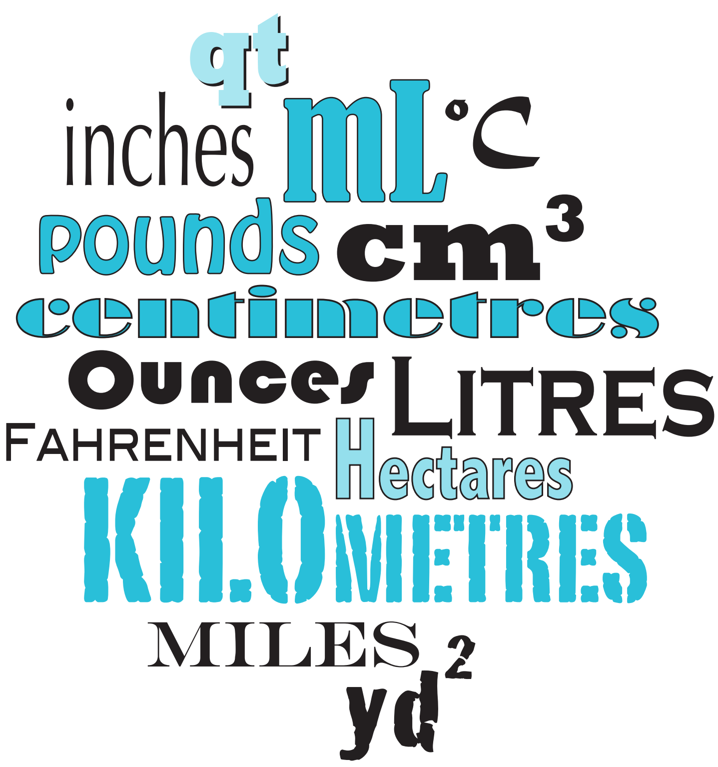
Version 01

Triangles and Other Polygons

Apprenticeship and Workplace Mathematics (Grade 10/Literacy Foundations Level 7)



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Course History

New, March 2012

Project Partners

This course was developed in partnership with the Distributed Learning Resources Branch of Alberta Education and the following organizations:

- Black Gold Regional Schools
- Calgary Board of Education
- Edmonton Public Schools
- Peace Wapiti School Division No. 76
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Viewing Your PDF Learning Package

This PDF Learning Package is designed to be viewed in Acrobat. If you are using the optional media resources, you should be able to link directly to the resource from the pdf viewed in Acrobat Reader. The links may not work as expected with other pdf viewers.

Download Adobe Acrobat Reader:
http://get.adobe.com/reader/

Section Organization

This section on Triangles and Other Polygons is made up of several lessons.

Lessons

Lessons have a combination of reading and hands-on activities to give you a chance to process the material while being an active learner. Each lesson is made up of the following parts:

Essential Questions

The essential questions included here are based on the main concepts in each lesson. These help you focus on what you will learn in the lesson.

Focus

This is a brief introduction to the lesson.

Get Started

This is a quick refresher of the key information and skills you will need to be successful in the lesson.

Activities

Throughout the lesson you will see three types of activities:

- Try This activities are hands-on, exploratory activities.
- Self-Check activities provide practice with the skills and concepts recently taught.
- Mastering Concepts activities extend and apply the skills you learned in the lesson.

You will mark these activities using the solutions at the end of each section.

Explore

Here you will explore new concepts, make predictions, and discover patterns.

Bringing Ideas Together

This is the main teaching part of the lesson. Here, you will build on the ideas from the Get Started and the Explore. You will expand your knowledge and practice your new skills.

Lesson Summary

This is a brief summary of the lesson content as well as some instructions on what to do next.

At the end of each section you will find:

Solutions
This contains all of the solutions to the Activities.

Appendix
Here you will find the Data Pages along with other extra resources that you need to complete the section. You will be directed to these as needed.

Glossary
This is a list of key terms and their definitions.

Throughout the section, you will see the following features:

Icons
Throughout the section you will see a few icons used on the left-hand side of the page. These icons are used to signal a change in activity or to bring your attention to important instructions.

AWM online resource (optional)
This indicates a resource available on the internet. If you do not have access, you may skip these sections.

Solutions

My Notes

The column on the outside edge of most pages is called "My Notes". You can use this space to:

- write questions about things you don't understand.
- note things that you want to look at again.
- draw pictures that help you understand the math.
- identify words that you don't understand.
- connect what you are learning to what you already know.
- make your own notes or comments.

Materials and Resources

There is no textbook required for this course.

You will be expected to have certain tools and materials at your disposal while working on the lessons. When you begin a lesson, have a look at the list of items you will need. You can find this list on the first page of the lesson, right under the lesson title.

In general, you should have the following things handy while you work on your lessons:

- a scientific calculator
- a ruler
- a geometry set
- Data Pages (found in the Appendix)

Triangles and Other Polygons

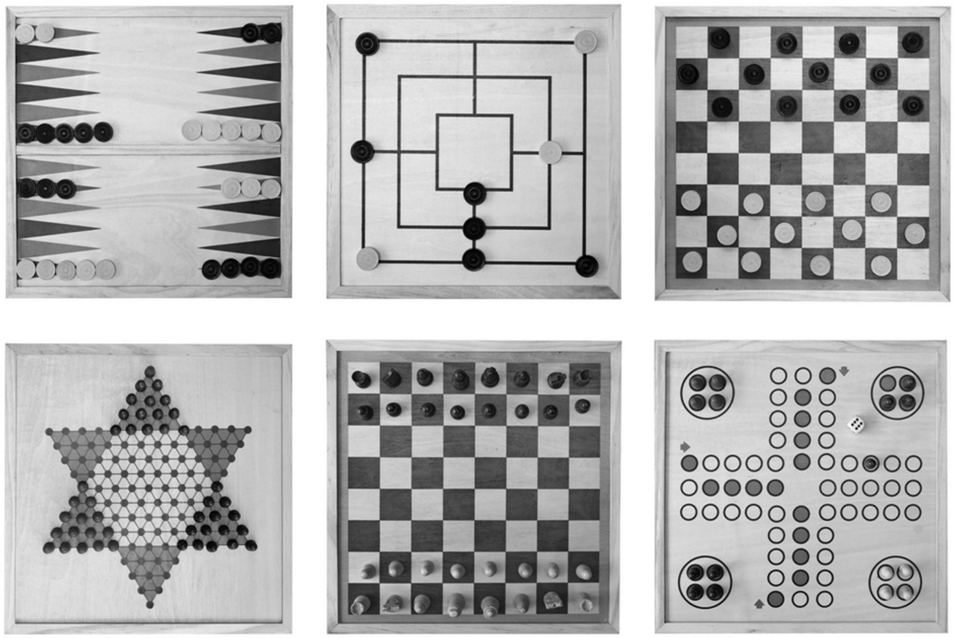


Photo by Elena Schweitzer © 2010

Board games are popular among all peoples of the world. Some of the games in the photograph have a long history. Backgammon, shown on the upper left of the photograph, originated from the Royal Game of Ur, played in Mesopotamia 5000 years ago. Chess originated in India in the sixth century and became popular in Europe 1000 years ago.

A common feature of these board games is the geometry, which is an integral part of the play. On the boards you can see repeated patterns of triangles, squares, and hexagons, just to name a few.

In this section you will explore the geometry of the triangle and of other polygons. You will begin by examining similar polygons and how their sides and angles are related. Then, you will turn your focus to triangles and how the relationships among similar triangles can be used to model and solve a variety of practical problem situations. The section concludes with an exploration of the right triangle and the Pythagorean Theorem.

In this section you will:

- identify similar polygons among regular and irregular polygons.
- use the properties of similarity to solve problems.
- identify situations that involve right triangles.
- verify and apply the Pythagorean Theorem.
- use the Pythagorean Theorem to solve problems.

Lesson A
Similar Polygons

To complete this lesson, you will need:

- a protractor
- an SI ruler
- several blank sheets of paper
- a calculator
- scissors
- a glue stick
- two rubber bands

In this lesson, you will complete:

- 4 activities

Essential Questions

- What techniques can be used to draw similar polygons?
- What are the relationships among the sides of similar polygons?
- What are the relationships among angles of similar polygons?

My Notes

Focus

Photo by shadow216 © 2010

Have you ever tried to draw a picture of your favourite cartoon character? For most people, it can be a difficult task, especially if the final drawing is to be larger or smaller than the original. Your goal is to end up with a shape that is as similar as possible to the original.

Geometry is the study of shapes and their relationships. An important class of shapes is similar polygons. This lesson deals with similar polygons and the relationships among their angles and among their sides.

Did You Know?

BC, a popular comic strip, has been run daily since 1958. That's about 18 250 comic strips!

Get Started

My Notes

In this lesson we'll look at enlarging and reducing the size of designs and shapes. We will revisit a bit of what you already know about scale factors. Recall that the number of times the length and the width is increased or decreased is called the **scale factor**. For example, if both the length and the width of an object are tripled in size, the scale factor is three.

Let's try drawing a scale enlargement.

Activity 1
Try This

In this activity you will draw a cartoon character. Your final drawing will be larger than the original, but geometrically similar in shape.

The activity is described using a cartoon of a happy cow. However, you may use any cartoon character or art design you prefer to enlarge.

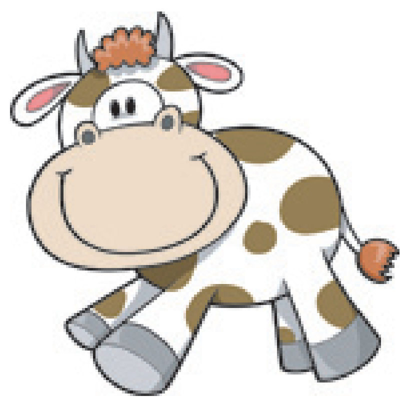


Photo by MisterElements © 2010

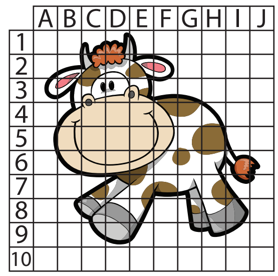
Step 1:

- If you are using the "happy cow" cartoon image, please proceed directly to Step 2.
- If you are using a different image, print it out and cut the figure out. It should be approximately one square inch in size.

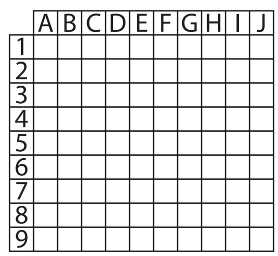
Step 2:

- If you are using "happy cow," look at the following image. The cow has been placed on an eighth-inch grid. The grid has been labelled so that it's easier to identify each square of the figure to be enlarged.

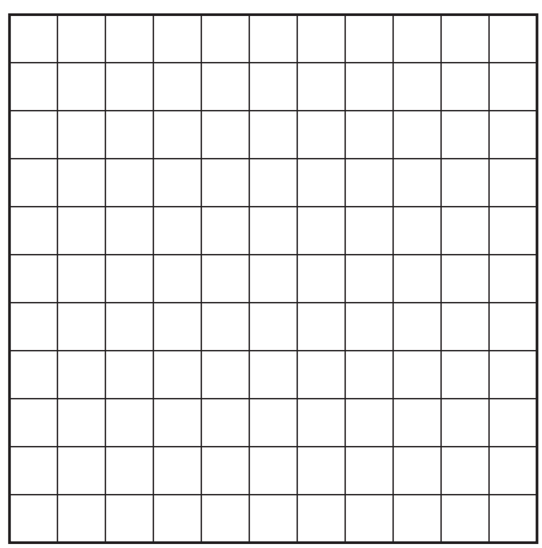
My Notes

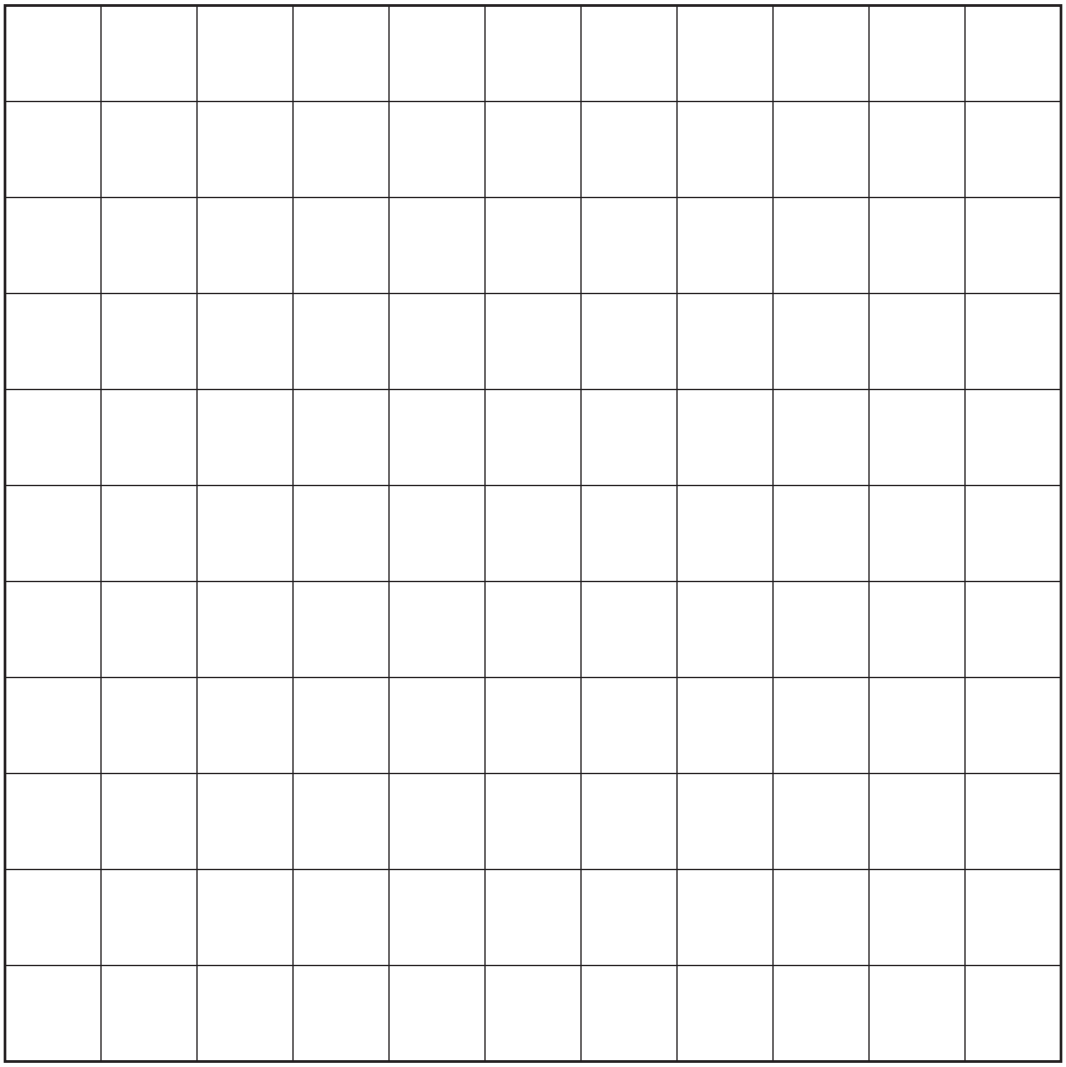


- If you are using a different image, paste the figure on the eighth-inch grid paper below and extend the grid lines over the cartoon figure. The grid has been labelled so that it's easier to identify each square of the figure to be enlarged.



Step 3: Label the quarter-inch and the half-inch grids below the same way that the eighth-inch grid was labelled.



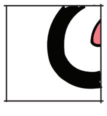


Step 4: In each square on the quarter-inch grid above, draw what you see in the corresponding square of the eighth-inch grid. In each square on the half-inch grid above, draw what you see in the corresponding square of the eighth-inch grid.

For example:

 This is a sample of what you see in A3 on the eigth-inch grid.

 This is a sample of what you draw in A3 on the quarter-inch grid.

 This is a sample of what you draw in A3 on the half-inch grid.

My Notes

My Notes

When you are done, you will have two enlargements.

Questions

1. What is the scale factor by which the length and the width were increased from the original figure on the eighth-inch grid to your first enlargement on the quarter-inch grid? How do you know?

2. How many times larger in area is the enlargement on the quarter-inch grid than the original on the eighth-inch grid? How do you know?

3. What is the scale factor by which the length and the width were increased from the original figure on the eighth-inch grid to your second enlargement on the half-inch grid? How do you know?

My Notes

4. How many times larger in area is the enlargement on the half-inch grid than the original on the eighth-inch grid? How do you know?

5. Suggest how you could reduce an image to one-half its length and one-half its width?

Turn to the solutions at the end of the section and mark your work.

Explore

The cartoon enlargements you have just drawn are geometrically **similar figures** or geometrically similar designs.

You may have also made enlargements and reductions on your computer screen by using the zoom control in a web browser, a drawing program, or other programs. Sometimes size is selected automatically, such as when you see the message on your screen: "This computer game has been adjusted to fit your screen."

My Notes

When zooming in or out from one size to another on your screen, an original figure and its enlargement are geometrically similar. It does not matter how complicated the figure is.

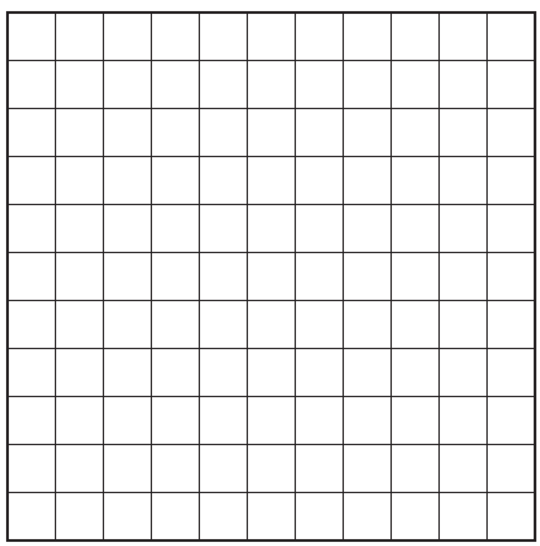
Compared to some images you come across on your computer screen, **polygons** are not very complicated. So, to draw similar polygons, you can still use the grid technique you used at the beginning of this lesson. By drawing similar polygons, you can explore these questions:

- What relationships exist among the sides of similar polygons?
- What relationships exist among the angles of similar polygons?

Activity 2
Try This

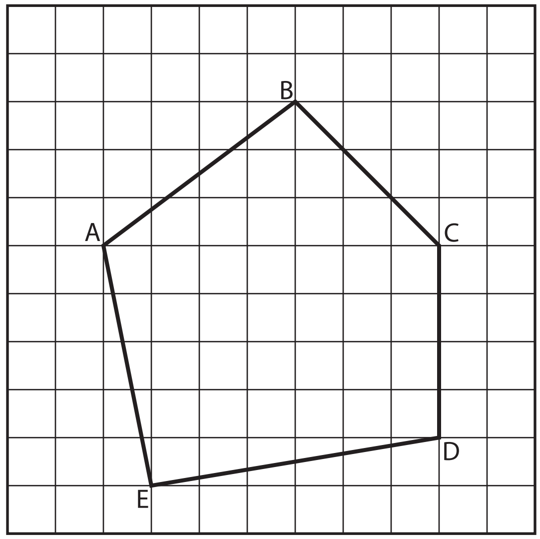
In this activity you will draw similar polygons on grid paper and compare their characteristics.

Step 1: Draw a pentagon—a five-sided polygon—on the quarter-inch grid paper provided below. Label the vertices A, B, C, D, and E going clockwise around the pentagon. An example follows.

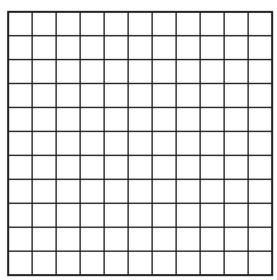


My Notes

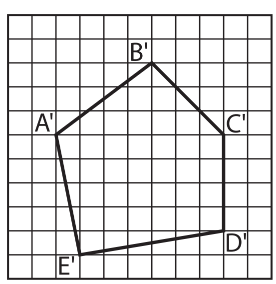
Example:



Step 2: Draw a similar pentagon on the blank eighth-inch grid provided below. Label the vertices A', B', C', D' and E'. The reduced polygon from the example is shown below.



Example:

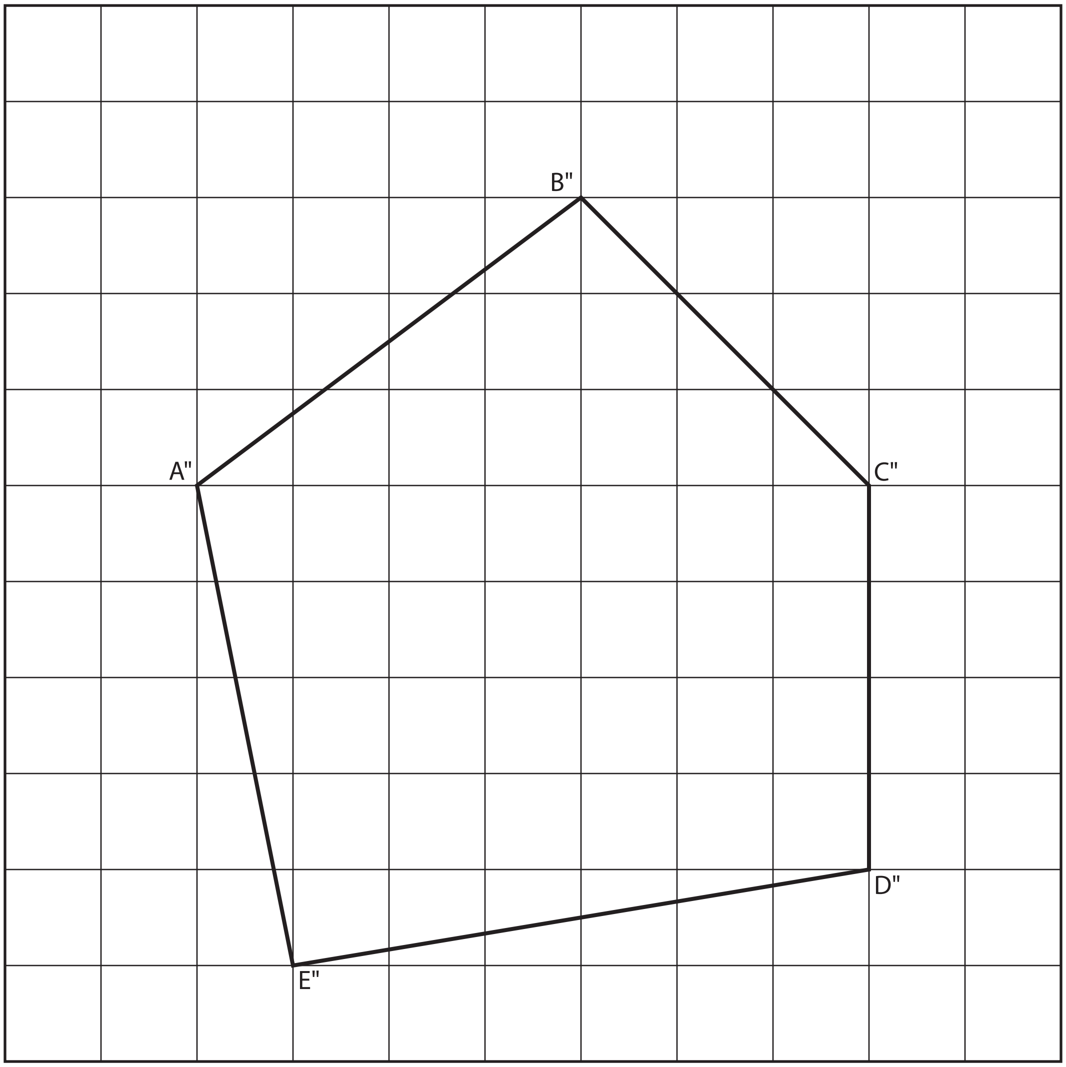


My Notes

Step 3: Draw a similar pentagon on half-inch grid paper. Label the vertices of the polygon A", B", C", D″ and E". The enlarged polygon from the example is shown on the next page.

My Notes

Example:



Step 4: Measure the angles of each pentagon. Record these measures on each diagram.

Step 5: Measure, to the nearest millimetre, the sides of each pentagon. Record these lengths on each diagram.

My Notes

Questions

1. How do the corresponding angles of the three pentagons compare?

2. Find the ratios listed in the table below for each pair of corresponding sides. Express your answers to one decimal place.

