1. Cheryl is going to apply some fertilizer to the lawn next to her house. The lawn is shaped like this.



What is the area of the lawn?

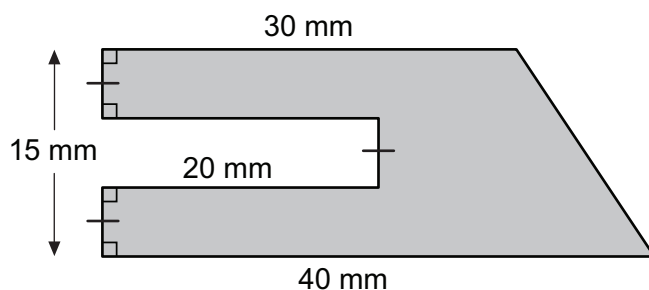
My Notes

2. A stop sign is a regular octagon. Each side is 10 in long. The distance from the top of the sign to the bottom is approximately 24.14 in.



What is the area of the stop sign to the nearest square inch?

3. Look at the composite shape.



You can infer that the top and bottom of the shape are parallel from the small squares in the left corners of the illustration. There is a rectangular cut-out 20 mm deep with a height one third of the total height of the shape.

Calculate the area of the composite shape.

My Notes



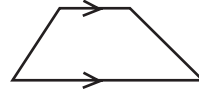
Turn to the solutions at the end of the section and mark your work.

My Notes

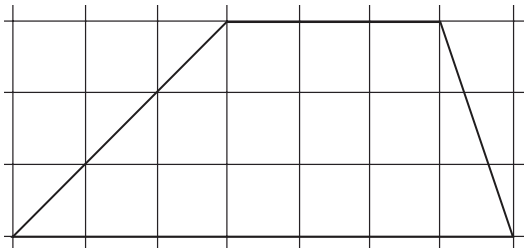
Activity 6 Mastering Concepts

The **trapezoid** is a shape with the following characteristics:

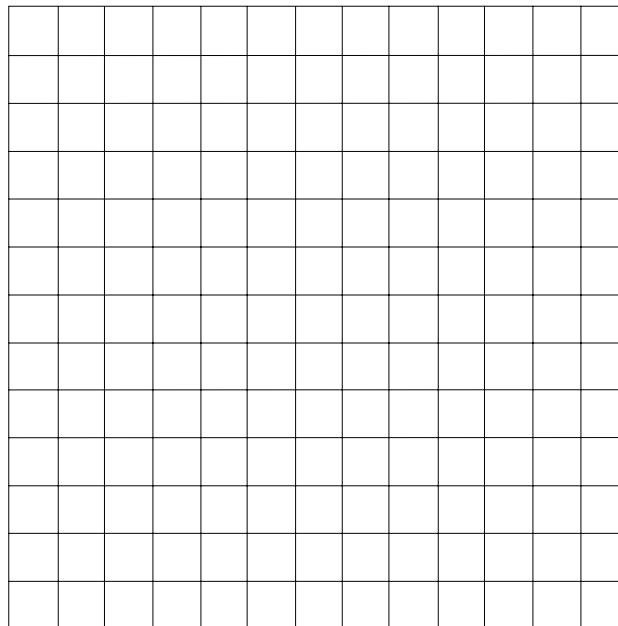
- it has four sides
- two of the sides are parallel



The trapezoid can be thought of as a composite shape. To find its area, you can divide it into simpler shapes.



1. Identify different shaped objects that are combined to form the trapezoid above. Sketch these shapes on the grid provided.



Lesson Summary



Photo by sgh © 2010

The Canadian flag is recognized by people all over the world. It is not only flown within our country, it is often printed on products that Canada exports around the world.

Did you know that the flag is twice as long as it is wide, and that the white square in the middle has the same area as the two vertical red stripes together?

This lesson dealt with areas of simple shapes and the design of composite figures—like the Canadian flag!

Lesson C

Area Formulas 2

To complete this lesson, you will need:

- a blank sheet of paper
- a compass or circular object
- scissors
- a measuring tape or ruler
- a calculator
- Data Pages found in the module appendix

In this lesson, you will complete:

- 5 activities

Essential Questions

- How are the areas of circles calculated?
- How can you find the areas of composite shapes involving circles?

My Notes

Focus



Photo by Michael Zysman © 2010

The First Nations dancer in the photograph uses hoops to tell stories through intricate designs. The dancer positions the hoops to display birds and animals such as the eagle, coyote, grouse, or butterfly. The hoops are circular, often handmade, and range in diameter. The image created when the dancer locks the hoops together, depends on the diameter and areas of the individual hoops.

You may remember learning about circles in Section 1, Lesson B. In that lesson you looked at the parts of a circle and investigated ways to determine the diameter of a given circle. In this lesson you will learn how to find the area of a circle as well as areas of composite figures that involve circles.

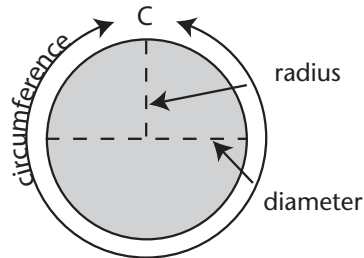
Get Started

Complete Activity 1 to review what you know about circles. Use the diagrams on the next page if you need a refresher on circles.

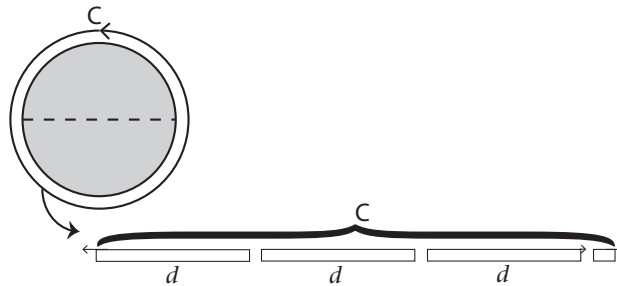


If you have access and need a refresher, watch the video, *Circles*, (http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/mod2_circles.html) before you begin the activity.

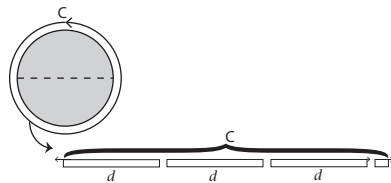
My Notes



If we unpeel the circumference or distance around the circle and lay it flat, we see it's just a little bit longer than three times the diameter of the circle.



It doesn't matter what size the circle is. The ratio is always the same: the circumference is 3.1415... times the length of the diameter.



We call this ratio "pi," and often abbreviate it to 3.14. We represent "pi" with the Greek letter pi (π).

$$\pi = 3.14$$

My Notes

Activity 1
Self-Check

1. The radius of a circle is _____ the length of the diameter.
2. The circumference of a circle is approximately _____ times longer than its diameter.
3. The symbol _____ is used in mathematics to represent the exact ratio of the circumference of a circle to its diameter.
4. The diameter of a circle always goes through the circle's _____.
5. The diameter of a circle is _____ the length of the radius.



Turn to the solutions at the end of the section and mark your work.

Explore

In the last lesson, you learned how to find the area of several different shapes. All of these shapes had straight edges. The circle is different because it has a curved edge: you can't fit square units neatly within the circular shape. So how can we find the area of a circle?

In Activity 2 you will explore a way to approximate the area of a circle. After the activity, we'll see if we can generalize a formula for finding the area of a circle.

Activity 2

Try This

My Notes

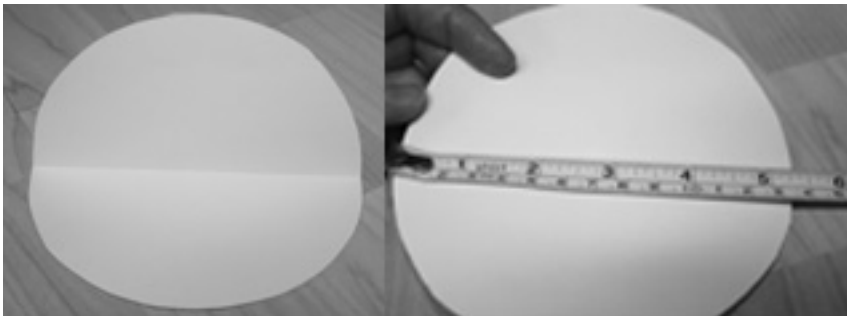
Please complete the steps below and answer the questions that follow. You will need some blank paper, a compass or circular object, scissors and a measuring tape or ruler.

Step 1: Use your compass or a round object to draw a circle on a blank sheet of paper. Then, carefully cut out the circle.



Reproduced with permission from Alberta Education.

Step 2: Fold the circle carefully in half to form a crease along a diameter. Use a ruler or tape to measure the diameter.



Reproduced with permission from Alberta Education.

Please answer questions 1 and 2.

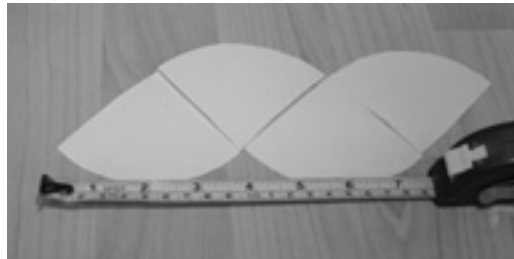
My Notes

Step 3: Fold your circle again so that the two diameters divide the circle into quarters. Cut along the creases to obtain four equal sectors of the circle. A sector is a pie-shaped piece of a circle.



Reproduced with permission from Alberta Education.

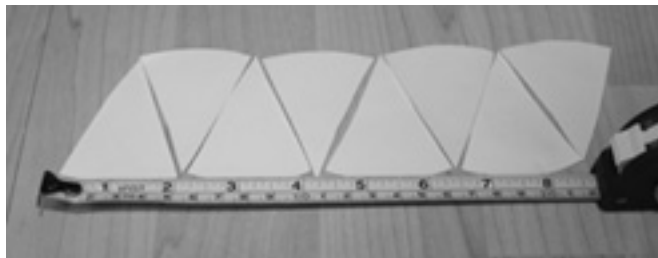
Step 4: Form a parallelogram-like shape with the pieces. Measure the base—the bottom side—of this shape.



Reproduced with permission from Alberta Education.

Please answer question 3.

Step 5: Fold each sector in two. Then cut along the crease to form a total of eight narrower sectors. Now form a parallelogram-like shape by arranging the pieces as you did in Step 4. Measure the base of this parallelogram.



Reproduced with permission from Alberta Education.

Please answer questions 4 and 5.

Questions:

1. a. What is the diameter of your circle to the nearest millimetre?

b. What is the radius to the nearest millimetre?

2. Using the formula below, determine the circumference of your circle to the nearest centimetre.

$$C = \pi d$$

3. a. What is the length of the base of the parallelogram?

b. How does the length of the base of the parallelogram compare to the circumference of the original circle?

My Notes

4. a. What is the length of the base of this parallelogram? Round to the nearest centimetre.

b. How does the length of the base of the parallelogram compare to the circumference of the original circle?

5. Measure the height of the parallelogram. The height of the parallelogram is similar to what part of the circle?

6. Calculate the area of the parallelogram. Remember,
 area of a parallelogram = base × height

7. How do you think the area of the parallelogram compares to the area of the circle?

8. How could you obtain a better approximation of the area of the circle?



Turn to the solutions at the end of the section and mark your work.

Using Technology



To summarize the work you have just completed, view *Circle Area* (<http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/glossary/Division03/Circle/CircleArea/index.html>). Scroll to the first applet and click the play button to begin the animation. You might want to keep this page open as you read through the rest of the Explore.

In question 8 of Activity 2, you were asked how you could obtain a better approximation of the area of a circle. One way to do this is by cutting the circle into smaller and smaller pieces. This can be difficult to do with your paper circle. However, technology allows us to see what would happen if we increased the number of sectors dramatically.

Open *Circle Area* (<http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/glossary/Division03/Circle/CircleArea/index.html>). This time, scroll down to the second applet. This applet lets you increase the number of sectors in the circle. Use the slider on the right to change the number of sectors. You should have noticed that, as you increase the number of sectors, the shape becomes closer and closer to that of a parallelogram.

If you used the media piece, see if you can write the general formula for the area of a circle with radius, r .

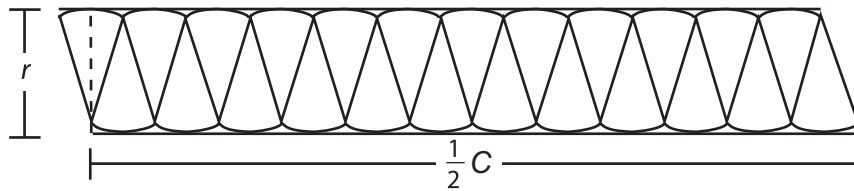
My Notes

My Notes

Bringing Ideas Together

In the Explore you discovered that the area of a circle is approximately the area of a parallelogram formed by fitting wedges or sectors of the circle together. This procedure can be used to develop the formula for the area of a circle in terms of its radius.

Formula for Area of a Circle



You can see that the base of the parallelogram is equal to half of the circumference. This is because half the sectors of the circle are across the top and half are across the base.

The height of the parallelogram is the radius. You know this because the radius is the distance from any point on the circumference to the centre-point of the circle. This is shown with a dashed line in the diagram above.

Now, let's see if we can extract a formula from this information.

area of a circle = area of the parallelogram shown above

$$A = \text{base} \times \text{height}$$

Substitute what we know about the base and height.

$$A = (\frac{1}{2} \text{ circumference}) \times r$$

Remember, $C = 2\pi r$.

$$A = (\frac{1}{2} \times 2\pi r) \times r$$

Now simplify.

$$A = \pi r^2$$

So, the formula for the area of a circle is:

$$A = \pi r^2$$

Now, have a look at the following examples before you try some problems on your own.

Example 1



Photo by Lynne Furrer © 2010

The diameter of a circular steel maintenance hole cover is 24 in. What is its area in square feet? Round your answer to two decimal places.

Solution

$$1 \text{ ft} = 12 \text{ in}$$

$$\begin{aligned} \text{So, } 24 \text{ in} &= \frac{24}{12} \text{ ft} \\ &= 2 \text{ ft} \end{aligned}$$

$$\text{The diameter} = 2 \text{ ft.}$$

$$\begin{aligned} \text{The radius} &= \frac{1}{2} \times \text{diameter} \\ &= \frac{1}{2} \times 2 \text{ ft} \\ &= 1 \text{ ft} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \pi r^2 \\ &= \pi \times 1 \text{ ft} \times 1 \text{ ft} \\ &= 3.141592654 \dots \text{ ft}^2 \\ &\approx 3.14 \text{ ft}^2 \end{aligned}$$

Remember, $r^2 = r \times r$.

The area of the maintenance hole cover is about 3.14 ft^2 .

My Notes

My Notes

Did You Know?

Maintenance hole covers are made circular so that they cannot slip down into the hole. A square, rectangular, or oval cover could fall into the hole it covers. Can you think of why?

**Example 2**

Photo by Edgewater Media © 2010

A circular punchbowl has a radius of 15 cm. If it is filled to the brim, what area of the punch is exposed to the air? Round to the nearest cm^2 .

Solution

$$A = \pi r^2$$

$$= \pi \times 15 \text{ cm} \times 15 \text{ cm}$$

$$= 706.8583471 \dots \text{ cm}^2$$

$$\approx 707 \text{ cm}^2$$

Remember, $r^2 = r \times r$.

Approximately 707 cm^2 of punch is exposed to the air.

Activity 3

Self-Check

My Notes

Please answer the following questions.

1. A circular cookie cutter has a diameter of 3 in. What is the area of dough this cookie cutter will cut? Round to one decimal place.

2. A water sprinkler covers a circular area 12 ft in radius. What area does it cover? Round to the nearest square foot.

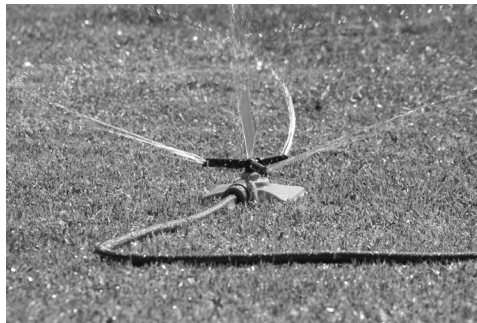


Photo by Eva Blanda © 2010

My Notes

3. If you didn't have a calculator, how might you estimate the area the sprinkler in question 2 covers?



Turn to the solutions at the end of the section and mark your work.

Composite Figures

In Lesson B you investigated areas of some composite figures. None of the figures in Lesson B contained circles. Now that you know how to find the areas of circles, you can try finding the areas of even more composite figures!



Open *Exploring Composite Figures* (http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/html/composite_figures/index.html). Try creating a few different composite figures that include circles. Look closely at how the area of each composite figure is calculated.

Example 3

The following is a plan for a backyard ice rink. The ends of the rink are semicircles.

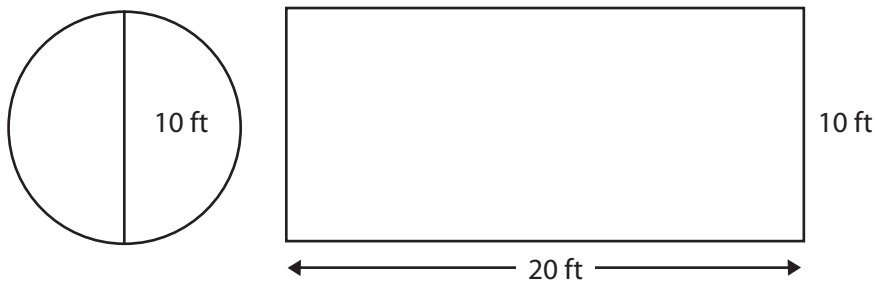


What is the area of the rink to the nearest square foot?

Solution

You may use *Exploring Composite Figures* (http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/html/composite_figures/index.html) to form the composite shape in this example. The media will show the formulas needed to calculate the area of each shape in the composite figure.

To find the area of the rink, separate the rink into a circle and a rectangle.



Find the area of the circle.

Its diameter is 10 ft. So, its radius is 5 ft.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 5 \text{ ft} \times 5 \text{ ft} \\ &= 78.53981634 \dots \text{ ft}^2 \end{aligned}$$

Find the area of the rectangle.

$$\begin{aligned} A &= lw \\ &= 20 \text{ ft} \times 10 \text{ ft} \\ &= 200 \text{ ft}^2 \end{aligned}$$

My Notes

The area of the rink = area of the circle + area of the rectangle
 $\approx 78.5398 \dots \text{ft}^2 + 200 \text{ft}^2$
 $\approx 279 \text{ft}^2$

The area of the rink is approximately 279ft^2 .

Area Summary

The steps in finding areas of composite figures are:

- Step 1. Break the figure into simple shapes.
- Step 2. Transfer the dimensions to the simple shapes.
- Step 3. Find the areas of each simple shape.
- Step 4. Combine the areas. Sometimes you will add areas and sometimes you will subtract areas.

Now, before you continue, get your Data Pages. Have a look at the “Geometric Formulae” table. You will see that some of the area formulas we’ve used are listed there.

Activity 4 Self-Check

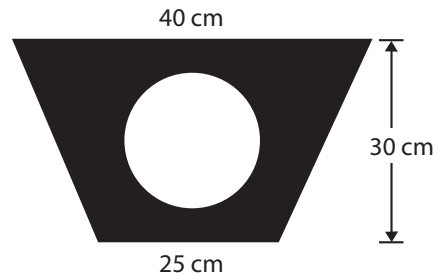
My Notes

Please answer the following questions.



You may find *Exploring Composite Figures* (http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/html/composite_figures/index.html) on helpful if you are stuck on a question.

1. Bernie is preparing a costume for a party. She has cut a circle 16 cm in diameter out of a trapezoidal piece of cloth.



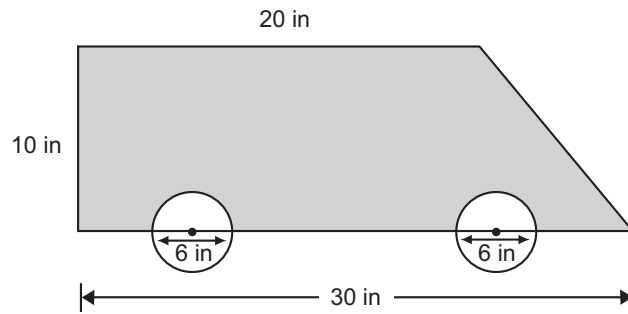
Determine the area of the remaining cloth. Round to the nearest square centimetre.

What two simple shapes are there in this composite figure?

My Notes

2. The following is a side panel for a toy car. The wheels have a diameter of 6 in.

What is the area of the coloured panel? Round to the nearest square inch.



Turn to the solutions at the end of the section and mark your work.

My Notes

Lesson Summary



Photo by Ian Bracegirdle © 2010

The design of traffic circles (also known as “roundabouts”) must take into account the volume and speed of traffic, as well as land available for development. If you examine the layout of the circle in the photo, you will notice that the paved section is a composite figure made up of circles with the same centre-point but different diameters. You could find the area of the paved section by finding the area of the larger circle and subtracting the area of the smaller circle.

In this lesson you explored a method of approximating the area of a circle. From this approximation, you saw how the general formula for the area of a circle was derived. You added circles to your repertoire of shapes and examined a few more composite figures.

Lesson D

Surface Area—Prisms and Pyramids

To complete this lesson, you will need:

- a cardboard box of any size
- the “Net for Activity 3” template from the appendix
- scissors
- tape
- a calculator
- Data Pages

In this lesson, you will complete:

- 6 activities

Essential Questions

- How are nets used to find the surface area of 3-D objects?
- How are formulas for the surface area of 3-D objects developed through examining nets?

My Notes

Focus

The cost of cardboard to manufacture containers like those pictured in the photograph depends on their total surface area. In this lesson you will investigate the surface area of three-dimensional objects with sides that involve common shapes such as rectangles, parallelograms, and triangles. A rectangular shipping container is just one example.



Photo by Franck Boston © 2010

Surface area is also an important consideration in renovations, construction, and decorating. Wallpaper, paint, sheet metal, wallboard, and insulation are just a few of many items the cost of which depends on the surface areas involved.

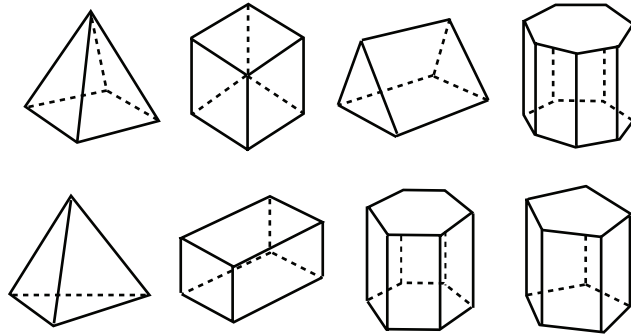
Get Started

So far in this section we've been looking at two-dimensional shapes. In this lesson we will explore some three-dimensional (3-D) shapes. Activity 1 will refresh your memory of some 3-D shapes.

Activity 1 Self-Check

My Notes

This illustration shows examples of prisms and **pyramids**.



1. How is a prism different from a pyramid? Fill in the blanks with the correct terms from the list below.

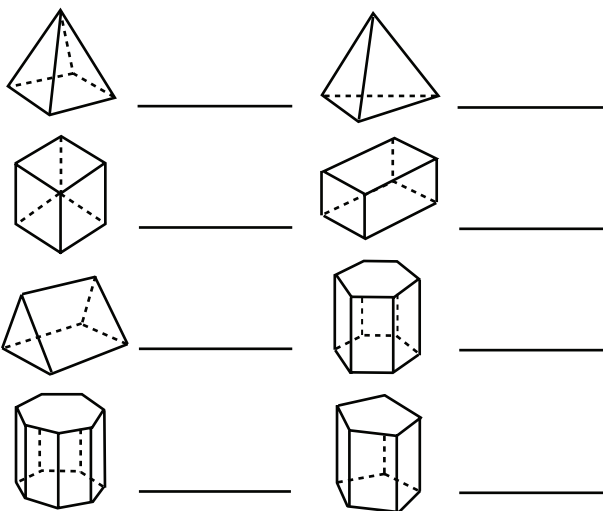
- apex
- lateral rectangular
- one base
- triangular
- two bases

A prism is a 3-D object with _____ joined by _____ **faces**.

A pyramid is a 3-D object with _____ whose _____ faces meet at a point called the _____.

My Notes

2. On the line beside each object, write *prism* or *pyramid*.



3. What does *area* mean?



Turn to the solutions at the end of the section and mark your work.

Investigating Nets

A **net** is a 2-D figure that makes a 3-D object when it's folded. We will be using the nets of 3-D objects to help us find surface areas.

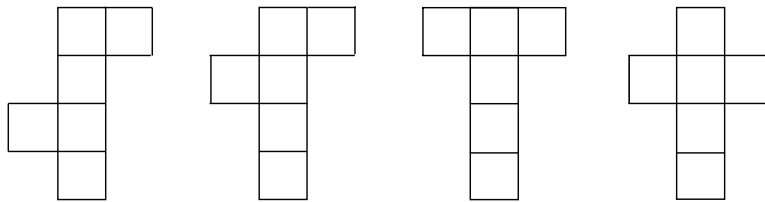
Activity 2 Try This

My Notes

Find a cardboard box—any **rectangular prism** or **cube** will do. For example a cereal box, small raisin box, or a shoe box.