$\frac{\text{length of } \overline{A'B'}}{\text{length of } \overline{AB}}$	
$\frac{\text{length of } \overline{B'C'}}{\text{length of } \overline{BC}}$	
$\frac{\text{length of } \overline{C'D'}}{\text{length of } \overline{CD}}$	
$\frac{\text{length of } \overline{D'E'}}{\text{length of } \overline{DE}}$	
$\frac{\text{length of } \overline{E'A'}}{\text{length of } \overline{EA}}$	

My Notes

3. Find the ratios listed in the table below for each pair of corresponding sides. Express your answers to one decimal place.

$\frac{\text{length of } \overline{A''B''}}{\text{length of } \overline{AB}}$	
$\frac{\text{length of } \overline{B''C''}}{\text{length of } \overline{BC}}$	
$\frac{\text{length of } \overline{C''D''}}{\text{length of } \overline{CD}}$	
$\frac{\text{length of } \overline{D''E''}}{\text{length of } \overline{DE}}$	
$\frac{\text{length of } \overline{E''A''}}{\text{length of } \overline{EA}}$	

4. What do you notice about the ratios in Question 2? Think about the squares on the two grids. Why did you get the ratios that you did in Question 2?

My Notes

5. What do you notice about the ratios in Question 3? Think about the squares on the two grids. Why did you get the ratios that you did in Question 3?

Turn to the solutions at the end of the section and mark your work.

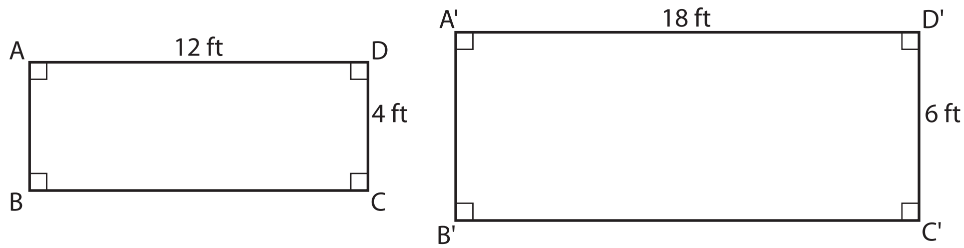
Bringing Ideas Together

In the Explore, you investigated similar polygons by changing the grid size upon which the polygons were drawn. You discovered that for those polygons, the corresponding angles were congruent—equal in measure. You also observed that the ratios of the corresponding sides are all equal and determined by the scale factor.

My Notes

Example 1

Jasmine is designing a rectangular patio for her neighbour. She has shown her neighbour two sketches. The sketches ABCD and A'B'C'D' are similar in shape but differ in size.



a. Are the corresponding angles congruent?

b. Determine the ratios of the lengths of the corresponding sides. Each ratio should have a side length from A'B'C'D' in the numerator and the corresponding side length from ABCD in the denominator. Are the ratios equal?

c. What scale factor did Jasmine use to draw A'B'C'D'?

d. How are the answers to b and c related?

Solution

a. All angles in the rectangles are right angles and are equal in measure.

So, $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, $\angle C \cong \angle C'$, and $\angle D \cong \angle D'$.

The corresponding angles are congruent.

b. Note: Use AB as the symbol for the length of side AB.

A'B' = 6 ft and AB = 4 ft, so $\frac{A'B'}{AB} = \frac{6 \text{ ft}}{4 \text{ ft}} = 1.5$.

B'C' = 18 ft and BC = 12 ft, so $\frac{B'C'}{BC} = \frac{18 \text{ ft}}{12 \text{ ft}} = 1.5$.

C'D' = 6 ft and CD = 4 ft, so $\frac{C'D'}{CD} = \frac{6 \text{ ft}}{4 \text{ ft}} = 1.5$.

A'D' = 18 ft and AD = 12 ft, so $\frac{A'D'}{AD} = \frac{18 \text{ ft}}{12 \text{ ft}} = 1.5$.

All the ratios are equal to 1.5.

My Notes

c. Jasmine used the scale factor 1.5 to draw A'B'C'D'.

She multiplied both the length and width of ABCD by 1.5 to draw A'B'C'D'.

The length of rectangle ABCD is 12 ft.
12 ft × 1.5 = 18 ft, the length of A'B'C'D'.

The width of rectangle ABCD is 4 ft.
4 ft × 1.5 = 6 ft, the width of A'B'C'D'.

d. The scale factor is equal to the ratios of the corresponding sides of the similar figures.

Proportional Reasoning

In the Example 1, you saw that the ratios of corresponding sides are equal.

In this example, $\frac{18 \text{ ft}}{12 \text{ ft}} = \frac{6 \text{ ft}}{4 \text{ ft}}$ is a **proportion**.

If the ratios of the sides of two polygons are equal, the sides of the two polygons are said to be *proportional*. We'll look further at proportional reasoning as it relates to similar polygons in Lesson B.

In the next activity you will investigate another method of drawing similar polygons. You will also determine if the sides are proportional and if the corresponding angles are congruent—equal in measure.

Activity 3
Try This

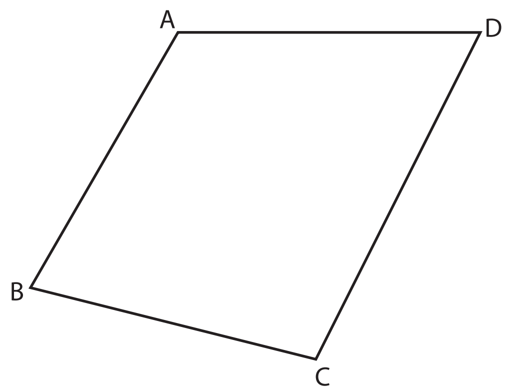
You will need two sheets of blank paper, two elastic bands, tape, a ruler, a protractor, and your calculator. You'll need to work with a partner for this activity, so ask a friend or family member to help out!

Note: you will be referring to this activity in Part 1 of your Section Assignment.

Purpose:

Use two elastic bands to draw a figure similar to quadrilateral ABCD.

My Notes

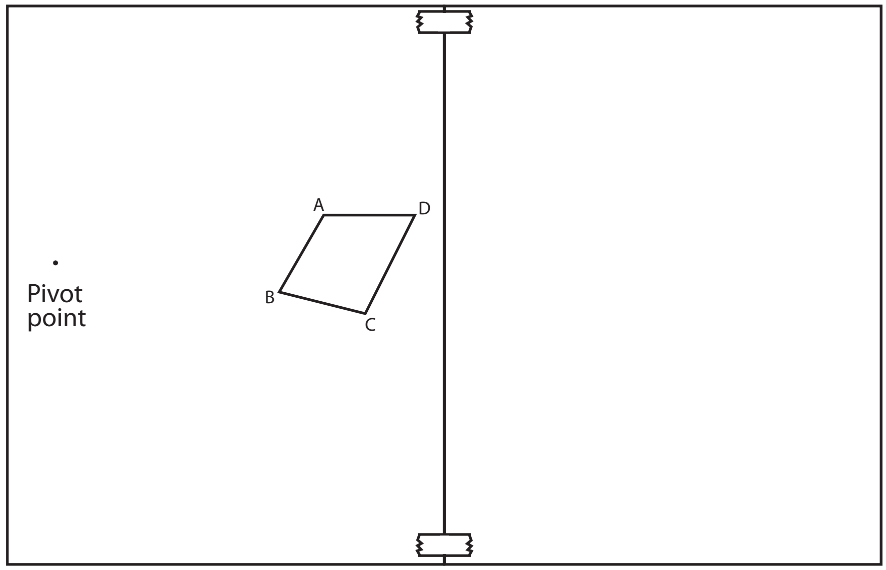


We'll use the quadrilateral above as an example throughout the instructions. You should complete the activity using your own quadrilateral, ABCD.

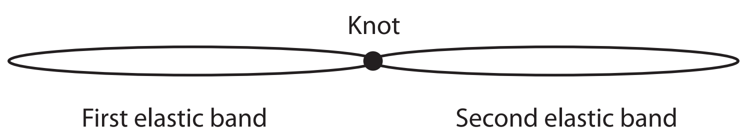
Step 1: Draw quadrilateral ABCD on the right side of a piece of paper as shown below.

Step 2: Tape the piece of paper with the quadrilateral on it to a blank sheet of paper as shown on the following page. Use masking tape so you can remove it easily. Spread the taped-together pages on a flat surface such as a desk or tabletop.

My Notes



Step 3: Tie two elastic bands together. If the bands are the same size, each must be shorter than the distance from the pivot point to the nearest point on ABCD. If the elastics are different sizes, one must be shorter than the distance from the pivot to ABCD.



Step 4:

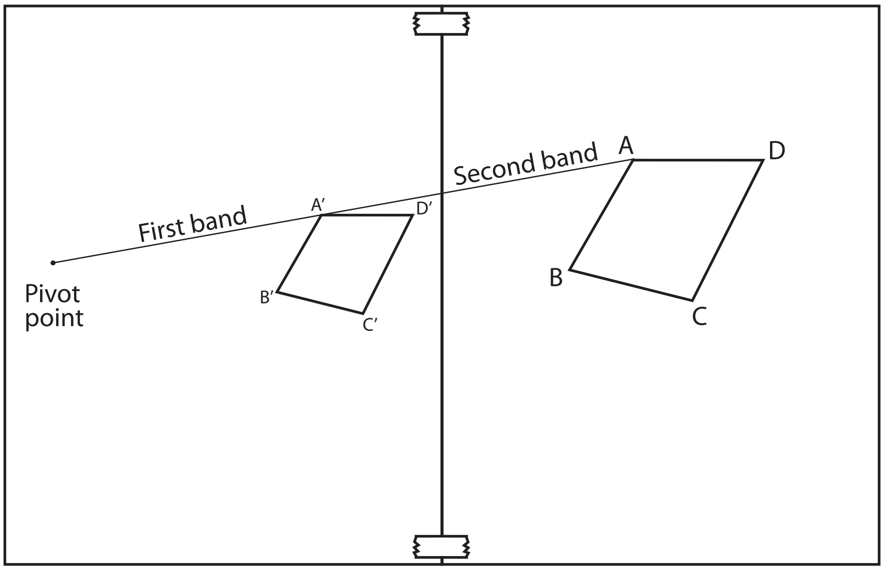
a. Have your partner hold the end of one elastic band on the pivot point. One method is to ask your partner to insert his pen in the loop of that band and place the tip of the pen on the pivot point.

b. Place the tip of your pen in the other loop and stretch the bands so that the knot is on point A.

c. Mark the point on the second sheet where your pen tip is. Call that point, *point A'*.

Step 5: Repeat the process described in Step 4 in order to locate points B', C', and D'.

Step 6: Join the four points to form A'B'C'D'.



My Notes

Question:
How do you know that A'B'C'D' is similar in shape to ABCD?

____

____

____

____

____

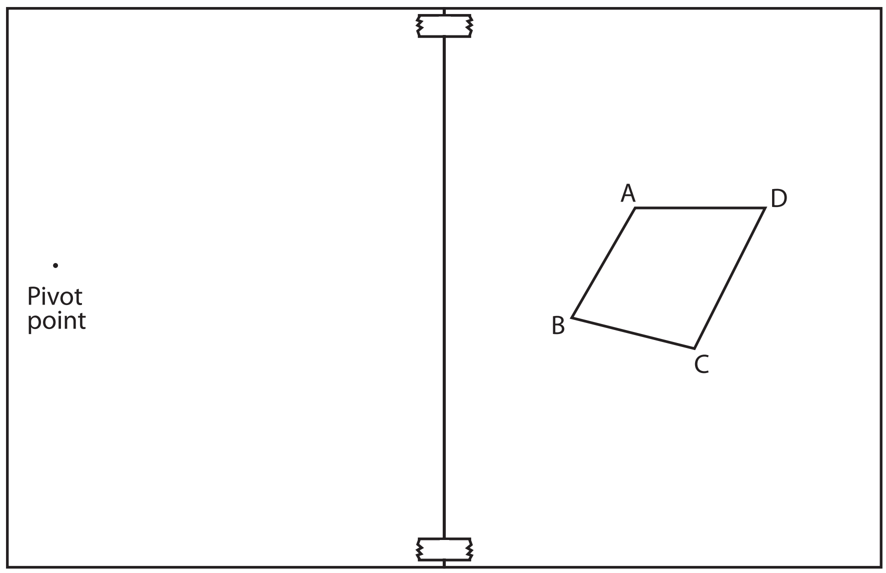
____

Turn to the solutions at the end of the section and mark your work.

My Notes

Activity 4
Mastering Concepts

Suppose you wanted to draw a polygon similar to, but smaller than, a given polygon. How might you use the elastic bands? Consider a quadrilateral ABCD placed on the right-hand sheet, and the pivot point on the left-hand sheet.



Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes

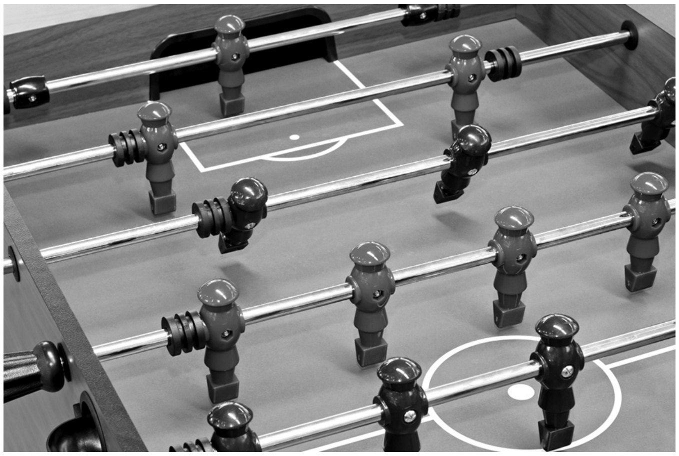


Photo by Baloncici © 2010

If you have played foosball or a table-top hockey game, the fields look similar to the real game. If you took the measurements of the length and the width, are they in fact proportional to an actual soccer pitch or hockey rink?

Did You Know?

Foosball was invented in 1922 in Britain. Today it is a highly competitive sport. However, unlike ping-pong, it is not yet an Olympic sport.

This lesson dealt with similar figures, how they are constructed, and the relationships among the sides and angles.

In this lesson you discovered, by taking measurements from similar figures you drew, that the corresponding angles of similar polygons are congruent and that the ratios of the corresponding sides are equal.

Lesson B
Ratios and Similar Polygons

To complete this lesson, you will need:

- a protractor
- a ruler
- several blank sheets of paper
- a calculator

In this lesson, you will complete:

- 5 activities

Essential Questions

- How can you determine if two polygons are similar?
- How are the relationships among the sides and angles of similar polygons used to solve problems?

My Notes

Focus

Photo by Fotonium © 2010

The matryoshka dolls or Russian nesting dolls are a series of hollow wooden dolls that fit one inside the other. This artistic tradition is over 120 years old. Similar Japanese dolls inspired artists in the 1890s. What makes this art form so appealing is the painted figures, which may be all geometrically similar as in this set, or they may portray different characters often ending with a baby as the smallest figurine.

In this lesson we'll continue to look at similarity. We'll investigate similar shapes and scale drawings, and solve problems using proportional reasoning.

Get Started

My Notes

You may remember using proportions to solve unit conversion problems in Module 1. A **proportion** is a statement showing one ratio equal to another. For example, the following mathematical equation is a proportion statement.

$$\frac{1}{12} = \frac{3}{36}$$

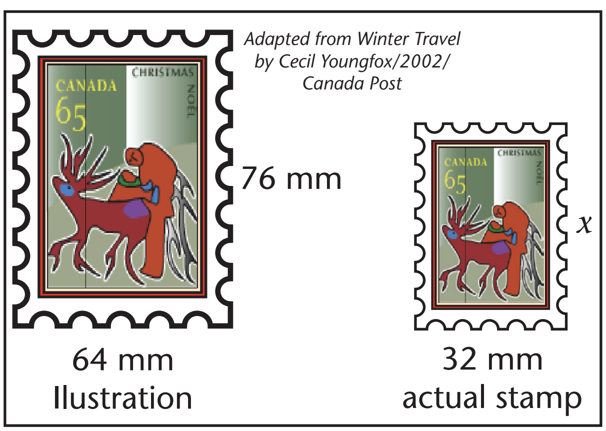
In this lesson, we will use the same type of reasoning—proportional reasoning—to solve problems involving similar shapes. Let's start by reviewing proportions.

Example 1

Angus is online looking for stamps to buy as a gift for his uncle who is a keen collector. One stamp that has caught Angus's attention is a 2002 issue celebrating First Nations and Inuit art.

Angus is wondering what the dimensions of the stamp are. The illustration says the stamp is 32 mm wide, but there is no mention of its height. Angus knows the picture of the stamp online is similar to the actual stamp. How can Angus determine the height?

Solution



Since the illustration is proportional to the actual stamp, the ratios of the corresponding sides are equal. Set up a proportion and fill in the dimensions you know. Then, solve for the unknown dimension.

My Notes

$$\frac{\text{height of actual stamp}}{\text{height of illustration}} = \frac{\text{widthof actual stamp}}{\text{width of illustration}}$$

$$\frac{x}{76} = \frac{32}{64}$$

$$(76)\frac{x}{76} = \frac{1}{2}(76)$$

$$x = 38$$

To solve for x, multiply both sides by 76.

The height of the actual stamp is 38 mm.

Note: There are many different ways you could have set up the proportion to solve this problem. Here is a different proportion that you could have used:

$$\frac{\text{height of illustration}}{\text{height of actual stamp}} = \frac{\text{width of illustration}}{\text{width of actual stamp}}$$

Can you think of another proportion you could have used?

To view the animated solution to this problem, go and look at *Finding Stamp Height* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/ma10_stamp_height_v2.html*).

Activity 1
Self-Check

My Notes

Use the method that Angus used in Example 1 to solve the following proportions.

1. $\frac{x}{12} = \frac{16}{18}$

2. $\frac{x}{13} = \frac{26}{39}$

My Notes

3. $\frac{x}{7} = \frac{18}{15}$

4. $\frac{7}{x} = \frac{14}{31}$

Turn to the solutions at the end of the section and mark your work.

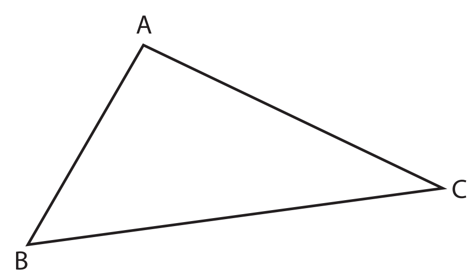
Explore

My Notes

In Lesson A you used grids and elastic bands to draw similar polygons. There are lots of ways you can draw similar polygons. Let's explore another method of sketching similar polygons. As you complete the next activity, you will also review the relationships between similar polygons.

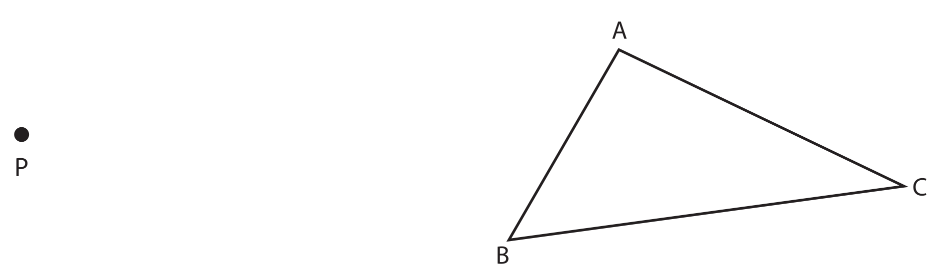
Activity 2
Try This

You will need a sheet of paper, a ruler, and a protractor to complete this activity.



We'll use the triangle above as an example throughout the instructions. You should complete the activity using your own triangle, ABC.

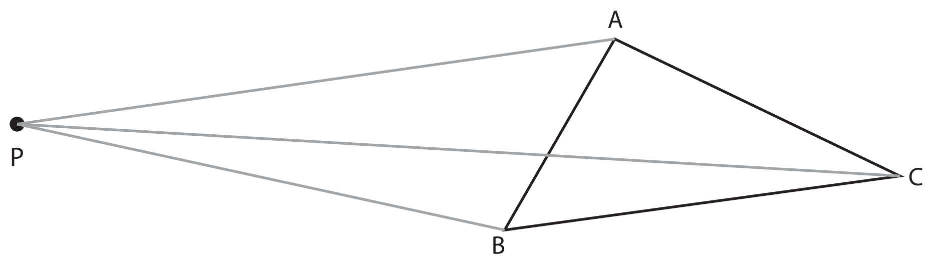
Step 1: Draw triangle ABC on the right side of a piece of paper as shown below.



My Notes

Step 2: You may be reminded of the elastic approach used in Lesson A. However, instead of using elastics, draw lines using your ruler that join the point P to each vertex of triangle ABC.

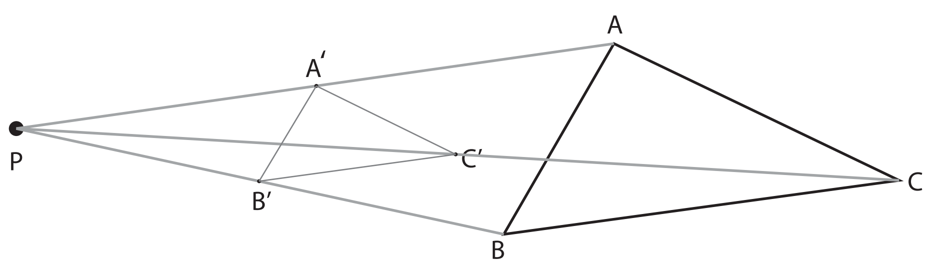
Example:



Step 3: Use your ruler (and a calculator if you need it) to determine the midpoint of $\overline{PA}$. Make a mark at the midpoint and call this point A'. Repeat this step for $\overline{PB}$ and $\overline{PC}$.

Step 4: Join points A', B' and C' to form triangle A'B'C'.

Example:



Questions

1. In Lesson A you learned two things that are true for all similar polygons, what were they?

My Notes

2. How can you prove that the two triangles you just drew are similar triangles? Record your work to prove your results.

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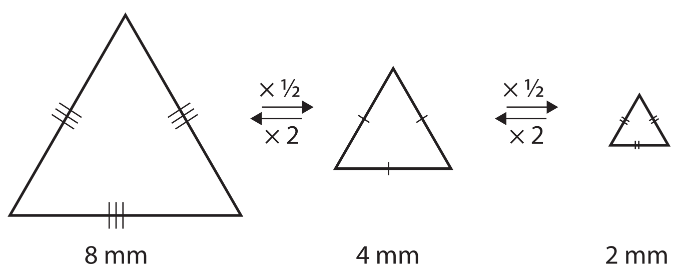
Turn to the solutions at the end of the section and mark your work.

Bringing Ideas Together

For two similar polygons—the angles are equal in measure and the sides are proportional; that is, the ratios of corresponding sides are equal.

Look at the similar polygons below.



When you move from a large polygon to the next smaller polygon in the diagram, the scale factor is one half. When you move from small polygon to the next larger polygon in the diagram the scale factor is two.

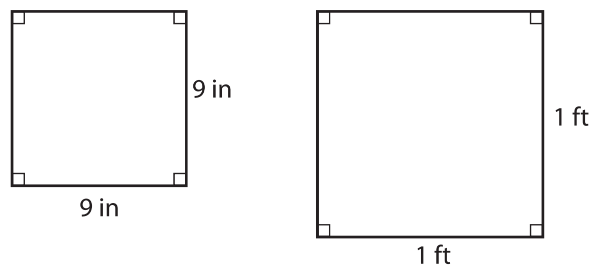
My Notes

Notice that whenever you move from a large polygon to a smaller polygon, the scale factor is less than one. Whenever you move from a small polygon to a large polygon, the scale factor is greater than one.

In the following example we'll determine if two figures are similar by comparing angle measurements and finding side ratios and scale factor.

Example 2

Is the small square floor tile below similar to the larger square floor tile below?

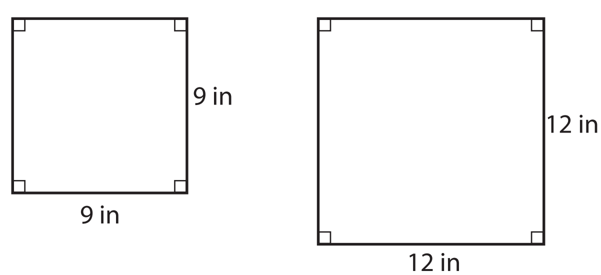


Solution

The angles in both squares are all right angles. So, the corresponding angles are equal in measure (congruent).

Now we'll look at the side lengths. When you are comparing side lengths, you must use the same unit in both parts of the ratio. Check to see if the units your sides are measured in are the same. If they aren't, you'll have to convert one unit.

Remember 1 ft = 12 in.



So, the ratio for each pair of corresponding sides is $\frac{12 \text{ in}}{9 \text{ in}}$.

My Notes

Reduced to lowest terms, this ratio is $\frac{4}{3}$.