Examine the 3-D object's sides and mark each side with an x. Pull the box apart and, if possible, keep all the sides with 'x's attached, and rip off any pieces without 'x's. Lay the pieces flat—what you have now is called a *net*.

Depending on how you unfold the cube or rectangular prism, the net could look like any of the following examples. Please note: the nets below are for a cube. If your box was a rectangular prism, the shapes that make up the net will be rectangles instead of squares.



There are 11 nets possible altogether. Sketch as many of the other nets as you can.

My Notes

You have just explored the net of a cube or rectangular prism. Now you'll look at the nets for other 3-D shapes.



Open *Exploring Composite Figures* (http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/html/composite_figures/index.html). Try to identify the correct net for each 3-D shape. Do not spend too much time on the cone and the cylinder. You will investigate the cone and the cylinder in the next lesson.



Turn to the solutions at the end of the section and mark your work.

Explore

Imagine taking a 3-D object and submerging it in a tub of water. The area of the object that is in contact with the water is called the **surface area**.

How would you determine the surface area of the object?

One way to do this is to peel off the outer layer of the object and then calculate the area of the “peel”. In math, the “peel” is called the net.

In Activity 3, you will use the net of an object to help you find surface area.

Activity 3

Try This

My Notes

Step 1: Go to the appendix at the end of the section and get the page titled “Net for Activity 3.” Cut out the net, fold it, and then tape this net to form a prism. The grid should be on the outside of the prism.

Now answer these questions.

1. a. What is the name of the 3-D object you created?

- b. Make a sketch of the 3-D object you made, and show its dimensions.

2. What are the areas of each face? Write the area on each face.

My Notes

3. What is the surface area of the prism based on the net?



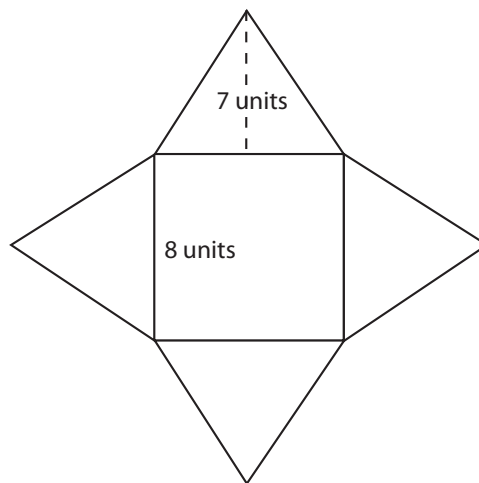
Turn to the solutions at the end of the section and mark your work.

Using Nets To Find Surface Area

In Activity 3 you used a net to help find the surface area of a rectangular prism. A similar approach can be used to find the surface area of a pyramid. Have a look at the following example.

Example 1

The net for a square-based pyramid is shown below. Find the surface area of this pyramid.



Solution

To find the surface area of this object, find the areas of all of the faces, and then add these areas up.

The base of the pyramid is a square 8 units on a side.

$$\begin{aligned} A_{\text{square}} &= s^2 \\ &= (8 \text{ units})^2 \\ &= 64 \text{ units}^2 \end{aligned}$$

$8^2 = 8 \times 8$

The pyramid has four identical triangular faces. The height of each triangular face, measured along the surface, is 7 units. Find the area of a single triangular face.

$$\begin{aligned} A_{\text{triangle}} &= \frac{bh}{2} \\ &= \frac{(8 \text{ units})(7 \text{ units})}{2} \\ &= \frac{56 \text{ units}^2}{2} \\ &= 28 \text{ units}^2 \end{aligned}$$

You'll find this formula on your Data Pages.

Now, total up the areas of all the faces.

$$\begin{aligned} SA_{\text{pyramid}} &= A_{\text{square}} + 4 A_{\text{triangle}} \\ &= 64 \text{ units}^2 + 4(28 \text{ units}^2) \\ &= 64 \text{ units}^2 + 112 \text{ units}^2 \\ &= 176 \text{ units}^2 \end{aligned}$$

There are 4 triangular faces.

The surface area of the square-based pyramid is 176 square units.



To see another example where we are asked to find the surface area of a square-based pyramid, go to *Surface Area of a Pyramid* (http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/html/surface_pyramid.htm).

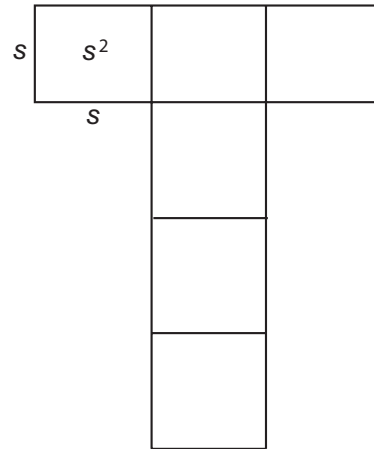
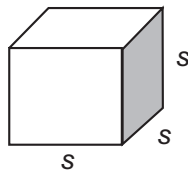
My Notes

Bringing Ideas Together

In Get Started and Explore you reviewed the nets of cubes, rectangular prisms, and pyramids. You can use nets to develop formulas for surface area.

Activity 4 Try This

The net for a cube, s units on a side, is shown below.



Please answer the following questions.

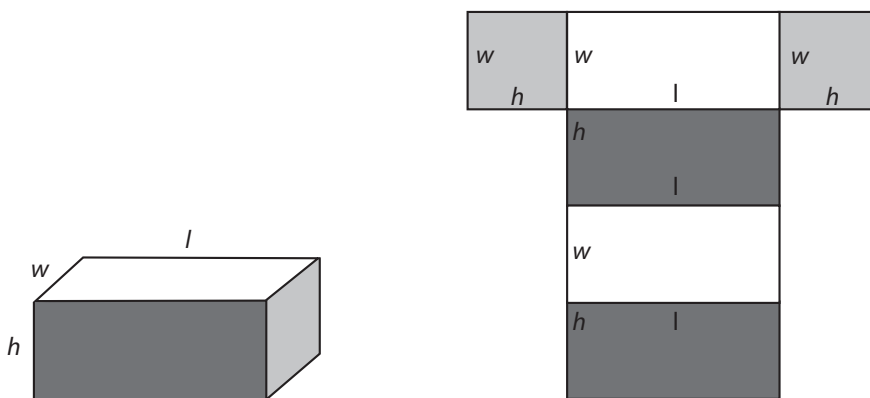
1. What do you notice about the areas of the squares in the net?

2. How many squares make up the net for a cube?

My Notes

3. What would you suggest for the formula for the surface area of a cube?

The net for a rectangular prism is shown below. The faces having the same areas are the same colour.



Please answer the following questions.

4. What are the areas of the three unique faces?

My Notes

5. What would you suggest for the formula for the surface area of a rectangular prism?



Turn to the solutions at the end of the section and mark your work.

Surface Area Formulas

In Activity 4, you came up with formulas for the surface area of a cube and for the surface area of a rectangular prism. You can find the formulas for the surface areas of several 3-D objects on your Data Pages. Have a look at your Data Pages and fill in the formulas below.

Surface area of a rectangular prism: _____

Surface area of a general right prism: _____

Surface area of a square-based pyramid: _____

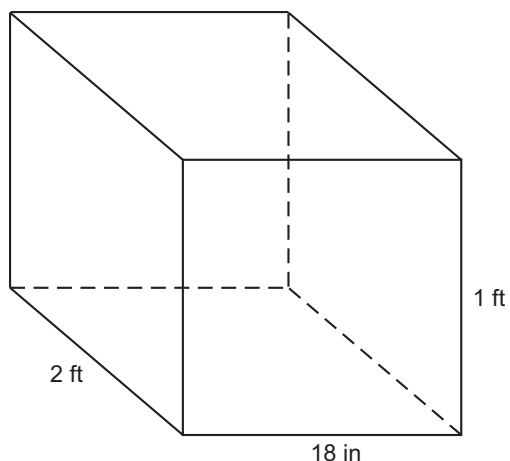
Surface area of a general right pyramid: _____

Example 2

What is the surface area of a cardboard container 2 ft long, 18 in wide, and 1 ft high?

Solution

Start by drawing the object.



All dimensions must be in the same units.

Since $1 \text{ ft} = 12 \text{ in}$,

$$\begin{aligned} 18 \text{ in} &= \frac{18}{12} \text{ ft} \\ &= 1.5 \text{ ft} \end{aligned}$$

So the dimensions are:

$$l = 2 \text{ ft}$$

$$w = 1.5 \text{ ft}$$

$$h = 1 \text{ ft}$$

$$\begin{aligned} SA &= 2 (wh + lw + lh) \\ &= 2 [(1.5 \text{ ft} \times 1 \text{ ft}) + (2 \text{ ft} \times 1.5 \text{ ft}) + (2 \text{ ft} \times 1 \text{ ft})] \\ &= 2 [1.5 \text{ ft}^2 + 3 \text{ ft}^2 + 2 \text{ ft}^2] \\ &= 2 [6.5 \text{ ft}^2] \\ &= 13 \text{ ft}^2 \end{aligned}$$

The surface area of the box is 13 ft^2 .

You could also have solved this question using inches.

Since 1 ft is 12 in ,

$$\begin{aligned} 2 \text{ ft} &= (2 \times 12) \text{ in} \\ &= 24 \text{ in} \end{aligned}$$

So the dimensions are:

$$l = 24 \text{ in}$$

$$w = 18 \text{ in}$$

$$h = 12 \text{ in}$$

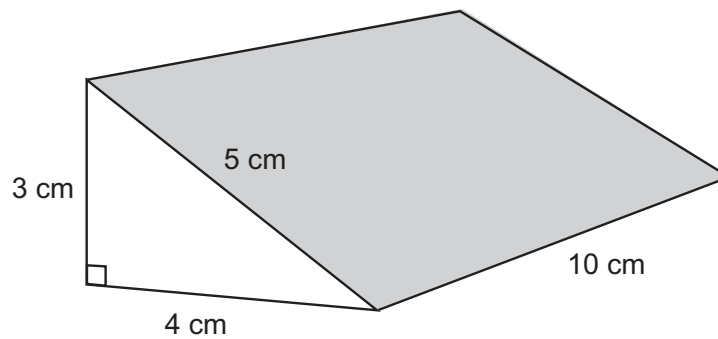
My Notes

$$\begin{aligned}
 SA &= 2 (wh + lw + lh) \\
 &= 2 [(18 \text{ in} \times 12 \text{ in}) + (24 \text{ in} \times 18 \text{ in}) + (24 \text{ in} \times 12 \text{ in})] \\
 &= 2 [216 \text{ in}^2 + 432 \text{ in}^2 + 288 \text{ in}^2] \\
 &= 2 [936 \text{ in}^2] \\
 &= 1872 \text{ in}^2
 \end{aligned}$$

The surface area of the box is 1872 in^2 .

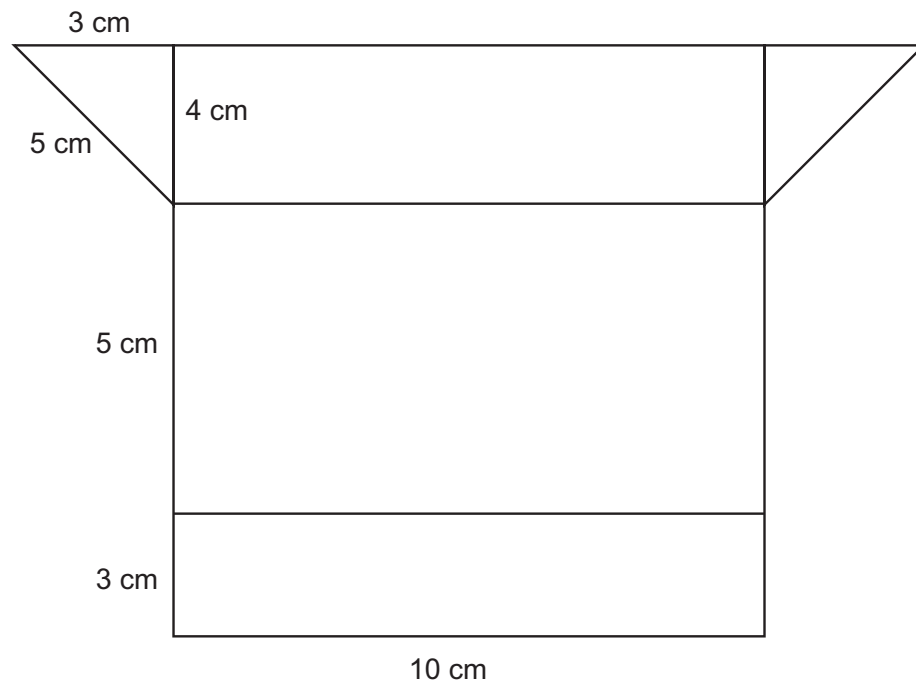
Example 3

Find the surface area of the triangular prism. Each triangular end is 4 cm along the base and 3 cm high, and 5 cm along its longest side. The prism is 10 cm long. Note: diagram is not drawn to scale.

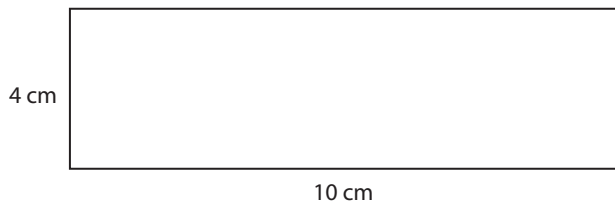


Solution

Sketch the net of the prism and then find the areas of all the faces. The surface area of the prism is the sum of the areas of the faces.

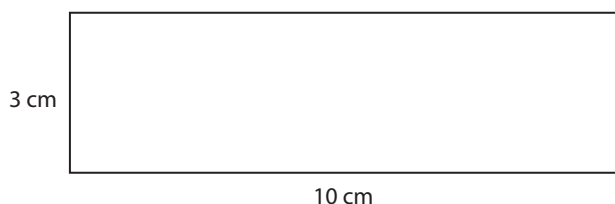


The base of the prism is a rectangle 10 cm by 4 cm.



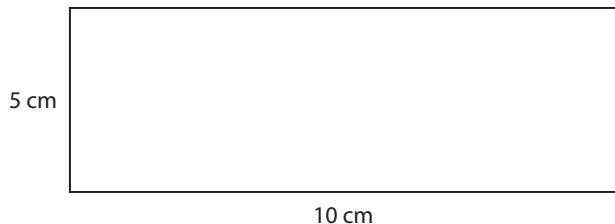
$$\begin{aligned}\text{Area of base} &= 10 \text{ cm} \times 4 \text{ cm} \\ &= 40 \text{ cm}^2\end{aligned}$$

The back of the prism is a rectangle 10 cm by 3 cm.



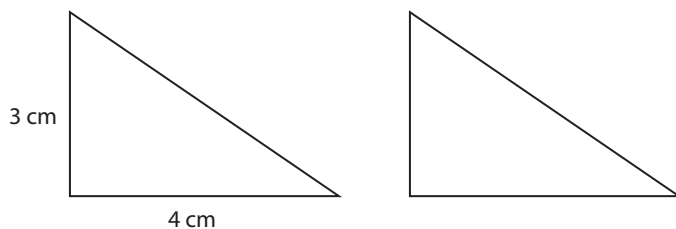
$$\begin{aligned}\text{Area of back} &= 10 \text{ cm} \times 3 \text{ cm} \\ &= 30 \text{ cm}^2\end{aligned}$$

The slant face of the prism is a rectangle 10 cm by 5 cm.



$$\begin{aligned}\text{Area of back} &= 10 \text{ cm} \times 5 \text{ cm} \\ &= 50 \text{ cm}^2\end{aligned}$$

Each end is a triangle with a base of 4 cm and a height of 3 cm.



$$\begin{aligned}\text{Area of each triangle} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 4 \text{ cm} \times 3 \text{ cm} \\ &= 6 \text{ cm}^2\end{aligned}$$

My Notes

$SA_{\text{prism}} = \text{area of base} + \text{area of back} + \text{area of slant face} + 2 (\text{area of triangle})$

$$\begin{aligned} SA &= 40 \text{ cm}^2 + 30 \text{ cm}^2 + 50 \text{ cm}^2 + 2 \times 6 \text{ cm}^2 \\ &= 40 \text{ cm}^2 + 30 \text{ cm}^2 + 50 \text{ cm}^2 + 12 \text{ cm}^2 \\ &= 132 \text{ cm}^2 \end{aligned}$$

The surface area of the prism is 132 cm^2 .

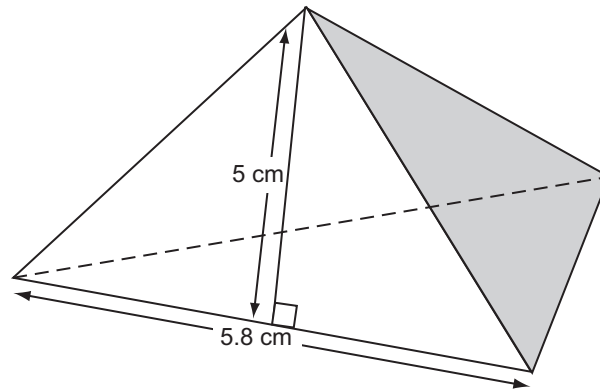
Activity 5
Self-Check

Answer the following questions. When you are finished, check your answers.

1. A cubical shipping carton is 30 in on a side. What is its surface area in cubic feet?

My Notes

4. A tetrahedron has 4 identical triangular faces. What is the surface area of the tetrahedron shown below?




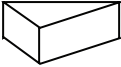




Turn to the solutions at the end of the section and mark your work.

Activity 6 Mastering Concepts

My Notes

So far, in this lesson, you’ve used nets to help you find the surface areas of objects. When you answer the question below, try using views instead of nets.

	3-D	Top	Front
a.			
b.			



If you’re not sure how to find the different views of an object, and you have access, practise by using the media piece *Exploring Composite Figures* (http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/html/composite_figures/index.html). Click on the tab titled “Use It 2: Views” to try the views.

Try the question below.

Star really liked the green fabric on her couch when she bought it. Now, she’s getting a bit tired of it. She plans to sew a slip-cover for her couch so that she can change the look of the couch whenever she wants! She needs to estimate the fabric required.

How can she minimize her calculations as she comes up with an estimate? Describe the process Star should use to estimate the amount of fabric needed.



Photo by Tr1 sha © 2010

My Notes



Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes



Photo by Vladimirs Koskins © 2010

The worker in the photograph is making repairs to the exterior of a home. The cost of similar repairs depends on the extent of the area damaged. Area is a factor in the cost of materials, repairs, and renovations.

In this lesson you reviewed the nets of several 3-D shapes. You investigated how nets can help you to develop formulas for finding surface area of 3-D shapes, and to calculate surface areas.

Lesson E

Surface Area—Cylinders and Cones

To complete this lesson, you will need:

- scissors
- string
- tape
- a cylindrical can with a removable label, such as a can of soup
- “Cone Net Template” from the appendix
- a calculator
- Data Pages

In this lesson, you will complete:

- 5 activities

Essential Questions

- How can you determine the surface areas of cones and cylinders?
- What are some everyday applications of surface area of these shapes?

My Notes

Focus



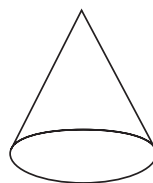
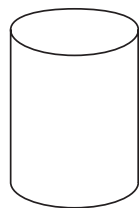
Photo by Losevsky Pavel © 2010

Cylindrical and conical shapes appear in modern architectural designs. Can you identify these shapes in the musical hall and surrounding buildings in the photograph?

In this lesson you will continue to investigate the surface area of 3-D shapes. You will explore the surface areas of **cylinders** and **cones**.

Get Started

To start this lesson, let's review what you know about cylinders and cones.





If you have access, open the following two applets:

- *The Cylinder* (<http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/glossary/Division02/Cylinder/index.html>)
- *The Cone* (<http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/glossary/Division03/Cone/index.html>)

My Notes

Use these applets to investigate some possible sizes and shapes of cylinders and cones. You can drag the moveable points to change the dimensions of the objects. You can also rotate the objects to view them from different perspectives.

When you are ready, please complete Activity 1.

Activity 1
Self-Check

1. a. Describe the characteristics of a cylinder.

- b. Describe the characteristics of a cone.

My Notes

2. a. Draw the net for a cylinder.

b. Draw the net for a cone.



Turn to the solutions at the end of the section and mark your work.

Explore

In Lesson D, you found the surface areas of a number of 3-D shapes. Regardless of what shape you were looking at, you found the surface area by first finding the area of each face and then adding those areas up. The process is no different for cylinders and cones—let's try it!

Activity 2

Try This

My Notes

You will need a ruler, a soup can, scissors, a piece of string, and a calculator to complete this activity.

Follow the steps listed under “Procedure,” and then complete the “Questions” portion of the activity. Use the table below to record your measurements.

Cylinder Measurements

Item	Measurement (cm)
height of can	
diameter of circular top of can	
circumference of circular top of can	
length of label	
width of label	

Procedure:

Examine a soup can or any other can with a removable label.

Step 1: Measure the height of the can, and record your measurement in the table above.

Step 2: Measure the diameter of the circular base of the can, and record your measurement in the table above.

3. What is the surface area of the can?

My Notes

4. What do you notice about the dimensions of the can's rectangular label?



Turn to the solutions at the end of the section and mark your work.

Cylinders

In Activity 2, you should have noticed that the length of the can's label was approximately the same as the circumference of the circular base.



To see this relationship, look at *Constructing Cylinders* (http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/html/ma0811b2f_cylinder.html).

We'll come back to this idea in Bringing Ideas Together. For now, move on to the next activity to explore cones.

My Notes

Activity 3

Try This

You will need scissors, a ruler, “Cone Net Template” from the appendix, and a calculator to complete this activity.

Follow the steps listed under “Procedure,” and then complete the “Questions” portion of the activity. Use the table below to record your measurements.

Cone Measurements

Item	Measurement (cm)
slant height	
radius of circular base	
arc length of partial circle	

Procedure:

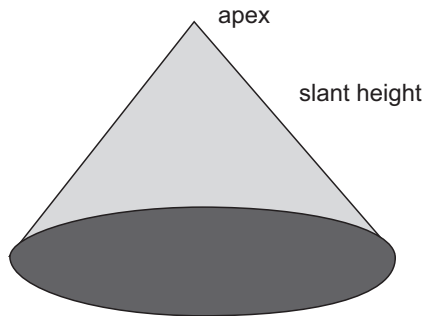
Step 1: Go to the appendix and get the “Cone Net Template.” Cut out the net by cutting along the solid lines. Do not cut through the dashed line.

Step 2: Create a cone by:

- pulling the straight line segments together, joining points A and B; and
- folding the smaller circle over to create a base.

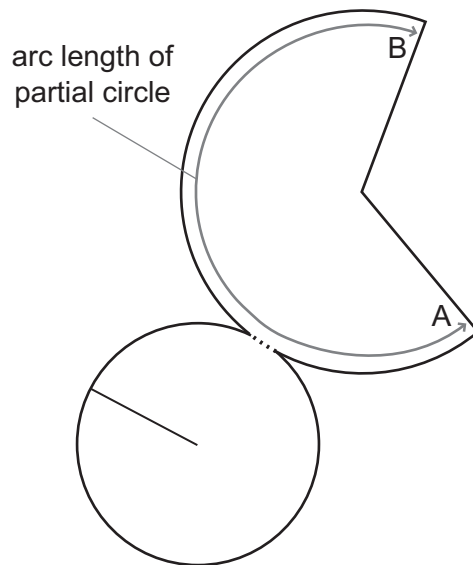
My Notes

Step 3: Measure the **slant height** of the cone by measuring along the side of the cone from the base to the **apex**. Record your measurement in the table. Note: the slant height of the 3-D cone is the same as the radius of the partial circle in the 2-D net. You might try unfolding the cone to see that this is true.



Step 4: Measure the radius of the circular base. (You may unfold the net at this point if it makes it easier.) Record your measurement in the table.

Step 5: Unfold the cone so that you have the flat net in front of you. Measure the arc length of the partial circle in the net using a piece of string and a ruler. Record your measurement in the table.



My Notes

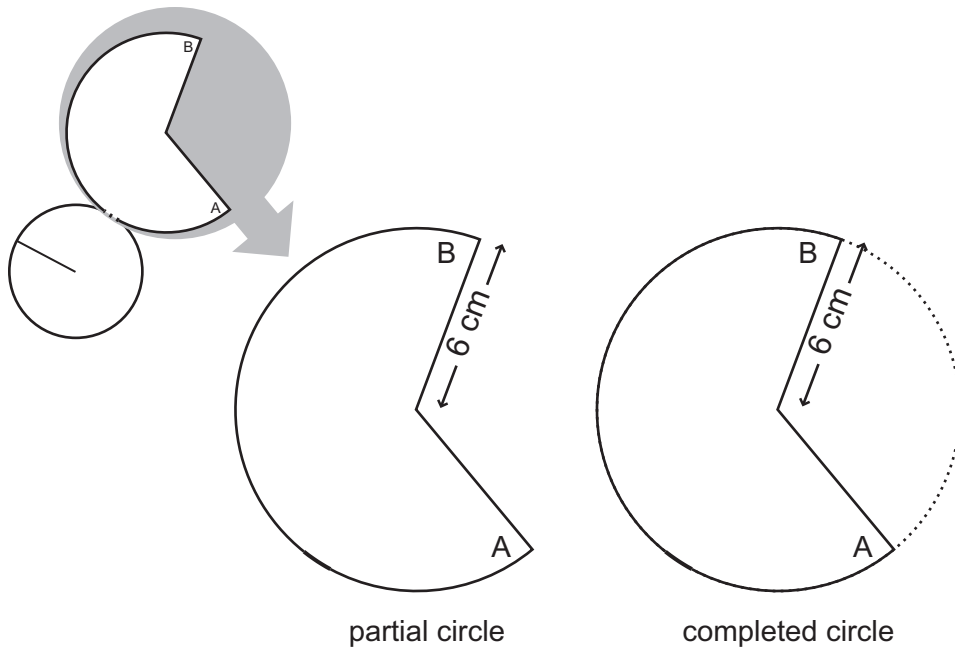
Questions:

1. Find the area of the circular base of the cone.

2. Use the information below to calculate the area of the curved surface of the cone.

We don't have a formula to find the area of a partial circle. We do, however, know how to find the area of a complete circle.

Let's look at the partial circle from the net. If we continue the arc clockwise from point B to point A, we will close the circle. Let's call this the "completed circle."



- a. Find the area and the circumference of the completed circle. Remember, the radius of this circle is the same as the slant height of the cone.

My Notes

The area of the partial circle is part of the area of the completed circle. We'll set up a proportion so that we can find the area.

$$\frac{\text{area of partial circle}}{\text{area of completed circle}} = \frac{\text{arc length of partial circle}}{\text{circumference of completed circle}}$$

- b. Use this proportion, and the information from your data table and your answer from question 2 (a) to find the area of the partial circle. Remember, this partial circle becomes the curved surface of the 3-D cone.

My Notes

3. Now that you have found the area of the circular base of the cone, and the area of the curved surface (the partial circle in the cone's net), find the total surface area of the cone.

4. a. Calculate the circumference of the circular base.

b. Compare the circumference of the circular base with the arc length of the partial circle. What do you notice?



Turn to the solutions at the end of the section and mark your work.

My Notes

Bringing Ideas Together

So far in this lesson, you’ve looked at the nets for cylinders and cones, and you’ve calculated the surface areas of one specific cylinder and one specific cone. Now it’s time to come up with general formulas for the surface areas of these two shapes.

A general formula is a formula that you can use in any situation. When you find the general formula for the surface area of a cylinder, it should work for all cylinders. Similarly, when you find the general formula for the surface area of a cone, it should work for all cones.

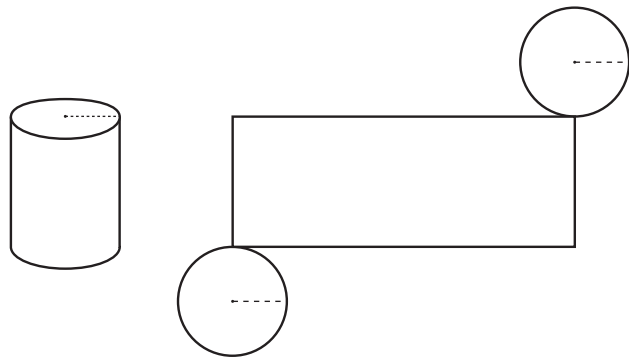
Cylinder Formula

Think back to the work you did in Activity 2.



If you want, review the nets and cylinder using the media piece *Constructing Cylinders* (http://media.openschool.bc.ca/osebmedia/math/mathawm10/html/ma0811b2f_cylinder.html).

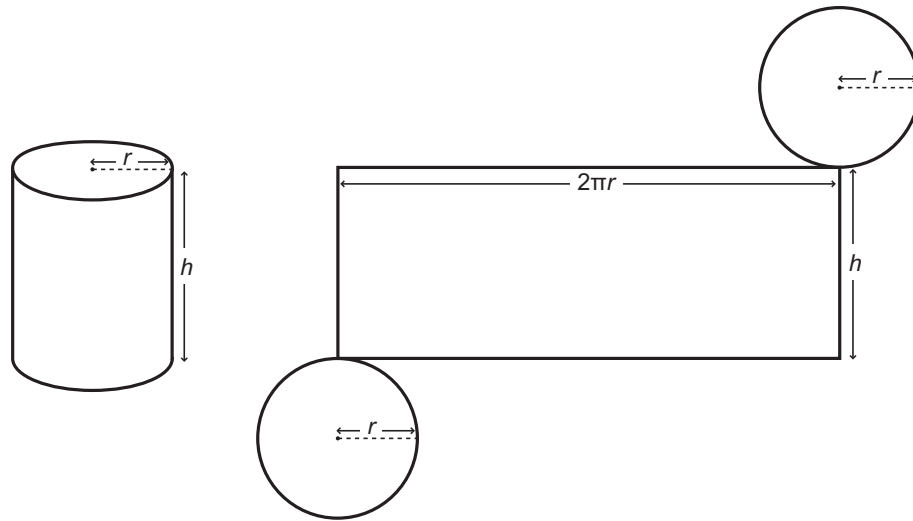
A cylinder and its net are shown below. If the radius of the cylinder’s circular base is r units in length, what is the length of the rectangle in the cylinder’s net?



Because the circumference of the circular base is $2\pi r$, the length of the rectangle will also be $2\pi r$.

My Notes

Let's label the diagram with the information that we know.



To find the surface area of any cylinder, you need to find the sum of the areas of the shapes that make up the cylinder's net.

$$SA_{\text{cylinder}} = 2A_{\text{base}} + A_{\text{rectangle}}$$

$$SA_{\text{cylinder}} = 2(\pi r^2) + (2\pi r)h$$

Multiply the length and width of the rectangle to find its area.

$$SA_{\text{cylinder}} = 2\pi r^2 + 2\pi rh$$

You can use this formula to find the surface area of any cylinder. Let's work through an example.

Example 1

The cost of aluminum in a soup can depends on the can's surface area. A cylindrical soup can is 6.5 cm in diameter and 9.5 cm tall. What is the can's surface area? Round your answer to one decimal place.

Solution

Determine the radius:

$$\begin{aligned} r &= \frac{1}{2} \times d \\ &= \frac{1}{2} \times 6.5 \text{ cm} \\ &= 3.25 \text{ cm} \end{aligned}$$

Don't round yet!

Use the formula to find the surface area of the can.

My Notes

$$\begin{aligned}
 SA &= 2\pi r^2 + 2\pi rh \\
 &= 2\pi(3.25 \text{ cm})(3.25 \text{ cm}) + 2\pi(3.25 \text{ cm})(9.5 \text{ cm}) \\
 &= 260.3594912 \dots \text{ cm}^2 \\
 &\approx 260.4 \text{ cm}^2
 \end{aligned}$$

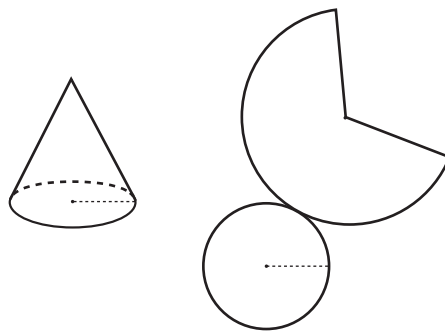
This is the answer, so round to one decimal place.

The surface area is 260.4 cm².

Cone Formula

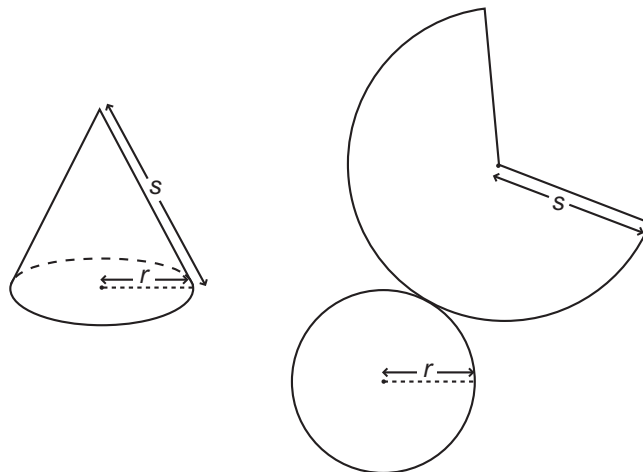
Think back to the work you did in Activity 3.

A cone and its net are shown below.



If the slant height of the cone is s units long, what is the radius of the partial circle in the cone's net?

Let's label the diagram with the information that we know.



My Notes

To find the surface area of any cone, you need to find the sum of the areas of the shapes that make up the cone’s net.

Remember, the area of the partial circle in the cone’s net is a bit tricky to find. In Activity 3 we set up a proportion to find the area. We can do the same thing here.

$$\frac{\text{area of partial circle}}{\text{area of completed circle}} = \frac{\text{arc length of partial circle}}{\text{circumference of completed circle}}$$

$$\text{area of partial circle} = \frac{\text{arc length of partial circle}}{\text{circumference of completed circle}} \times \text{area of completed circle}$$

In this case, we don’t have numbers to substitute in; we’ll have to use variables. Remember that the arc length of the partial circle is the same length as the circumference of the cone’s circular base.

This is equal to the circumference of the circular base.
 $C_{\text{base}} = 2\pi r$

$$\text{area of partial circle} = \frac{\text{arc length of partial circle}}{\text{circumference of completed circle}} \times \text{area of completed circle}$$

$$A_{\text{partial circle}} = \frac{\cancel{2} \pi r}{\cancel{2} \pi s} \times \pi s^2$$

$C_{\text{completed circle}} = 2\pi s$

$A_{\text{completed circle}} = \pi s^2$

Simplify.

$$A_{\text{partial circle}} = \pi r s$$

Now, let’s find the formula for the surface area of the cone.

$$SA_{\text{cone}} = A_{\text{base}} + A_{\text{partial circle}}$$

$$SA_{\text{cone}} = \pi r^2 + \pi r s$$

You can use this formula to find the surface area of any cone.



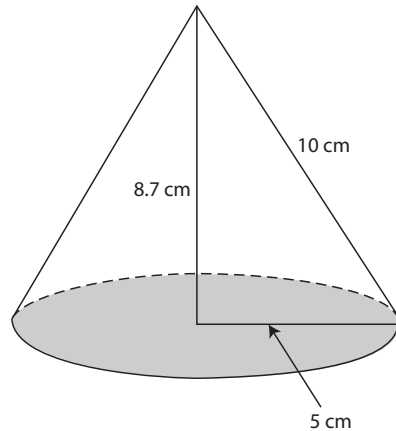
If you’d like to see a different way of finding the formula for the surface area of a cone, go to *Surface Area of a Cone* (http://www.web-formulas.com/Math_Formulas/Geometry_Surface_of_Cone.aspx).

My Notes

Now, let's work through an example.

Example 2

Determine the surface area of cone. Round to the nearest cm^2 .



Solution

$$\begin{aligned}
 SA &= \pi r^2 + \pi rs, \text{ where } r = 5 \text{ cm and } s = 10 \text{ cm} \\
 &= \pi(5 \text{ cm})(5 \text{ cm}) + \pi(5 \text{ cm})(10 \text{ cm}) \\
 &= 235.6 \text{ cm}^2
 \end{aligned}$$

$s =$ slant height
Don't use 8.7 cm -
that's the height of the cone.

Remember, $r^2 = (r)(r)$.

The surface area of the cone is 235.6 cm^2 .

Finding and Using Formulas

You can find the surface area formulas for cylinders and cones on your Data Pages. Go to your Data Pages now and find the formulas. Record them here.

Surface area of a cylinder: _____

Surface area of a cone: _____

My Notes



If you have access and if you would like to look at a few more examples, you can use *Surface Area and Volume* (http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/html/ma0812a1f_areavolume.html).

Open *Surface Area and Volume*.

- Select “Surface Area.”
- Select the cylinder on one side and the cone on the other.
- Use the sliders to adjust the sizes of your 3-D shapes and examine the surface area calculations.

When you’re ready, try the next activity.

Activity 4

Self-Check

Using your Data Pages, please answer the following questions.



If you have access, you may use *Surface Area and Volume* (http://media.openschool.bc.ca/osbcmmedia/math/mathawm10/html/ma0812a1f_areavolume.html).

1. During harvest on the prairies, you will often see wheat stored in conical piles in the fields. The wheat is protected from soil moisture by a plastic ground sheet. A conical pile of wheat 15 ft in diameter will be about 3.5 ft high and have a slant height of about 8.3 ft. What is the surface area of the pile exposed to the air? Round to one decimal place.

My Notes

4. The picture shows a view of an Alberta seed cleaning plant. The hoppers are shaped in the form of a cylinder on the top of a cone. If the diameter of the cylinder is 12 ft, the height of the cylindrical portion is 18 ft, and the slant height of the cone is 8 ft, what is the area of the outside of the cylinder and cone? Ignore the top surface of the cylinder in your calculation. Round to the nearest square foot.



Photo by Dan Leskiew



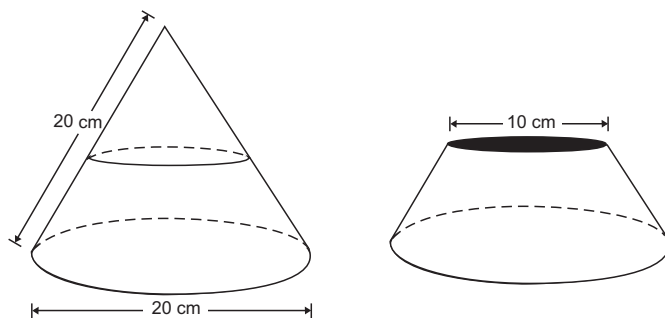
Turn to the solutions at the end of the section and mark your work.

Activity 5

Mastering Concepts

My Notes

A wooden cone with a diameter of 20 cm and a slant height of 20 cm is cut, halfway up the side, into two pieces. The top is discarded. What is the surface area of the remaining bottom piece shown on the right (including the newly flattened top)? Round your answer to the nearest cm^2 .



My Notes



Turn to the solutions at the end of the section and mark your work.

Lesson Summary



Photo by Philip Lange © 2010

Historically, the First Nations peoples of the Canadian Prairies built conical shaped teepees for shelter. Teepees were warm in the winter and cool in the summer. They were easy to dismantle and transport as the people followed the herds of buffalo, which provided food and shelter. Buffalo skins were sewn together to make the teepee covers. The size and shape of the buffalo-skin covers is an application of the surface area of a cone.

In this lesson you investigated the surface areas of cylinders and cones. You found general formulas and then applied those formulas and solved problems.

Lesson F

Perimeter and Area

To complete this lesson, you will need:

- grid paper (you will find some in the appendix)
- a ruler
- a calculator

In this lesson, you will complete:

- 6 activities

Essential Questions

- How does changing one or more dimensions of a rectangle affect the rectangle's perimeter?
- How does changing one or more dimensions of a rectangle affect the rectangle's area?

My Notes

Focus



Photo by Stephen Leech © 2010

Raveena has taken a photograph of her neighbour's deck. She wants to have a similar deck built at her home. However, as her backyard is not as large, she intends to scale it back. The shape of her deck will be the same, but it will cover a smaller area.

Raveena thinks that her deck will have to be half the area. How much shorter will the dimensions of Raveena's deck be than her neighbour's?

In this lesson we'll explore how changing the dimensions of a rectangle affects the perimeter and area of the rectangle.

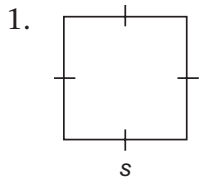
Get Started

To start, let's briefly review perimeter and area.

Perimeter is the distance around a shape. We measure perimeter using linear units such as centimetres (cm), metres (m), inches (in), and feet (ft). **Area** is the amount of space that a shape covers. We measure area using square units such as square centimetres (cm²), square metres (m²), square inches (in²), and square feet (ft²).

My Notes

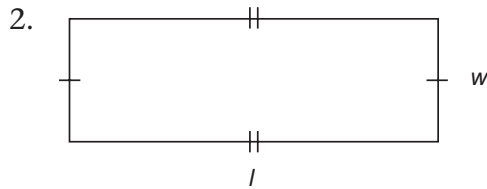
Activity 1
Self-Check



shape: _____

perimeter = _____

area = _____



shape: _____

perimeter = _____

area = _____



Turn to the solutions at the end of the section and mark your work.

Explore

To look at the relationship between the dimensions of a rectangle and its perimeter and area, you'll complete two activities. In the first, you'll work with a square, and see what happens when you increase the lengths of the sides. The second activity is similar, except you'll work with a rectangle.

Activity 2
Try This

In this activity you will examine how changing the length of the sides of a square affects its perimeter and area.

You will need a sheet of grid paper. You may use your own, or use the pages in the appendix.

My Notes



If you have access to the internet, you may want to use an online applet to help you complete this activity. Go to *Demonstration Applet Perimeter and Area* (<http://www.learnalberta.ca/content/memg/Division03/Rectangle/index.html>).

Follow the steps listed under “Procedure” and then complete the “Questions” portion of the activity. Use the table below to record your measurements.

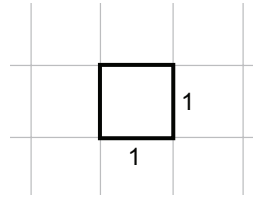
Perimeter and Area of Squares

Dimensions of Square	Perimeter in Units	perimeter of square	Area in Units ²	area of square
		perimeter of 1×1 square		area of 1×1 square
$s = 1$				
$s = 2$				
$s = 3$				
$s = 4$				
$s = 5$				
$s = 6$				

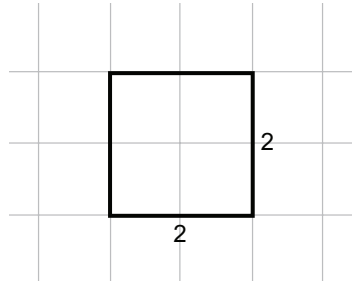
My Notes

Procedure:

Step 1: Draw a square that has sides one unit in length. Calculate the perimeter and area of the square and enter these items in the table provided.



Step 2: Draw a second square. This square should have sides 2 units long. Calculate the perimeter and area of the square and enter these items in the table provided.



Step 3: Draw four more squares using the dimensions listed in the first column of the table. Calculate the perimeter and area of the squares and enter these items in the table provided.

Step 4: Complete all other parts of the table. The first row is shown below as an example.

Perimeter and Area of Squares

Dimensions of Square	Perimeter in Units	perimeter of square	Area in Units ²	area of square
		perimeter of 1 × 1 square		area of 1 × 1 square
$s = 1$	4	$\frac{4}{4} = 1$	1	$\frac{1}{1} = 1$

Questions:

As you answer the questions below, think of the 1 × 1 square as your starting point.

- How much larger is the **perimeter** of the square with a length of 3 than the original 1 × 1 square?

My Notes

2. How much larger is the **area** of the square with a length of 3 than the original 1×1 square?

3. Based on the trend you have noticed in this table, if you made the sides of a square 7 times as long, how would the **perimeter** of the square change?

4. If you made the sides of a square 7 times as long, how would the **area** of the square change?

5. If you made the sides of a square k times as long, how would the **perimeter** of the square change?

6. If you made the sides of a square k times as long, how would the **area** of the square change?



Turn to the solutions at the end of the section and mark your work.

Activity 3

Try This

My Notes

In this activity you will explore how the perimeter and the area of a rectangle change when the dimensions of the rectangle change.

You will need a sheet of grid paper. You may use your own, or use the pages in the appendix.



If you have access to the internet, you may want to use an online applet to help you complete this activity. Go to *Demonstration Applet Perimeter and Area* (<http://www.learnalberta.ca/content/memg/Division03/Rectangle/index.html>).

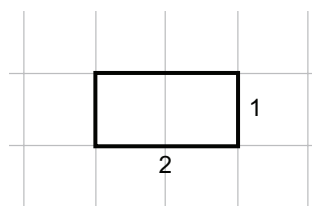
Follow the steps listed under “Procedure” and then complete the “Questions” portion of the activity. Use the table below to record your measurements.

Perimeter and Area of Rectangles

Scale Factor	Dimensions of Rectangle	Perimeter in Units	perimeter of rectangle	Area in Units ²	area of rectangle
			perimeter of 1 × 2 rectangle		area of 1 × 2 rectangle
1	$l = 2, w = 1$				
2	$l = 4, w = 2$				
	$l = 6, w = 3$				
	$l = 8, w = 4$				
	$l = 10, w = 5$				
	$l = 12, w = 6$				

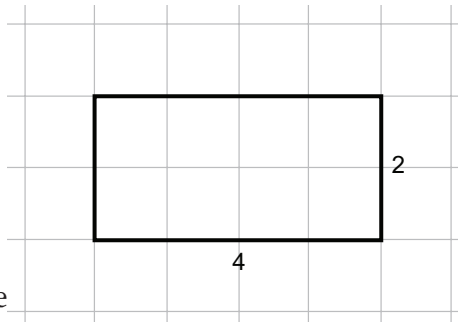
Procedure:

Step 1: Draw a rectangle that has dimensions of 1 unit in width and 2 units in length (1×2). Calculate the perimeter and area of the rectangle and enter these items in the table provided above.



My Notes

Step 2: Draw a second rectangle. To obtain the dimensions of the second rectangle, multiply the dimensions of the first rectangle by two. This new rectangle should have the dimensions 2 units by 4 units. Calculate the perimeter and area of the rectangle and enter these items in the table provided.



Step 3: Draw four more rectangles using the dimensions listed in the second column of the table. Calculate the perimeter and area of the rectangles and enter these items in the table provided.

Step 4: Complete all other parts of the table. The first row is shown below as an example.

Perimeter and Area of Rectangles

Scale Factor	Dimensions of Rectangle	Perimeter in Units	perimeter of rectangle	Area in Units ²	area of rectangle
			perimeter of 1 × 2 rectangle		area of 1 × 2 rectangle
1	$l = 2, w = 1$	6	$\frac{6}{6} = 1$	2	$\frac{2}{2} = 1$

Questions:

*Note: The numbers by which you multiply the dimensions of the original rectangles are called **scale factors**.*

1. What happens to the perimeter when the scale factor is increased by 1?

2. If the scale factor were 7, how many times greater would the perimeter of the new rectangle be compared to the perimeter of the original rectangle?

My Notes

3. If the scale factor were 7, how many times larger would the area of the new rectangle be compared to the area of the original rectangle?

4. If the scale factor were k , how many times longer would the perimeter of the new rectangle be compared to the perimeter of the original rectangle?

5. If the scale factor were k , how many times larger would the area of the new rectangle be compared to the area of the original rectangle?



Turn to the solutions at the end of the section and mark your work.

Bringing Ideas Together

In Getting Started, Explore, and Share you discovered the following:

- If both the length and width of a square or rectangle are changed, then the perimeter and area change.
- If the scale factor for changing a figure is k , the perimeter of the new figure is k times the perimeter of the original figure.
- If the scale factor for changing a figure is k , the area is k^2 times the area of the original figure.

Please work through the following examples.

Example 1

My Notes

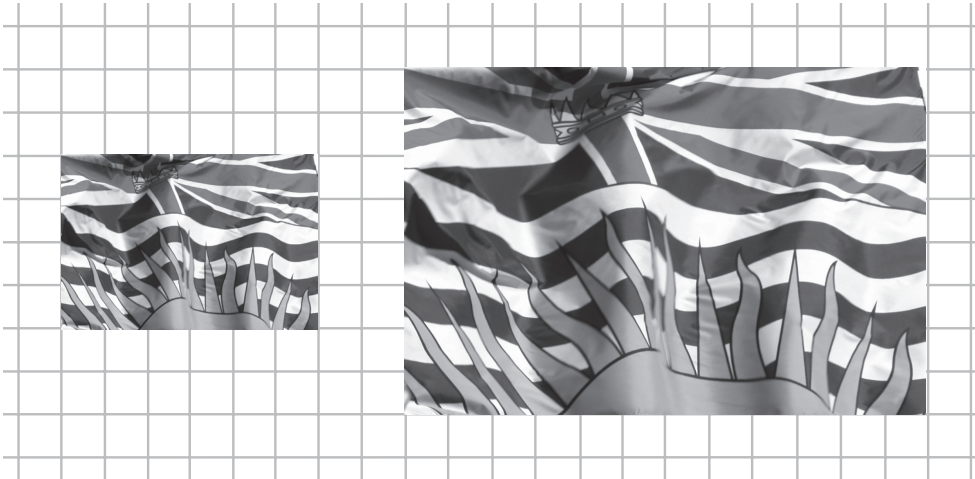


Photo of BC flag by Tupungato © 2010

The photograph of the British Columbia flag on the right has been enlarged.

Part 1:

- By what scale factor has the small flag been enlarged?
- What is the perimeter, in units, of the original photograph?
- What is the perimeter of the enlargement?
- How many times longer is the perimeter of the enlargement than the perimeter of the original photograph?
- How does the scale factor predict the change in perimeter?