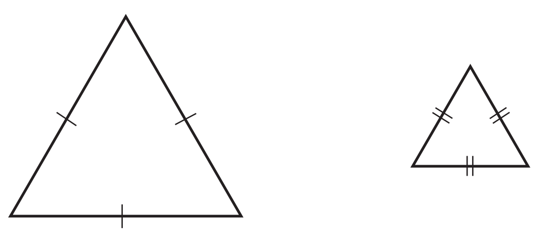
Because corresponding angles are equal in measure and the sides are proportional, the squares are similar polygons.

Note that the scale factor describing the change from a 9 in square into a 12 in (1 ft) square is $\frac{4}{3}$. The fraction $\frac{4}{3}$ is the scale factor, because 9 in × $\frac{4}{3}$ = 12 in.

Regular Polygons

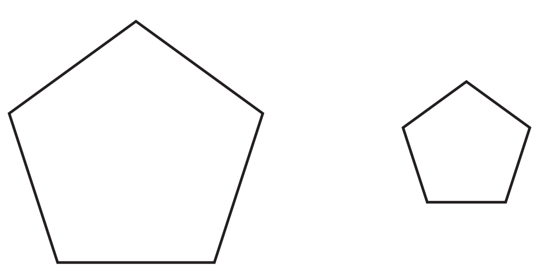
Did you know that some shapes are always similar to each other? All squares are similar. Squares are examples of **regular polygons**.

Another example of a regular polygon is an equilateral triangle. Each angle of an equilateral triangle is 60°



Can you think of another example of a class of shapes that are always similar?

All regular pentagons are similar to each other. Each angle in a regular pentagon is 108°.



My Notes

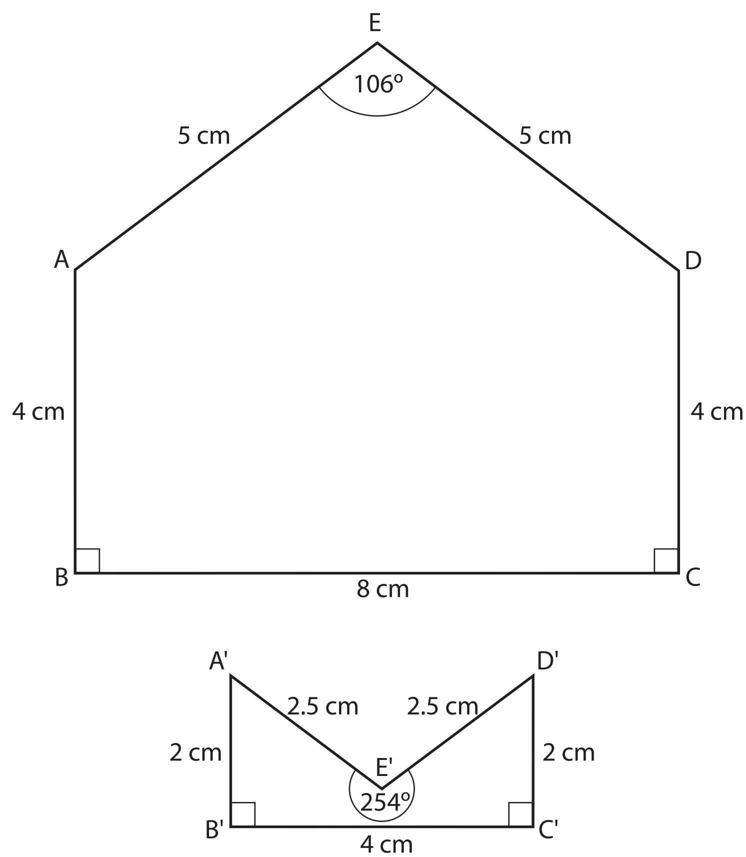
Like squares, all equilateral triangles are similar; all regular pentagons are similar, and so on. You may also have thought of all regular octagons, hexagons, heptagons, and so on.

For irregular shapes—shapes whose sides vary in length—you have to compare corresponding angles and side lengths to determine if they are similar.

In the next example, you will investigate whether the given pair of pentagons are similar.

Example 3

Determine the ratios of the pairs of sides of the pentagons ABCDE and A'B'C'D'E'. Are the pentagons similar? Why or why not?



Solution

You can see that the shapes are not similar because not all of their corresponding angles are equal in measure. For example, the given angles E and E' are 106° and 254° respectively.

You may have noticed that the side lengths are, in fact, proportional.

$$\frac{AB}{A'B'} = \frac{4 \text{ cm}}{2 \text{ cm}} = 2$$

$$\frac{BC}{B'C'} = \frac{8 \text{ cm}}{4 \text{ cm}} = 2$$

$$\frac{CD}{C'D'} = \frac{4 \text{ cm}}{2 \text{ cm}} = 2$$

$$\frac{DE}{D'E'} = \frac{5 \text{ cm}}{2.5 \text{ cm}} = 2$$

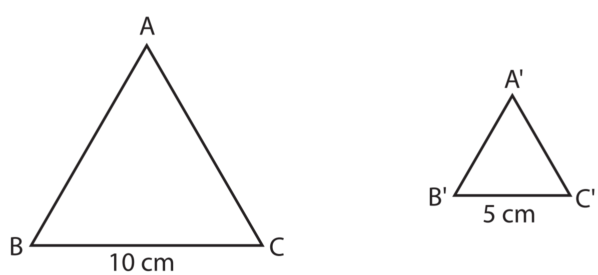
$$\frac{AE}{A'E'} = \frac{5 \text{ cm}}{2.5 \text{ cm}} = 2$$

However, because the corresponding angles are not equal, these pentagons are not similar.

My Notes

Activity 3
Self-Check

1. Look at the diagram below. By what scale factor would you have to multiply the sides of equilateral triangle ABC to obtain triangle A'B'C'?



My Notes

2. The kitchen in Jasper's home is rectangular in shape and is 12 ft by 9 ft. Jasper's bedroom is also rectangular and is 8 ft by 10 ft. Are the two rooms similar polygons?

Turn to the solutions at the end of the section and mark your work.

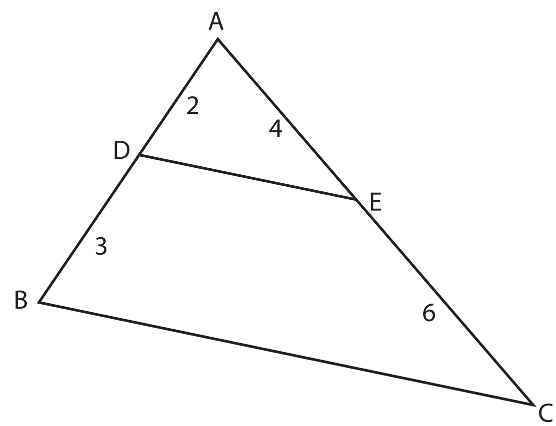
Similarity and Side Lengths

In the next examples you will find the missing sides of two similar figures by solving proportions.

My Notes

Example 4

In the diagram of ΔADE and ΔABC below, the measurements of side lengths are shown in units. For example, line segment AD has a measure of 2 units.



Is ΔABC ~ ΔADE? Give reasons for your answer to this question.

Solution

Notice that ΔABC and ΔADE share a common angle, ∠A. So, both triangles have one angle that is identical. The triangles will be similar if the sides that form ∠A are proportional.

$$\frac{AD}{AB}=\frac{2}{2+3}=\frac{2}{5}$$

$$\frac{AE}{AC}=\frac{4}{4+6}=\frac{4}{10}=\frac{2}{5}$$

Notice that the legs of the smaller triangle, ΔADE, are in the numerator of both ratios. The legs of the larger triangle, ΔABC, are in the denominator of both ratios.

Since both ratios are equal, the side-lengths are proportional and thus the triangles are similar. We can write this as follows.

Since ∠A is shared, and $\frac{AD}{AB}=\frac{AE}{AC}$, then ΔABC ~ ΔADE.

To watch the animated solution for this example, go and look at *Similar Triangles* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/ma10_similar_triangles.html*).

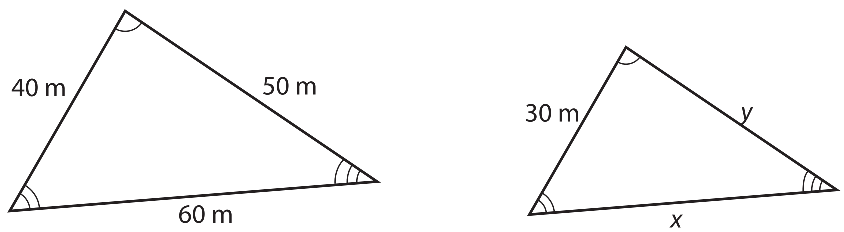
My Notes

You can identify the corresponding angles by looking for matching symbols.

 *corresponds to* 

Example 5

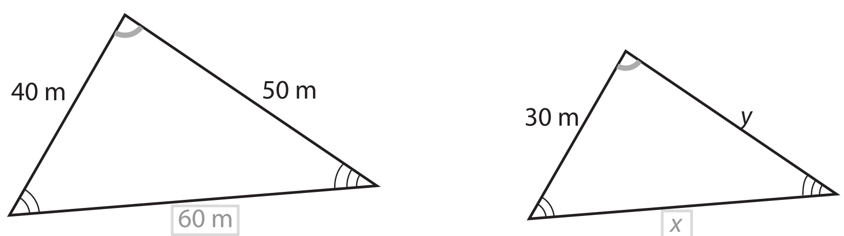
Two similar triangular city lots are pictured below.



Find the missing measures x and y.

Solution

We'll solve for x first. We'll start by finding the corresponding side to side x. Remember, corresponding sides are always across from equal angles



The corresponding side to side x is 60 m in length.

Set up a proportion. In the first ratio, put x in the numerator and 60 in the denominator. For the second ratio, use another pair of corresponding sides.

$$\frac{x}{60} = \frac{30}{40}$$

Notice that the numbers on the tops of the fractions, in the numerators, are from the small triangle and the numbers in the denominators are from the large triangle.

Now, solve for x.

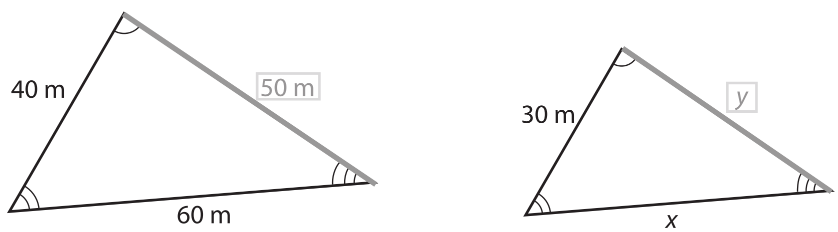
My Notes

$$\frac{x}{60} = \frac{30}{40}$$
$$\frac{x}{60} = \frac{3}{4}$$
$$(60)\frac{x}{60} = \frac{3}{4}(60)$$
$$x = 45$$

Make the calculation simpler by reducing the fraction.

So, side x is 45 m long.

Now we can solve for y the same way. Set up a new proportion. The first ratio contains y and its corresponding side.



For the second ratio, use another pair of corresponding sides.

$$\frac{y}{50} = \frac{30}{40}$$

Now, solve for y.

$$\frac{y}{50} = \frac{30}{40}$$
$$\frac{y}{50} = \frac{3}{4}$$
$$(50)\frac{y}{50} = \frac{3}{4}(50)$$
$$y = 37.5$$

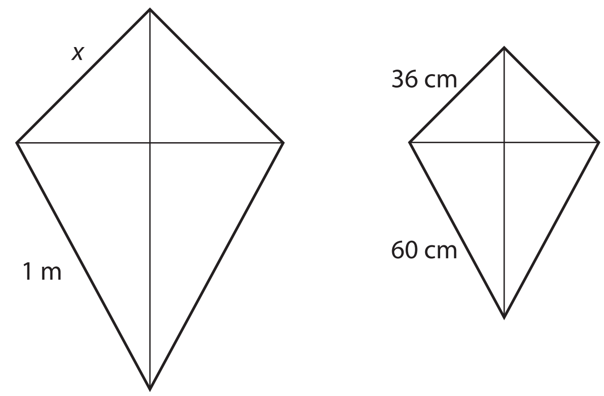
So, side y is 37.5 m long.

To watch the animated solution for this example, go and look at *Finding Lengths in Similar Triangles* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/ma10_similar_triangles.html*).

My Notes

Example 6

The outlines of two kites are similar polygons. Determine x.



Solution

Remember that 1 m = 100 cm.

Set up the proportion.

$$\frac{x}{36} = \frac{100 \text{ cm}}{60 \text{ cm}}$$
$$\frac{x}{36} = \frac{5}{3}$$
$$36 \times \frac{x}{36} = 36 \times \frac{5}{3}$$
$$x = 60$$

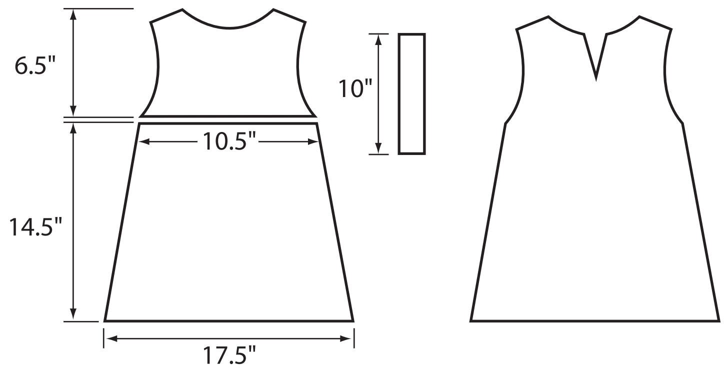
So, x = 60 cm.

Now, it's your turn.

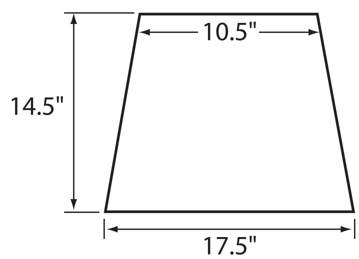
Activity 4
Self-Check

My Notes

1. Dorothy is preparing to sew an outfit for her toddler. She wants to adapt the following pattern.



To fit her child, Dorothy plans to reduce the dimensions of the pattern by 10%. What is the scale factor? What will the dimensions of this section be if Dorothy decides to round each measurement to the nearest quarter inch?



My Notes

2. Shawn was writing a report about the Nisga'a First Nation of the Nass River Valley in north-western British Columbia. Shawn had to decide between two sizes of the Nisga'a First Nation's flag for the report.

6 cm

12 cm

5 cm

x

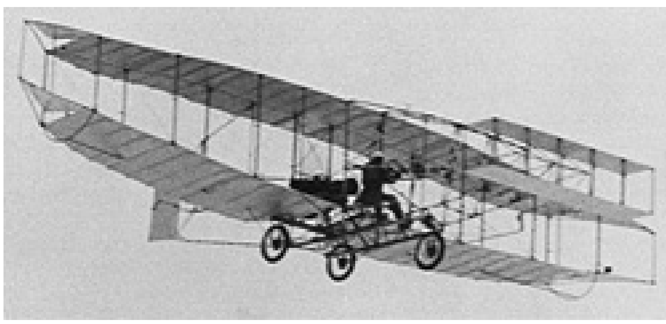
The length of the smaller flag is missing. Calculate the length to one decimal place.

3. Maxine is interested in model aircraft. She wants to build a 1/32 scale model of the *Silver Dart*, the first powered airplane to fly in Canada.

   If the wingspan of the *Silver Dart* was 15 m, what will the wingspan of Maxine's model be? Round to two decimal places.

Did You Know?

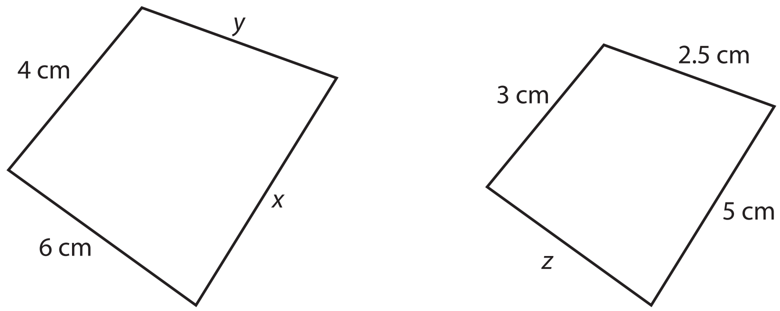
The original *Silver Dart* was made of steel tubing, bamboo, wire, and wood.



My Notes

My Notes

4. The following polygons are similar. Find the missing measures (*x*, *y*, and *z*). Round to one decimal point.

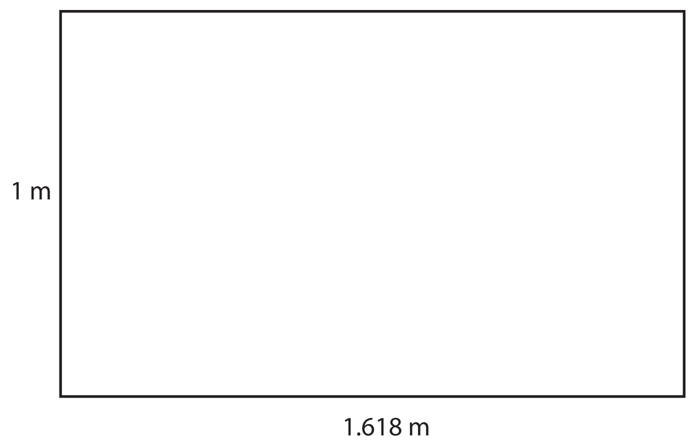


Turn to the solutions at the end of the section and mark your work.

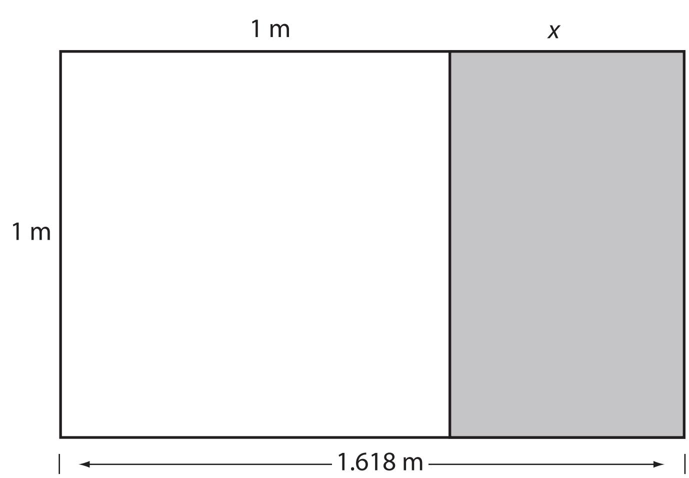
My Notes

Activity 5
Mastering Concepts

One rectangle that often appears in art and architecture is the *golden rectangle*. The proportions of the golden rectangle are considered pleasing to the eye. The length of the golden rectangle is approximately 1.618 times as long as its width. So, if the width is 1 m, the length would be 1.618 m.



One of the interesting properties of the golden rectangle is that if you draw a square inside with the width as one side, the rectangle remaining is also a golden rectangle.



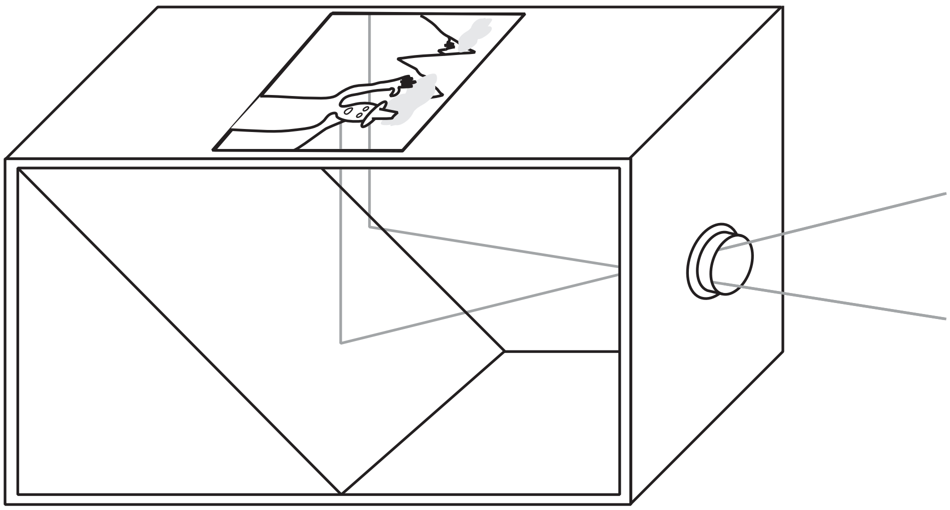
My Notes

Assuming the large golden rectangle is similar to the smaller golden rectangle on the right, use a proportion to find x correct to 3 decimal places. Does this value of x make sense in the diagram? Why?

Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes



The *camera obscura* was a forerunner of the photographic camera. This device consisted of a lens and mirror that projected an image onto a plate, which could be used to sketch or paint a realistic picture. The proportions of the objects displayed on the screen are preserved. This is another example of similar figures—the focus of this lesson.

In this lesson you applied the principles for similar polygons:

- corresponding angles are equal in measure
- ratios of corresponding sides are equal

Equal ratios are proportions, and you used proportional reasoning to solve a variety of practical problem situations.

Lesson C
Similar Triangles

To complete this lesson, you will need:

- a protractor
- a ruler
- several blank sheets of paper
- a calculator
- a compass

In this lesson, you will complete:

- 5 activities
- Part 3 of your Section Assignment

Essential Questions

- How can you identify similar triangles?
- How can you use the relationships among similar triangles to solve problem situations?

My Notes

Focus

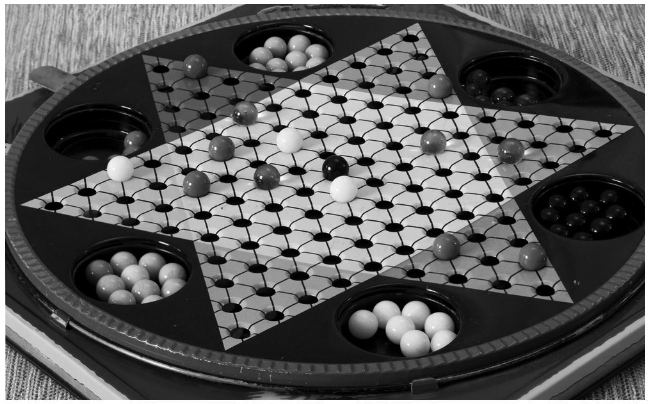


Photo by NicolasMcComber © 2010

Many board games, such as chess and Chinese checkers, involve a variety of geometric shapes that play an important part of the game. In Chinese checkers, each player places ten pieces in a coloured triangle. The object of the game is to transfer these pieces by jumps or moves to the triangle opposite. The first player to reach this goal is the winner.

If you look at the board, there are numerous triangles. Can you see some of them? Look at the Chinese checkers board again. The smallest triangle has one unit per side. Are all of these triangles similar?

In previous lessons you explored a variety of similar polygons. In this lesson you'll focus on triangles.

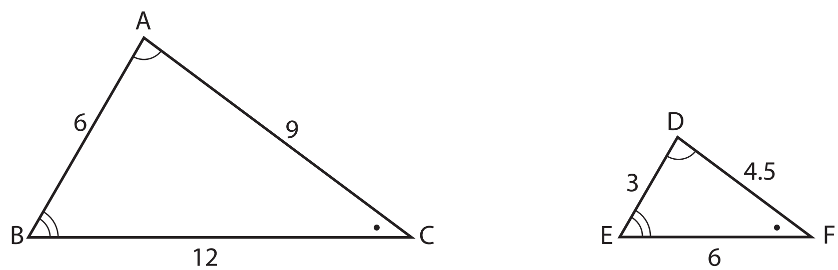
Get Started

To start this lesson, you will review the relationships among the corresponding sides and angles of similar triangles. You'll also practise your skills with ratios.

Activity 1
Self-Check

My Notes

1. Consider the following two triangles.



The two triangles are similar.

Write down the side ratios you could use to help prove they are similar. What does the size of the ratio indicate about the two triangles?

My Notes

2. Take the side ratios you used in Question 1 and flip each fraction over. What do you notice about the new ratios? What does the size of the ratio tell you about the triangles?

Turn to the solutions at the end of the section and mark your work.

Working with Ratios

In Question 2 of Activity 1 you should have noticed that flipping both ratios in a proportion results in the same ratio. This idea can help you to work quickly and easily with proportions—but you have to follow the rules!

- You can flip the fractions in a proportion. Remember, a proportion is a set of equivalent ratios.

  $\frac{AB}{CD} = \frac{7}{3}$ is the same as $\frac{CD}{AB} = \frac{3}{7}$

- If you flip one ratio in a proportion, you MUST flip the other one too.

Let's look at an example.

Example 1

Solve $\frac{6}{x} = \frac{3}{5}$.

Solution

Many people prefer to see the variable in the numerator (on top of the fraction) rather than in the denominator, where it is in this case.

Whenever you have a proportion (two equal ratios) you can flip the ratios in the equation.

So,

$$\frac{6}{x} = \frac{3}{5}$$

Flip both ratios to bring x to the numerator.

$$\frac{x}{6} = \frac{5}{3}$$

Multiply both sides by 6 to isolate x.

$$(6)\frac{x}{6} = \frac{5}{3}(6)$$

$$x = 10$$

You can check your answer by substituting for x in the original equation.

Check: $\frac{6}{10} = \frac{3}{5}$.

It works!

You may solve proportions using a different method. One alternate method is *cross multiplication*.

$$\frac{6}{x} = \frac{3}{5}$$

Find the cross product.

$$(6)(5) = 3x$$

$$\frac{30}{3} = \frac{3x}{3}$$

$$10 = x$$

You get the same answer!

Now practice a few on your own.

My Notes

My Notes

Activity 2
Self-Check

Solve each equation.

1. $\frac{7}{x} = \frac{21}{11}$

2. $\frac{4}{x} = \frac{8}{3}$

3. $\frac{16}{x} = \frac{8}{7}$

My Notes

Turn to the solutions at the end of the section and mark your work.

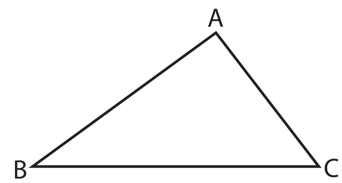
Explore

In the previous lessons you explored a variety of methods for drawing similar polygons. Now you will explore three additional methods for drawing similar triangles. As you read through the methods, you should follow the steps to draw your own similar triangle on a separate sheet of paper. You will practise each method in Activity 3 later in the lesson.

My Notes

Method 1: Constructing Congruent Angles

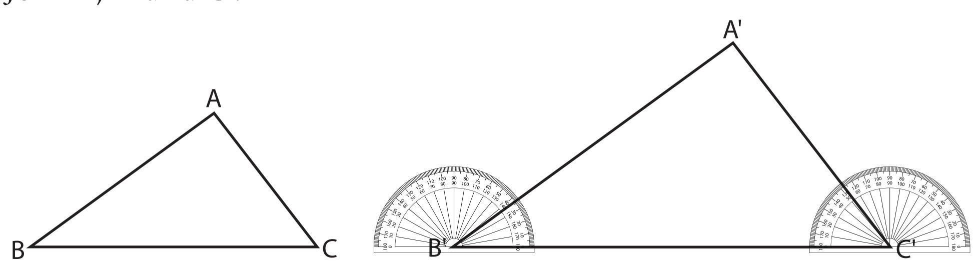
Step 1: Draw any triangle ABC.



Step 2: Use your ruler to draw the base, $\overline{B'C'}$, of ΔA'B'C'. This base can be any length. If you want the sides of ΔA'B'C' to **be twice as long** as those of ΔABC, you would make B'C' = 2(BC). But any length will do.

Note: Using the names A', B', C' (instead of completely different letters) to label the new triangle, makes it easy to see the pairs of corresponding angles and the pairs of corresponding sides.

Step 3: Use your protractor to draw angles at B' and C' congruent to ∠B and to ∠C. Call the point where these angles' arms cross point A'. Join A', B' and C'.



Now, you can check to make sure the two triangles are similar.

You can use a protractor to make sure the corresponding angles are congruent.

$\angle A \cong \angle A' = 92°$

$\angle B \cong \angle B' = 36°$

$\angle C \cong \angle C' = 52°$

You can use a ruler to measure the side lengths of each triangle. Then, compare the ratios of the pairs of corresponding side lengths.

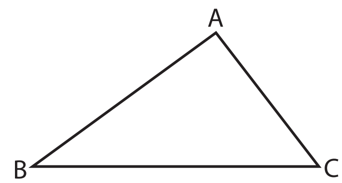
$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'} \approx 0.6$$

You have constructed a pair of similar triangles. Now try Method 2.

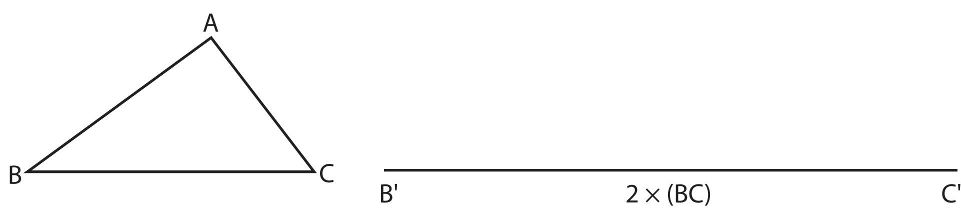
My Notes

Method 2: Constructing Proportional Sides

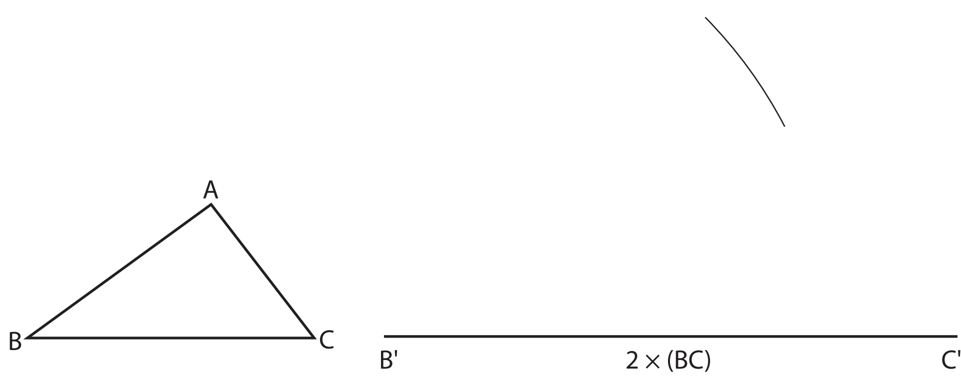
Step 1: Draw any triangle, ABC.



Step 2: Decide what ratio you wish to use—it can be any ratio. For this example we'll create a similar triangle with sides twice as long as the original triangle. Measure the base, $\overline{BC}$. Then, draw base $\overline{B'C'}$ twice as long as $\overline{BC}$.

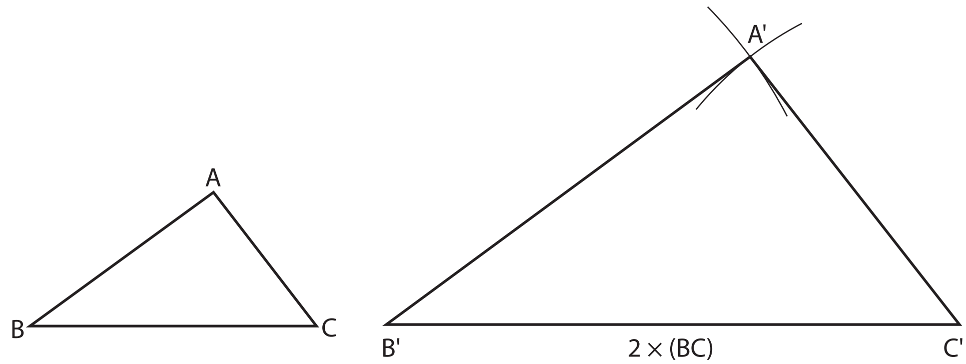


Step 3: Measure AB. Then set your compass to a radius twice as large. With B' as the centre, draw an arc where A' is likely to be.



My Notes

Step 4: Measure AC. Then set your compass to a radius twice as large. With C' as centre, draw an arc intersecting the first arc. Call this point A'. Join A', B' and C'.



We know the sides are proportional because we constructed them that way, but are the corresponding angles congruent? Measure the angle pairs.

$\angle A \cong \angle A' = 92°$

$\angle B \cong \angle B' = 36°$

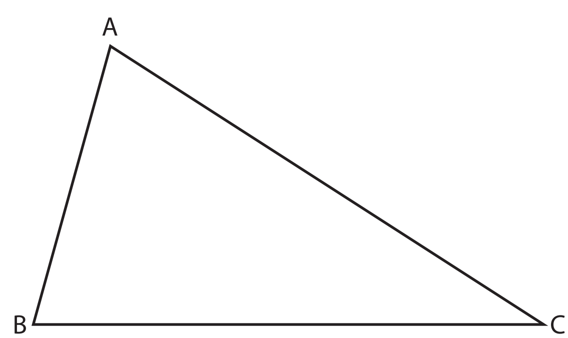
$\angle C \cong \angle C' = 52°$

The triangles are similar.

Method 3: Constructing Two Pairs of Proportional Sides That Form a Shared Angle

You can use this method to create two similar triangles that share an angle.

Step 1: Draw any triangle ABC.

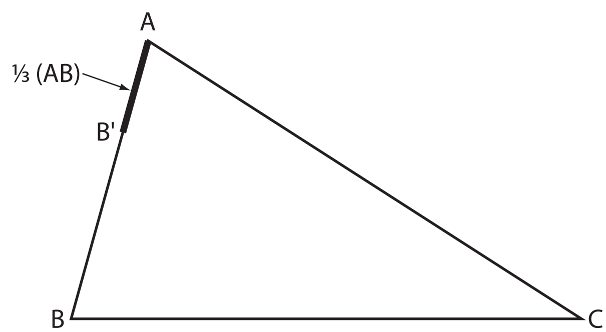


My Notes

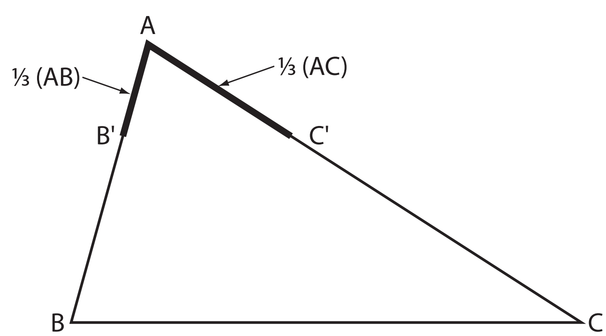
Suppose you want the second triangle to share ∠A. Also, suppose that you want the second triangle sides of the second triangle to be one-third as long as the sides of ΔABC.

Step 2: Measure AB. Divide that length by three. Measure out the calculated distance from point A along AB. Call that point B'.

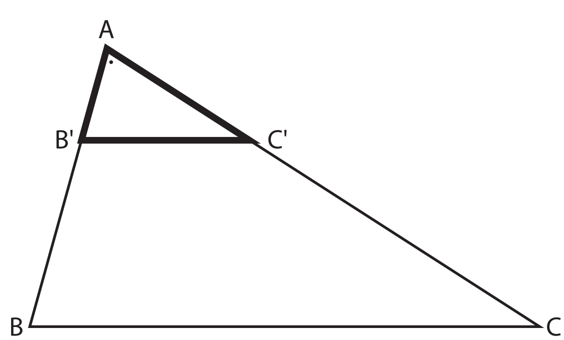
So, $AB' = \frac{1}{3}(AB)$.



Step 3: Measure AC. Divide that length by 3. Measure out that distance from point A along AC. Call that point C'. So, $AC' = \frac{1}{3}(AC)$.



Step 4: Join B'C'.



My Notes

So, ΔABC and $\Delta A'B'C'$ share $\angle A$ and $\frac{AB'}{AB} = \frac{AC'}{AC} = \frac{1}{3}$.

But are ΔABC and $\Delta A'B'C'$ similar? If they are, the following should be true:

$\angle B \cong \angle B'$

$\angle C \cong \angle C'$

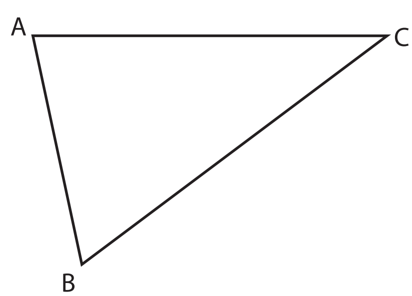
$\frac{B'C'}{BC} = \frac{1}{3}$

Use your protractor and ruler to check that the triangles are similar.

Now you know three methods for constructing similar triangles. You can practice these methods of construction in Activity 3.

Activity 3
Try This

1. a. Use Method 1 to construct a triangle, A'B'C', that is similar to triangle ABC shown below.



My Notes

b. You used your protractor to construct two pairs of congruent corresponding angles. How do you know, without measuring, that the third pair of corresponding angles are congruent?

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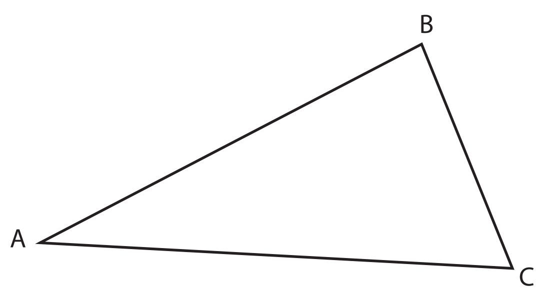
2. a. Use Method 2 to construct a triangle, A'B'C', that is similar to triangle ABC shown below.



My Notes

b. Measure the three pairs of corresponding angles in your similar triangles. Are the angles in each pair congruent? If not, can you identify any sources of error?

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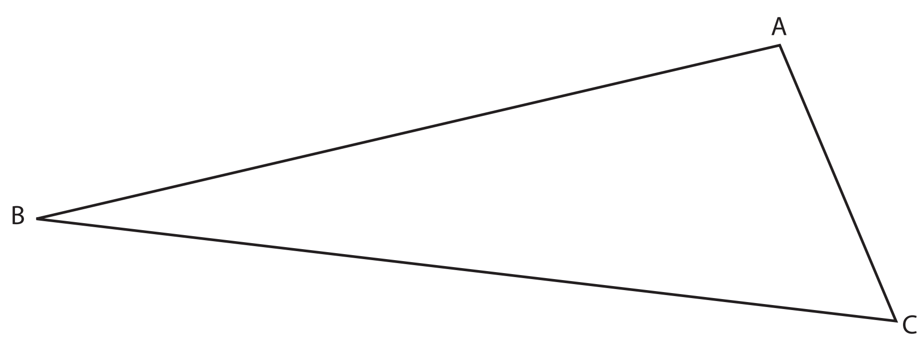
3. a. Use Method 3 to construct a triangle, AB'C', that is similar to triangle ABC shown below.



My Notes

b. Are $\overline{B'C'}$ and $\overline{BC}$ parallel? Justify your answer.

4. Which method of drawing similar triangles makes most sense to you? Why?

Turn to the solutions at the end of the section and mark your work.

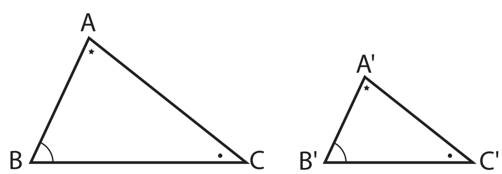
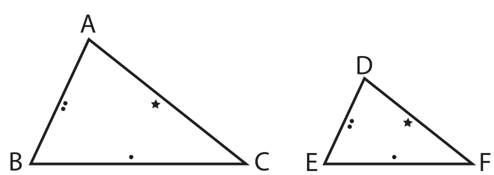
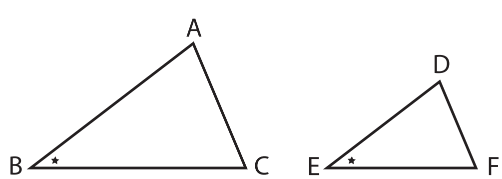
Bringing Ideas Together

You don't actually have to measure all the corresponding side lengths and angle measures to decide if a pair of triangles are similar. There are certain conditions that you can use to check if a pair of triangles are similar. In fact, the methods of construction you used in Explore are based on these sets of conditions.

My Notes

Three conditions for triangle similarity are listed in the following table. Nicknames for the conditions may help you remember the conditions more easily. Some common nicknames are provided in the table, but feel free to create your own.

Conditions for Triangle Similarity

Graphic Representation	Description in Words	Symbolic Description	Nickname
Condition 1  	If the corresponding angles of two triangles are equal in measure, then the triangles are similar.	In the diagram, if $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, and $\angle C \cong \angle C'$, then $\Delta ABC \sim \Delta A'B'C'$.	AAA Similarity
Condition 2  	If the corresponding sides of two triangles are proportional, the triangles are similar.	In the diagram, if $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{EF}$, then $\Delta ABC \sim \Delta DEF$.	SSS Similarity
Condition 3  	If two pairs of sides of two triangles are proportional, and the angles between those pairs of sides are congruent, the triangles are similar.	If, in the diagram, $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A \cong \angle D$, then $\Delta ABC \sim \Delta DEF$.	SAS Similarity

Note: When two figures are similar, you can use the symbol ~ to replace the words "is similar to."

Using the Similarity Conditions

The following examples show how these conditions can be used to solve problems involving similar triangles.

My Notes

Example 2: Using Condition 1 (AAA Similarity)

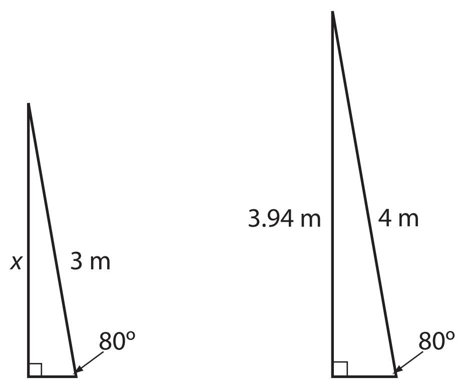
A 3 m pole and a 4 m pole are leaning against a vertical wall. Each pole makes an angle of 80° with the ground.

a. Are the triangles formed by the poles similar? Why?

b. If the 4 m pole reaches 3.94 m up the wall, how far up the wall does the 3 m pole reach? Round your answer to two decimal places.

Solution

a. Start by drawing a diagram. Assume the vertical wall is perpendicular to the ground. All the corresponding angles are equal.



The two 80° angles are equal in measure.

The two right angles are equal in measure.

So the third angle in each triangle must be 10°, since $10° + 80° + 90° = 180°$.

Note: This example confirms the fact that if two pairs of angles are equal in measure (congruent), so is the third pair.

Since the three pairs of corresponding angles are congruent, these triangles are similar.

My Notes

b. Since the triangles are similar, the sides must be proportional.

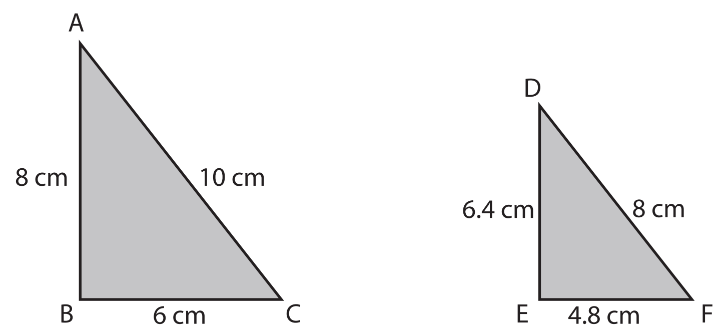
Let x be the height up the wall the 3 m pole reaches.

$$\frac{x}{3.94} = \frac{3}{4}$$
$$(3.94)\frac{x}{3.94} = \frac{3}{4}(3.94)$$
$$x = 2.955$$
$$x \cong 2.96$$

The 3 m pole reaches approximately 2.96 m up the wall.

Example 3: Using Condition 2 (SSS Similarity)

Two triangular sails from a model ship have the following dimensions.



Are the corresponding angles congruent?

Solution

Use the indicated lengths of the sides to see if corresponding sides are proportional.

$$\frac{AB}{DE} = \frac{8}{6.4} = 1.25$$

$$\frac{AC}{DF} = \frac{10}{8} = 1.25$$

$$\frac{BC}{EF} = \frac{6}{4.8} = 1.25$$

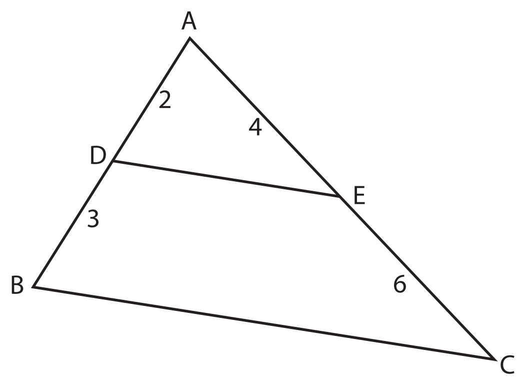
Because the ratios are equal, the triangles are similar.

Since $\Delta ABC \sim \Delta DEF$, the corresponding angles are congruent.

My Notes

Example 4: Using Condition 3 (SAS Similarity)

In the following figure there are two triangles—one is inside the other.



Is ΔABC ~ ΔADE?

Solution

Notice that ΔABC and ΔADE share a common angle, ∠A. So, both triangles have one angle that is identical. The triangles will be similar if the sides that form ∠A are proportional.

$$\frac{AD}{AB} = \frac{2}{2+3} = \frac{2}{5}$$

$$\frac{AE}{AC} = \frac{4}{4+6} = \frac{4}{10} = \frac{2}{5}$$

Notice that the legs of the smaller triangle, ΔADE, are in the numerator of both ratios. The legs of the larger triangle, ΔABC, are in the denominator of both ratios.

Since both ratios are equal, the side-lengths are proportional and thus the triangles are similar. We can write this as follows.

Since ∠A is shared, and $\frac{AD}{AB} = \frac{AE}{AC}$, then ΔABC ~ ΔADE.

To view the animated solution, go and look at *Similar Triangles* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/ma10_similar_triangles.html*).

My Notes

Activity 4
Self-Check

1. a. Jill told her friend the following:

   "For any two triangles, if two pairs of corresponding angles are given to be equal in measure, the triangles are similar."

   Is Jill's statement correct? Explain why or why not.

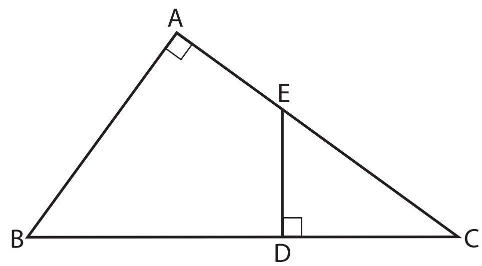
   b. Harlon said that from Jill's statement he could conclude the following about right triangles.

   "For right triangles, if just one pair of corresponding acute angles are given to be equal in measure, the right triangles are similar."

   How could Harlon make this conclusion about right triangles?

My Notes

2. Look at ΔABC and ΔDEC in the following diagram.

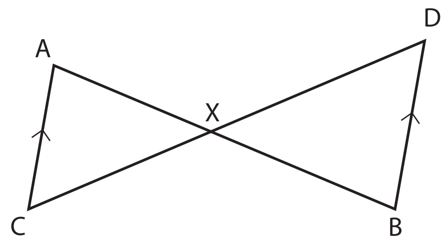


Are ΔABC and ΔDEC similar? Why or why not?

3. Name the corresponding sides in the pair of triangles in Question 2. If there are none, explain why there are none.

My Notes

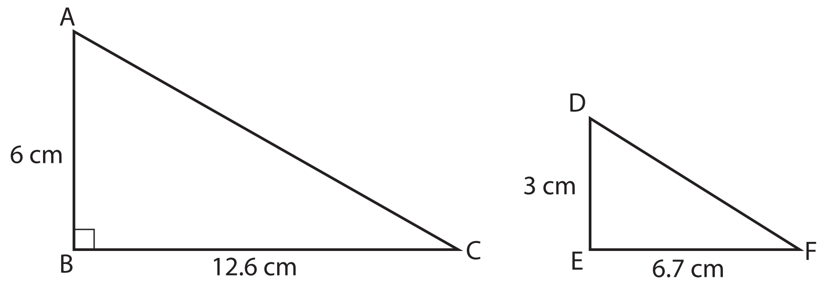
4. In the following diagram, $\overline{AB}$ crosses $\overline{CD}$ at X. $\overline{AC} \parallel \overline{DB}$. (Remember, || means "parallel to.")



Are ΔACX and ΔBDX similar? Why or why not?

My Notes

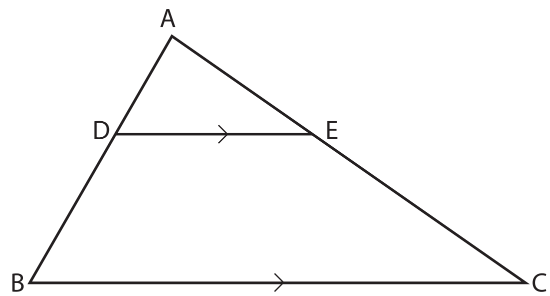
5. Look at the following triangles.