Solution

- The length of the enlargement is 12 units. The length of the original photograph is 6 units.

$$\frac{12 \text{ units}}{6 \text{ units}} = 2$$

The width of the enlargement is 8 units. The width of the original photograph is 4 units.

$$\frac{8 \text{ units}}{4 \text{ units}} = 2$$

Both the length and width were increased 2 times.

The scale factor is 2.

My Notes

b. The perimeter of the original photograph
 $= 2 \times \text{length} + 2 \times \text{width}$
 $= 2 \times 6 \text{ units} + 2 \times 4 \text{ units}$
 $= 20 \text{ units}$

c. The perimeter of the enlarged photograph
 $= 2 \times \text{length} + 2 \times \text{width}$
 $= 2 \times 12 \text{ units} + 2 \times 8 \text{ units}$
 $= 40 \text{ units}$

d.
$$\frac{\text{Perimeter of enlargement}}{\text{Perimeter of original}} = \frac{40 \text{ units}}{20 \text{ units}}$$

$$= 2$$

Perimeter of enlargement = $2 \times$ perimeter of original.

The perimeter of the enlargement is 2 times the perimeter of the original.

e. The number of times the perimeter increases = scale factor.

Part 2:

- What is the area, in units², of the original photograph?
- What is the area of the enlargement?
- How many times larger is the area of the enlargement than the area of the original photograph?
- How does the scale factor predict the change in area?

Solution

a. The area of the original photograph = length \times width.
 $= 6 \text{ units} \times 4 \text{ units}$
 $= 24 \text{ units}^2$

b. The area of the enlarged photograph = length \times width.
 $= 12 \text{ units} \times 8 \text{ units}$
 $= 96 \text{ units}^2$

c.
$$\frac{\text{area of enlargement}}{\text{area of original}} = \frac{96 \text{ units}^2}{24 \text{ units}^2}$$

$$= 4$$

d. The number of times the area increases
 $=$ square of the scale factor
 $= 2^2$ or 4

Example 2

The dimensions of the square window in Jerritt’s bathroom are one third the length of each side of his bedroom window.



Photo of bathroom by David Hughes © 2010 and Photo of bedroom by Joseph Calev © 2010

1. How much shorter is the trim on the bathroom window than the same trim on the bedroom window?
2. The amount of light a window lets in depends on its area. How many times less light does the bathroom window let in than the bedroom window?

Solution

1. The scale factor = $\frac{1}{3}$.

The perimeter of Jerritt’s bathroom window = $\frac{1}{3}$ × perimeter of the bedroom window.

Jerritt’s bathroom window has $\frac{1}{3}$ times the length of trim of the bedroom window.

2. The scale factor = $\frac{1}{3}$.

The area of Jerritt’s bathroom window = $\left(\frac{1}{3}\right)^2$ × area of bedroom window.

Jerritt’s bathroom window lets in $\left(\frac{1}{3}\right)^2$ or $\frac{1}{9}$ times as much light as the bedroom window.

My Notes

Example 3

Two rectangular blankets are the same shape but different sizes. The perimeter of the larger blanket is four times the perimeter of the smaller blanket. How many times larger is the area of the bigger blanket than the smaller?

Solution

The scale factor = 4.

$$\begin{aligned}\text{Area of larger blanket} &= 4^2 \times \text{area of smaller.} \\ &= 16 \times \text{area of the smaller blanket}\end{aligned}$$

Example 4

The area of a square mirror is 25 times the area of a smaller square mirror. How many times longer is each side of the larger square than the smaller?

Solution

Number of times the area changes = (scale factor)².

Since $25 = 5^2$, the scale factor = 5.

The sides of the larger mirror are 5 times as long as the sides of the smaller mirror.

Example 5



Original Image

Photo by Tischenko Irina © 2010



Photocopied Image

The area of a photocopied image of sunflowers is a quarter of the area of the original. How many times shorter is each side of the photocopied image compared to the corresponding side in the original?

Solution

Number of times the area changes = (scale factor)².

Since $\frac{1}{4} = (\frac{1}{2})^2$, the scale factor = $\frac{1}{2}$

Each side of the photocopied image is $\frac{1}{2}$ of the length of the corresponding side in the original.

My Notes

6. The dimensions of a rectangular deck are increased by a factor of 2.5. The original perimeter of the deck was 30 ft. What is the perimeter of the enlarged deck?

My Notes

7. How many times was the area of the deck in question 6 increased?



Turn to the solutions at the end of the section and mark your work.

My Notes

Irregular Changes to Squares and Rectangles

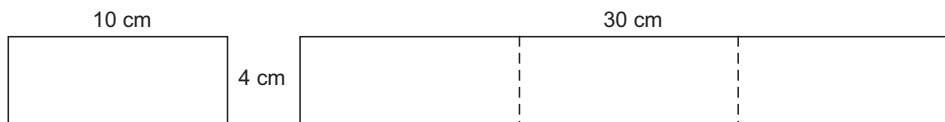
The Examples and the Self Check dealt with squares and rectangles for which both dimensions were increased by a given scale factor. If both length and width are not increased by the same factor, it is best to solve each problem without using a formula involving a scale factor.

Example 6

A rectangle measures 10 cm by 4 cm. A second rectangle measures 30 cm by 4 cm. Compare the perimeters and areas of the two figures.

Solution

The length of the second rectangle is 3 times the length of the first rectangle. The widths are the same.



Step 1: Compare the areas.

From the diagram, the area of the second rectangle appears to be 3 times the area of the first rectangle.

Check

$$\begin{aligned}\text{Area of the first rectangle} &= 10 \text{ cm} \times 4 \text{ cm} \\ &= 40 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{The area of the second rectangle} &= 30 \text{ cm} \times 4 \text{ cm} \\ &= 120 \text{ cm}^2\end{aligned}$$

Since $3 \times 40 \text{ cm}^2 = 120 \text{ cm}^2$, the second rectangle is 3 times the area of the first.

Step 2: Compare the perimeters.

$$\begin{aligned}\text{Perimeter of the first rectangle} &= 2 \times \text{length} + 2 \times \text{width} \\ &= 2 \times 10 \text{ cm} + 2 \times 4 \text{ cm} \\ &= 28 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Perimeter of the second rectangle} &= 2 \times \text{length} + 2 \times \text{width} \\ &= 2 \times 30 \text{ cm} + 2 \times 4 \text{ cm} \\ &= 68 \text{ cm}\end{aligned}$$

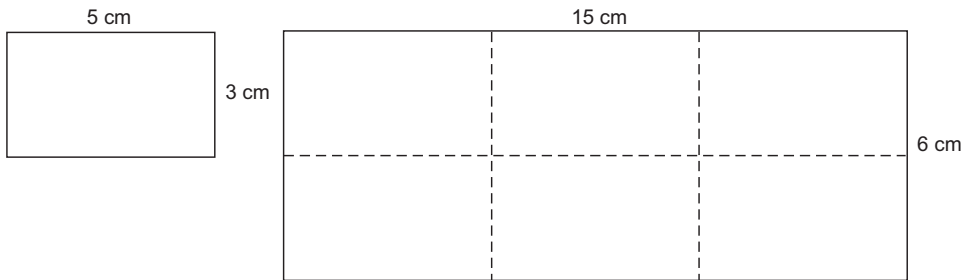
The perimeter of the second rectangle is not 3 times the perimeter of the first rectangle. Three times the perimeter of the first rectangle would be 84 cm. The second rectangle's perimeter is only 68 cm.

Example 7

A rectangle is 3 cm wide and 5 cm long. If it were made twice as wide and 3 times as long, how would the area and perimeter change?

Solution

The new rectangle is $3 \times 5 \text{ cm} = 15 \text{ cm}$ long and $2 \times 3 \text{ cm} = 6 \text{ cm}$ wide.



Step 1: Compare the areas.

From the diagram, it appears that the new rectangle is 3×2 , or 6 times the area of the original rectangle.

Check

$$\begin{aligned} \text{Area of original rectangle} &= \text{length} \times \text{width} \\ &= 5 \text{ cm} \times 3 \text{ cm} \\ &= 15 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of new rectangle} &= \text{length} \times \text{width} \\ &= 15 \text{ cm} \times 6 \text{ cm} \\ &= 90 \text{ cm}^2 \end{aligned}$$

Since $6 \times 15 \text{ cm}^2 = 90 \text{ cm}^2$, the area was increased 3×2 , or 6 times.

Step 2: Compare the perimeters.

$$\begin{aligned} \text{Perimeter of original rectangle} &= 2 \times \text{length} + 2 \times \text{width} \\ &= 2 \times 5 \text{ cm} + 2 \times 3 \text{ cm} \\ &= 16 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Perimeter of new rectangle} &= 2 \times \text{length} + 2 \times \text{width} \\ &= 2 \times 15 \text{ cm} + 2 \times 6 \text{ cm} \\ &= 42 \text{ cm} \end{aligned}$$

Notice that the perimeter is not 2 times, or 3 times, or even 6 times the perimeter of the original figure. We have to determine the perimeters from the dimensions of each rectangle. There is no shortcut for perimeter.

My Notes

Factors Affecting Area

From the previous two examples, notice that if the length of a rectangle is increased m times and its width is increased n times, then the area increases $m \times n$ times. This formula applies to area only. (There is no shortcut for perimeter.)

Activity 5
Self-Check

Please answer the following questions.

1. The length of a certain rectangle is changed by a factor of 5 and its width by a factor of 2. If the area of the original rectangle were 23 ft^2 , what is the area of the new rectangle?

2. The width of a square is enlarged 7 times. The length is increased by a factor of 9. How many of the original squares would fit into the new figure? Include a diagram in your solution.

My Notes



Turn to the solutions at the end of the section and mark your work.

My Notes

Activity 6
Mastering Concepts

Try these questions.

1. Show that if both the length l and width w of a rectangle are scaled by a factor of k , the area of the new rectangle is k^2 times the area of the original, and the new perimeter is k times the original perimeter.

2. A rectangle is enlarged to give an area that is twice the original area. What changes to the length and/or width could have caused the doubling of area?

My Notes



Turn to the solutions at the end of the section and mark your work.

Lesson Summary

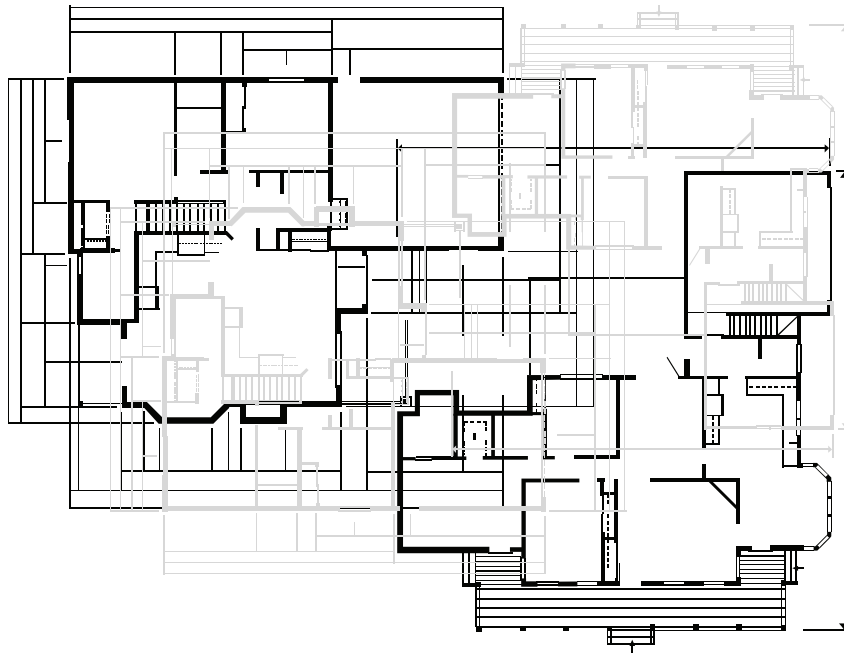


Photo by Vector Mushrooms © 2010

House plans are scale drawings. If you knew the scale factor for the plans, you could calculate the dimensions or area of any part of the house by first taking measurements from the plans themselves. Also, any modifications to this plan must be drawn to the same scale.

In this lesson you explored scale factors and their relationship to the perimeter and area of squares and rectangles, similar to the squares and rectangles illustrated on the technical diagram.

Area— Appendix

Data Pages	133
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Parallelogram Template.	179
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TABLE OF CONVERSIONS

1 inch	≈	2.54 centimetres
1 foot	≈	30.5 centimetres
1 foot	≈	0.305 metres
1 foot	=	12 inches
1 yard	=	3 feet
1 yard	≈	0.915 metres
1 mile	=	1760 yards
1 mile	≈	1.6 kilometres
1 kilogram	≈	2.2 pounds
1 litre	≈	1.06 US quarts
1 litre	≈	0.26 US gallons
1 gallon	≈	4 quarts
1 British gallon	≈	$\frac{6}{5}$ US gallon

FORMULAE**Temperature**

$$C = \frac{5}{9}(F - 32)$$

Trigonometry

(Put your calculator in Degree Mode)

- Right triangles

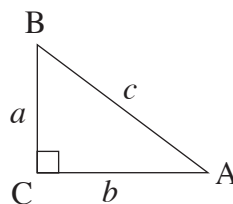
Pythagorean Theorem

$$a^2 + b^2 = c^2$$

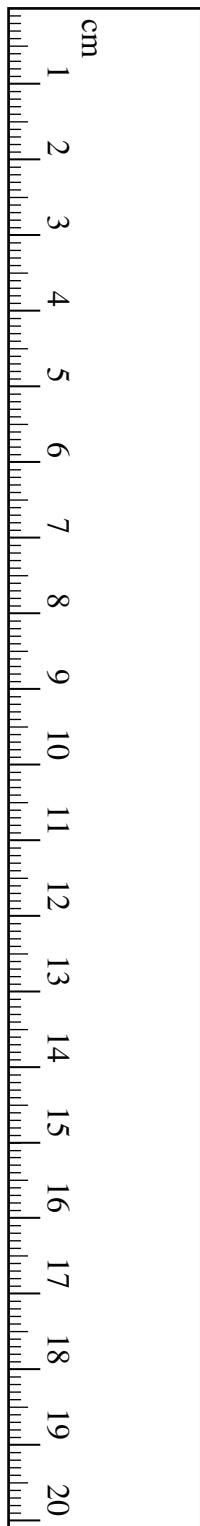
$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$



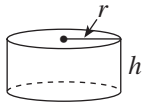
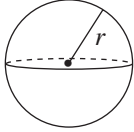
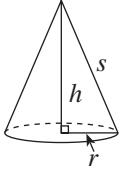
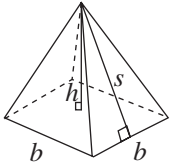
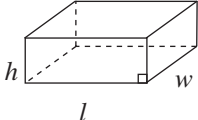
GEOMETRIC FORMULAE

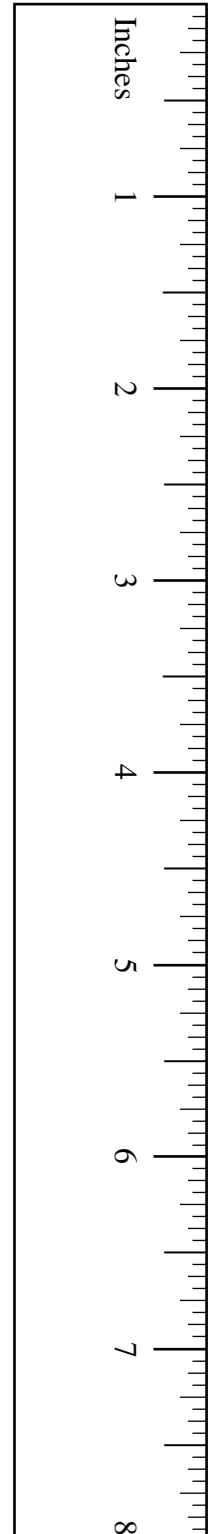


Key Legend	
l = length	P = perimeter
w = width	C = circumference
b = base	A = area
h = height	SA = surface area
s = slant height	V = volume
r = radius	
d = diameter	

Geometric Figure	Perimeter	Area
Rectangle 	$P = 2l + 2w$ or $P = 2(l + w)$	$A = lw$
Triangle 	$P = a + b + c$	$A = \frac{bh}{2}$
Circle 	$C = \pi d$ or $C = 2\pi r$	$A = \pi r^2$

Note: Use the value of π programmed in your calculator rather than the approximation of 3.14.

Geometric Figure	Surface Area
<p>Cylinder</p> 	$A_{top} = \pi r^2$ $A_{base} = \pi r^2$ $A_{side} = 2\pi rh$ $SA = 2\pi r^2 + 2\pi rh$
<p>Sphere</p> 	$SA = 4\pi r^2$ <p>or</p> $SA = \pi d^2$
<p>Cone</p> 	$A_{side} = \pi rs$ $A_{base} = \pi r^2$ $SA = \pi r^2 + \pi rs$
<p>Square-Based Pyramid</p> 	$A_{triangle} = \frac{1}{2}bs \text{ (for each triangle)}$ $A_{base} = b^2$ $SA = 2bs + b^2$
<p>Rectangular Prism</p> 	$SA = wh + wh + lw + lw + lh + lh$ <p>or</p> $SA = 2(wh + lw + lh)$
<p>General Right Prism</p>	$SA = \text{the sum of the areas of all the faces}$
<p>General Pyramid</p>	$SA = \text{the sum of the areas of all the faces}$



Note: Use the value of π programmed in your calculator rather than the approximation of 3.14.

**Canada Pension Plan Contributions
Weekly (52 pay periods a year)**

**Cotisations au Régime de pensions du Canada
Hebdomadaire (52 périodes de paie par année)**

Pay Rémunération		CPP RPC	Pay Rémunération		CPP RPC	Pay Rémunération		CPP RPC	Pay Rémunération		CPP RPC
From - De	To - À		From - De	To - À		From - De	To - À		From - De	To - À	
358.11 - 358.31	14.40	372.66 - 372.85	15.12	387.20 - 387.40	15.84	401.75 - 401.94	16.56				
358.32 - 358.51	14.41	372.86 - 373.05	15.13	387.41 - 387.60	15.85	401.95 - 402.14	16.57				
358.52 - 358.71	14.42	373.06 - 373.25	15.14	387.61 - 387.80	15.86	402.15 - 402.35	16.58				
358.72 - 358.91	14.43	373.26 - 373.46	15.15	387.81 - 388.00	15.87	402.36 - 402.55	16.59				
358.92 - 359.11	14.44	373.47 - 373.66	15.16	388.01 - 388.20	15.88	402.56 - 402.75	16.60				
359.12 - 359.32	14.45	373.67 - 373.86	15.17	388.21 - 388.41	15.89	402.76 - 402.95	16.61				
359.33 - 359.52	14.46	373.87 - 374.06	15.18	388.42 - 388.61	15.90	402.96 - 403.15	16.62				
359.53 - 359.72	14.47	374.07 - 374.26	15.19	388.62 - 388.81	15.91	403.16 - 403.36	16.63				
359.73 - 359.92	14.48	374.27 - 374.47	15.20	388.82 - 389.01	15.92	403.37 - 403.56	16.64				
359.93 - 360.12	14.49	374.48 - 374.67	15.21	389.02 - 389.21	15.93	403.57 - 403.76	16.65				
360.13 - 360.33	14.50	374.68 - 374.87	15.22	389.22 - 389.42	15.94	403.77 - 403.96	16.66				
360.34 - 360.53	14.51	374.88 - 375.07	15.23	389.43 - 389.62	15.95	403.97 - 404.16	16.67				
360.54 - 360.73	14.52	375.08 - 375.27	15.24	389.63 - 389.82	15.96	404.17 - 404.37	16.68				
360.74 - 360.93	14.53	375.28 - 375.48	15.25	389.83 - 390.02	15.97	404.38 - 404.57	16.69				
360.94 - 361.13	14.54	375.49 - 375.68	15.26	390.03 - 390.22	15.98	404.58 - 404.77	16.70				
361.14 - 361.34	14.55	375.69 - 375.88	15.27	390.23 - 390.43	15.99	404.78 - 404.97	16.71				
361.35 - 361.54	14.56	375.89 - 376.08	15.28	390.44 - 390.63	16.00	404.98 - 405.17	16.72				
361.55 - 361.74	14.57	376.09 - 376.28	15.29	390.64 - 390.83	16.01	405.18 - 405.38	16.73				
361.75 - 361.94	14.58	376.29 - 376.49	15.30	390.84 - 391.03	16.02	405.39 - 405.58	16.74				
361.95 - 362.14	14.59	376.50 - 376.69	15.31	391.04 - 391.23	16.03	405.59 - 405.78	16.75				
362.15 - 362.35	14.60	376.70 - 376.89	15.32	391.24 - 391.44	16.04	405.79 - 405.98	16.76				
362.36 - 362.55	14.61	376.90 - 377.09	15.33	391.45 - 391.64	16.05	405.99 - 406.18	16.77				
362.56 - 362.75	14.62	377.10 - 377.29	15.34	391.65 - 391.84	16.06	406.19 - 406.39	16.78				
362.76 - 362.95	14.63	377.30 - 377.50	15.35	391.85 - 392.04	16.07	406.40 - 406.59	16.79				
362.96 - 363.15	14.64	377.51 - 377.70	15.36	392.05 - 392.24	16.08	406.60 - 406.79	16.80				
363.16 - 363.36	14.65	377.71 - 377.90	15.37	392.25 - 392.45	16.09	406.80 - 406.99	16.81				
363.37 - 363.56	14.66	377.91 - 378.10	15.38	392.46 - 392.65	16.10	407.00 - 407.19	16.82				
363.57 - 363.76	14.67	378.11 - 378.31	15.39	392.66 - 392.85	16.11	407.20 - 407.40	16.83				
363.77 - 363.96	14.68	378.32 - 378.51	15.40	392.86 - 393.05	16.12	407.41 - 407.60	16.84				
363.97 - 364.16	14.69	378.52 - 378.71	15.41	393.06 - 393.25	16.13	407.61 - 407.80	16.85				
364.17 - 364.37	14.70	378.72 - 378.91	15.42	393.26 - 393.46	16.14	407.81 - 408.00	16.86				
364.38 - 364.57	14.71	378.92 - 379.11	15.43	393.47 - 393.66	16.15	408.01 - 408.20	16.87				
364.58 - 364.77	14.72	379.12 - 379.32	15.44	393.67 - 393.86	16.16	408.21 - 408.41	16.88				
364.78 - 364.97	14.73	379.33 - 379.52	15.45	393.87 - 394.06	16.17	408.42 - 408.61	16.89				
364.98 - 365.17	14.74	379.53 - 379.72	15.46	394.07 - 394.26	16.18	408.62 - 408.81	16.90				
365.18 - 365.38	14.75	379.73 - 379.92	15.47	394.27 - 394.47	16.19	408.82 - 409.01	16.91				
365.39 - 365.58	14.76	379.93 - 380.12	15.48	394.48 - 394.67	16.20	409.02 - 409.21	16.92				
365.59 - 365.78	14.77	380.13 - 380.33	15.49	394.68 - 394.87	16.21	409.22 - 409.42	16.93				
365.79 - 365.98	14.78	380.34 - 380.53	15.50	394.88 - 395.07	16.22	409.43 - 409.62	16.94				
365.99 - 366.18	14.79	380.54 - 380.73	15.51	395.08 - 395.27	16.23	409.63 - 409.82	16.95				
366.19 - 366.39	14.80	380.74 - 380.93	15.52	395.28 - 395.48	16.24	409.83 - 410.02	16.96				
366.40 - 366.59	14.81	380.94 - 381.13	15.53	395.49 - 395.68	16.25	410.03 - 410.22	16.97				
366.60 - 366.79	14.82	381.14 - 381.34	15.54	395.69 - 395.88	16.26	410.23 - 410.43	16.98				
366.80 - 366.99	14.83	381.35 - 381.54	15.55	395.89 - 396.08	16.27	410.44 - 410.63	16.99				
367.00 - 367.19	14.84	381.55 - 381.74	15.56	396.09 - 396.28	16.28	410.64 - 410.83	17.00				
367.20 - 367.40	14.85	381.75 - 381.94	15.57	396.29 - 396.49	16.29	410.84 - 411.03	17.01				
367.41 - 367.60	14.86	381.95 - 382.14	15.58	396.50 - 396.69	16.30	411.04 - 411.23	17.02				
367.61 - 367.80	14.87	382.15 - 382.35	15.59	396.70 - 396.89	16.31	411.24 - 411.44	17.03				
367.81 - 368.00	14.88	382.36 - 382.55	15.60	396.90 - 397.09	16.32	411.45 - 411.64	17.04				
368.01 - 368.20	14.89	382.56 - 382.75	15.61	397.10 - 397.29	16.33	411.65 - 411.84	17.05				
368.21 - 368.41	14.90	382.76 - 382.95	15.62	397.30 - 397.50	16.34	411.85 - 412.04	17.06				
368.42 - 368.61	14.91	382.96 - 383.15	15.63	397.51 - 397.70	16.35	412.05 - 412.24	17.07				
368.62 - 368.81	14.92	383.16 - 383.36	15.64	397.71 - 397.90	16.36	412.25 - 412.45	17.08				
368.82 - 369.01	14.93	383.37 - 383.56	15.65	397.91 - 398.10	16.37	412.46 - 412.65	17.09				
369.02 - 369.21	14.94	383.57 - 383.76	15.66	398.11 - 398.31	16.38	412.66 - 412.85	17.10				
369.22 - 369.42	14.95	383.77 - 383.96	15.67	398.32 - 398.51	16.39	412.86 - 413.05	17.11				
369.43 - 369.62	14.96	383.97 - 384.16	15.68	398.52 - 398.71	16.40	413.06 - 413.25	17.12				
369.63 - 369.82	14.97	384.17 - 384.37	15.69	398.72 - 398.91	16.41	413.26 - 413.46	17.13				
369.83 - 370.02	14.98	384.38 - 384.57	15.70	398.92 - 399.11	16.42	413.47 - 413.66	17.14				
370.03 - 370.22	14.99	384.58 - 384.77	15.71	399.12 - 399.32	16.43	413.67 - 413.86	17.15				
370.23 - 370.43	15.00	384.78 - 384.97	15.72	399.33 - 399.52	16.44	413.87 - 414.06	17.16				
370.44 - 370.63	15.01	384.98 - 385.17	15.73	399.53 - 399.72	16.45	414.07 - 414.26	17.17				
370.64 - 370.83	15.02	385.18 - 385.38	15.74	399.73 - 399.92	16.46	414.27 - 414.47	17.18				
370.84 - 371.03	15.03	385.39 - 385.58	15.75	399.93 - 400.12	16.47	414.48 - 414.67	17.19				
371.04 - 371.23	15.04	385.59 - 385.78	15.76	400.13 - 400.33	16.48	414.68 - 414.87	17.20				
371.24 - 371.44	15.05	385.79 - 385.98	15.77	400.34 - 400.53	16.49	414.88 - 415.07	17.21				
371.45 - 371.64	15.06	385.99 - 386.18	15.78	400.54 - 400.73	16.50	415.08 - 415.27	17.22				
371.65 - 371.84	15.07	386.19 - 386.39	15.79	400.74 - 400.93	16.51	415.28 - 415.48	17.23				
371.85 - 372.04	15.08	386.40 - 386.59	15.80	400.94 - 401.13	16.52	415.49 - 415.68	17.24				
372.05 - 372.24	15.09	386.60 - 386.79	15.81	401.14 - 401.34	16.53	415.69 - 415.88	17.25				
372.25 - 372.45	15.10	386.80 - 386.99	15.82	401.35 - 401.54	16.54	415.89 - 416.08	17.26				
372.46 - 372.65	15.11	387.00 - 387.19	15.83	401.55 - 401.74	16.55	416.09 - 416.28	17.27				