Are ΔACX and ΔBDX similar? Explain why or why not.

My Notes

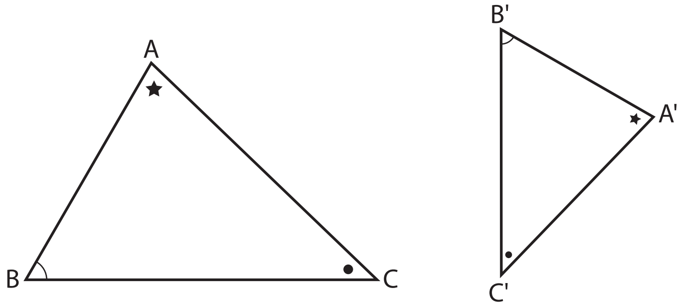
6. Identify ΔADE and ΔABC in the diagram.



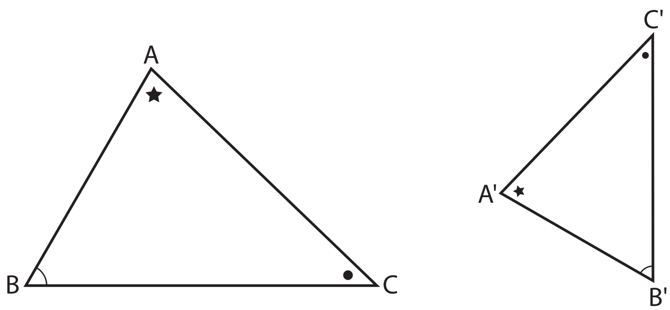
Suppose $\overline{DE} \parallel \overline{BC}$. Then are ΔADE and ΔABC similar triangles? Why or why not?

7. Which diagram makes it easiest to see the corresponding parts of the two triangles? Explain your answer.

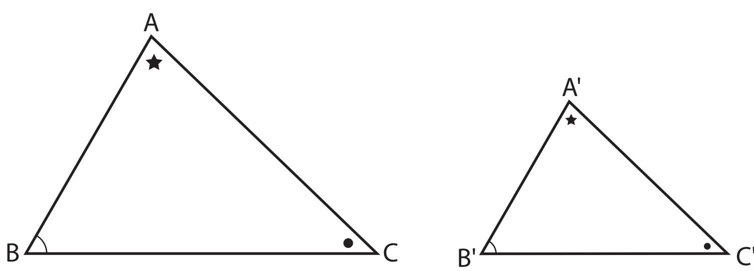
a.



b.



c.



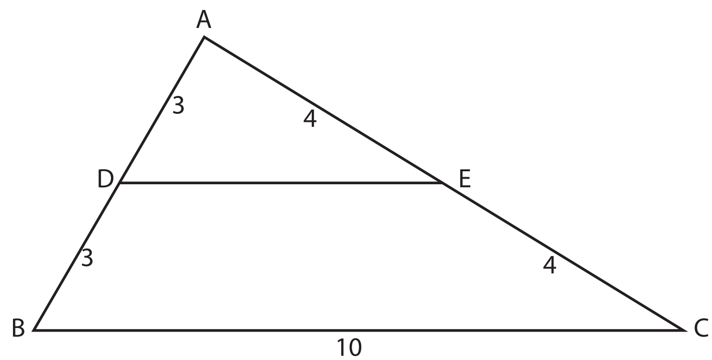
Turn to the solutions at the end of the section and mark your work.

My Notes

My Notes

Activity 5
Mastering Concepts

The diagram shows triangle ABC. Line segment DE joins point D on side AB with point E on side AC.



1. In the diagram, is $\overline{DE} \parallel \overline{BC}$? Why or why not?

2. What is the length of $\overline{DE}$?

My Notes

Turn to the solutions at the end of the section and mark your work.

My Notes

Lesson Summary

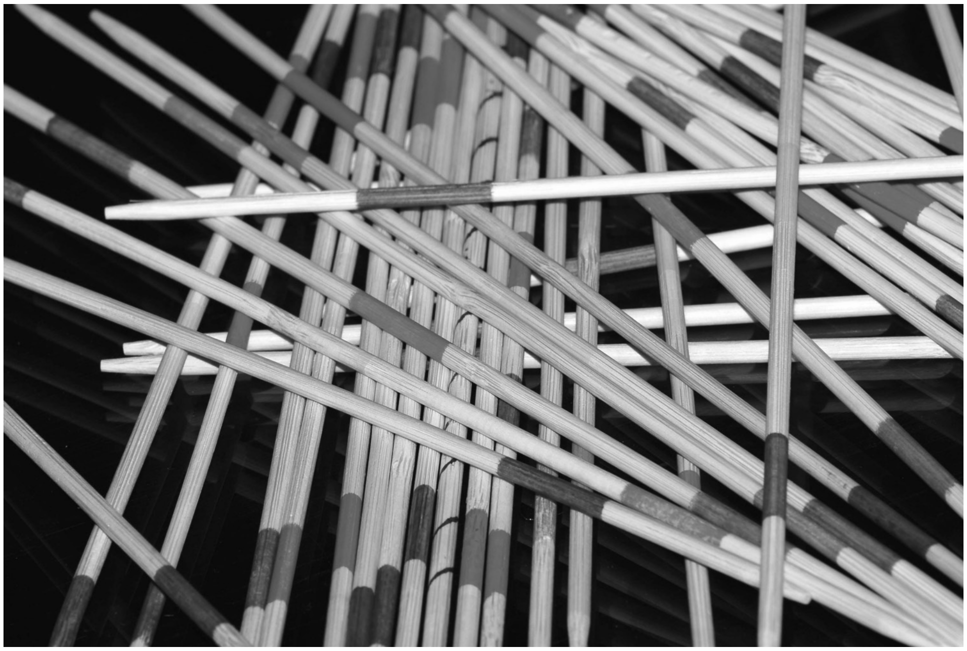


Photo by Marcel Jancovic © 2010

Have you ever played pickup sticks? The sticks in the photograph, scattered on the dark surface. form a variety of geometric shapes. These shapes include various triangles. Can you identify any similar triangles?

In this lesson you explored three methods for assessing whether or not two triangles are similar. Two triangles are similar if:

- the corresponding angles are congruent (AAA)
- the corresponding sides are proportional (SSS)
- one pair of angles are congruent and the two pairs of corresponding sides, which form those angles, are proportional (SAS)

Lesson D
Applying Similar Triangles

To complete this lesson, you will need:

- a measuring tape or ruler
- a calculator

In this lesson, you will complete:

- 4 activities

Essential Questions

- How are the measures of relationships among similar right triangles used in problem solving?

My Notes

Focus

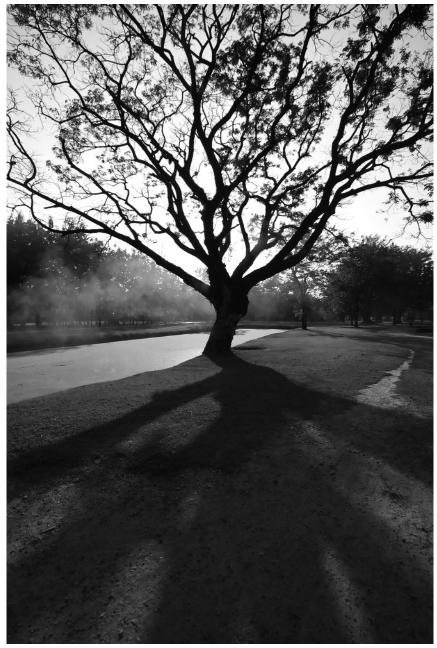


Photo by platongkoh © 2010

How could you determine the height of the tree in the photograph? I appears to be quite tall and it wouldn't be easy to measure the height using a tape measure or metre-stick. In fact, any type of direct measurement would be very difficult.

Can you come up with any strategies to find the height of the tree without measuring it directly?

Did any of your strategies involve using similar triangles?

In this lesson you will apply what you know about similar triangles. You'll use the relationships between corresponding parts of similar triangles to solve a variety of problems–including finding the height of a tree!

Get Started

My Notes

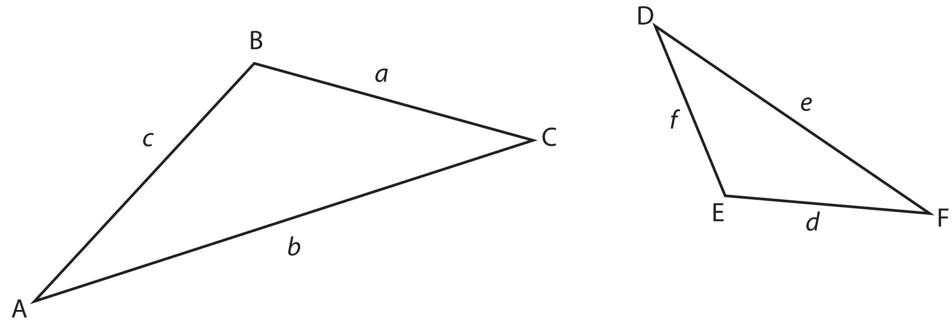
To be successful at solving problems using the properties of similar triangles, you'll need to be very comfortable with the Conditions of Triangle Similarity (AAA, SSS, and SAS) and with proportional reasoning. You can review some of this in Activity 1. If you need to review further, go back to Lesson C and read over the concepts you are uncertain about, or contact your teacher for help.

Activity 1
Self-Check

1. Describe three Conditions of Triangle Similarity. You may describe them in words, symbols and/or pictures.

My Notes

2. The two triangles shown below are similar.



a. Identify the corresponding angles in the two triangles. You may use a protractor.

b. Identify the corresponding sides in the two triangles.

c. Set up a proportion that you could use to determine the length of side *a*.

Turn to the solutions at the end of the section and mark your work.

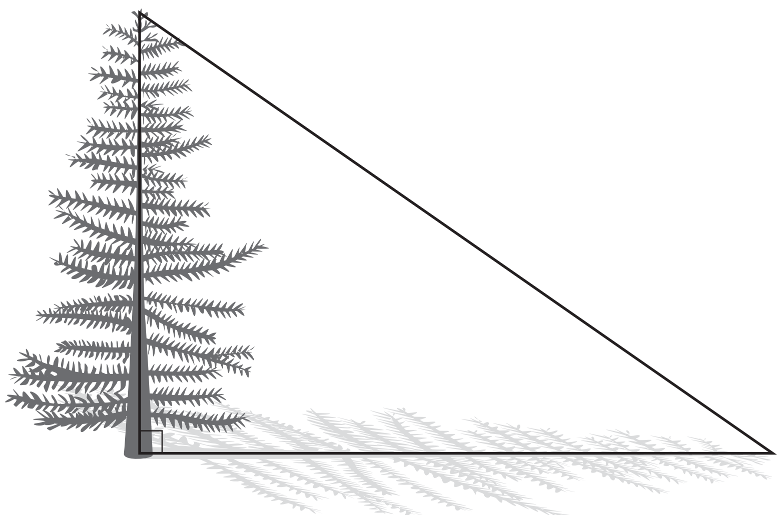
Explore

My Notes

In the lesson Focus you made some suggestions as to how you would determine the height of a tree without measuring it directly. There are many strategies that you could use. In Activity 2, you'll try a technique that uses the properties of similar triangles.

Shadows and Triangles

Think about the shadow that a tree will cast on the ground on a sunny day. The diagram below shows how a tree and its shadow can form the arms of a right triangle.



In fact, any object will cast a shadow on a sunny day. We can use the triangle formed by a small object and its shadow as well as the triangle formed by a tree and its shadow to find the height of a tree without measuring it directly. Try this method in Activity 2.

My Notes

Activity 2
Try This

To complete this activity, you will need a measuring tape; an object big enough to cast a shadow, but small enough to measure; a tree; and a sunny day!

Procedure

Note: You will have to take all measurements in as short a time-span as possible. The angle of the sunlight changes throughout the day, so it's important to measure the shadows at close to the same time.

Step 1: Go outside when the sun is shining measure, along a flat horizontal stretch of ground, the length of the shadow of a tall tree. (If you can't find a suitable tree, you can use a building, a vertical pole, or some other similar object the height of which is unknown.)

Step 2: Measure the height of a short vertical object, such as a fence post or garbage can, that casts a shadow along a level stretch of ground. Also, measure the length of its shadow along the ground.

Step 3: Answer the questions to determine the height of the tree.

Questions

1. Draw a diagram consisting of the similar right triangles created by each object and its shadow.

2. How do you know the two right triangles in your diagram are similar?

3. Set up a proportion and solve for x.

4. Is your answer to Question 3 reasonable? Explain your answer.

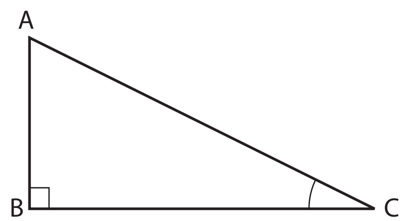
Turn to the solutions at the end of the section and mark your work.

My Notes

My Notes

Bringing Ideas Together

In the Explore section you investigated a practical problem involving similar right triangles. Remember, two right triangles are similar if there is a pair of congruent corresponding acute angles.



$\Delta ABC \sim \Delta DEF$, since they are right triangles and $\angle C \cong \angle F$.

Of course, not all similar triangles are right triangles. In the previous lesson, you explored the following requirements or conditions for stating that any pair of triangles are similar:

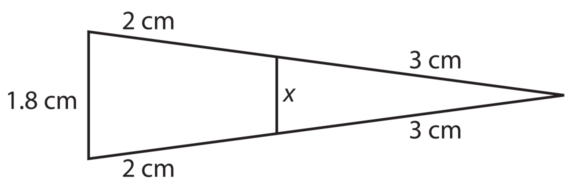
- The corresponding angles are congruent. (AAA)
- The corresponding sides are proportional. (SSS)
- One pair of angles is congruent and the two pairs of corresponding sides, which form those angles, are proportional. (SAS)

The following examples and Activity 3 questions involve identifying similar triangles in a variety of contexts contexts and then applying proportional reasoning to solve problems.

Example 1

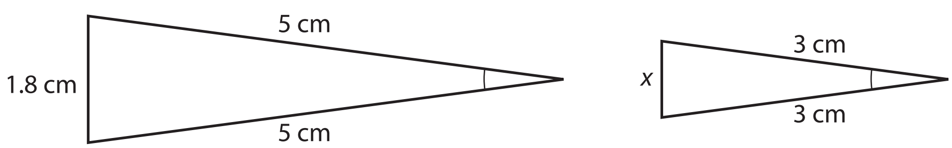
Photo by Kletr © 2010

Lucinda is building a scale model of a 1910 Curtiss biplane. She is working on the tail section and has taken measurements from an online graphic, but forgot to measure the length labelled x on the diagram below. What is its measure correct to one decimal place?



Solution

First, separate the triangles.



Now determine whether or not the triangles are similar.

- The two triangles share an angle at the tail section.
- The ratios of the sides forming the shared angle are equal, $\frac{3}{5} = \frac{3}{5}$.

Since a pair of angles is congruent and the two pairs of corresponding sides that form those angles are proportional, the triangles are similar.

My Notes

My Notes

Since the triangles are similar, we can solve the problem using a proportion.

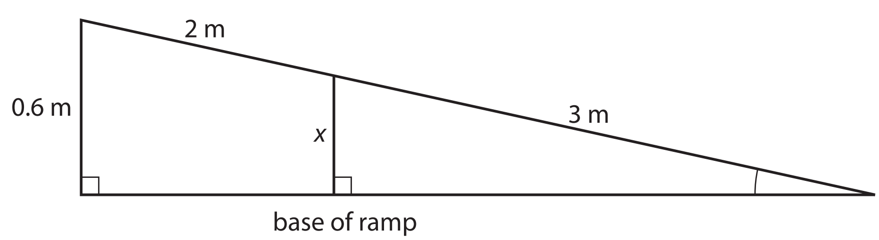
$$\begin{aligned}\frac{x}{1.8} &= \frac{3}{5}\\(1.8)\frac{x}{1.8} &= \frac{3}{5}(1.8)\\x &= 1.08\\x &\approx 1.1\end{aligned}$$

The missing measure is approximately 1.1 cm.

Example 2

Photo by prism68 © 2010

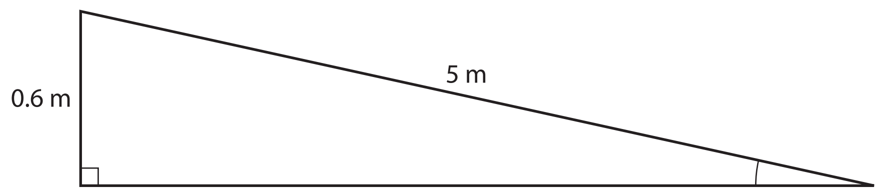
The bottom of a wheelchair ramp 5 m long is 0.6 m below the top of the ramp. How far is a wheelchair above the base of the ramp when it has moved 2 m down the ramp?

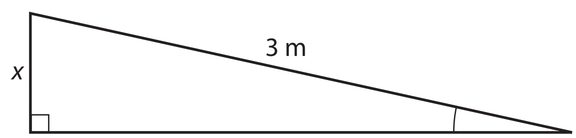


My Notes

Solution

First, separate the triangles.





Now determine whether or not the triangles are similar.

> Since the two right triangles in the original drawing share an acute angle, the triangles are similar. Note that the congruent angles are indicated in the diagram above by matching arcs.

Since the triangles are similar, we can solve the problem using the proportional side lengths.

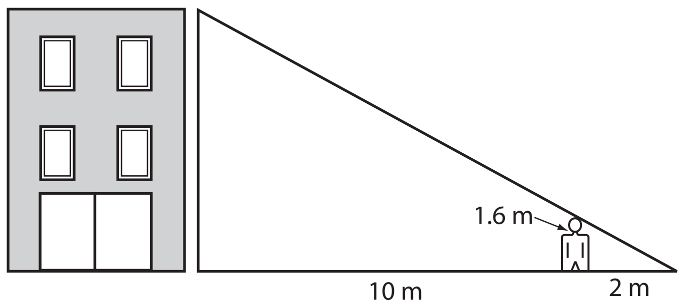
$$\frac{x}{0.6} = \frac{3}{5}$$
$$(0.6)\frac{x}{0.6} = \frac{3}{5}(0.6)$$
$$x = 0.36$$

The wheelchair is 0.36 m above the foot of the ramp.

My Notes

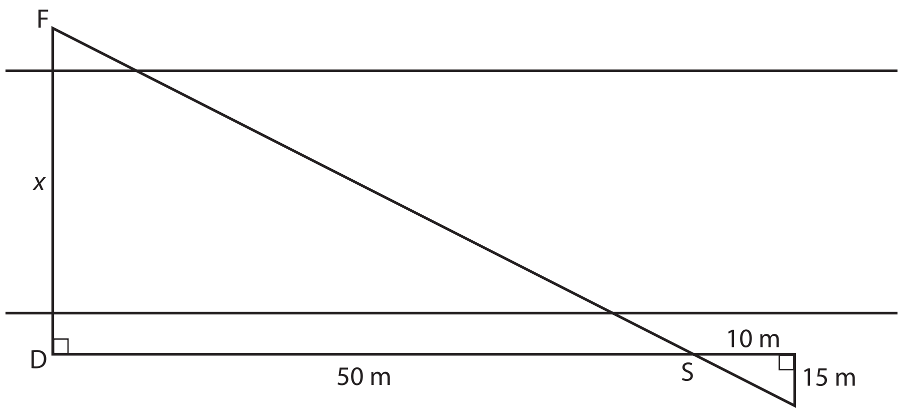
Activity 3
Self-Check

1. Jon walked 10 m away from a wall outside of his school. At that point, he noticed that his shadow reached the same point on the ground as the school's shadow. If Jon is 1.6 m tall, and his shadow is 2 m long, how high is the school? Round to one decimal place.



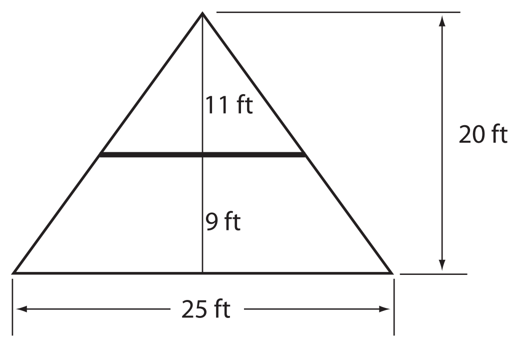
My Notes

2. Dace is standing on the shore of the river (marked "D" on the diagram). She sees a fisherman on the opposite bank (marked "F" on the diagram). She would like to know how far away the fisherman is, but can't measure across the water. Dace walks 50 m downstream along the riverbank. She stops and pushes a stick into the bank (marked "S" on the diagram). She walks another 10 m downstream, and then turns 90°. She now walks 15 m away from the river until she sees that the stick she pushed into the bank lines up with the fisherman across the river. Calculate the distance between Dace and the fisherman (marked "*x*" on the diagram).



My Notes

3. A chalet in the mountains has a triangular profile.



There is a balcony 9 ft above the ground. If the chalet is 25 ft across the base and 20 ft high, what is the width of the balcony?

My Notes

4. The crossed legs of an ironing board are illustrated below.

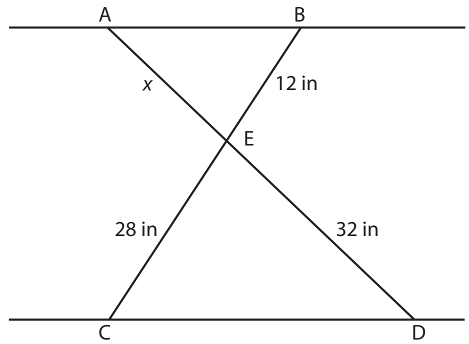


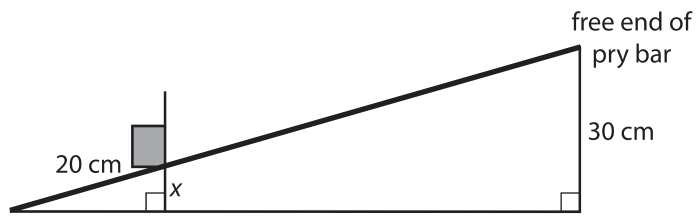
Photo by Lane V. Erickson © 2010

a. If the top of the ironing board is parallel to the floor, is $\Delta ABE \sim \Delta DCE$? Justify your answer.

b. Determine the value of x to the nearest tenth of an inch.

My Notes

5. A 2 m pry bar is placed under a timber 20 cm from the end of the bar. The free end of the bar is lifted 30 cm. How high off the ground is the timber?

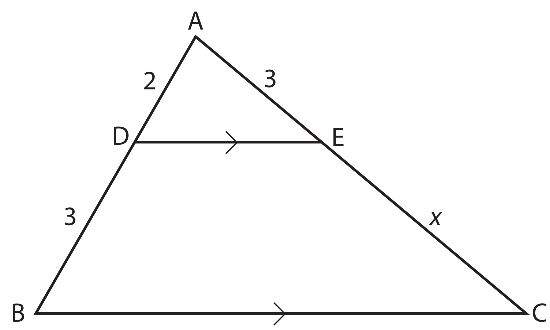


Turn to the solutions at the end of the section and mark your work.

Activity 4
Mastering Concepts

My Notes

$\overline{DE} \parallel \overline{BC}$



1. Why are ΔABC and ΔADE similar?

My Notes

2. Determine the value of *x*.

Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes

Photo by Tischenko Irina © 2010

The properties of similar triangles can help you determine lengths and distances when direct measurement is difficult.

In this lesson you explored a variety of real-world contexts involving similar triangles and proportional reasoning. You practised identifying similar triangles and then using the relationships between the corresponding parts of the triangles to solve problems.

Lesson E
Pythagorean Theorem

To complete this lesson, you will need:

- a protractor
- string
- a calculator
- scissors
- tape
- a marker
- a carpenter's square or set square from your geometry kit
- "Pythagorean Theorem Proof Template" from the Appendix

In this lesson, you will complete:

- 6 activities
- Part 5 of your Section Assignment

Essential Questions

- What is the Pythagorean Theorem?
- How can you verify the Pythagorean Theorem?

My Notes

Focus

Photo by Shawn Zhang © 2010

A baseball diamond is a square 30 yards on each side. If you were on second base and you wanted to throw the ball to the catcher standing at home plate, how far would you have to throw the ball?

A similar problem was outlined on a clay tablet from ancient Babylon over 3600 years ago. Cuneiform tablets, found in present-day Iraq and Iran, reveal that ancient mathematicians had knowledge of the Pythagorean relationship and employed sophisticated methods for solving problems involving square roots.

In this lesson you'll review the basics of the Pythagorean theorem, explore some historical proofs, and investigate some current applications.

Get Started

You have likely learned about Pythagoras and the Pythagorean theorem in previous courses. Take a minute to jot down what you already know about this concept.

My Notes

This Get Started will be a bit longer than usual because you must solidify some basic concepts related to the Pythagorean theorem before moving on in the lesson.

Egyptian Rope Stretchers

Five thousand years ago or more, the annual flooding of the Nile made it necessary to resurvey property lines and replace boundary markers. Early Egyptian surveyors were called *rope stretchers* because they used knotted ropes to measure distances and determine angles.

One method these surveyors used was to stretch a looped rope with 12 equally spaced knots.

In the next activity, you will use the ideas of the "Egyptian rope stretchers" to create a very special triangle: the triangle that has sides measuring 3 units, 4 units, and 5 units.

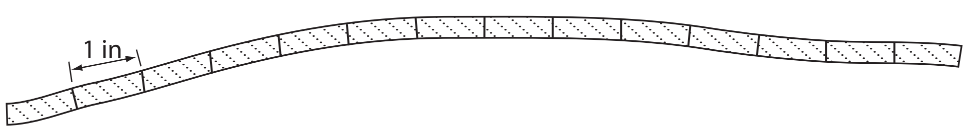
Activity 1
Self-Check

You'll need help from a partner to complete this activity. Ask a friend or family member.

You'll also need string, scissors, tape, a marker, a ruler, and a protractor.

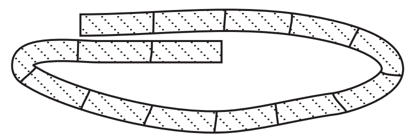
Step 1: Cut a piece of string 14 inches long.

Step 2: With your marker, carefully mark the string at 1-inch increments. You should have 13 marks on the string as shown in the diagram below.



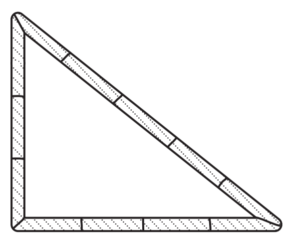
Step 3: Pick up the string and join the ends together to form a loop. You'll need to overlap the ends of the string so that the marks at each end match up as shown in the next diagram. Have your partner tape the ends together by wrapping a small piece of tape around the overlapped string.

My Notes



When finished, you will have created a loop with 12 equally spaced, dark, vertical lines—similar to the Egyptian looped rope with 12 equally spaced knots. Think of your loop as 12 units in circumference with the distance between two consecutive marks being 1 unit.

Step 4: Help your partner to stretch the loop on the table (or other flat surface) to form a triangle measuring 3 units on one side, 4 units on a second side, and 5 units on the third side.



Step 5: While your partner holds the vertices of the triangle, measure the angle created by the 3-unit side and the 4-unit side. Record your measurement.

____

Step 6: Help your partner try to create a different triangle with the same side lengths. Your partner must hold tight to the vertices to ensure the side lengths stay the same (3 units, 4 units, and 5 units) as he/she tries to move the vertices into different positions.

Questions

1. Does your triangle look like the one shown in Step 4?

   ____

2. What was the measure of the angle you measured in Step 5?

   ____

My Notes

3. Were you able to create any other triangles with the same side lengths?

____

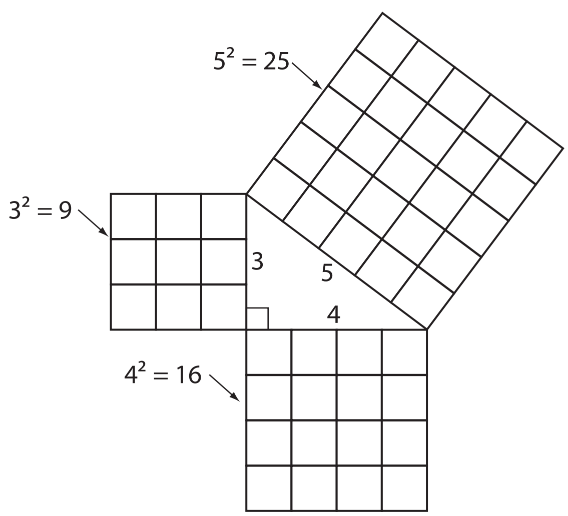
Turn to the solutions at the end of the section and mark your work.

Pythagorean Triples

The ancient Egyptians, as well as the Babylonians and Chinese, among others, knew that a 3-4-5 triangle formed a right angle. Today, we call such a *triple*, a **Pythagorean triple**.

Another thing the Egyptians, and others, knew is: if squares were constructed on the sides of a right triangle, the area of the largest square would be exactly equal to the areas of the two smaller squares added together.

Look at the following squares constructed on the sides of a 3-4-5 right triangle.



My Notes

As you can see,

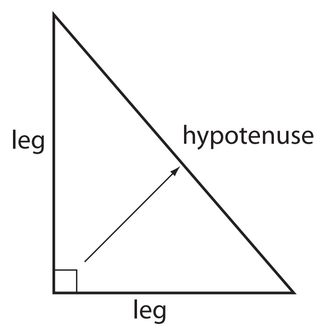
Area of the two small squares = Area of the large square

Area of smallest square	+	Area of mid-size square	=	Area of large square

$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$

The formula for Area of a square is $A = s^2$.

The **Pythagorean Theorem** states that for any right triangle, the square on the **hypotenuse** is equal to the sum of the squares on the two **legs**. Remember that the hypotenuse is the longest side of a right triangle. It is always located across from the right angle. The legs are the other two sides of a right triangle.



As you saw above, the 3-4-5 right triangle side relationship can be represented by:

$$3^2 + 4^2 = 5^2$$

This is just one instance of the Pythagorean Theorem. In the next activity you will look at several triangles and see how the Pythagorean Theorem applies.

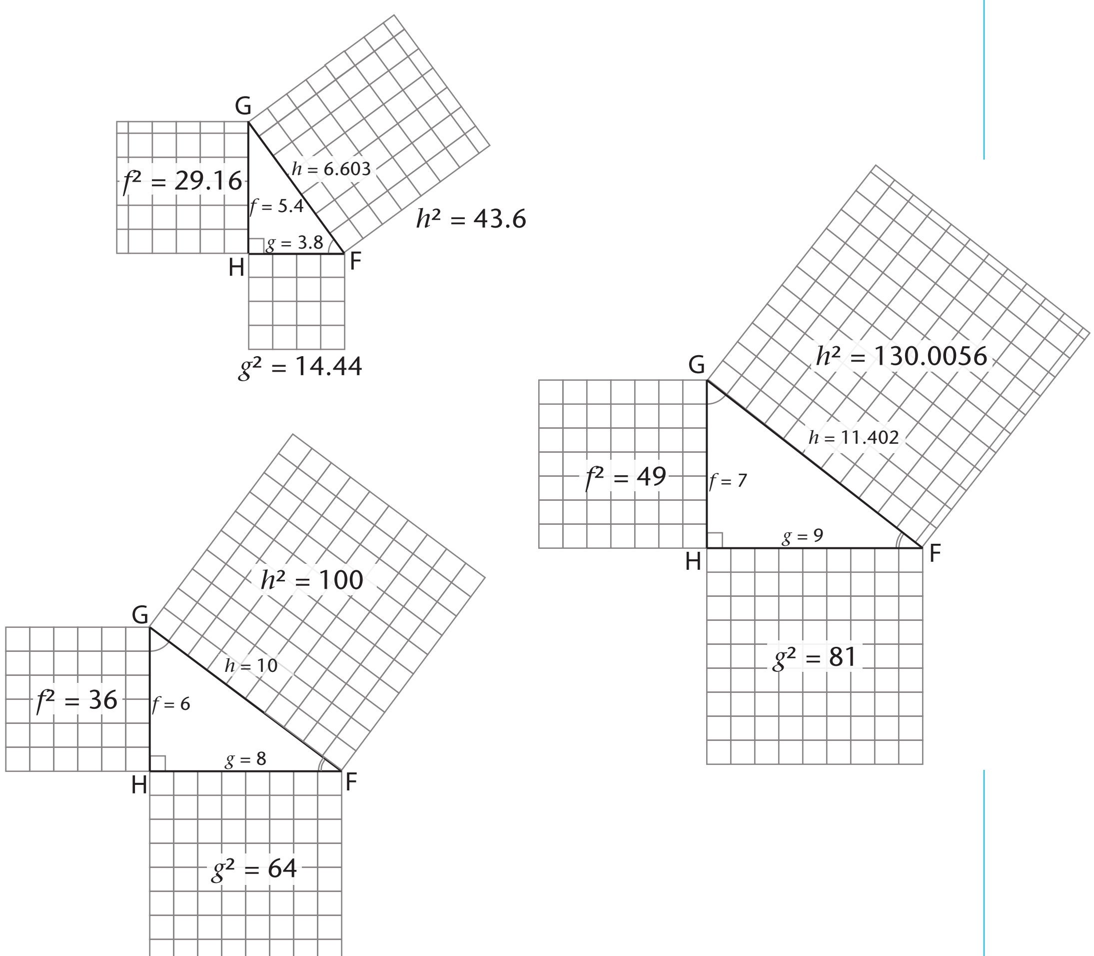
Activity 2
Try This

My Notes

In this activity you'll see how the Pythagorean Theorem works.

If you have access to the Internet go and open *Pythagorean Theorem* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/glossary/Division03/Pythagorean%20Theorem/index.html*). Scroll down to the demonstration applet. Use the "hide areas" and "show areas" buttons to help fill in the table below for any three different triangles you make.

If you don't have access to the Internet, you can fill in the table on the next page using information from the following three diagrams.



My Notes

Lengths			Area of Squares			Sum of Squares of Legs
Leg (f)	Leg (g)	Hypotenuse (longest side) (h)	Leg (f^2)	Leg (g^2)	Hypotenuse (longest side) (h^2)	$f^2 + g^2$

Questions

1. Do the squares of the legs always add up to equal the square of the hypotenuse? In other words, is $f^2 + g^2 = h^2$ true in each case?

   ____

   ____

2. Draw a triangle, by hand, where the Pythagorean Theorem would NOT apply. Add the measurements of this triangle and the calculations to the chart too.

   a. Is this a right triangle?

   ____

   b. Is $f^2 + g^2 = h^2$ true for this triangle?

   ____

Turn to the solutions at the end of the section and mark your work.

My Notes

Squares and Square Roots

In Activity 2, you examined several possibilities for the dimensions of a right triangle. For some triangles the sides were whole numbers, as in the 3-4-5 right triangle. In other triangles the sides were decimal approximations. In all cases, the calculations were done for you. However, later in this lesson, you will be required to perform the calculations yourself. In the next activity, you'll practise squares and square roots.

Activity 3
Self-Check

Use your calculator to answer these questions.

1. a. $4^2 =$ _______________

   b. $15.3^2 =$ _______________

   c. $\sqrt{49} =$ _______________

   d. $\sqrt{885.72} =$ _______________

2. Solve for x.

   $x^2 = 144$

Turn to the solutions at the end of the section and mark your work.

My Notes

Explore

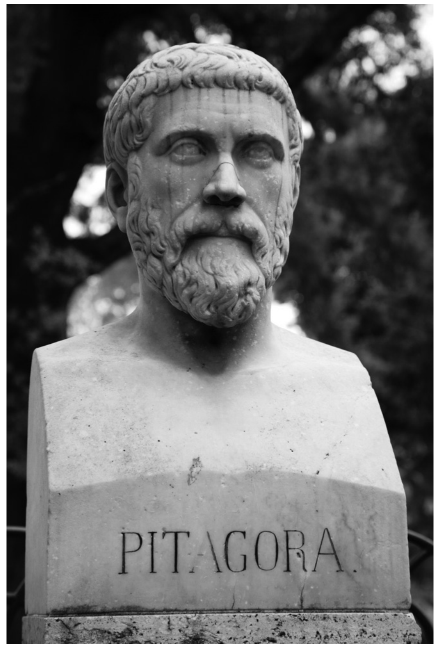


Photo by Konovalikov Andrey © 2010

In the Get Started section, you reviewed the essentials of the Pythagorean Theorem. This theorem is named after Pythagoras of Samos, a Greek philosopher and mathematician who lived from about 570 BCE to 490 BCE. Even though he spent a great deal of his life on the Greek island of Samos, he did study in Egypt and would have been familiar with the work of the rope-stretchers. He is credited with deriving a formal proof for the relationship among the sides of the right triangle.

Despite the fact that this theorem bears his name, mathematicians in China, India, and Babylon applied this relationship almost two thousand years earlier. Ideas flowed from culture to culture. Mathematics is not unique to a specific people or time, but is a human endeavour.

In this activity you will explore a proof from China, which was known at least 500 years before Pythagoras.

Activity 4
Try This

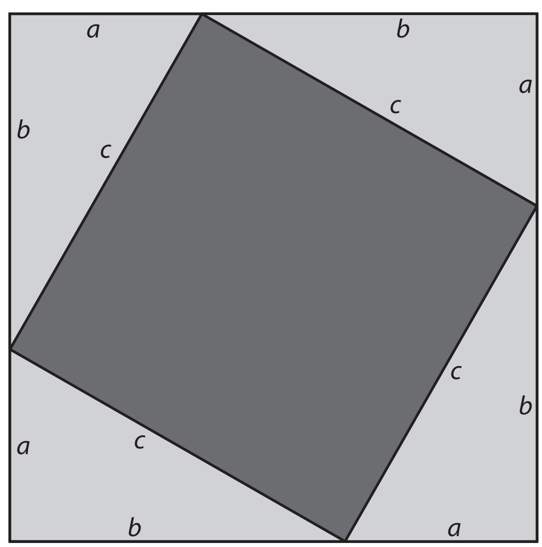
My Notes

You'll need to get the "Pythagorean Theorem Proof Template" from the Templates section of the Appendix.

Note: c = length of hypotenuse
a = length of shorter leg
b = length of longer leg

Step 1: Cut out the four identical grey right triangles and the black square. Place the four triangles on top of the black square as shown below.

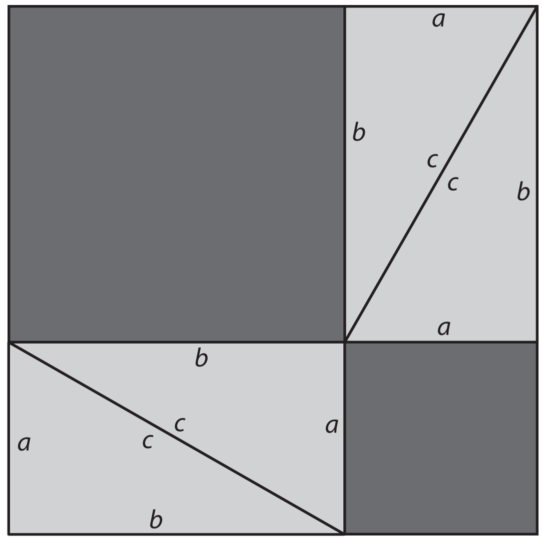
Notice that the lengths of the sides of the original big black square equals $a + b$, the sum of the legs of each triangle.



Step 3: Answer Questions 1 and 2 and then proceed to Step 4.

My Notes

Step 4: Rearrange the four grey right triangles on the original, larger black square as shown below.



Step 5: Answer Questions 3–8.

Questions

1. The black figure in the middle of the triangle arrangement appears to be a square. Without measuring, explain how you know that each of the four angles of the black figure is a right angle and how you know the sides are all equal.

2. Is the area of the small black square showing in the middle of the arrangement a^2, b^2, or c^2? Why?

My Notes

3. Why must the total area of the black areas, showing in this arrangement, be the same as the black area showing in the first arrangement?

4. The two black figures shown in this arrangement look like squares. However, how do you know that the two black figures are actually squares?

5. Is the area of the larger black square a^2, b^2, or c^2? Why?

6. Is the area of the smaller black square a^2, b^2, or c^2? Why?

7. What is the total of the areas of the two black squares?

8. How is your answer to Question 7 related to your answer to Question 2? Write out the relationship. Where have you seen this relationship before?

My Notes

You have just proven the Pythagorean Theorem!

Turn to the solutions at the end of the section and mark your work.

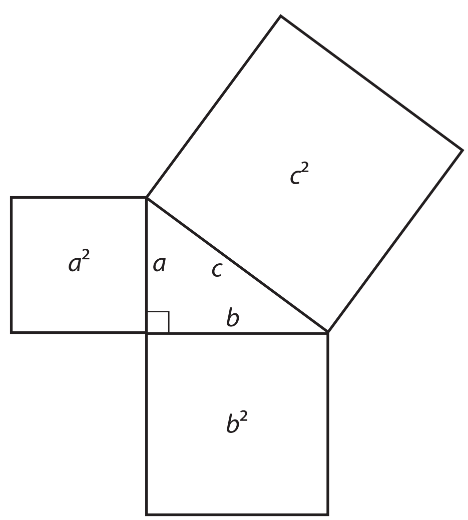
Bringing Ideas Together

In Explore you investigated an ancient proof of the Pythagorean Theorem. There are many proofs of the Pythagorean Theorem that involve cutting out pieces and rearranging them as you did in Explore.

You can find more proofs on the internet. Start your search at the Companion Website (*http://www.openschool.bc.ca/courses/math/awm10/mod3.html*) and look at *Proof of the Pythagorean Theorem.*

Remember, the Pythagorean Theorem only applies to right triangles, as you discovered in the Get Started. It states that for any right triangle, the square of the hypotenuse equals the sum of the squares of the two legs.

The letters used in the statement depend on the labels on the sides of the triangle.



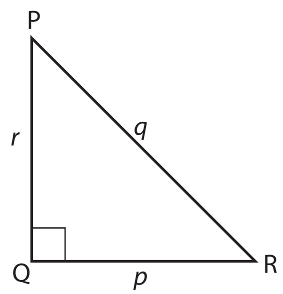
$a^2 + b^2 = c^2$ OR $c^2 = a^2 + b^2$

Example 1

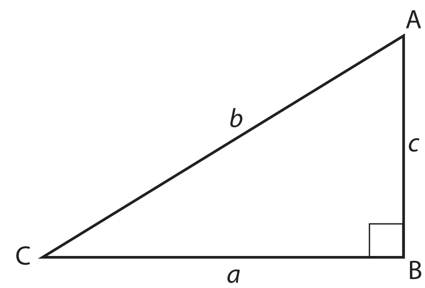
My Notes

Write the statement of the Pythagorean Theorem for each right triangle.

1.



2.



Solution

1. Notice that, here, the lengths of the sides of the triangle are named using lowercase (small) letters. The letter used depends on the opposite angle.

   The length of side QR is p, because this side lies opposite angle P. The length of side PR is q, because this side lies opposite angle Q. The length of side PQ is r, because this side lies opposite angle R.

   Because q is the length of the hypotenuse, $q^2 = p^2 + r^2$ or $p^2 + r^2 = q^2$.

2. Once again, the lengths of the sides of the triangle are named using lowercase (small) letters. Again, the letter used depends on the opposite angle.

   The length of side BC is a, because this side lies opposite angle A. The length of side AC is b, because this side lies opposite angle B. The length of side AB is c, because this side lies opposite angle C.

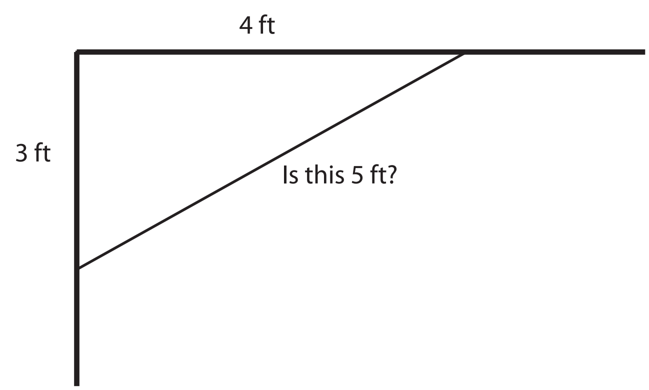
   Because b is the length of the hypotenuse, $b^2 = a^2 + c^2$ or $a^2 + c^2 = b^2$.

My Notes

Carpenters' Corner

In Get Started, you stretched a loop of string to form a 3-4-5 right triangle. This Pythagorean triple is used by carpenters to check if corners are square.

First, the carpenter marks points 3 ft and 4 ft from the corner. Then he or she measures the distance between the two points. If the distance between the marks is 5 ft, the corner is square.



Because triangles with proportional sides are similar, the following side length ratios will all be Pythagorean triples!

3:4:5
6:8:10
9:12:15
12:16:20

Do you see the pattern here? How would you find another Pythagorean triple with this same ratio?

____

____

You can multiply each term in the ratio by any whole number to get an equivalent ratio. The equivalent ratio will be another Pythagorean triple.

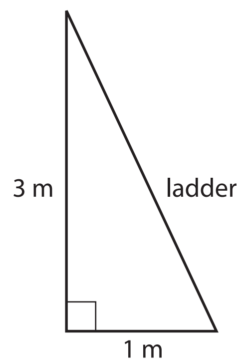
multiply by 2 to get 6:8:10
multiply by 3 to get 9:12:15
multiply by 4 to get 12:16:20

Using the Pythagorean Theorem

Pythagoras' theorem is a very useful tool for solving problems. In fact, you will apply the theorem to many problems in Lesson F. For now, have a look at the example below.

Example 2

A ladder is leaning against a vertical wall. The foot of the ladder is 1 m from the wall and the ladder reaches 3 m up the wall. How long is the ladder? Round to 1 decimal place.



Solution

Let the length of the ladder be x.

(hypotenuse)^2 = (leg 1)^2 + (leg 2)^2

$x^2 = 3^2 + 1^2$
$x^2 = 9 + 1$
$x^2 = 10$

$x = \sqrt{10}$ — Use your calculator.
$x = 3.16227766\ldots$
$x \approx 3.2$

The ladder is about 3.2 m in length.

My Notes

My Notes

Activity 5
Self-Check

1. a. Show that 5, 12, 13 is a Pythagorean triple.

   b. Sketch a triangle with these sides.

   c. From 5, 12, 13, write three more Pythagorean triples.

   ____

   ____

   ____

2. a. Is 4, 7, 9 a Pythagorean triple?

My Notes

b. Sketch a triangle with these sides.

c. Is the triangle a right triangle? Why or why not?

My Notes

3. Kale walked 300 m due north, turned, and walked 200 m due east. How far is he from his starting point? Round your answer to the nearest metre.

Turn to the solutions at the end of the section and mark your work.

Activity 6
Mastering Concepts

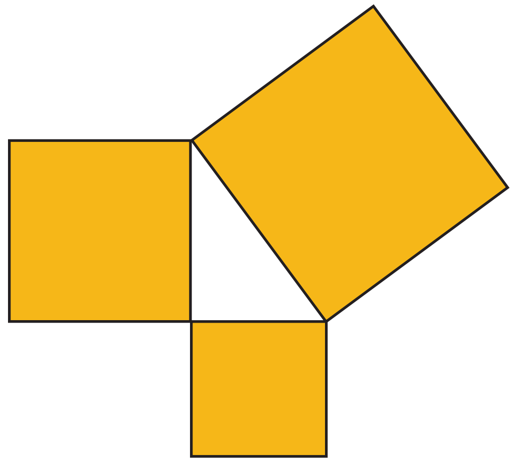
The numbers 3, 4, 5 form a Pythagorean triple. Prove that the triplet $3n$, $4n$, $5n$, where n is any positive whole number ($n = 1, 2, 3$, or 4, and so on), is also a Pythagorean triple.

Turn to the solutions at the end of the section and mark your work.

My Notes

My Notes

Lesson Summary



In his book *5000 BC and Other Philosophical Fantasies*, the mathematician Raymond Smullyan describes a puzzle he would put to his students. Squares of beaten gold are placed on the three sides of a right triangle as shown in the illustration. You can either choose the large square or you can choose both of the two smaller squares.

Which would you choose?

In this lesson, you investigated the 3-4-5 right triangle, its application as a surveyor's tool in Ancient Egypt, and its use today in modern carpentry. You discovered that this triangle is a special instance of the Pythagorean Theorem. You explored one proof attributed to the Chinese of over three thousand years ago! In the next lesson you will explore this theorem further in its application to a variety of everyday problems.

Lesson F
Applying the Pythagorean Theorem

To complete this lesson, you will need:

- a protractor
- a ruler
- a compass
- a calculator

In this lesson, you will complete:

- 4 activities

Essential Questions

- How is the Pythagorean Theorem applied to solve a variety of practical problem situations?

My Notes

Focus

Photo by Galina Barskaya © 2010

Snowboarding is a popular winter sport in Canada. Skilled boarders will argue it is both a sport and an art. Part of the thrill is becoming airborne. The time in the air and distance travelled down the slope depends on the boarder's speed and technique. If the snowboarder in the photograph jumped 5 m horizontally and landed 1.5 m lower vertically, from the point she became airborne, how far down slope did she travel?