Employee's maximum CPP contribution for the year 2009 is \$2,118.60

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La cotisation maximale de l'employé au RPC pour l'année 2009 est de 2 118,60 \$

Employment Insurance Premiums

Cotisations à l'assurance-emploi

Insurable Earnings Rémunération assurable		EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable		EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable		EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable		EI premium Cotisation d'AE
From - De	To - À		From - De	To - À		From - De	To - À		From - De	To - À	
333.24	- 333.81	5.77	374.86	- 375.43	6.49	416.48	- 417.05	7.21	458.10	- 458.67	7.93
333.82	- 334.39	5.78	375.44	- 376.01	6.50	417.06	- 417.63	7.22	458.68	- 459.24	7.94
334.40	- 334.97	5.79	376.02	- 376.58	6.51	417.64	- 418.20	7.23	459.25	- 459.82	7.95
334.98	- 335.54	5.80	376.59	- 377.16	6.52	418.21	- 418.78	7.24	459.83	- 460.40	7.96
335.55	- 336.12	5.81	377.17	- 377.74	6.53	418.79	- 419.36	7.25	460.41	- 460.98	7.97
336.13	- 336.70	5.82	377.75	- 378.32	6.54	419.37	- 419.94	7.26	460.99	- 461.56	7.98
336.71	- 337.28	5.83	378.33	- 378.90	6.55	419.95	- 420.52	7.27	461.57	- 462.13	7.99
337.29	- 337.86	5.84	378.91	- 379.47	6.56	420.53	- 421.09	7.28	462.14	- 462.71	8.00
337.87	- 338.43	5.85	379.48	- 380.05	6.57	421.10	- 421.67	7.29	462.72	- 463.29	8.01
338.44	- 339.01	5.86	380.06	- 380.63	6.58	421.68	- 422.25	7.30	463.30	- 463.87	8.02
339.02	- 339.59	5.87	380.64	- 381.21	6.59	422.26	- 422.83	7.31	463.88	- 464.45	8.03
339.60	- 340.17	5.88	381.22	- 381.79	6.60	422.84	- 423.41	7.32	464.46	- 465.02	8.04
340.18	- 340.75	5.89	381.80	- 382.36	6.61	423.42	- 423.98	7.33	465.03	- 465.60	8.05
340.76	- 341.32	5.90	382.37	- 382.94	6.62	423.99	- 424.56	7.34	465.61	- 466.18	8.06
341.33	- 341.90	5.91	382.95	- 383.52	6.63	424.57	- 425.14	7.35	466.19	- 466.76	8.07
341.91	- 342.48	5.92	383.53	- 384.10	6.64	425.15	- 425.72	7.36	466.77	- 467.34	8.08
342.49	- 343.06	5.93	384.11	- 384.68	6.65	425.73	- 426.30	7.37	467.35	- 467.91	8.09
343.07	- 343.64	5.94	384.69	- 385.26	6.66	426.31	- 426.87	7.38	467.92	- 468.49	8.10
343.65	- 344.21	5.95	385.27	- 385.83	6.67	426.88	- 427.45	7.39	468.50	- 469.07	8.11
344.22	- 344.79	5.96	385.84	- 386.41	6.68	427.46	- 428.03	7.40	469.08	- 469.65	8.12
344.80	- 345.37	5.97	386.42	- 386.99	6.69	428.04	- 428.61	7.41	469.66	- 470.23	8.13
345.38	- 345.95	5.98	387.00	- 387.57	6.70	428.62	- 429.19	7.42	470.24	- 470.80	8.14
345.96	- 346.53	5.99	387.58	- 388.15	6.71	429.20	- 429.76	7.43	470.81	- 471.38	8.15
346.54	- 347.10	6.00	388.16	- 388.72	6.72	429.77	- 430.34	7.44	471.39	- 471.96	8.16
347.11	- 347.68	6.01	388.73	- 389.30	6.73	430.35	- 430.92	7.45	471.97	- 472.54	8.17
347.69	- 348.26	6.02	389.31	- 389.88	6.74	430.93	- 431.50	7.46	472.55	- 473.12	8.18
348.27	- 348.84	6.03	389.89	- 390.46	6.75	431.51	- 432.08	7.47	473.13	- 473.69	8.19
348.85	- 349.42	6.04	390.47	- 391.04	6.76	432.09	- 432.65	7.48	473.70	- 474.27	8.20
349.43	- 349.99	6.05	391.05	- 391.61	6.77	432.66	- 433.23	7.49	474.28	- 474.85	8.21
350.00	- 350.57	6.06	391.62	- 392.19	6.78	433.24	- 433.81	7.50	474.86	- 475.43	8.22
350.58	- 351.15	6.07	392.20	- 392.77	6.79	433.82	- 434.39	7.51	475.44	- 476.01	8.23
351.16	- 351.73	6.08	392.78	- 393.35	6.80	434.40	- 434.97	7.52	476.02	- 476.58	8.24
351.74	- 352.31	6.09	393.36	- 393.93	6.81	434.98	- 435.54	7.53	476.59	- 477.16	8.25
352.32	- 352.89	6.10	393.94	- 394.50	6.82	435.55	- 436.12	7.54	477.17	- 477.74	8.26
352.90	- 353.46	6.11	394.51	- 395.08	6.83	436.13	- 436.70	7.55	477.75	- 478.32	8.27
353.47	- 354.04	6.12	395.09	- 395.66	6.84	436.71	- 437.28	7.56	478.33	- 478.90	8.28
354.05	- 354.62	6.13	395.67	- 396.24	6.85	437.29	- 437.86	7.57	478.91	- 479.47	8.29
354.63	- 355.20	6.14	396.25	- 396.82	6.86	437.87	- 438.43	7.58	479.48	- 480.05	8.30
355.21	- 355.78	6.15	396.83	- 397.39	6.87	438.44	- 439.01	7.59	480.06	- 480.63	8.31
355.79	- 356.35	6.16	397.40	- 397.97	6.88	439.02	- 439.59	7.60	480.64	- 481.21	8.32
356.36	- 356.93	6.17	397.98	- 398.55	6.89	439.60	- 440.17	7.61	481.22	- 481.79	8.33
356.94	- 357.51	6.18	398.56	- 399.13	6.90	440.18	- 440.75	7.62	481.80	- 482.36	8.34
357.52	- 358.09	6.19	399.14	- 399.71	6.91	440.76	- 441.32	7.63	482.37	- 482.94	8.35
358.10	- 358.67	6.20	399.72	- 400.28	6.92	441.33	- 441.90	7.64	482.95	- 483.52	8.36
358.68	- 359.24	6.21	400.29	- 400.86	6.93	441.91	- 442.48	7.65	483.53	- 484.10	8.37
359.25	- 359.82	6.22	400.87	- 401.44	6.94	442.49	- 443.06	7.66	484.11	- 484.68	8.38
359.83	- 360.40	6.23	401.45	- 402.02	6.95	443.07	- 443.64	7.67	484.69	- 485.26	8.39
360.41	- 360.98	6.24	402.03	- 402.60	6.96	443.65	- 444.21	7.68	485.27	- 485.83	8.40
360.99	- 361.56	6.25	402.61	- 403.17	6.97	444.22	- 444.79	7.69	485.84	- 486.41	8.41
361.57	- 362.13	6.26	403.18	- 403.75	6.98	444.80	- 445.37	7.70	486.42	- 486.99	8.42
362.14	- 362.71	6.27	403.76	- 404.33	6.99	445.38	- 445.95	7.71	487.00	- 487.57	8.43
362.72	- 363.29	6.28	404.34	- 404.91	7.00	445.96	- 446.53	7.72	487.58	- 488.15	8.44
363.30	- 363.87	6.29	404.92	- 405.49	7.01	446.54	- 447.10	7.73	488.16	- 488.72	8.45
363.88	- 364.45	6.30	405.50	- 406.06	7.02	447.11	- 447.68	7.74	488.73	- 489.30	8.46
364.46	- 365.02	6.31	406.07	- 406.64	7.03	447.69	- 448.26	7.75	489.31	- 489.88	8.47
365.03	- 365.60	6.32	406.65	- 407.22	7.04	448.27	- 448.84	7.76	489.89	- 490.46	8.48
365.61	- 366.18	6.33	407.23	- 407.80	7.05	448.85	- 449.42	7.77	490.47	- 491.04	8.49
366.19	- 366.76	6.34	407.81	- 408.38	7.06	449.43	- 449.99	7.78	491.05	- 491.61	8.50
366.77	- 367.34	6.35	408.39	- 408.95	7.07	450.00	- 450.57	7.79	491.62	- 492.19	8.51
367.35	- 367.91	6.36	408.96	- 409.53	7.08	450.58	- 451.15	7.80	492.20	- 492.77	8.52
367.92	- 368.49	6.37	409.54	- 410.11	7.09	451.16	- 451.73	7.81	492.78	- 493.35	8.53
368.50	- 369.07	6.38	410.12	- 410.69	7.10	451.74	- 452.31	7.82	493.36	- 493.93	8.54
369.08	- 369.65	6.39	410.70	- 411.27	7.11	452.32	- 452.89	7.83	493.94	- 494.50	8.55
369.66	- 370.23	6.40	411.28	- 411.84	7.12	452.90	- 453.46	7.84	494.51	- 495.08	8.56
370.24	- 370.80	6.41	411.85	- 412.42	7.13	453.47	- 454.04	7.85	495.09	- 495.66	8.57
370.81	- 371.38	6.42	412.43	- 413.00	7.14	454.05	- 454.62	7.86	495.67	- 496.24	8.58
371.39	- 371.96	6.43	413.01	- 413.58	7.15	454.63	- 455.20	7.87	496.25	- 496.82	8.59
371.97	- 372.54	6.44	413.59	- 414.16	7.16	455.21	- 455.78	7.88	496.83	- 497.39	8.60
372.55	- 373.12	6.45	414.17	- 414.73	7.17	455.79	- 456.35	7.89	497.40	- 497.97	8.61
373.13	- 373.69	6.46	414.74	- 415.31	7.18	456.36	- 456.93	7.90	497.98	- 498.55	8.62
373.70	- 374.27	6.47	415.32	- 415.89	7.19	456.94	- 457.51	7.91	498.56	- 499.13	8.63
374.28	- 374.85	6.48	415.90	- 416.47	7.20	457.52	- 458.09	7.92	499.14	- 499.71	8.64

Yearly maximum insurable earnings are \$42,300
 Yearly maximum employee premiums are \$731.79
 The premium rate for 2009 is 1.73 %

Le maximum annuel de la rémunération assurable est de 42 300 \$
 La cotisation maximale annuelle de l'employé est de 731,79 \$
 Le taux de cotisation pour 2009 est de 1,73 %

AREA—APPENDIX

Federal tax deductions
 Effective January 1, 2009
 Weekly (52 pay periods a year)
 Also look up the tax deductions
 in the provincial table

Retenues d'impôt fédéral
 En vigueur le 1^{er} janvier 2009
 Hebdomadaire (52 périodes de paie par année)
 Cherchez aussi les retenues d'impôt
 dans la table provinciale

Pay Rémunération	Federal claim codes/Codes de demande fédéraux										
	0	1	2	3	4	5	6	7	8	9	10
From Less than De Moins de	Deduct from each pay Retenez sur chaque paie										
335 - 339	44.65	15.55	12.70	7.00	1.30						
339 - 343	45.20	16.10	13.25	7.55	1.85						
343 - 347	45.80	16.65	13.80	8.10	2.45						
347 - 351	46.35	17.20	14.35	8.65	3.00						
351 - 355	46.90	17.75	14.90	9.25	3.55						
355 - 359	47.45	18.35	15.50	9.80	4.10						
359 - 363	48.00	18.90	16.05	10.35	4.65						
363 - 367	48.60	19.45	16.60	10.90	5.25						
367 - 371	49.15	20.00	17.15	11.45	5.80	.10					
371 - 375	49.70	20.55	17.70	12.05	6.35	.65					
375 - 379	50.25	21.15	18.30	12.60	6.90	1.20					
379 - 383	50.80	21.70	18.85	13.15	7.45	1.80					
383 - 387	51.40	22.25	19.40	13.70	8.00	2.35					
387 - 391	51.95	22.80	19.95	14.25	8.60	2.90					
391 - 395	52.50	23.35	20.50	14.85	9.15	3.45					
395 - 399	53.05	23.95	21.10	15.40	9.70	4.00					
399 - 403	53.60	24.50	21.65	15.95	10.25	4.60					
403 - 407	54.20	25.05	22.20	16.50	10.80	5.15					
407 - 411	54.75	25.60	22.75	17.05	11.40	5.70					
411 - 415	55.30	26.15	23.30	17.65	11.95	6.25	.55				
415 - 419	55.85	26.75	23.90	18.20	12.50	6.80	1.15				
419 - 423	56.40	27.30	24.45	18.75	13.05	7.40	1.70				
423 - 427	57.00	27.85	25.00	19.30	13.60	7.95	2.25				
427 - 431	57.55	28.40	25.55	19.85	14.20	8.50	2.80				
431 - 435	58.10	28.95	26.10	20.45	14.75	9.05	3.35				
435 - 439	58.65	29.50	26.70	21.00	15.30	9.60	3.95				
439 - 443	59.20	30.10	27.25	21.55	15.85	10.20	4.50				
443 - 447	59.80	30.65	27.80	22.10	16.40	10.75	5.05				
447 - 451	60.35	31.20	28.35	22.65	17.00	11.30	5.60				
451 - 455	60.90	31.75	28.90	23.25	17.55	11.85	6.15	.50			
455 - 459	61.45	32.30	29.50	23.80	18.10	12.40	6.75	1.05			
459 - 463	62.00	32.90	30.05	24.35	18.65	12.95	7.30	1.60			
463 - 467	62.60	33.45	30.60	24.90	19.20	13.55	7.85	2.15			
467 - 471	63.15	34.00	31.15	25.45	19.80	14.10	8.40	2.70			
471 - 475	63.70	34.55	31.70	26.05	20.35	14.65	8.95	3.30			
475 - 479	64.25	35.10	32.30	26.60	20.90	15.20	9.55	3.85			
479 - 483	64.80	35.70	32.85	27.15	21.45	15.75	10.10	4.40			
483 - 487	65.40	36.25	33.40	27.70	22.00	16.35	10.65	4.95			
487 - 491	65.95	36.80	33.95	28.25	22.60	16.90	11.20	5.50			
491 - 495	66.50	37.35	34.50	28.85	23.15	17.45	11.75	6.10	.40		
495 - 499	67.05	37.90	35.10	29.40	23.70	18.00	12.35	6.65	.95		
499 - 503	67.60	38.50	35.65	29.95	24.25	18.55	12.90	7.20	1.50		
503 - 507	68.20	39.05	36.20	30.50	24.80	19.15	13.45	7.75	2.05		
507 - 511	68.75	39.60	36.75	31.05	25.40	19.70	14.00	8.30	2.65		
511 - 515	69.30	40.15	37.30	31.65	25.95	20.25	14.55	8.90	3.20		
515 - 519	69.85	40.70	37.90	32.20	26.50	20.80	15.15	9.45	3.75		
519 - 523	70.40	41.30	38.45	32.75	27.05	21.35	15.70	10.00	4.30		
523 - 527	71.00	41.85	39.00	33.30	27.60	21.95	16.25	10.55	4.85		
527 - 531	71.55	42.40	39.55	33.85	28.20	22.50	16.80	11.10	5.45		
531 - 535	72.10	42.95	40.10	34.45	28.75	23.05	17.35	11.70	6.00	.30	
535 - 539	72.65	43.50	40.70	35.00	29.30	23.60	17.90	12.25	6.55	.85	
539 - 543	73.20	44.10	41.25	35.55	29.85	24.15	18.50	12.80	7.10	1.40	
543 - 547	73.80	44.65	41.80	36.10	30.40	24.75	19.05	13.35	7.65	2.00	
547 - 551	74.35	45.20	42.35	36.65	31.00	25.30	19.60	13.90	8.25	2.55	
551 - 555	74.90	45.75	42.90	37.25	31.55	25.85	20.15	14.50	8.80	3.10	

British Columbia provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions
in the federal table

Retenues d'impôt provincial de la Colombie-Britannique
En vigueur le 1^{er} janvier 2009
Hebdomadaire (52 périodes de paie par année)
Cherchez aussi les retenues d'impôt
dans la table fédérale

Pay Rémunération	Provincial claim codes/Codes de demande provinciaux											
	0	1	2	3	4	5	6	7	8	9	10	
From Less than De Moins de	Deduct from each pay Retenez sur chaque paie											
343 - 343	*	.00										*You normally use claim code "0" only for non-resident employees. However, if you have non-resident employees who earn less than the minimum amount shown in the "Pay" column, you may not be able to use these tables. Instead, refer to the "Step-by-step calculation of tax deductions" in Section "A" of this publication. *Le code de demande «0» est normalement utilisé seulement pour les non-résidents. Cependant, si la rémunération de votre employé non résidant est inférieure au montant minimum indiqué dans la colonne «Rémunération», vous ne pourrez peut-être pas utiliser ces tables. Reportez-vous alors au «Calcul des retenues d'impôt, étape par étape» dans la section «A» de cette publication.
343 - 345	9.30	.20										
345 - 347	9.45	.35										
347 - 349	9.60	.50										
349 - 351	9.80	.65										
351 - 353	9.95	.80										
353 - 355	10.10	.95										
355 - 357	10.25	1.15	.10									
357 - 359	10.40	1.30	.25									
359 - 361	10.55	1.45	.40									
361 - 363	10.75	1.60	.60									
363 - 365	10.90	1.75	.75									
365 - 367	11.05	1.90	.90									
367 - 369	11.20	2.10	1.05									
369 - 371	11.35	2.25	1.20									
371 - 373	11.50	2.40	1.35									
373 - 375	11.70	2.55	1.55									
375 - 377	11.85	2.70	1.70									
377 - 379	12.00	2.90	1.85									
379 - 381	12.15	3.05	2.00									
381 - 383	12.30	3.20	2.15	.10								
383 - 385	12.45	3.35	2.30	.25								
385 - 387	12.65	3.50	2.50	.45								
387 - 389	12.80	3.65	2.65	.60								
389 - 391	12.95	3.85	2.80	.75								
391 - 393	13.10	4.00	2.95	.90								
393 - 395	13.25	4.15	3.10	1.05								
395 - 397	13.40	4.30	3.30	1.20								
397 - 399	13.60	4.45	3.45	1.40								
399 - 401	13.75	4.60	3.60	1.55								
401 - 403	13.90	4.80	3.75	1.70								
403 - 405	14.05	4.95	3.90	1.85								
405 - 407	14.20	5.10	4.05	2.00								
407 - 409	14.35	5.25	4.25	2.15	.10							
409 - 411	14.55	5.40	4.40	2.35	.30							
411 - 413	14.70	5.55	4.55	2.50	.45							
413 - 415	14.85	5.75	4.70	2.65	.60							
415 - 417	15.00	5.90	4.85	2.80	.75							
417 - 419	15.15	6.05	5.00	2.95	.90							
419 - 421	15.30	6.20	5.20	3.10	1.05							
421 - 423	15.50	6.35	5.35	3.30	1.25							
423 - 425	15.65	6.50	5.50	3.45	1.40							
425 - 427	15.80	6.70	5.65	3.60	1.55							
427 - 429	15.95	6.85	5.80	3.75	1.70							
429 - 431	16.10	7.00	5.95	3.90	1.85							
431 - 433	16.25	7.15	6.15	4.10	2.00							
433 - 435	16.45	7.30	6.30	4.25	2.20	.15						
435 - 437	16.60	7.45	6.45	4.40	2.35	.30						
437 - 439	16.75	7.65	6.60	4.55	2.50	.45						
439 - 441	16.90	7.80	6.75	4.70	2.65	.60						
441 - 443	17.05	7.95	6.90	4.85	2.80	.75						
443 - 445	17.20	8.10	7.10	5.05	2.95	.90						
445 - 447	17.40	8.25	7.25	5.20	3.15	1.10						
447 - 449	17.55	8.40	7.40	5.35	3.30	1.25						
449 - 451	17.70	8.60	7.55	5.50	3.45	1.40						

Solutions

Section—Lesson A: Estimating Area

Lesson A: Activity 1: Self-Check

Unit Name	Symbol	SI or Imperial?
square foot	ft ²	imperial
square metre	m ²	SI
square inch	in ²	imperial
acre	ac	imperial
hectare	ha	SI
square centimetre	cm²	SI
square yard	yd ²	imperial
square kilometre	km²	SI
square mile	mi²	imperial

Lesson A: Activity 2: Self-Check

1. in²
2. ft²
3. km²
4. cm²

Lesson A: Activity 3: Self-Check

1. a. Remember that 3 ft = 1 yd.

The driveway is 12 ½ ft wide and 59 ft long.

Convert each dimension to an approximate dimension in yards.

$$12\frac{1}{2} \text{ ft} = \frac{12.5}{3} \text{ yd} \quad \text{Think: } 12 \div 3 = 4$$

$$= 4 \text{ yd}$$

$$59 \text{ ft} = \frac{59}{3} \text{ yd} \quad \text{Think: } 60 \div 3 = 20$$

$$= 20 \text{ yd}$$

Estimate the area using the approximate dimensions in yards.

$$A = l \times w$$

$$= 4 \text{ yd} \times 20 \text{ yd}$$

$$80 \text{ yd}^2$$

The area of the driveway is approximately 80 square yards.

- b. Convert each dimension to yards.

$$12\frac{1}{2} \text{ ft} = \frac{12.5}{3} \text{ yd}$$

$$= 4.1666 \dots \text{ yd}$$

$$59 \text{ ft} = \frac{59}{3} \text{ yd}$$

$$= 19.6666 \dots \text{ yd}$$

Now, calculate the area of the driveway in square yards.

$$A = l \times w$$

$$= 4.1666 \dots \text{ yd} \times 19.6666 \dots \text{ yd}$$

$$= 81.9444 \dots \text{ yd}^2$$

The area of the driveway is 81.9 yd².

- c. The estimate is close to the calculated value. The estimate tells you if your calculated answer is reasonable. In this case, our answer is reasonable because it is close to the estimate.

2. a. A binder cover is 27 cm by 29 cm.

Round each dimension to the nearest 10 cm.

27 cm rounds to 30 cm.

29 cm rounds to 30 cm.

Find the approximate area.

$$\begin{aligned} A &= l \times w \\ &= 30 \text{ cm} \times 30 \text{ cm} \\ &= 900 \text{ cm}^2 \end{aligned}$$

The estimated area of the binder cover is 900 cm².