This problem can be solved by applying the theorem you explored in Lesson E: The Pythagorean Theorem.

Get Started

In this activity you will examine a method of drawing a triangle given the lengths of its three sides.

Activity 1
Try This

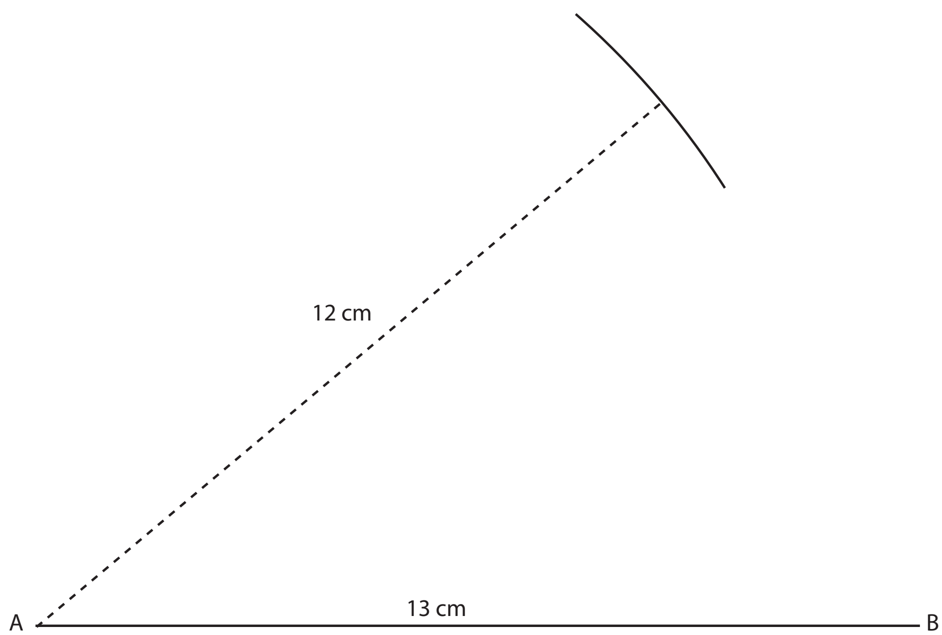
My Notes

Follow the instructions to construct a triangle. Please note: the diagrams within the instructions are drawn to scale, but they are not shown in actual size. Once you've constructed the triangle, answer the questions that follow.

We'll construct a triangle with sides 13 cm, 12 cm, and 5 cm.

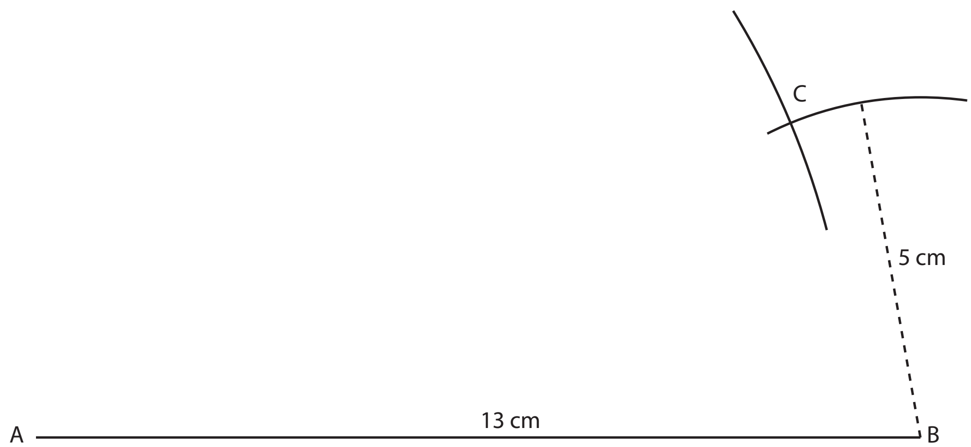
Step 1: With your ruler, draw a line segment, $\overline{AB}$, 13 cm long. You could also have drawn the segment 12 cm or 5 cm long, but generally, it is easier in subsequent steps if you chose the longest side.

Step 2: Open your compasses to a radius of 12 cm. Note: You could use 5 cm in this step, but again, to simplify the procedure, the longer of the two remaining sides is chosen. With centre A and the radius of 12 cm, draw an arc where you judge the third vertex of the triangle to be.

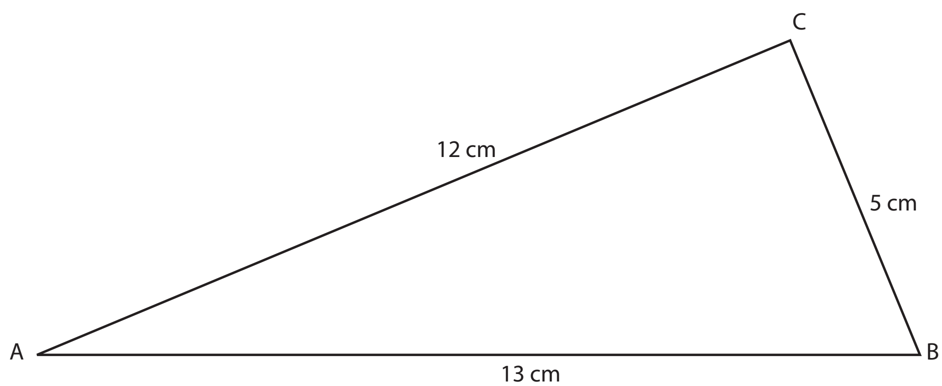


My Notes

Step 3: Open your compass to a radius of 5 cm. With centre B and the radius of 5 cm, draw an arc intersecting the first arc. Call this point C.



Step 4: Join A, B, and C.



Questions

1. Use your protractor to measure ∠C. What is its measure?

2. Suppose you didn't have a protractor, how could you prove that $\angle C$ is a right angle—show your steps.

3. What can we say about the three numbers 5, 12, 13?

Turn to the solutions at the end of the section and mark your work.

Explore

In Get Started you examined how to draw a triangle given the lengths of its sides. In the next activity you will explore further how the Pythagorean Theorem is used to determine whether or not a triangle is a right triangle given the lengths of its sides.

My Notes

My Notes

Activity 2
Self-Check

Follow these steps to complete the table below. Then answer the questions that follow.

Step 1: Use the method outlined in Get Started to draw the triangles in the table. It will probably be easier to draw the sides, *a*, *b*, and *c*, in centimetres rather than in inches. But any unit would do.

Step 2: After you draw each triangle, estimate, measure, and record the size of ∠C. Round angle measures to the nearest degree.

Step 3: Complete each row of the table to see which values of *a*, *b*, and *c* form Pythagorean triples.

Triangle	a	b	c	a^2	b^2	c^2	$a^2 + b^2$	Estimate ∠C	Measured ∠C
#1	3	4	5						
#2	4	5	5						
#3	5	6	7						
#4	6	8	10						
#5	8	15	17						
#6	6	16	17						

Questions

1. How do you know that ∠C would be the right angle in these triangles? Why didn't you measure ∠A or ∠B to see if they were the right angle in the triangle?

2. Which triangles were right triangles? How do you know?

3. Which values of *a*, *b*, and *c* form Pythagorean triples?

4. Using the ratio 8, 15, 17, form four more sets of Pythagorean triples.

My Notes

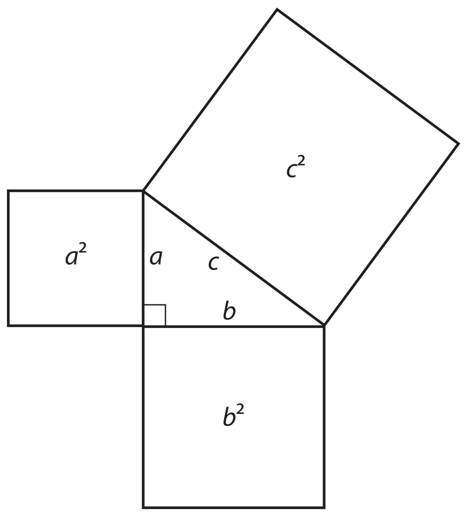
My Notes

Turn to the solutions at the end of the section and mark your work.

Bringing Ideas Together

In Explore you investigated how the Pythagorean Theorem may be used to verify whether a given triangle is a right triangle.

The Pythagorean Theorem states that for any right triangle, the square on the hypotenuse is equal to the sum of the squares on the two legs.



$a^2 + b^2 = c^2$ OR $c^2 = a^2 + b^2$

The Pythagorean Theorem also states that if the square on the longest side of a triangle is equal to the sum of the squares on the other two sides, then the triangle is a right triangle. The longest side will be the hypotenuse. The angle opposite the longest side will be a right angle. So the following are both true and useful when problem solving.

If $\angle C$ is a right angle in ΔABC, then $a^2 + b^2 = c^2$.

If $a^2 + b^2 = c^2$, then ΔABC is a right triangle and $\angle C$ is a right angle if c is the hypotenuse.

Now let's apply the Pythagorean Theorem in problem situations.

My Notes

Example 1

The sides of ΔABC are a = 5 cm, b = 8 cm, and c = 7 cm.

1. If ΔABC was a right triangle, which side would the hypotenuse be? Why?

2. Is ΔABC a right triangle? Why or why not?

Solution

1. The hypotenuse would be the longest side, b.

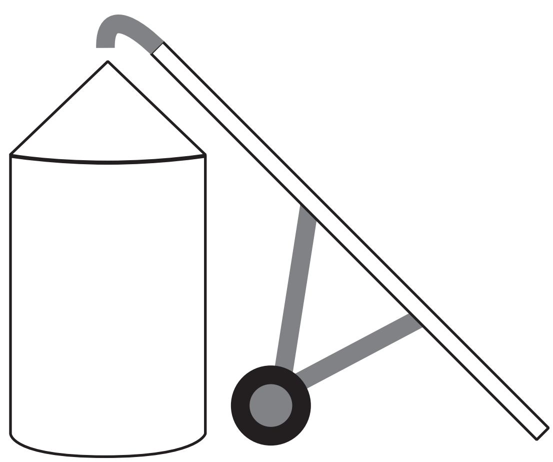
2. $b^2 = 8^2$
   $= 64$

   $a^2 + c^2 = 5^2 + 7^2$
   $= 25 + 49$
   $= 74$

   Since the square on the longest side is not equal to the sum of the squares on the two other sides, ΔABC is not a right triangle.

Example 2

A 51 ft grain auger is backed up to a 30 ft tall grain bin as in the illustration.

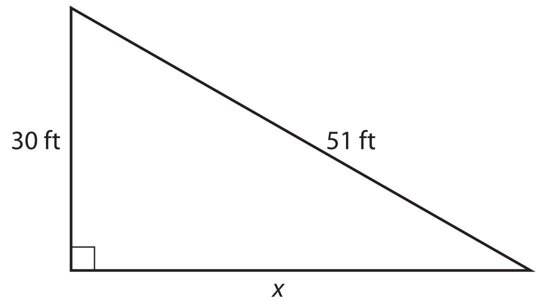


How far from the bin is the foot of the auger? Round your answer to the nearest foot.

My Notes

Solution

Let the distance to the bin be x.



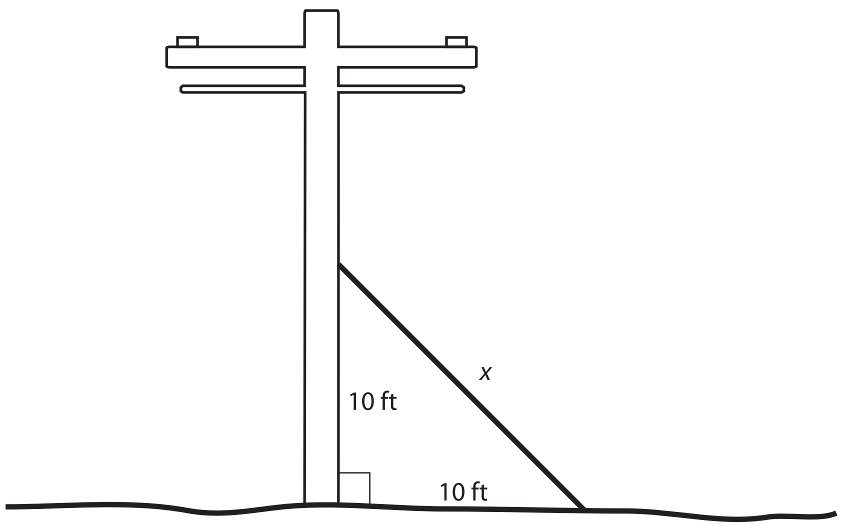
$$x^2 + 30^2 = 51^2$$
$$x^2 + 900 = 2601$$
$$x^2 = 2601 - 900$$
$$x^2 = 1701$$

$$x = \sqrt{1701}$$
$$x = 41.24318125\ldots$$
$$x \approx 41$$

The foot of the auger is approximately 41 ft from the bin.

Example 3

A guy wire supporting a power pole is anchored 10 ft up the pole, and, at the ground, 10 ft from the pole.



Correct to one decimal point, what is the length of the guy wire?

Solution

Let x be the length of the guy wire.

$$
\begin{aligned}
x^2 &= 10^2 + 10^2 \\
x^2 &= 100 + 100 \\
x^2 &= 200 \\
x &= \sqrt{200} \\
x &= 14.1421\ldots \\
x &\approx 14.1
\end{aligned}
$$

The guy wire is approximately 14.1 ft in length.

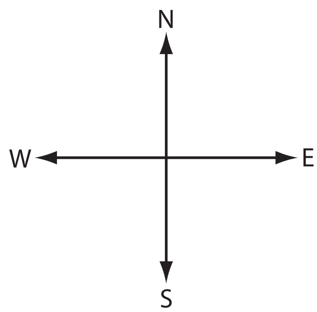
My Notes

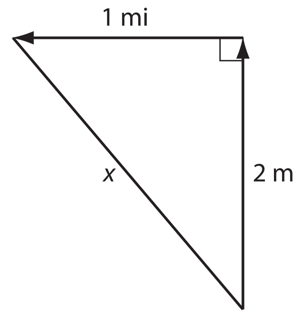
My Notes

Activity 3
Self-Check

1. What is the longest metal rod that will lie flat on the bottom of a rectangular box 20 cm wide and 30 cm long? Express your answer to the nearest centimetre.

2. Maxim is cycling in the country. He travels 2 miles north along a range road, and then 1 mile west along a township road.





To the nearest tenth of a mile, how far is Maxim from his starting point?

My Notes

My Notes

3. A kite is being flown over the school 100 m away. In reaching this position, 130 m of string has been let out. How high above the ground level is the kite? Round to the nearest metre.

My Notes

4. The interior of a rectangular sewing box is 15 cm wide, 20 cm long, and 16 cm high. Can a knitting needle 28 cm long be placed in the box without sticking out of the top?

Turn to the solutions at the end of the section and mark your work.

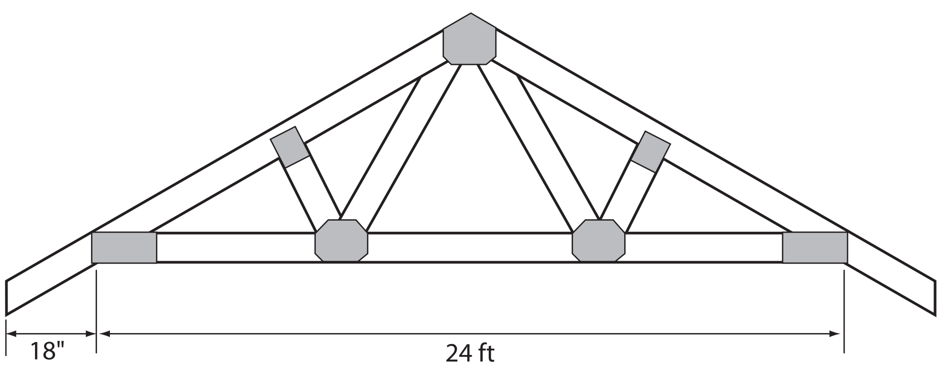
My Notes

Activity 4
Mastering Concepts

Anya is helping her father build a garage. They have seen W-trusses used in the construction of several garages in the neighbourhood. They plan also to use W-trusses to support their roof.

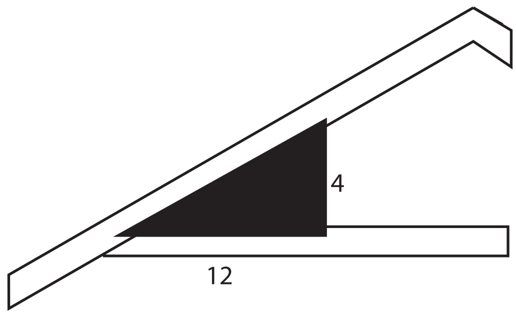
Photo by Brandon Bourdages © 2010

Anya wants to calculate the length the rafters (the sloping beams of the trusses) will have to be. Each truss must span 24 ft—the width of the garage—and have an 18-inch overhang beyond each wall, as shown in the diagram below.



My Notes

The roof has a slope of 4 in for a horizontal run of 12 in. This slope is represented by the triangle in the following diagram.

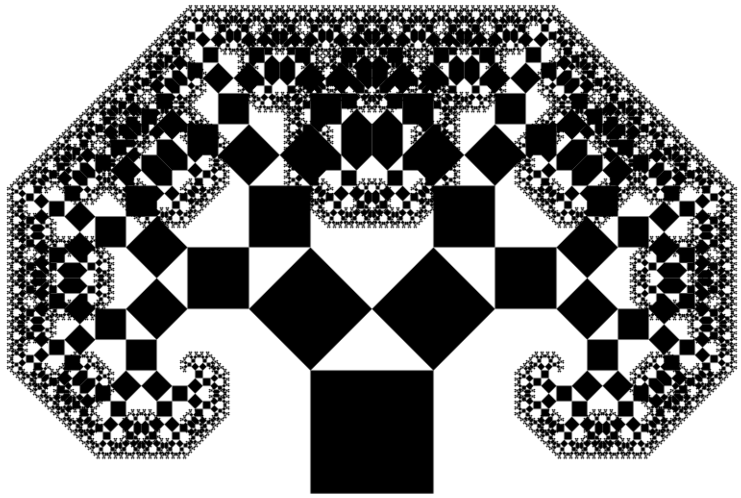


Help Anya find the length of each truss to the nearest sixteenth of an inch.

Turn to the solutions at the end of the section and mark your work.

My Notes

Lesson Summary



Fractal art is based on similarity. The "trees" in the illustrations were created by drawing similar Pythagorean relationships on progressively smaller scales. As you can see, each square on the legs of a right triangle becomes the square on the hypotenuse of a smaller but similar right triangle, and so on, and so on!

You can create trees like this too! Go and open *Pythagorean Tree* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/GrandPythagoreanTree/m10_3_m6_020.html*). To make the tree grow, press More. To start over, press Reset. For a variety of trees, adjust the Slant slider.

The Pythagorean Tree is just one of many applications of the Pythagorean Theorem.

In this lesson you used the Pythagorean Theorem in various situations to find the missing side of a right triangle from the other two sides. You also applied the Pythagorean Theorem to check if the lengths of three sides could form a right triangle. In every case, you applied the fact that the area of the square constructed on the longest side of a right triangle equals the sum of the areas of the squares on the two remaining sides.

Triangles And Other Polygons —Appendix

TABLE OF CONVERSIONS

1 inch	≈	2.54 centimetres
1 foot	≈	30.5 centimetres
1 foot	≈	0.305 metres
1 foot	=	12 inches
1 yard	=	3 feet
1 yard	≈	0.915 metres
1 mile	=	1760 yards
1 mile	≈	1.6 kilometres
1 kilogram	≈	2.2 pounds
1 litre	≈	1.06 US quarts
1 litre	≈	0.26 US gallons
1 gallon	≈	4 quarts
1 British gallon	≈	$\frac{6}{5}$ US gallon

FORMULAE

Temperature
$C = \frac{5}{9}(F - 32)$

Trigonometry

(Put your calculator in Degree Mode)

- Right triangles

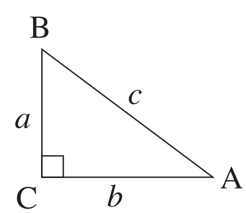
  Pythagorean Theorem

  $$a^2 + b^2 = c^2$$

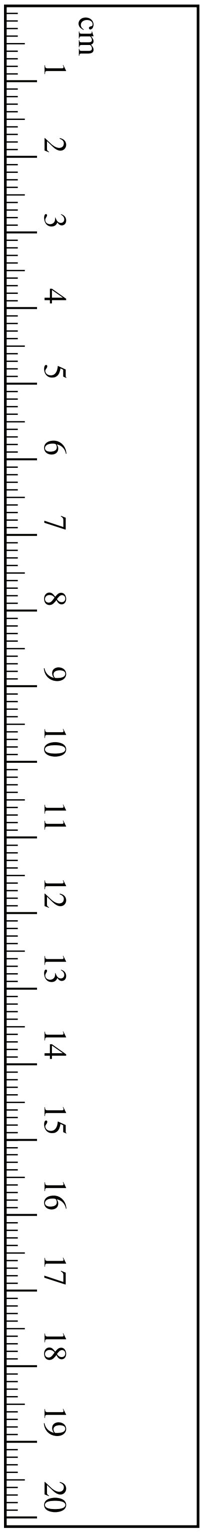
  $$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

  $$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

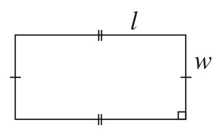
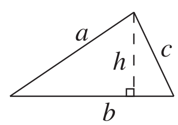
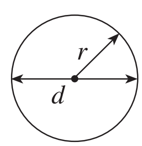
  $$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$



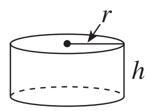
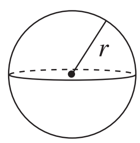
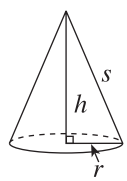
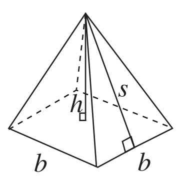
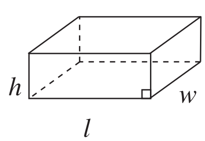
GEOMETRIC FORMULAE



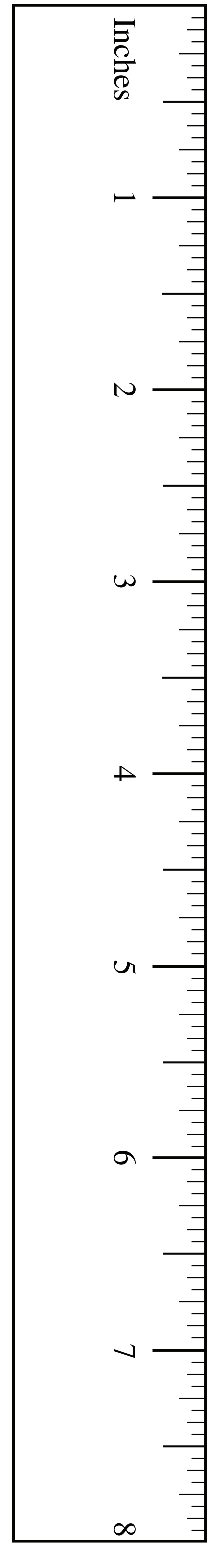
Key Legend	
l = length w = width b = base h = height s = slant height r = radius d = diameter	P = perimeter C = circumference A = area SA = surface area V = volume

Geometric Figure	Perimeter	Area
Rectangle 	$P = 2l + 2w$ **or** $P = 2(l + w)$	$A = lw$
Triangle 	$P = a + b + c$	$A = \frac{bh}{2}$
Circle 	$C = \pi d$ **or** $C = 2\pi r$	$A = \pi r^2$

Note: Use the value of π programmed in your calculator rather than the approximation of 3.14.

Geometric Figure	Surface Area
Cylinder 	$A_{top} = \pi r^2$ $A_{base} = \pi r^2$ $A_{side} = 2\pi rh$ $SA = 2\pi r^2 + 2\pi rh$
Sphere 	$SA = 4\pi r^2$ **or** $SA = \pi d^2$
Cone 	$A_{side} = \pi rs$ $A_{base} = \pi r^2$ $SA = \pi r^2 + \pi rs$
Square-Based Pyramid 	$A_{triangle} = \frac{1}{2}bs$ (for each triangle) $A_{base} = b^2$ $SA = 2bs + b^2$
Rectangular Prism 	$SA = wh + wh + lw + lw + lh + lh$ **or** $SA = 2(wh + lw + lh)$
General Right Prism	$SA =$ the sum of the areas of all the faces
General Pyramid	$SA =$ the sum of the areas of all the faces

Note: Use the value of π programmed in your calculator rather than the approximation of 3.14.