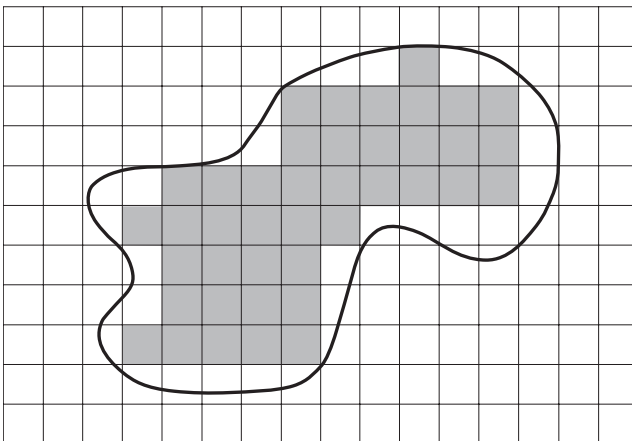
- b. A binder cover is 27 cm by 29 cm.

$$\begin{aligned} A &= l \times w \\ &= 27 \text{ cm} \times 29 \text{ cm} \\ &= 783 \text{ cm}^2 \end{aligned}$$

The calculated answer less than the estimate. You would expect the calculated answer to be less than 900 cm² since each dimension was rounded up when finding the estimate. Therefore the answer is reasonable.

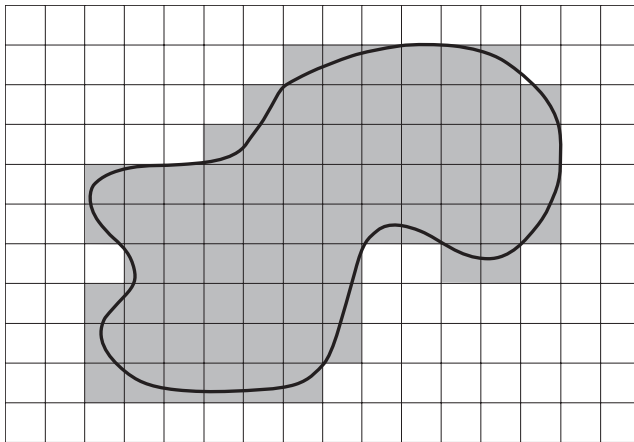
Lesson A: Activity 4: Self-Check

1. Count the squares entirely within the figure.



There are 41 unit squares inside the figure.

Now count the squares that cover the figure.



There are 75 squares that cover the figure.

Find the average of the two counts.

$$\begin{aligned} \text{Average} &= \frac{41 + 75}{2} \\ &= \frac{116}{2} \\ &= 58 \end{aligned}$$

The area of the figure is about 58 units².

- If the squares were 1 cm on a side, the estimated area would be 58 cm².

Lesson A: Activity 5: Mastering Concepts

- 1 square = 100 ft²

$$\begin{aligned} 1 \text{ bundle} &= \frac{1}{3} \text{ of a square} \\ &= \frac{1}{3} \times 100 \text{ ft}^2 \\ &= 33\frac{1}{3} \text{ ft}^2 \end{aligned}$$

There are $33\frac{1}{3}$ square feet in a bundle.

$$\begin{aligned}
 2. \text{ Waste per bundle} &= 33\frac{1}{3} \text{ ft}^2 - 32 \text{ ft}^2 \\
 &= 1\frac{1}{3} \text{ ft}^2
 \end{aligned}$$

There are about $1\frac{1}{3}$ square feet of wasted shingles per bundle.

1 square = 3 bundles

Waste per square = $3 \times$ (waste per bundle)

$$\begin{aligned}
 &= 3 \times 1\frac{1}{3} \text{ ft}^2 \\
 &= 3 \times \frac{4}{3} \text{ ft}^2 \\
 &= 4 \text{ ft}^2
 \end{aligned}$$

There are about 4 ft^2 of wasted shingles per square.

$$\begin{aligned}
 3. \text{ So, } 4 \text{ ft}^2 \text{ per square or } 100 \text{ ft}^2 &= \frac{4}{100} \\
 &= 4\%
 \end{aligned}$$

A contractor allows for 4% waste.

4. 1 bundle covers about 32 ft^2 .

For 1000 ft^2 , a contractor would need $1000 \div 32 = 31.25$ bundles. So, likely, she would order 32 bundles of shingles.

Section—Lesson B: Area Formulas 1

Lesson B: Activity 1: Self-Check

- | | | | |
|------|------|------|------|
| 1. B | 2. H | 3. G | 4. E |
| 5. D | 6. C | 7. F | 8. A |

Lesson B: Activity 2: Try This

- The base is 9 units long.
 - Its height is 6 units.
- The length of the rectangle is 9 units.
 - Its width is 6 units.
 - The dimensions are the same.
- The area is 6 units \times 9 units, which equals 54 square units.
 - This area is the same as the area of the original parallelogram.

Lesson B: Activity 3: Try This

- Each base is 11 units long.
 - Each height is 7 units.
- The base is 11 units long.
 - Its height is 7 units.
- Area of a parallelogram = base \times height

$$A = bh$$

$$= 11 \text{ units} \times 7 \text{ units}$$

$$= 77 \text{ square units}$$
 - Since the area of two triangles (of the same area) make up the area of the parallelogram, the area of one triangle is $\frac{1}{2}$ the area of the parallelogram.

$$\frac{1}{2} \times 77 = 38.5$$

The area of each triangle is 38.5 square units.

Lesson B: Activity 4: Self-Check

- The base and height are:

$$b = 3 \text{ ft}$$

$$h = 4 \text{ ft}$$

Note: you don't need to know the length of the third side to find the area—this is extra information.

$$A = \frac{1}{2} bh$$

$$= \frac{1}{2} \times 3 \text{ ft} \times 4 \text{ ft}$$

$$= 6 \text{ ft}^2$$

2. All measures must be in the same units before you can calculate the area.

$$b = 30 \text{ ft}$$

$$h = 4 \text{ ft } 8 \frac{1}{2} \text{ in}$$

$$= 4 \text{ ft} + (8.5 \div 12) \text{ ft}$$

$$= 4 \text{ ft} + 0.7083 \dots \text{ ft}$$

$$= 4.7083 \dots \text{ ft}$$

Don't round until you get the final answer.

$$A = bh$$

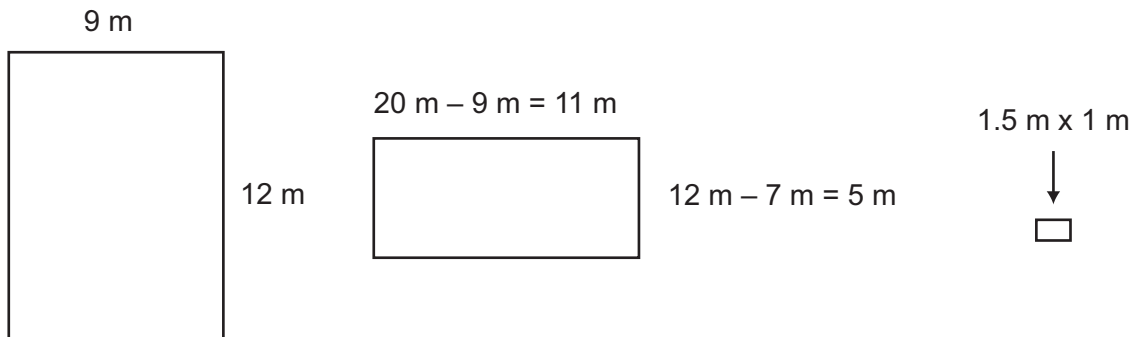
$$= 30 \text{ ft} \times 4.7083 \dots \text{ ft}$$

$$= 141.25 \text{ ft}^2$$

An area of 141.25 ft^2 lies between the tracks.

Lesson B: Activity 5: Self-Check

1. Separate the simple shapes and transfer the dimensions.



Calculate the area of each shape.

$$\begin{aligned} \text{Area of large rectangle} &= \text{length} \times \text{width} \\ &= 9 \text{ m} \times 12 \text{ m} \\ &= 108 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of small rectangle} &= \text{length} \times \text{width} \\ &= 11 \text{ m} \times 5 \text{ m} \\ &= 55 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of doghouse} &= \text{length} \times \text{width} \\ &= 1.5 \text{ m} \times 1 \text{ m} \\ &= 1.5 \text{ m}^2 \end{aligned}$$

Combine the areas.

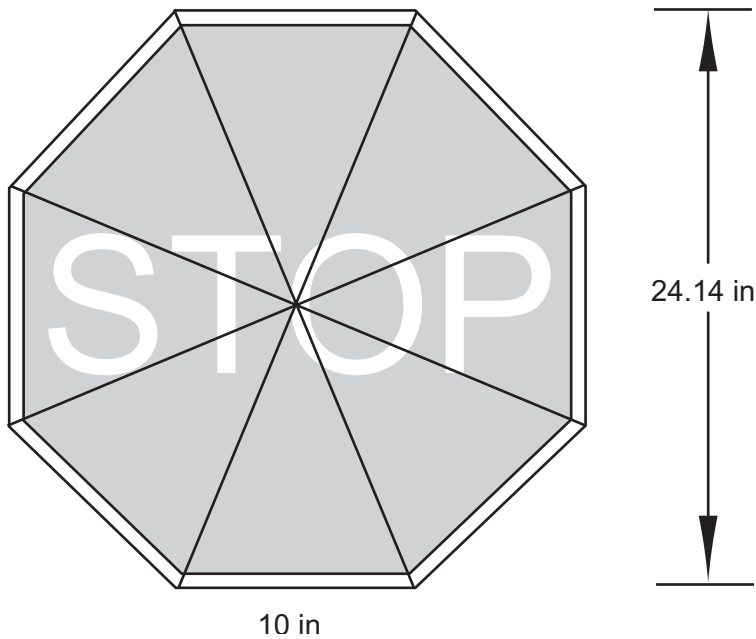
You will have to subtract the area of the doghouse.

Area of lawn = area of large rectangle + area of small rectangle – area of doghouse

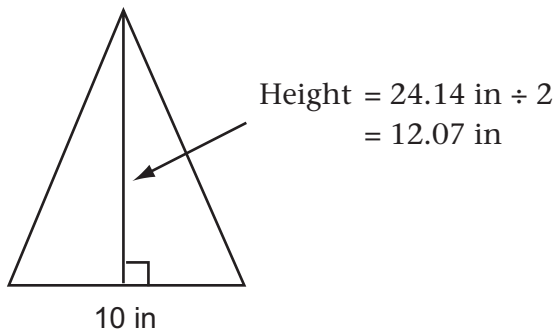
$$= 108 \text{ m}^2 + 55 \text{ m}^2 - 1.5 \text{ m}^2$$

$$= 161.5 \text{ m}^2$$

2. You could divide the octagon in many different ways. One way is to divide the octagon into eight triangles.



Determine the dimensions of each triangle.



$$\text{Area of a triangle} = \frac{1}{2} bh$$

$$= \frac{1}{2} \times 10 \text{ in} \times 12.07 \text{ in}$$

$$= 60.35 \text{ in}^2$$

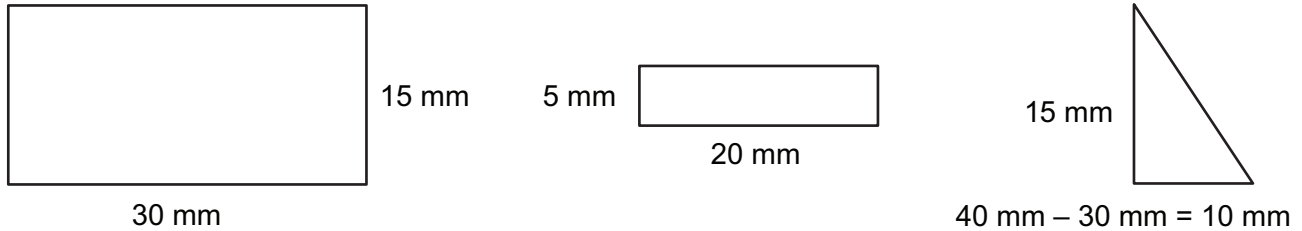
Don't round yet!

$$\begin{aligned} \text{Area of stop sign} &= 8 \times 60.35 \text{ in}^2 \\ &= 482.8 \text{ in}^2 \\ &= 483 \text{ in}^2 \end{aligned}$$

Round after the last calculation.

The area of the stop sign is about 483 in^2 .

3. Separate the figure into simple shapes. There are several possibilities. One is shown.



Calculate the area of each shape.

$$\begin{aligned} \text{Area of large rectangle} &= \text{length} \times \text{width} \\ &= 30 \text{ mm} \times 15 \text{ mm} \\ &= 450 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of small rectangle} &= \text{length} \times \text{width} \\ &= 20 \text{ mm} \times 5 \text{ mm} \\ &= 100 \text{ mm}^2 \end{aligned}$$

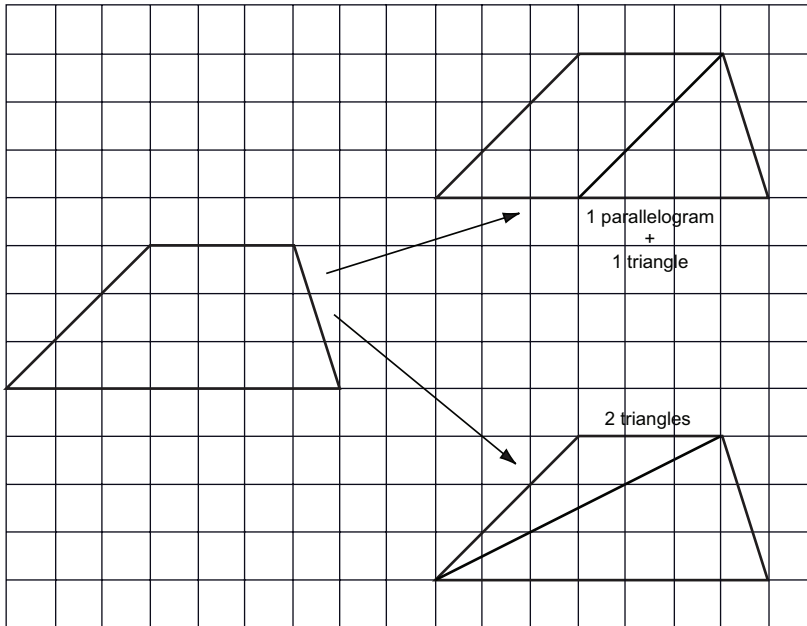
$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 10 \text{ mm} \times 15 \text{ mm} \\ &= 75 \text{ mm}^2 \end{aligned}$$

Area of shaded region = area of large rectangle – area of small rectangle + area of triangle

$$\begin{aligned} &= 450 \text{ mm}^2 - 100 \text{ mm}^2 + 75 \text{ mm}^2 \\ &= 425 \text{ mm}^2 \end{aligned}$$

Lesson B: Activity 6: Mastering Concepts

1. Answers will vary. Two possibilities are shown below.



2. Answers will vary.

One way to calculate the area of the trapezoid is based on splitting its area into just two triangles, as shown in previous diagram. This way you find the areas of the lower and upper triangles and add these areas.

Area of the lower triangle

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 7 \text{ units} \times 3 \text{ units} \\ &= 10.5 \text{ square units} \end{aligned}$$

Area of the upper triangle

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 3 \text{ units} \times 3 \text{ units} \\ &= 4.5 \text{ square units} \end{aligned}$$

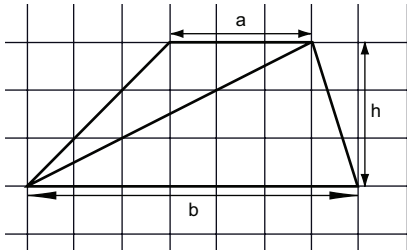
Area of the trapezoid

$$\begin{aligned} \text{Area of trapezoid} &= \text{Area of lower triangle} + \text{area of upper triangle} \\ &= 10.5 \text{ square units} + 4.5 \text{ square units} \\ &= 15 \text{ square units} \end{aligned}$$

The area of the trapezoid is 15 square units.

3. Answers will vary. We'll show the two-triangles method.

First, cut the trapezoid into two triangles and label the diagram.



Now, use the labels to find the area. This will give you a formula with variables instead of a numerical answer.

$$\begin{aligned} \text{Area of trapezoid} &= \text{area of lower triangle} + \text{area of upper triangle} \\ &= \frac{1}{2}bh + \frac{1}{2}ah \end{aligned}$$

This formula can be used for any trapezoid. Note, a and b represent the lengths of the parallel sides. You often see this formula written $A = \frac{1}{2}(a + b)h$. This is the same formula, it's just been rearranged.



To see a solution that is different from the one given in the solutions at the end of the section, go to your Media CD and open *Area of a Trapezoid*.

Section—Lesson C: Area Formulas 2

Lesson C: Activity 1: Self-Check

1. half
2. three
3. pi (π)
4. centre
5. twice

Lesson C: Activity 2: Try This

Answers will vary. Sample answers are given.

1. a. The diameter of my circle is 16 cm.

b. $\text{radius} = \frac{1}{2} \times 16 \text{ cm}$
 $= 8 \text{ cm}$

2. $C = \pi d$

$= \pi \times 16 \text{ cm}$
 $= 50.26548246 \dots \text{ cm}$
 $\approx 50 \text{ cm}$

3. a. Base $\approx 24 \text{ cm}$

b. The base is close to half the circumference.

4. a. Base $\approx 25 \text{ cm}$.

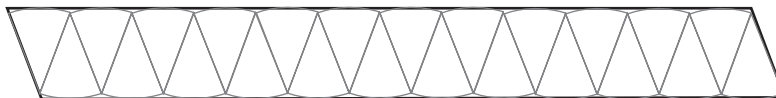
b. Rounded to the nearest centimetre, the base is half the circumference.

5. height = 8 cm

The height of the parallelogram is similar to the radius of the circle.

6. Area of the parallelogram = base \times height
 $= 25 \text{ cm} \times 8 \text{ cm}$
 $= 200 \text{ cm}^2$

7. They are close to the same. The parallelogram is made up of sectors of the circle. The sectors don't make a perfect parallelogram, so the area of the parallelogram should be a bit bigger than the area of the circle. You can see in the graphic below, that the parallelogram covers a bit more area than the sectors of the circle. There are some "empty spaces" between the sectors and the outline of the parallelogram.



8. To obtain a better approximation, divide the circle onto more sectors so that the base of the parallelogram becomes more like a straight line.

Lesson C: Activity 3: Self-Check

1. Diameter = 3 in

$$\begin{aligned}\text{Radius} &= \frac{1}{2} \text{ diameter} \\ &= \frac{1}{2} \times 3 \text{ in} \\ &= 1.5 \text{ in}\end{aligned}$$

$$\begin{aligned}A &= \pi r^2 \\ &= \pi \times 1.5 \text{ in} \times 1.5 \text{ in} \\ &= 7.068583471 \dots \text{ in}^2 \\ &\approx 7.1 \text{ in}^2\end{aligned}$$

The area of the dough is approximately 7.1 in².

2. $A = \pi r^2$
 $= \pi \times 12 \text{ ft} \times 12 \text{ ft}$
 $= 452.3893421 \dots \text{ ft}^2$
 $\approx 452 \text{ ft}^2$

The area the sprinkler covers is approximately 452 ft².

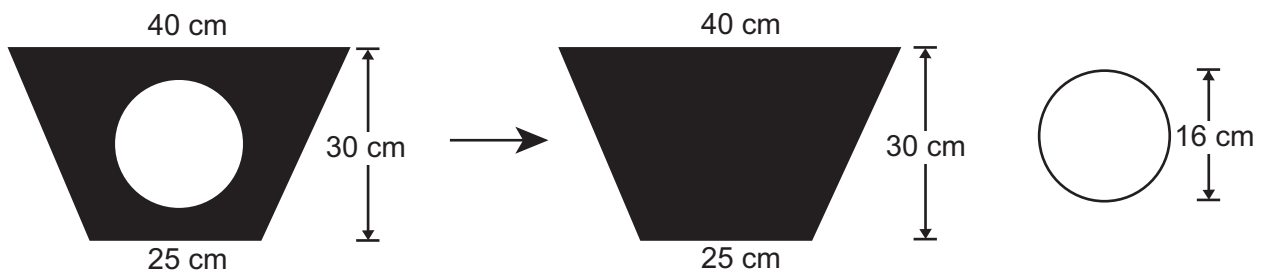
3. Pi is close to 3. We can use this approximation to estimate the area.

$$\begin{aligned}A &= \pi r^2 \\ &\approx 3 \times r^2 \\ &\approx 3 \times 12 \text{ ft} \times 12 \text{ ft} \\ &\approx 432 \text{ ft}^2\end{aligned}$$

An approximation of the area of any circle is $3 \times (\text{radius})^2$.

Lesson C: Activity 4: Self-Check

1. The two simple shapes in the composite figure are a trapezoid and a circle. Separate the shapes.



First, find the area of the trapezoid. If you remember the formula for area of a trapezoid you can find the area as follows:

$$\begin{aligned} \text{Area of the trapezoid} &= \frac{1}{2} (a + b)h \\ &= \frac{1}{2} (40 \text{ cm} + 25 \text{ cm})(30 \text{ cm}) \\ &= \frac{1}{2} (65 \text{ cm})(30 \text{ cm}) \\ &= 975 \text{ cm}^2 \end{aligned}$$

If you don't remember the formula, you may have chosen other ways to find the area of the trapezoid. Perhaps you used two triangles of a height of 30 cm—one with a base of 25 and the other with a base of 40. You could find the area of the trapezoid as follows:

$$\begin{aligned} \text{Area of the trapezoid} &= \text{area of triangle 1} + \text{area of triangle 2} \\ &= \frac{1}{2} ah + \frac{1}{2} bh \\ &= \frac{1}{2} (40 \text{ cm})(30 \text{ cm}) + \frac{1}{2} (25 \text{ cm})(30 \text{ cm}) \\ &= 600 \text{ cm}^2 + 375 \text{ cm}^2 \\ &= 975 \text{ cm}^2 \end{aligned}$$

Now find the area of the circle.

$$\begin{aligned} \text{Area of circle} &= \pi r^2 \\ &= \pi \times 8 \text{ cm} \times 8 \text{ cm} \end{aligned}$$

$r = \frac{1}{2} (16 \text{ cm}) = 8 \text{ cm}$

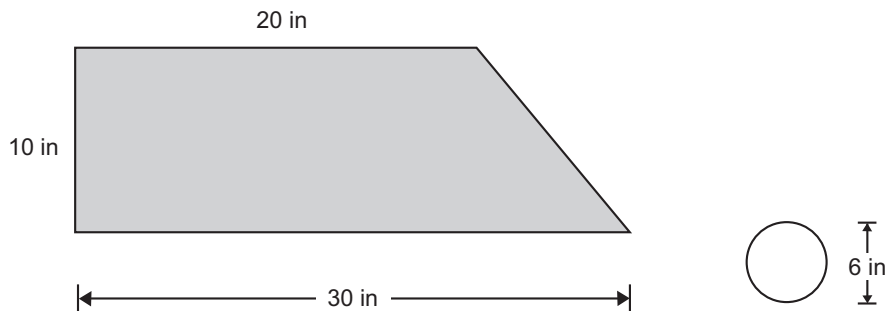
$$\begin{aligned} &= 201.0619298 \dots \text{ cm}^2 \\ &\approx 201 \text{ cm}^2 \end{aligned}$$

Now you can calculate the area of the remaining cloth.

$$\begin{aligned} \text{Area of remaining cloth} &= \text{area of trapezoid} - \text{area of circle} \\ &\approx 975 \text{ cm}^2 - 201 \text{ cm}^2 \\ &\approx 774 \text{ cm}^2 \end{aligned}$$

Approximately 774 cm² of cloth remains.

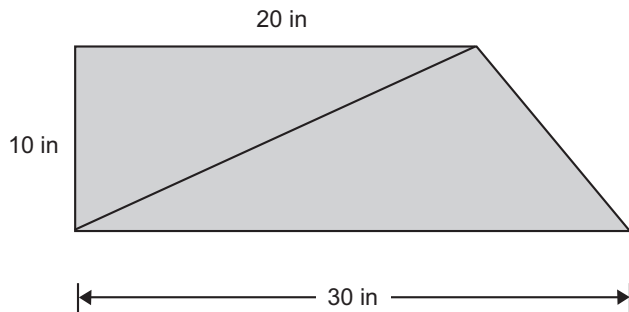
2. Separate the figures.



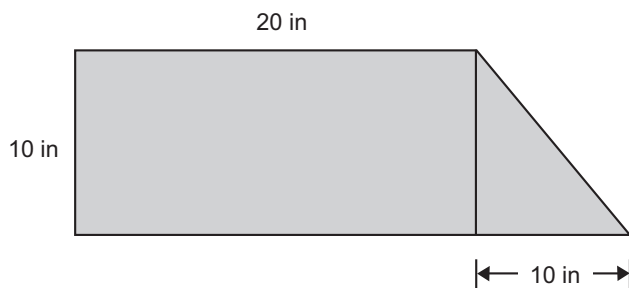
$$\begin{aligned} \text{Area of the trapezoid} &= \frac{1}{2} (a + b)h \\ &= \frac{1}{2} \times (20 \text{ in} + 30 \text{ in}) \times 10 \text{ in} \\ &= \frac{1}{2} \times 50 \text{ in} \times 10 \text{ in} \\ &= 250 \text{ in}^2 \end{aligned}$$

(You could also find the area of the trapezoid by dividing the shape into two smaller shapes. There are several ways to do this. Two are shown below:

You could divide the trapezoid into two triangles. Then you would use the formulas from your Data Pages to find the areas.



You could divide the trapezoid into a rectangle and a triangle. Then you would use the formulas from your Data Pages to find the areas.



The two half circles, cut away, form a whole circle. Find the area of this circle.

$$\text{Diameter} = 6 \text{ in}$$

$$\begin{aligned} \text{Radius} &= \frac{1}{2} \times 6 \text{ in} \\ &= 3 \text{ in} \end{aligned}$$

Area of circle

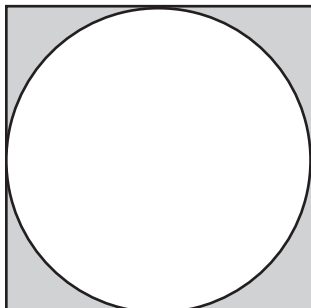
$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 3 \text{ in} \times 3 \text{ in} \\ &= 28.27433388 \dots \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \text{The area of the panel} &= \text{area of the trapezoid} - \text{area of the circle} \\ &= 250 \text{ in}^2 - 28.2743 \dots \text{ in}^2 \\ &\approx 222 \text{ in}^2 \end{aligned}$$

The area of the panel is approximately 222 in².)

Lesson C: Activity 5: Mastering Concepts

1. Start by drawing a diagram to help you visualize the problem.



$$1 \text{ ft} = 12 \text{ in}$$

Each side of the square is 12 inches long.

The diameter of the circle is 12 in.

Area of circle

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 6 \text{ in} \times 6 \text{ in} \\ &= 113.0973355 \dots \text{ in}^2 \\ &\approx 113 \text{ in}^2 \end{aligned}$$

The area of the largest circle is 113 in^2 .

2. The area of the tile = 1 ft^2 .

$$\begin{aligned} &= 12 \text{ in} \times 12 \text{ in} \\ &= 144 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \text{Waste} &= 144 \text{ in}^2 - 113 \text{ in}^2 \\ &= 31 \text{ in}^2 \end{aligned}$$

The area of the tile that is waste is 31 in^2 .

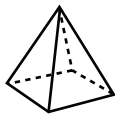
Section—Lesson D: Surface Area—Prisms and Pyramids

Lesson D: Activity 1: Self-Check

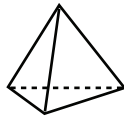
1. A prism is a 3-D object with **two bases** joined by **lateral rectangular** faces.

A pyramid is a 3-D object with **one base** whose **triangular** faces meet at a point called the **apex**.

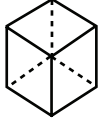
2.



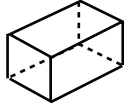
pyramid



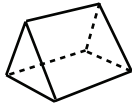
pyramid



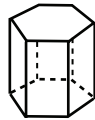
prism



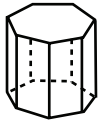
prism



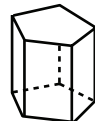
prism



prism



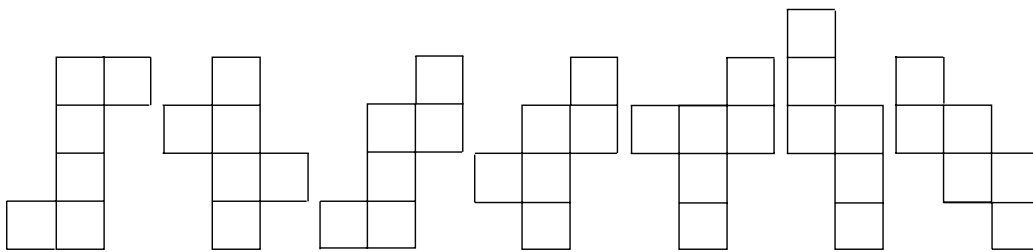
prism



prism

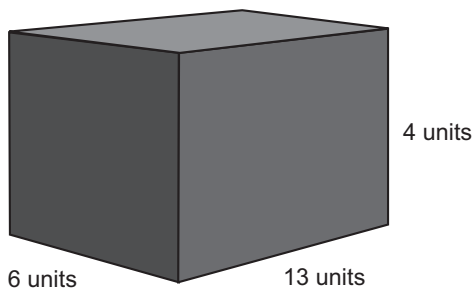
3. Area is an amount of space enclosed by a closed 2-D figure.

Lesson D: Activity 2: Try This



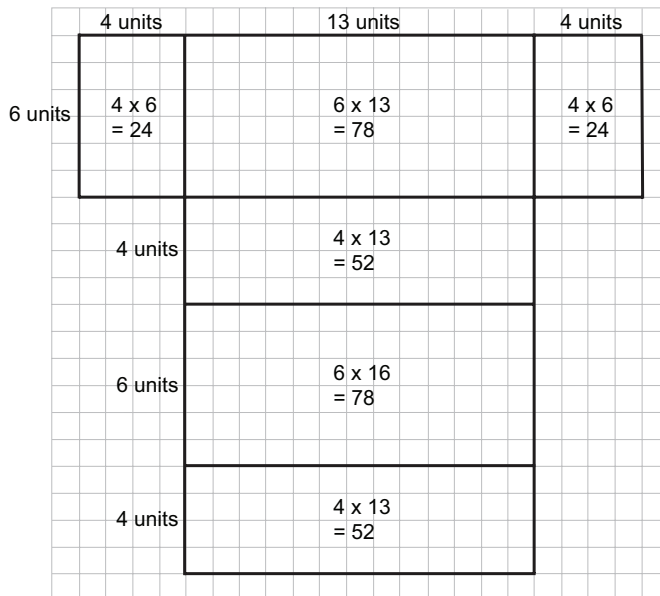
Lesson D: Activity 3: Try This

1. a. rectangular prism
- b. Its dimensions are 13 units long, 6 units wide, and 4 units high.



2. There are two faces—each $13 \text{ units} \times 6 \text{ units} = 78$ square units in area. There are two faces—each $13 \text{ units} \times 4 \text{ units} = 52$ square units in area.

There are two faces—each $6 \text{ units} \times 4 \text{ units} = 24$ square units in area.

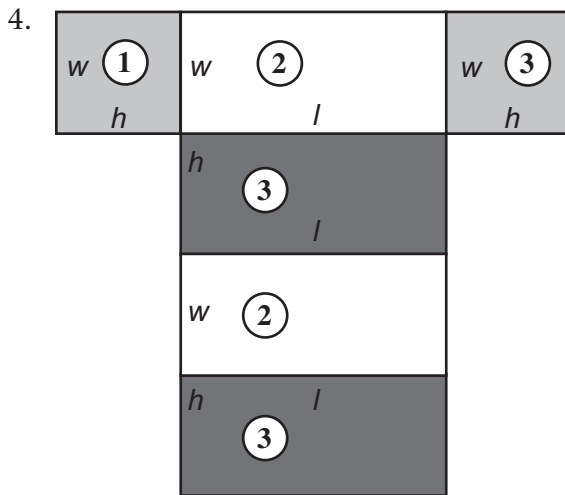


3. $SA = 2 \times 78 \text{ units}^2 + 2 \times 52 \text{ units}^2 + 2 \times 24 \text{ units}^2$
 $= 156 \text{ units}^2 + 104 \text{ units}^2 + 48 \text{ units}^2$
 $= 308 \text{ units}^2$

The surface area of the prism (based on the net) is 308 square units.

Lesson D: Activity 4: Try This

1. The squares all have the same area. They have the same length of sides.
2. There are six squares in the net for a cube.
3. Each of the six faces has an area of s^2 . Therefore, the surface area of cube is $6s^2$.



$$A_1 = wh$$

$$A_2 = lw$$

$$A_3 = lh$$

5. Add up the areas of all the faces.

$$\begin{aligned} SA &= A_1 + A_1 + A_2 + A_2 + A_3 + A_3 \\ &= wh + wh + lw + lw + lh + lh \\ &= 2wh + 2lw + 2lh \end{aligned}$$

OR

$$= 2(wh + lw + lh)$$

Lesson D: Activity 5: Self-Check

1. 1 ft = 12 in

$$\begin{aligned} 30 \text{ in} &= \frac{30}{12} \text{ ft} \\ &= 2.5 \text{ ft} \end{aligned}$$

$$\begin{aligned} SA &= 6 s^2 \\ &= 6 \times 2.5 \text{ ft} \times 2.5 \text{ ft} \\ &= 37.5 \text{ ft}^2 \end{aligned}$$

The surface area of the carton is 37.5 ft².

2. $l = 48 \text{ ft}$

$w = 8 \text{ ft}$

$h = 8 \text{ ft}$

$$SA = 2 (wh + lw + lh)$$

$$= 2 [(8 \text{ ft} \times 8 \text{ ft}) + (48 \text{ ft} \times 8 \text{ ft}) + (48 \text{ ft} \times 8 \text{ ft})]$$

$$= 2 [64 \text{ ft}^2 + 384 \text{ ft}^2 + 384 \text{ ft}^2]$$

$$= 1664 \text{ ft}^2$$

The surface area of the container is 1664 ft^2 .

3. There are 2 walls 12 ft long and 8 ft high.

There are 2 walls 9 ft long and 8 ft high.

The ceiling is 12 ft by 9 ft.

$$SA = 2 (12 \text{ ft} \times 8 \text{ ft}) + 2 (9 \text{ ft} \times 8 \text{ ft}) + (12 \text{ ft} \times 9 \text{ ft})$$

$$= 192 \text{ ft}^2 + 144 \text{ ft}^2 + 108 \text{ ft}^2$$

$$= 444 \text{ ft}^2$$

Jim's estimate is 444 ft^2 .

4. $SA = 4 \times \text{area of triangular face}$

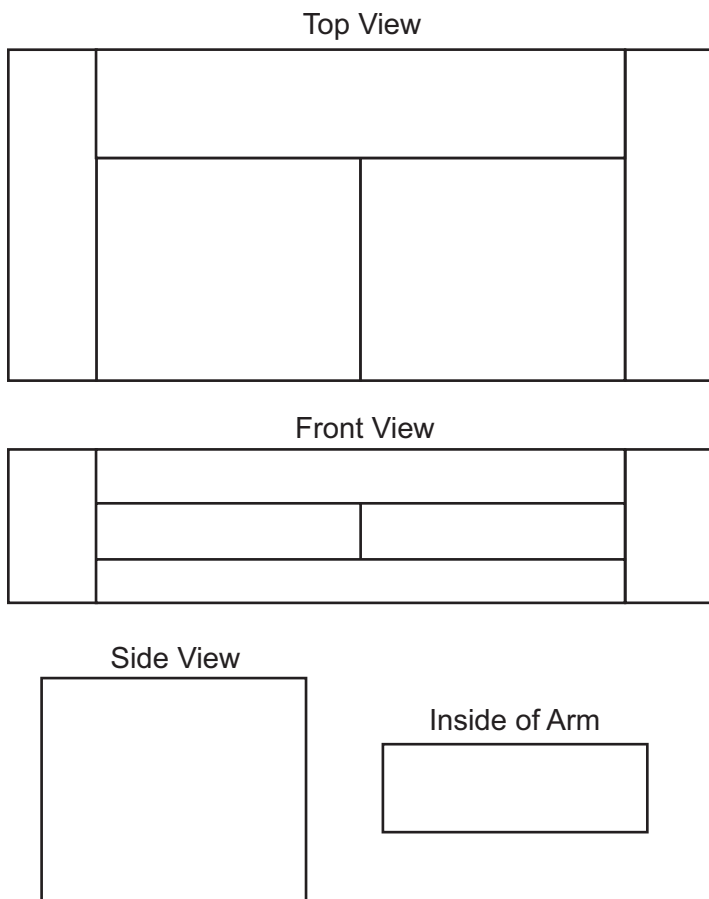
$$= 4 (\frac{1}{2} bh)$$

$$= 4 (\frac{1}{2} \times 5.8 \text{ cm} \times 5 \text{ cm})$$

$$= 58 \text{ cm}^2$$

The surface area of the tetrahedron is 58 cm^2 .

Lesson D: Activity 6: Mastering Concepts



Total area of fabric required = area of top view + 2 (area of front view) + 2 (area of side view) + 2 (area of inside arm)

Section—Lesson E: Surface Area—Cylinders and Cones

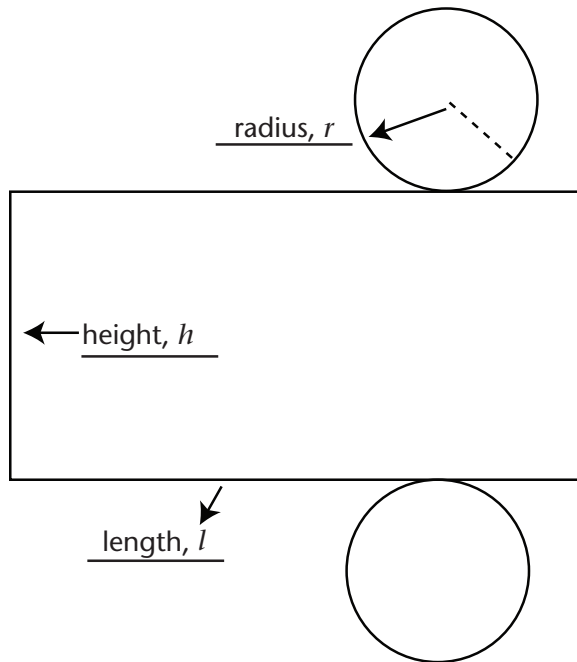
Lesson E: Activity 1: Self-Check

1. a. A cylinder is a three-dimensional shape which has two congruent circular bases that are parallel to each other.
 - two circular bases joined by a curved surface
 - can be short and fat or long and skinny (and all sizes in between!)

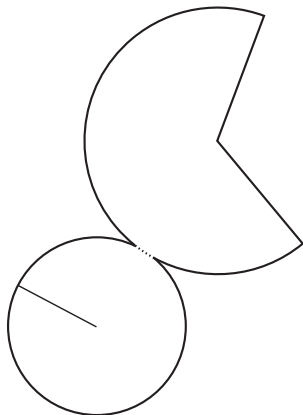
b. A cone is a three-dimensional object that has one circular base and an attached curved surface that comes to a point.

- one circular base
- the sides taper up to a point
- can be short and fat or tall and skinny (and all sizes in between!)

2. a.



b.



Lesson E: Activity 2: Try This

Cylinder Measurements

Item	Measurement (cm)
height of can	11
diameter of circular top of can	7.5
circumference of circular top of can	24
length of label	23.5
width of label	10.5

Answers will vary. Sample answers based on the sample data are given below.

$$\begin{aligned}
 1. \quad A &= lw \\
 &= (23.5 \text{ cm})(10.5 \text{ cm}) \\
 &= 246.75 \text{ cm}^2
 \end{aligned}$$

2. First find the radius of the can.

$$\begin{aligned}
 r &= \frac{d}{2} \\
 &= \frac{7.5 \text{ cm}}{2} \\
 &= 3.75 \text{ cm}
 \end{aligned}$$

Now find the area of the circular base.

$$\begin{aligned}
 A &= \pi r^2 \\
 &= \pi(3.75 \text{ cm})^2 \\
 &= 44.2 \text{ cm}^2
 \end{aligned}$$

3. To find the surface area, add up the areas of all the faces of the can. There are two circular bases and a rectangular label.

$$\begin{aligned}
 SA &= 2A_{\text{base}} + A_{\text{label}} \\
 &= 2(44.2 \text{ cm}^2) + 246.75 \text{ cm}^2 \\
 &= 335.1 \text{ cm}^2
 \end{aligned}$$

4. The width of the label is almost the same as the height of the can. The length of the label is almost the same as the length of the circumference of the can.

Lesson E: Activity 3: Try This

Cone Measurements

Item	Measurement (cm)
slant height	6
radius of circular base	4
arc length of partial circle	25

$$\begin{aligned}
 1. \quad A &= \pi r^2 \\
 &= \pi(4 \text{ cm})^2 \\
 &= 50.3 \text{ cm}^2
 \end{aligned}$$

2. a. Area of completed circle:

$$\begin{aligned}
 A &= \pi r^2 \\
 &= \pi(6 \text{ cm})^2 \\
 &= 113.1 \text{ cm}^2
 \end{aligned}$$

Circumference of completed circle:

$$\begin{aligned}
 C &= 2\pi r \\
 &= 2\pi(6 \text{ cm}) \\
 &= 37.7 \text{ cm}
 \end{aligned}$$

$$\text{b.} \quad \frac{\text{area of partial circle}}{\text{area of completed circle}} = \frac{\text{arc length of partial circle}}{\text{circumference of completed circle}}$$

$$\frac{\text{area of partial circle}}{113.1 \text{ cm}^2} = \frac{25 \text{ cm}}{37.7 \text{ cm}}$$

$$(\text{area of partial circle})(37.7 \text{ cm}) = (25 \text{ cm})(113.1 \text{ cm}^2)$$

$$\text{area of partial circle} = \frac{(25 \text{ cm})(113.1 \text{ cm}^2)}{37.7 \text{ cm}}$$

$$\text{area of partial circle} = 75 \text{ cm}^2$$

$$\begin{aligned}
 3. \quad SA_{\text{cone}} &= A_{\text{circular base}} + A_{\text{curved surface}} \\
 &= 50.3 \text{ cm}^2 + 75 \text{ cm}^2 \\
 &= 125.3 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \text{a.} \quad C &= 2\pi r \\
 &= 2\pi(4 \text{ cm}) \\
 &= 25.1 \text{ cm}
 \end{aligned}$$

- b. The circumference of the circular base of the cone is the same length as the arc length of the partial circle in the cone's net. This is because, when you fold the net to create a 3-D shape, the arc of the partial circle becomes attached to the circular base. Therefore, the circumference of the base should be the same as the length of the arc on the net's partial circle.

Lesson E: Activity 4: Self-Check

1. In this question you do not need to include the area of the base. The base of the pile rests on a ground sheet.

$$\begin{aligned}\text{Radius} &= \frac{1}{2} \times \text{diameter} \\ &= \frac{1}{2} \times 15 \text{ ft} \\ &= 7.5 \text{ ft}\end{aligned}$$

$$\begin{aligned}\text{SA exposed to the air} &= \pi r s \\ &= \pi(7.5 \text{ ft})(8.3 \text{ ft}) \\ &= 195.564127 \dots \text{ft}^2 \\ &\approx 195.6 \text{ ft}^2\end{aligned}$$

About 195.6 ft^2 is exposed to the air.

2. Diameter = 20 ft

$$\text{Radius} = 10 \text{ ft}$$

$$\begin{aligned}\text{SA} &= \text{area of side} + \text{area of top} \\ &= 2\pi r h + \pi r^2 \\ &= 2\pi(10 \text{ ft})(65 \text{ ft}) + \pi(10 \text{ ft})(10 \text{ ft}) \\ &= 4398.229715 \dots \text{ft}^2 \\ &\approx 4400 \text{ ft}^2\end{aligned}$$

The painted area of the silo is approximately 4400 ft^2 .

3. Diameter = 4 ft

$$\text{Radius} = 2 \text{ ft}$$

$$\begin{aligned}\text{SA} &= \text{area of side} + \text{area of both circular ends} \\ &= 2\pi r^2 + 2\pi r h \\ &= 2\pi(2 \text{ ft})(2 \text{ ft}) + 2\pi(2 \text{ ft})(6 \text{ ft}) \\ &= 100.5309649 \dots \text{ft}^2 \\ &\approx 101 \text{ ft}^2\end{aligned}$$

The surface area of the tank is approximately 101 ft^2 .

4. Area of cylindrical side

$$\text{Diameter} = 12 \text{ ft}$$

$$\begin{aligned} \text{Radius} &= \frac{1}{2} d \\ &= \frac{1}{2} (12 \text{ ft}) \\ &= 6 \text{ ft} \end{aligned}$$

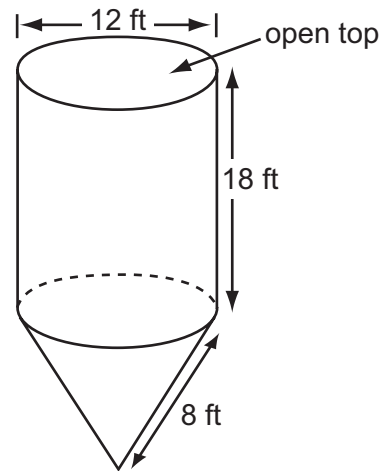
$$\text{Height} = 18 \text{ ft}$$

$$\begin{aligned} \text{Area} &= 2\pi rh \\ &= 2\pi(6 \text{ ft})(18 \text{ ft}) \\ &= 678.5840132 \dots \text{ft}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of conical surface} &= \pi rs \\ &= \pi(6 \text{ ft})(8 \text{ ft}) \\ &= 150.7964474 \dots \text{ft}^2 \end{aligned}$$

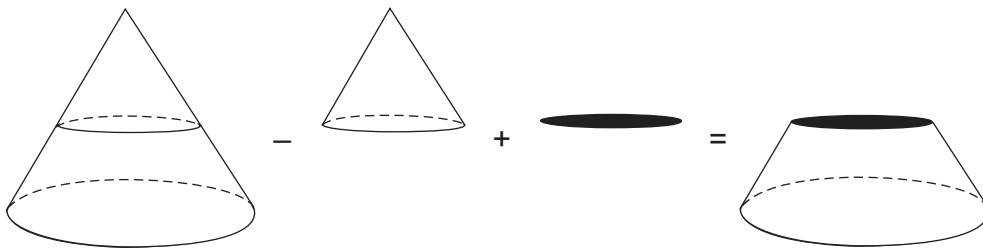
$$\begin{aligned} \text{SA} &= \text{area of cylindrical side} + \text{area of conical side} \\ &= 678.5840132 \dots \text{ft}^2 + 150.7964474 \dots \text{ft}^2 \\ &\approx 829 \text{ ft}^2 \end{aligned}$$

The outside area of the hopper is about 829 ft².



Lesson E: Activity 5: Mastering Concepts

Since you are removing a part of the original cone, finding the surface area will involve subtracting. First, you need to find the surface area of the whole conical surface. Then, you subtract the surface area of the part that was removed.



Step 1: Find the area of the conical surface of the original cone.

$$\text{Diameter} = 20 \text{ cm}$$

$$\text{Radius} = 10 \text{ cm}$$

$$\text{Slant height} = 20 \text{ cm}$$

$$\begin{aligned} \text{Area} &= \pi rs \\ &= \pi(10 \text{ cm})(20 \text{ cm}) \\ &= 628.3185307 \dots \text{cm}^2 \\ &\approx 628 \text{ cm}^2 \end{aligned}$$

Step 2: Find the area of the conical surface of the top piece that is cut off.

Diameter = 10 cm

Radius = 5 cm

Slant height = 10 cm

$$\begin{aligned} \text{Area} &= \pi r s \\ &= \pi(5 \text{ cm})(10 \text{ cm}) \\ &= 157.0796327 \dots \text{ cm}^2 \\ &\approx 157 \text{ cm}^2 \end{aligned}$$

Step 3: Find the surface area of the bottom of the conical piece.

$$\begin{aligned} \text{Area} &= \text{area of conical surface of the entire cone} - \text{area of conical surface of the top} \\ &\approx 628 \text{ cm}^2 - 157 \text{ cm}^2 \\ &\approx 471 \text{ cm}^2 \end{aligned}$$

Step 4: Find the total surface area by adding the areas of the circular top and bottom of the remaining piece.

Radius of bottom circle = 10 cm

Radius of top circle = 5 cm

$$\begin{aligned} \text{SA} &= \text{area of conical surface} + \text{area of top circle} + \text{area of bottom circle} \\ &\approx 471 \text{ cm}^2 + \pi(10 \text{ cm})(10 \text{ cm}) + \pi(5 \text{ cm})(5 \text{ cm}) \\ &\approx 863.6990817 \dots \text{ cm}^2 \\ &\approx 864 \text{ cm}^2 \end{aligned}$$

The surface area of the remaining piece is approximately 864 cm².

Section—Lesson F: Perimeter and Area

Lesson F: Activity 1: Self-Check

1. shape: square

$$\text{Perimeter} = 4s$$

$$\text{Area} = s^2$$

2. shape: rectangle

$$\text{Perimeter} = 2l + 2w$$

$$\text{Area} = lw$$

Lesson F: Activity 2: Try This

Perimeter and Area of Squares

Dimensions of Square	Perimeter in Units	$\frac{\text{perimeter of square}}{\text{perimeter of } 1 \times 1 \text{ square}}$	Area in Units ²	$\frac{\text{area of square}}{\text{area of } 1 \times 1 \text{ square}}$
$s = 1$	4	$\frac{4}{4} = 1$	1	$\frac{1}{1} = 1$
$s = 2$	8	$\frac{8}{4} = 2$	4	$\frac{4}{1} = 4$
$s = 3$	12	$\frac{12}{4} = 3$	9	$\frac{9}{1} = 9$
$s = 4$	16	$\frac{16}{4} = 4$	16	$\frac{16}{1} = 16$
$s = 5$	20	$\frac{20}{4} = 5$	25	$\frac{25}{1} = 25$
$s = 6$	24	$\frac{24}{4} = 6$	36	$\frac{36}{1} = 36$

1. The perimeter would be 3 times larger.
2. The area would be 3^2 (or 9) times larger.
3. The perimeter would change by a factor of 7.
4. The area would change by a factor of 7^2 (or 49).
5. The perimeter would change by a factor of k .
6. The area would change by a factor of k^2 .

Lesson F: Activity 3: Try This

Perimeter and Area of Rectangles

Scale Factor	Dimensions of Rectangle	Perimeter in Units	perimeter of rectangle	Area in Units ²	area of rectangle
			perimeter of 1×2 rectangle		area of 1×2 rectangle
1	$l = 2, w = 1$	6	$\frac{6}{6} = 1$	2	$\frac{2}{2} = 1$
2	$l = 4, w = 2$	12	$\frac{12}{6} = 2$	8	$\frac{8}{2} = 4$
3	$l = 6, w = 3$	18	$\frac{18}{6} = 3$	18	$\frac{18}{2} = 9$
4	$l = 8, w = 4$	24	$\frac{24}{6} = 4$	32	$\frac{32}{2} = 16$
5	$l = 10, w = 5$	30	$\frac{30}{6} = 5$	50	$\frac{50}{2} = 25$
6	$l = 12, w = 6$	36	$\frac{36}{6} = 6$	72	$\frac{72}{2} = 36$

1. If the scale factor is increased by 1, the perimeter increases by 6 units.
2. The perimeter would change by a factor of 7.
3. The area would change by a factor of 7^2 or 49.
4. The perimeter would change by a factor of k .
5. The area would change by a factor of k^2 .

Lesson F: Activity 4: Self-Check

1. First, find the scale factor.

$$\frac{\text{perimeter of larger enclosure}}{\text{perimeter of smaller enclosure}} = \frac{300 \text{ m}}{100 \text{ m}}$$

$$= 3$$

The scale factor is 3.

So, the larger enclosure would be 3^2 or 9 times the area.

2. **Method 1**

Find the area of each tin.

$$\begin{aligned} \text{Smaller tin} &= 8 \text{ in} \times 8 \text{ in} \\ &= 64 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \text{Larger tin} &= 12 \text{ in} \times 12 \text{ in} \\ &= 144 \text{ in}^2 \end{aligned}$$

Compare the two areas:

$$\frac{\text{Area of larger tin}}{\text{Area of smaller tin}} = \frac{144}{64}$$

$$= 2.25$$

The top of the larger cake is 2.25 times the area of the top of the smaller cake.

Method 2

First, find the scale factor.

$$\frac{\text{Side of larger tin}}{\text{Side of smaller tin}} = \frac{12 \text{ in}}{8 \text{ in}}$$

$$= 1.5$$

$$\begin{aligned} \text{Number of times the area increases} &= (\text{scale factor})^2 \\ &= 1.5^2 \\ &= 2.25 \end{aligned}$$

The top of the larger cake is 2.25 times the area of the top of the smaller cake.

3. a. Number of times the area changes = (scale factor)².

$$2 = (\text{scale factor})^2$$

$$(\text{scale factor})^2 = 2$$

$$\text{scale factor} = \sqrt{2}$$

$$= 1.414213562 \dots$$

$$\approx 1.414$$

$$\begin{aligned}
 \text{b. Length of enlargement} &= \text{scale factor} \times \text{length of the original.} \\
 &= 1.414 \times 7 \text{ in} \\
 &= 9.898 \text{ in} \\
 &\approx 10 \text{ in}
 \end{aligned}$$

$$\begin{aligned}
 \text{Width of enlargement} &= \text{scale factor} \times \text{width of the original.} \\
 &= 1.414 \times 5 \text{ in} \\
 &= 7.07 \text{ in} \\
 &\approx 7 \text{ in}
 \end{aligned}$$

The enlargement is approximately 7 inches by 10 inches.

$$4. \quad 1 \text{ ft} = 12 \text{ in}$$

$$\begin{aligned}
 29 \text{ ft } 6 \text{ in} &= 29 \times 12 \text{ in} + 6 \text{ in} \\
 &= 354 \text{ in}
 \end{aligned}$$

$$\text{Scale factor} = \frac{1}{32}$$

$$\begin{aligned}
 \text{Length of scale model} &= \text{scale factor} \times \text{length of Harvard} \\
 &= \frac{1}{32} \times 354 \text{ in} \\
 &= 11.0625 \text{ in} \\
 &\approx 11 \text{ in}
 \end{aligned}$$

Renée's model Harvard will be 11 inches long.

$$5. \quad \text{Find the scale factor.}$$

$$\begin{aligned}
 2 \text{ mi} \div \frac{1}{2} \text{ mi} &= 2 \times 2 \\
 &= 4
 \end{aligned}$$

So, the scale factor is 4.

The area of a square 2 miles on a side is

$$\begin{aligned}
 (\text{scale factor})^2 &= 4^2 \\
 &= 16 \text{ times as large as a square only } \frac{1}{2} \text{ mi on a side}
 \end{aligned}$$

Number of acres in the square 2 mi on a side = 16×160 acres.

$$= 2560 \text{ acres}$$

$$\begin{aligned}
 6. \quad \text{New perimeter} &= \text{scale factor} \times \text{original perimeter} \\
 &= 2.5 \times 30 \text{ ft} \\
 &= 75 \text{ ft}
 \end{aligned}$$

The perimeter of the enlarged deck is 75 ft.

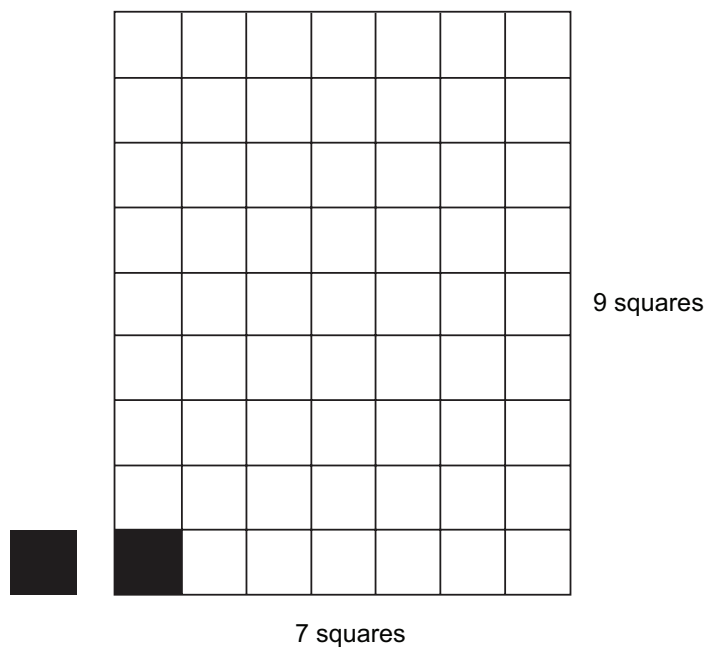
7. New area = (scale factor)² × original area
 = (2.5)² × original area
 = 6.25 × original area

Lesson F: Activity 5: Self-Check

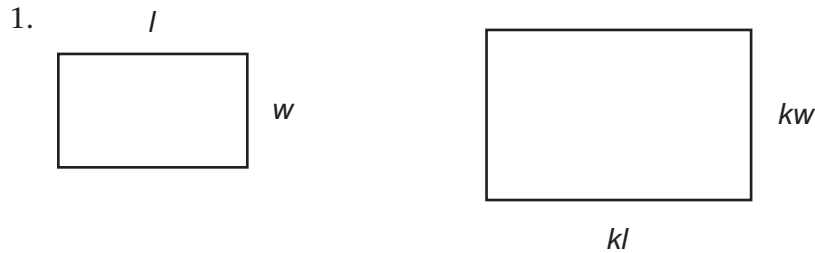
1. Length increases 5 times. Width increases 2 times. So the area of the new rectangle will be $5 \times 2 = 10$ times as large.

The area of the new rectangle is $10 \times 23 \text{ ft}^2 = 230 \text{ ft}^2$.

2. $7 \times 9 = 63$ of the original squares would fit into the enlarged figure.



Lesson F: Activity 6: Mastering Concepts



Step 1: Compare areas.

$$\text{Area of original rectangle} = lw$$

$$\begin{aligned} \text{Area of new rectangle} &= \text{length} \times \text{width} \\ &= kl \times kw \\ &= k^2 \times lw \\ &= k^2 \times \text{Area of original rectangle} \end{aligned}$$

Step 2: Compare perimeters.

$$\text{Perimeter of original rectangle} = 2l + 2w.$$

$$\begin{aligned} \text{Perimeter of new rectangle} &= 2kl + 2kw. \\ &= k(2l + 2w) \\ &= k \times \text{perimeter of original rectangle} \end{aligned}$$

2. There are many possibilities changing dimensions that would double the area of a rectangle. The following are the most straight forward:

- If the side lengths were increased by a factor of $\sqrt{2}$, the area would double.
- If the length was doubled and the width remained the same, the area would double.
- If the width was doubled and the length remained the same, the area would double.

There are other possibilities though.

- If the width was divided by 2 and the length was multiplied by 4, the area would double.

The most general answer is that the product of the change of length and the change of width must be 2.

Glossary

apex

the point furthest from a base in a cone

area

a measurement of how many square units into which a surface may be divided

For example, if your living room carpet can be divided into 10 square metres, its area is 10 m².

centre

a point of a circle that is at the circle's middle

circumference

the perimeter of, or the distance around, a circle

composite figure

any shape formed by combining simpler shapes

Combining simpler shapes such as the triangle, rectangle, parallelogram, and circle results in a composite figure.

cone

a three-dimensional object made up of a flat circular base and an attached curved surface that comes to a point called the *apex*

cube

a rectangular prism for which all faces are identical squares

cylinder

a three-dimensional object that has two congruent circular bases attached by a curved surface

diagonal

a straight line joining any two non-adjacent vertices of a polygon

Non-adjacent vertices are vertices that are not end points of the same side. For example, a diagonal of a rectangle is a line joining opposite corners.

diameter

a line segment that divides a circle in half

A diameter of a circle passes through the circle's centre point.

face

a 2-D side of a prism

horizontal

parallel to the level of the ground

On a sheet of paper, the horizontal direction is parallel to the top and bottom sides of the paper.

linear measurement

the measurement of length

Units of linear measure include centimetre, metre, kilometre, inch, foot, and yard.

midpoint

the point at the middle or centre of an object

net

a two-dimensional (2-D) pattern used to create, by folding and joining, a three-dimensional (3-D) object

parallelogram

a quadrilateral in which opposite sides are parallel and are the same length

perimeter

the linear distance around the outside of a shape

pyramid

a 3-D object having a polygonal base and triangular sides with a common vertex

quadrilateral

a four-sided polygon

Quad represents four. *Lateral* represents side.

radius

a line segment that joins the centre of a circle to a point on the diameter of the circle

rectangular prism

a 3-D object for which all the sides are rectangles

A cardboard box is an example of a rectangular prism.

referent

an object or part of the human body you can refer to when estimating length or distance

scale factor

a number used to multiply the dimensions of a 2-D object such as a rectangle

semicircle

half of a circle

slant height

the shortest distance from the apex of a cone to its base along its curved surface

surface area

the measure of how much exposed area a solid object has, expressed in square units; the combined area of all of the surfaces of a three-dimensional object

trapezoid

a quadrilateral with one pair of parallel sides

two-dimensional (2-D) object

an object having just two dimensions

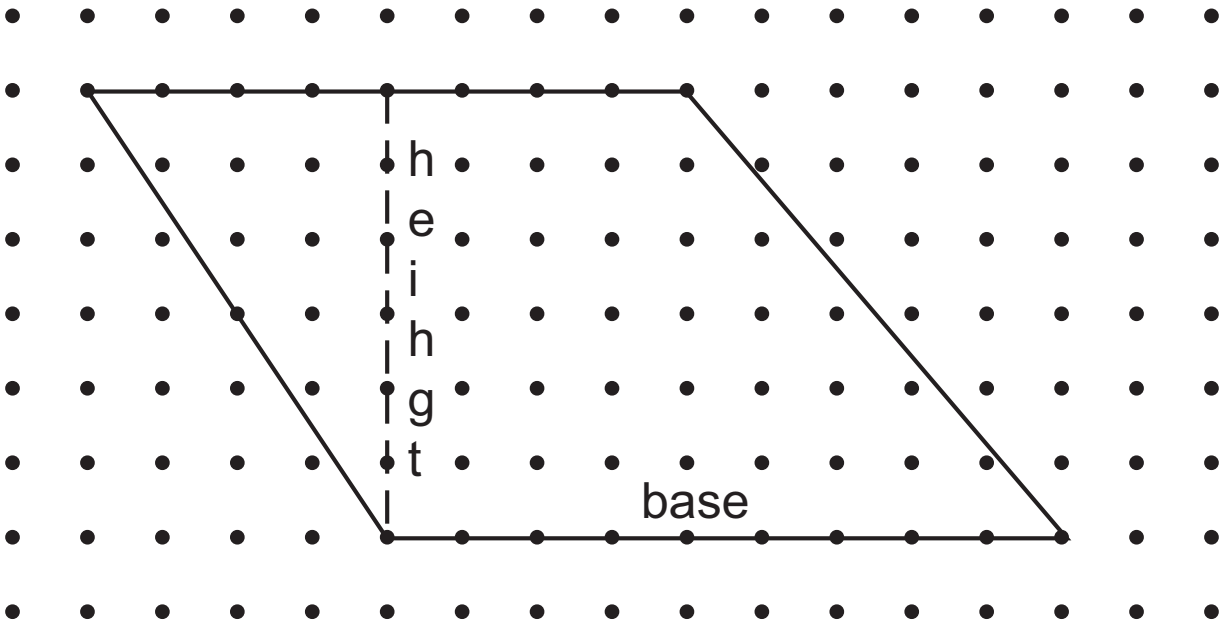
A 2-D object has length and width but no depth or thickness.

vertical

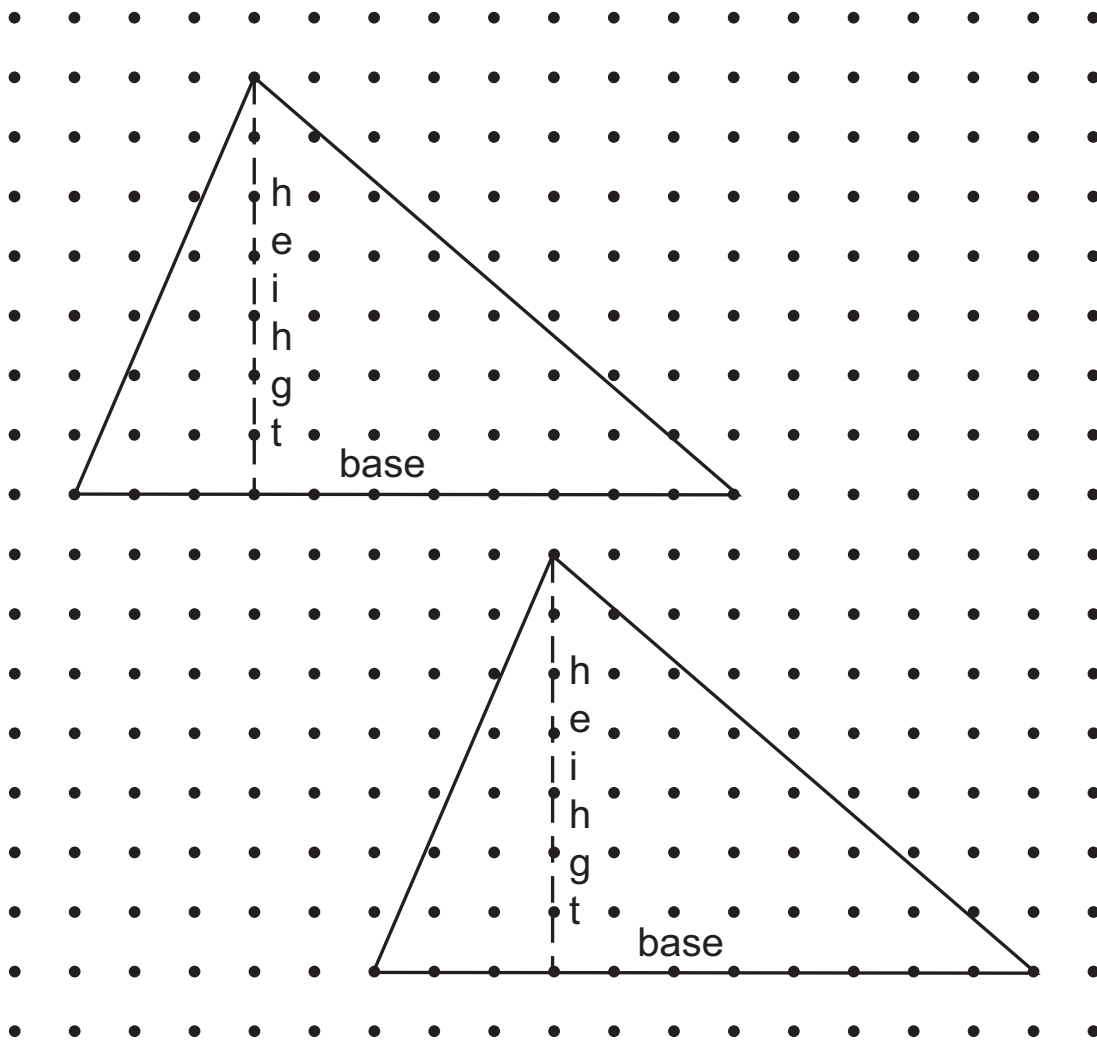
perpendicular to the level of the ground

On a sheet of paper, the vertical direction is parallel to the left and right sides of the paper.

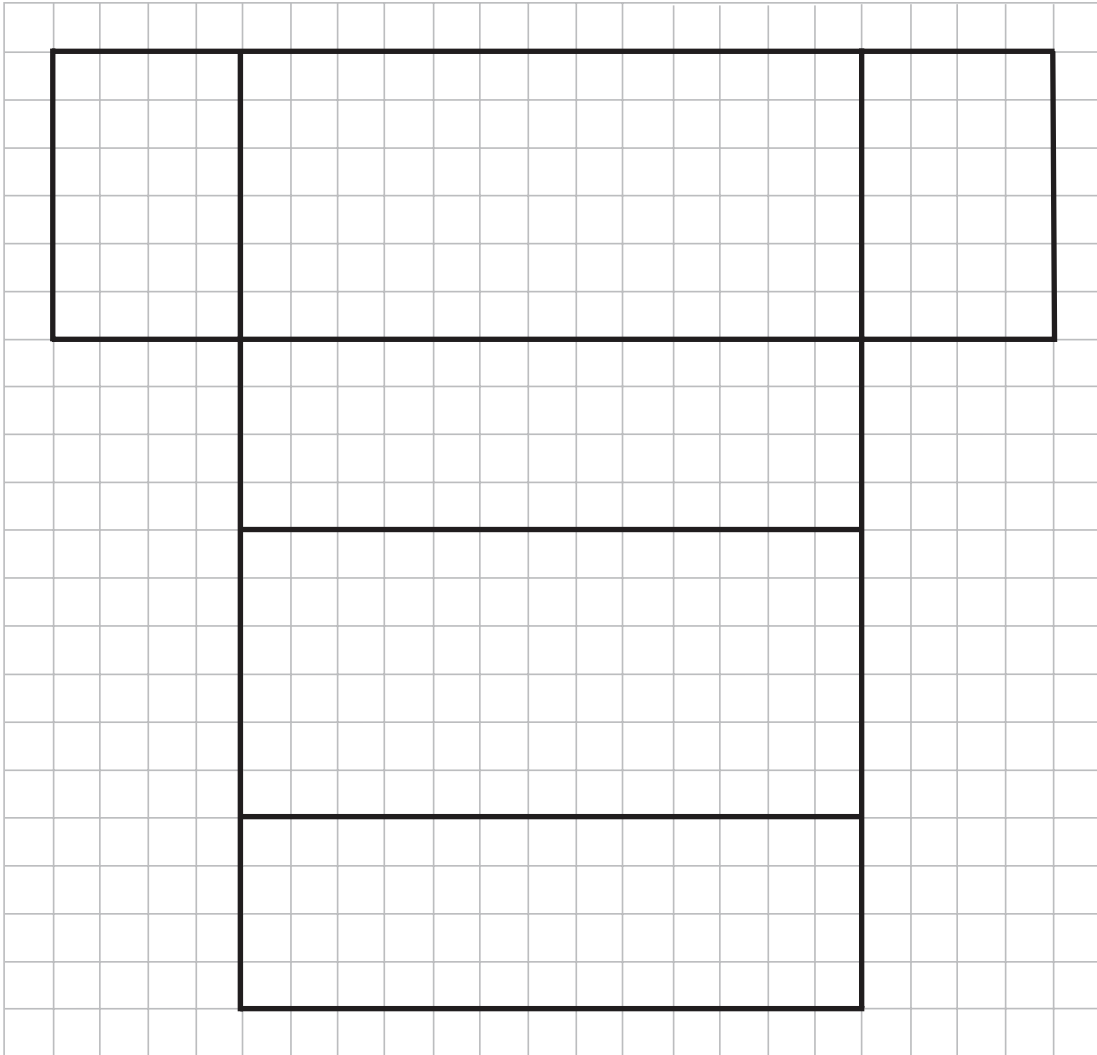
Parallelogram Template



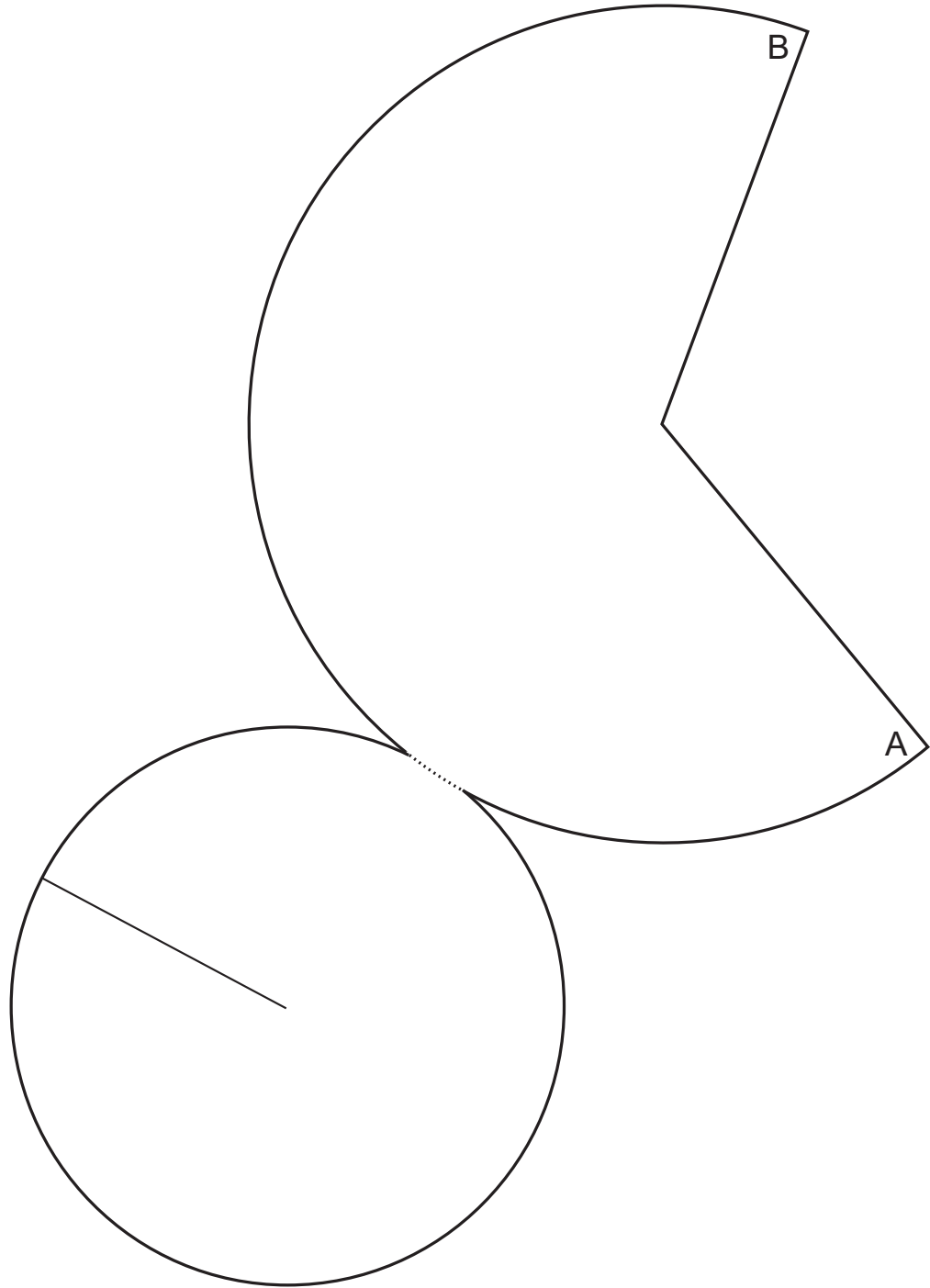
Triangle Template



Net for Activity 3



Cone Net Template



Grid Paper