Canada Pension Plan Contributions | **Cotisations au Régime de pensions du Canada**

Weekly (52 pay periods a year) | Hebdomadaire (52 périodes de paie par année)

Pay / Rémunération From - De	Pay / Rémunération To - À	CPP RPC	Pay / Rémunération From - De	Pay / Rémunération To - À	CPP RPC	Pay / Rémunération From - De	Pay / Rémunération To - À	CPP RPC	Pay / Rémunération From - De	Pay / Rémunération To - À	CPP RPC
358.11	358.31	14.40	372.66	372.85	15.12	387.20	387.40	15.84	401.75	401.94	16.56
358.32	358.51	14.41	372.86	373.05	15.13	387.41	387.60	15.85	401.95	402.14	16.57
358.52	358.71	14.42	373.06	373.25	15.14	387.61	387.80	15.86	402.15	402.35	16.58
358.72	358.91	14.43	373.26	373.46	15.15	387.81	388.00	15.87	402.36	402.55	16.59
358.92	359.11	14.44	373.47	373.66	15.16	388.01	388.20	15.88	402.56	402.75	16.60
359.12	359.32	14.45	373.67	373.86	15.17	388.21	388.41	15.89	402.76	402.95	16.61
359.33	359.52	14.46	373.87	374.06	15.18	388.42	388.61	15.90	402.96	403.15	16.62
359.53	359.72	14.47	374.07	374.26	15.19	388.62	388.81	15.91	403.16	403.36	16.63
359.73	359.92	14.48	374.27	374.47	15.20	388.82	389.01	15.92	403.37	403.56	16.64
359.93	360.12	14.49	374.48	374.67	15.21	389.02	389.21	15.93	403.57	403.76	16.65
360.13	360.33	14.50	374.68	374.87	15.22	389.22	389.42	15.94	403.77	403.96	16.66
360.34	360.53	14.51	374.88	375.07	15.23	389.43	389.62	15.95	403.97	404.16	16.67
360.54	360.73	14.52	375.08	375.27	15.24	389.63	389.82	15.96	404.17	404.37	16.68
360.74	360.93	14.53	375.28	375.48	15.25	389.83	390.02	15.97	404.38	404.57	16.69
360.94	361.13	14.54	375.49	375.68	15.26	390.03	390.22	15.98	404.58	404.77	16.70
361.14	361.34	14.55	375.69	375.88	15.27	390.23	390.43	15.99	404.78	404.97	16.71
361.35	361.54	14.56	375.89	376.08	15.28	390.44	390.63	16.00	404.98	405.17	16.72
361.55	361.74	14.57	376.09	376.28	15.29	390.64	390.83	16.01	405.18	405.38	16.73
361.75	361.94	14.58	376.29	376.49	15.30	390.84	391.03	16.02	405.39	405.58	16.74
361.95	362.14	14.59	376.50	376.69	15.31	391.04	391.23	16.03	405.59	405.78	16.75
362.15	362.35	14.60	376.70	376.89	15.32	391.24	391.44	16.04	405.79	405.98	16.76
362.36	362.55	14.61	376.90	377.09	15.33	391.45	391.64	16.05	405.99	406.18	16.77
362.56	362.75	14.62	377.10	377.29	15.34	391.65	391.84	16.06	406.19	406.39	16.78
362.76	362.95	14.63	377.30	377.50	15.35	391.85	392.04	16.07	406.40	406.59	16.79
362.96	363.15	14.64	377.51	377.70	15.36	392.05	392.24	16.08	406.60	406.79	16.80
363.16	363.36	14.65	377.71	377.90	15.37	392.25	392.45	16.09	406.80	406.99	16.81
363.37	363.56	14.66	377.91	378.10	15.38	392.46	392.65	16.10	407.00	407.19	16.82
363.57	363.76	14.67	378.11	378.31	15.39	392.66	392.85	16.11	407.20	407.40	16.83
363.77	363.96	14.68	378.32	378.51	15.40	392.86	393.05	16.12	407.41	407.60	16.84
363.97	364.16	14.69	378.52	378.71	15.41	393.06	393.25	16.13	407.61	407.80	16.85
364.17	364.37	14.70	378.72	378.91	15.42	393.26	393.46	16.14	407.81	408.00	16.86
364.38	364.57	14.71	378.92	379.11	15.43	393.47	393.66	16.15	408.01	408.20	16.87
364.58	364.77	14.72	379.12	379.32	15.44	393.67	393.86	16.16	408.21	408.41	16.88
364.78	364.97	14.73	379.33	379.52	15.45	393.87	394.06	16.17	408.42	408.61	16.89
364.98	365.17	14.74	379.53	379.72	15.46	394.07	394.26	16.18	408.62	408.81	16.90
365.18	365.38	14.75	379.73	379.92	15.47	394.27	394.47	16.19	408.82	409.01	16.91
365.39	365.58	14.76	379.93	380.12	15.48	394.48	394.67	16.20	409.02	409.21	16.92
365.59	365.78	14.77	380.13	380.33	15.49	394.68	394.87	16.21	409.22	409.42	16.93
365.79	365.98	14.78	380.34	380.53	15.50	394.88	395.07	16.22	409.43	409.62	16.94
365.99	366.18	14.79	380.54	380.73	15.51	395.08	395.27	16.23	409.63	409.82	16.95
366.19	366.39	14.80	380.74	380.93	15.52	395.28	395.48	16.24	409.83	410.02	16.96
366.40	366.59	14.81	380.94	381.13	15.53	395.49	395.68	16.25	410.03	410.22	16.97
366.60	366.79	14.82	381.14	381.34	15.54	395.69	395.88	16.26	410.23	410.43	16.98
366.80	366.99	14.83	381.35	381.54	15.55	395.89	396.08	16.27	410.44	410.63	16.99
367.00	367.19	14.84	381.55	381.74	15.56	396.09	396.28	16.28	410.64	410.83	17.00
367.20	367.40	14.85	381.75	381.94	15.57	396.29	396.49	16.29	410.84	411.03	17.01
367.41	367.60	14.86	381.95	382.14	15.58	396.50	396.69	16.30	411.04	411.23	17.02
367.61	367.80	14.87	382.15	382.35	15.59	396.70	396.89	16.31	411.24	411.44	17.03
367.81	368.00	14.88	382.36	382.55	15.60	396.90	397.09	16.32	411.45	411.64	17.04
368.01	368.20	14.89	382.56	382.75	15.61	397.10	397.29	16.33	411.65	411.84	17.05
368.21	368.41	14.90	382.76	382.95	15.62	397.30	397.50	16.34	411.85	412.04	17.06
368.42	368.61	14.91	382.96	383.15	15.63	397.51	397.70	16.35	412.05	412.24	17.07
368.62	368.81	14.92	383.16	383.36	15.64	397.71	397.90	16.36	412.25	412.45	17.08
368.82	369.01	14.93	383.37	383.56	15.65	397.91	398.10	16.37	412.46	412.65	17.09
369.02	369.21	14.94	383.57	383.76	15.66	398.11	398.31	16.38	412.66	412.85	17.10
369.22	369.42	14.95	383.77	383.96	15.67	398.32	398.51	16.39	412.86	413.05	17.11
369.43	369.62	14.96	383.97	384.16	15.68	398.52	398.71	16.40	413.06	413.25	17.12
369.63	369.82	14.97	384.17	384.37	15.69	398.72	398.91	16.41	413.26	413.46	17.13
369.83	370.02	14.98	384.38	384.57	15.70	398.92	399.11	16.42	413.47	413.66	17.14
370.03	370.22	14.99	384.58	384.77	15.71	399.12	399.32	16.43	413.67	413.86	17.15
370.23	370.43	15.00	384.78	384.97	15.72	399.33	399.52	16.44	413.87	414.06	17.16
370.44	370.63	15.01	384.98	385.17	15.73	399.53	399.72	16.45	414.07	414.26	17.17
370.64	370.83	15.02	385.18	385.38	15.74	399.73	399.92	16.46	414.27	414.47	17.18
370.84	371.03	15.03	385.39	385.58	15.75	399.93	400.12	16.47	414.48	414.67	17.19
371.04	371.23	15.04	385.59	385.78	15.76	400.13	400.33	16.48	414.68	414.87	17.20
371.24	371.44	15.05	385.79	385.98	15.77	400.34	400.53	16.49	414.88	415.07	17.21
371.45	371.64	15.06	385.99	386.18	15.78	400.54	400.73	16.50	415.08	415.27	17.22
371.65	371.84	15.07	386.19	386.39	15.79	400.74	400.93	16.51	415.28	415.48	17.23
371.85	372.04	15.08	386.40	386.59	15.80	400.94	401.13	16.52	415.49	415.68	17.24
372.05	372.24	15.09	386.60	386.79	15.81	401.14	401.34	16.53	415.69	415.88	17.25
372.25	372.45	15.10	386.80	386.99	15.82	401.35	401.54	16.54	415.89	416.08	17.26
372.46	372.65	15.11	387.00	387.19	15.83	401.55	401.74	16.55	416.09	416.28	17.27

Employee's maximum CPP contribution for the year 2009 is $2,118.60

B-6

La cotisation maximale de l'employé au RPC pour l'année 2009 est de 2 118,60 $

Employment Insurance Premiums

Cotisations à l'assurance-emploi

Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE
333.24	333.81	5.77	374.86	375.43	6.49	416.48	417.05	7.21	458.10	458.67	7.93
333.82	334.39	5.78	375.44	376.01	6.50	417.06	417.63	7.22	458.68	459.24	7.94
334.40	334.97	5.79	376.02	376.58	6.51	417.64	418.20	7.23	459.25	459.82	7.95
334.98	335.54	5.80	376.59	377.16	6.52	418.21	418.78	7.24	459.83	460.40	7.96
335.55	336.12	5.81	377.17	377.74	6.53	418.79	419.36	7.25	460.41	460.98	7.97
336.13	336.70	5.82	377.75	378.32	6.54	419.37	419.94	7.26	460.99	461.56	7.98
336.71	337.28	5.83	378.33	378.90	6.55	419.95	420.52	7.27	461.57	462.13	7.99
337.29	337.86	5.84	378.91	379.47	6.56	420.53	421.09	7.28	462.14	462.71	8.00
337.87	338.43	5.85	379.48	380.05	6.57	421.10	421.67	7.29	462.72	463.29	8.01
338.44	339.01	5.86	380.06	380.63	6.58	421.68	422.25	7.30	463.30	463.87	8.02
339.02	339.59	5.87	380.64	381.21	6.59	422.26	422.83	7.31	463.88	464.45	8.03
339.60	340.17	5.88	381.22	381.79	6.60	422.84	423.41	7.32	464.46	465.02	8.04
340.18	340.75	5.89	381.80	382.36	6.61	423.42	423.98	7.33	465.03	465.60	8.05
340.76	341.32	5.90	382.37	382.94	6.62	423.99	424.56	7.34	465.61	466.18	8.06
341.33	341.90	5.91	382.95	383.52	6.63	424.57	425.14	7.35	466.19	466.76	8.07
341.91	342.48	5.92	383.53	384.10	6.64	425.15	425.72	7.36	466.77	467.34	8.08
342.49	343.06	5.93	384.11	384.68	6.65	425.73	426.30	7.37	467.35	467.91	8.09
343.07	343.64	5.94	384.69	385.26	6.66	426.31	426.87	7.38	467.92	468.49	8.10
343.65	344.21	5.95	385.27	385.83	6.67	426.88	427.45	7.39	468.50	469.07	8.11
344.22	344.79	5.96	385.84	386.41	6.68	427.46	428.03	7.40	469.08	469.65	8.12
344.80	345.37	5.97	386.42	386.99	6.69	428.04	428.61	7.41	469.66	470.23	8.13
345.38	345.95	5.98	387.00	387.57	6.70	428.62	429.19	7.42	470.24	470.80	8.14
345.96	346.53	5.99	387.58	388.15	6.71	429.20	429.76	7.43	470.81	471.38	8.15
346.54	347.10	6.00	388.16	388.72	6.72	429.77	430.34	7.44	471.39	471.96	8.16
347.11	347.68	6.01	388.73	389.30	6.73	430.35	430.92	7.45	471.97	472.54	8.17
347.69	348.26	6.02	389.31	389.88	6.74	430.93	431.50	7.46	472.55	473.12	8.18
348.27	348.84	6.03	389.89	390.46	6.75	431.51	432.08	7.47	473.13	473.69	8.19
348.85	349.42	6.04	390.47	391.04	6.76	432.09	432.65	7.48	473.70	474.27	8.20
349.43	349.99	6.05	391.05	391.61	6.77	432.66	433.23	7.49	474.28	474.85	8.21
350.00	350.57	6.06	391.62	392.19	6.78	433.24	433.81	7.50	474.86	475.43	8.22
350.58	351.15	6.07	392.20	392.77	6.79	433.82	434.39	7.51	475.44	476.01	8.23
351.16	351.73	6.08	392.78	393.35	6.80	434.40	434.97	7.52	476.02	476.58	8.24
351.74	352.31	6.09	393.36	393.93	6.81	434.98	435.54	7.53	476.59	477.16	8.25
352.32	352.89	6.10	393.94	394.50	6.82	435.55	436.12	7.54	477.17	477.74	8.26
352.90	353.46	6.11	394.51	395.08	6.83	436.13	436.70	7.55	477.75	478.32	8.27
353.47	354.04	6.12	395.09	395.66	6.84	436.71	437.28	7.56	478.33	478.90	8.28
354.05	354.62	6.13	395.67	396.24	6.85	437.29	437.86	7.57	478.91	479.47	8.29
354.63	355.20	6.14	396.25	396.82	6.86	437.87	438.43	7.58	479.48	480.05	8.30
355.21	355.78	6.15	396.83	397.39	6.87	438.44	439.01	7.59	480.06	480.63	8.31
355.79	356.35	6.16	397.40	397.97	6.88	439.02	439.59	7.60	480.64	481.21	8.32
356.36	356.93	6.17	397.98	398.55	6.89	439.60	440.17	7.61	481.22	481.79	8.33
356.94	357.51	6.18	398.56	399.13	6.90	440.18	440.75	7.62	481.80	482.36	8.34
357.52	358.09	6.19	399.14	399.71	6.91	440.76	441.32	7.63	482.37	482.94	8.35
358.10	358.67	6.20	399.72	400.28	6.92	441.33	441.90	7.64	482.95	483.52	8.36
358.68	359.24	6.21	400.29	400.86	6.93	441.91	442.48	7.65	483.53	484.10	8.37
359.25	359.82	6.22	400.87	401.44	6.94	442.49	443.06	7.66	484.11	484.68	8.38
359.83	360.40	6.23	401.45	402.02	6.95	443.07	443.64	7.67	484.69	485.26	8.39
360.41	360.98	6.24	402.03	402.60	6.96	443.65	444.21	7.68	485.27	485.83	8.40
360.99	361.56	6.25	402.61	403.17	6.97	444.22	444.79	7.69	485.84	486.41	8.41
361.57	362.13	6.26	403.18	403.75	6.98	444.80	445.37	7.70	486.42	486.99	8.42
362.14	362.71	6.27	403.76	404.33	6.99	445.38	445.95	7.71	487.00	487.57	8.43
362.72	363.29	6.28	404.34	404.91	7.00	445.96	446.53	7.72	487.58	488.15	8.44
363.30	363.87	6.29	404.92	405.49	7.01	446.54	447.10	7.73	488.16	488.72	8.45
363.88	364.45	6.30	405.50	406.06	7.02	447.11	447.68	7.74	488.73	489.30	8.46
364.46	365.02	6.31	406.07	406.64	7.03	447.69	448.26	7.75	489.31	489.88	8.47
365.03	365.60	6.32	406.65	407.22	7.04	448.27	448.84	7.76	489.89	490.46	8.48
365.61	366.18	6.33	407.23	407.80	7.05	448.85	449.42	7.77	490.47	491.04	8.49
366.19	366.76	6.34	407.81	408.38	7.06	449.43	449.99	7.78	491.05	491.61	8.50
366.77	367.34	6.35	408.39	408.95	7.07	450.00	450.57	7.79	491.62	492.19	8.51
367.35	367.91	6.36	408.96	409.53	7.08	450.58	451.15	7.80	492.20	492.77	8.52
367.92	368.49	6.37	409.54	410.11	7.09	451.16	451.73	7.81	492.78	493.35	8.53
368.50	369.07	6.38	410.12	410.69	7.10	451.74	452.31	7.82	493.36	493.93	8.54
369.08	369.65	6.39	410.70	411.27	7.11	452.32	452.89	7.83	493.94	494.50	8.55
369.66	370.23	6.40	411.28	411.84	7.12	452.90	453.46	7.84	494.51	495.08	8.56
370.24	370.80	6.41	411.85	412.42	7.13	453.47	454.04	7.85	495.09	495.66	8.57
370.81	371.38	6.42	412.43	413.00	7.14	454.05	454.62	7.86	495.67	496.24	8.58
371.39	371.96	6.43	413.01	413.58	7.15	454.63	455.20	7.87	496.25	496.82	8.59
371.97	372.54	6.44	413.59	414.16	7.16	455.21	455.78	7.88	496.83	497.39	8.60
372.55	373.12	6.45	414.17	414.73	7.17	455.79	456.35	7.89	497.40	497.97	8.61
373.13	373.69	6.46	414.74	415.31	7.18	456.36	456.93	7.90	497.98	498.55	8.62
373.70	374.27	6.47	415.32	415.89	7.19	456.94	457.51	7.91	498.56	499.13	8.63
374.28	374.85	6.48	415.90	416.47	7.20	457.52	458.09	7.92	499.14	499.71	8.64

Yearly maximum insurable earnings are $42,300
Yearly maximum employee premiums are $731.79
The premium rate for 2009 is 1.73 %

Le maximum annuel de la rémunération assurable est de 42 300 $
La cotisation maximale annuelle de l'employé est de 731,79 $
Le taux de cotisation pour 2009 est de 1,73 %

C-3

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the provincial table

Retenues d'impôt fédéral
En vigueur le 1er janvier 2009
Hebdomadaire (52 périodes de paie par année)
Cherchez aussi les retenues d'impôt dans la table provinciale

Pay Rémunération		Federal claim codes/Codes de demande fédéraux										
		0	1	2	3	4	5	6	7	8	9	10
From De	Less than Moins de	Deduct from each pay Retenez sur chaque paie										
335	339	44.65	15.55	12.70	7.00	1.30						
339	343	45.20	16.10	13.25	7.55	1.85						
343	347	45.80	16.65	13.80	8.10	2.45						
347	351	46.35	17.20	14.35	8.65	3.00						
351	355	46.90	17.75	14.90	9.25	3.55						
355	359	47.45	18.35	15.50	9.80	4.10						
359	363	48.00	18.90	16.05	10.35	4.65						
363	367	48.60	19.45	16.60	10.90	5.25						
367	371	49.15	20.00	17.15	11.45	5.80	.10					
371	375	49.70	20.55	17.70	12.05	6.35	.65					
375	379	50.25	21.15	18.30	12.60	6.90	1.20					
379	383	50.80	21.70	18.85	13.15	7.45	1.80					
383	387	51.40	22.25	19.40	13.70	8.00	2.35					
387	391	51.95	22.80	19.95	14.25	8.60	2.90					
391	395	52.50	23.35	20.50	14.85	9.15	3.45					
395	399	53.05	23.95	21.10	15.40	9.70	4.00					
399	403	53.60	24.50	21.65	15.95	10.25	4.60					
403	407	54.20	25.05	22.20	16.50	10.80	5.15					
407	411	54.75	25.60	22.75	17.05	11.40	5.70					
411	415	55.30	26.15	23.30	17.65	11.95	6.25	.55				
415	419	55.85	26.75	23.90	18.20	12.50	6.80	1.15				
419	423	56.40	27.30	24.45	18.75	13.05	7.40	1.70				
423	427	57.00	27.85	25.00	19.30	13.60	7.95	2.25				
427	431	57.55	28.40	25.55	19.85	14.20	8.50	2.80				
431	435	58.10	28.95	26.10	20.45	14.75	9.05	3.35				
435	439	58.65	29.50	26.70	21.00	15.30	9.60	3.95				
439	443	59.20	30.10	27.25	21.55	15.85	10.20	4.50				
443	447	59.80	30.65	27.80	22.10	16.40	10.75	5.05				
447	451	60.35	31.20	28.35	22.65	17.00	11.30	5.60				
451	455	60.90	31.75	28.90	23.25	17.55	11.85	6.15	.50			
455	459	61.45	32.30	29.50	23.80	18.10	12.40	6.75	1.05			
459	463	62.00	32.90	30.05	24.35	18.65	12.95	7.30	1.60			
463	467	62.60	33.45	30.60	24.90	19.20	13.55	7.85	2.15			
467	471	63.15	34.00	31.15	25.45	19.80	14.10	8.40	2.70			
471	475	63.70	34.55	31.70	26.05	20.35	14.65	8.95	3.30			
475	479	64.25	35.10	32.30	26.60	20.90	15.20	9.55	3.85			
479	483	64.80	35.70	32.85	27.15	21.45	15.75	10.10	4.40			
483	487	65.40	36.25	33.40	27.70	22.00	16.35	10.65	4.95			
487	491	65.95	36.80	33.95	28.25	22.60	16.90	11.20	5.50			
491	495	66.50	37.35	34.50	28.85	23.15	17.45	11.75	6.10	.40		
495	499	67.05	37.90	35.10	29.40	23.70	18.00	12.35	6.65	.95		
499	503	67.60	38.50	35.65	29.95	24.25	18.55	12.90	7.20	1.50		
503	507	68.20	39.05	36.20	30.50	24.80	19.15	13.45	7.75	2.05		
507	511	68.75	39.60	36.75	31.05	25.40	19.70	14.00	8.30	2.65		
511	515	69.30	40.15	37.30	31.65	25.95	20.25	14.55	8.90	3.20		
515	519	69.85	40.70	37.90	32.20	26.50	20.80	15.15	9.45	3.75		
519	523	70.40	41.30	38.45	32.75	27.05	21.35	15.70	10.00	4.30		
523	527	71.00	41.85	39.00	33.30	27.60	21.95	16.25	10.55	4.85		
527	531	71.55	42.40	39.55	33.85	28.20	22.50	16.80	11.10	5.45		
531	535	72.10	42.95	40.10	34.45	28.75	23.05	17.35	11.70	6.00	.30	
535	539	72.65	43.50	40.70	35.00	29.30	23.60	17.90	12.25	6.55	.85	
539	543	73.20	44.10	41.25	35.55	29.85	24.15	18.50	12.80	7.10	1.40	
543	547	73.80	44.65	41.80	36.10	30.40	24.75	19.05	13.35	7.65	2.00	
547	551	74.35	45.20	42.35	36.65	31.00	25.30	19.60	13.90	8.25	2.55	
551	555	74.90	45.75	42.90	37.25	31.55	25.85	20.15	14.50	8.80	3.10	

British Columbia provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the federal table

Retenues d'impôt provincial de la Colombie-Britannique
En vigueur le 1er janvier 2009
Hebdomadaire (52 périodes de paie par année)
Cherchez aussi les retenues d'impôt dans la table fédérale

Pay Rémunération		Provincial claim codes/Codes de demande provinciaux										
		0	1	2	3	4	5	6	7	8	9	10
From De	Less than Moins de	Deduct from each pay Retenez sur chaque paie										
	343	*	.00									
343 -	345	9.30	.20									
345 -	347	9.45	.35									
347 -	349	9.60	.50									
349 -	351	9.80	.65									
351 -	353	9.95	.80									
353 -	355	10.10	.95									
355 -	357	10.25	1.15	.10								
357 -	359	10.40	1.30	.25								
359 -	361	10.55	1.45	.40								
361 -	363	10.75	1.60	.60								
363 -	365	10.90	1.75	.75								
365 -	367	11.05	1.90	.90								
367 -	369	11.20	2.10	1.05								
369 -	371	11.35	2.25	1.20								
371 -	373	11.50	2.40	1.35								
373 -	375	11.70	2.55	1.55								
375 -	377	11.85	2.70	1.70								
377 -	379	12.00	2.90	1.85								
379 -	381	12.15	3.05	2.00								
381 -	383	12.30	3.20	2.15	.10							
383 -	385	12.45	3.35	2.30	.25							
385 -	387	12.65	3.50	2.50	.45							
387 -	389	12.80	3.65	2.65	.60							
389 -	391	12.95	3.85	2.80	.75							
391 -	393	13.10	4.00	2.95	.90							
393 -	395	13.25	4.15	3.10	1.05							
395 -	397	13.40	4.30	3.30	1.20							
397 -	399	13.60	4.45	3.45	1.40							
399 -	401	13.75	4.60	3.60	1.55							
401 -	403	13.90	4.80	3.75	1.70							
403 -	405	14.05	4.95	3.90	1.85							
405 -	407	14.20	5.10	4.05	2.00							
407 -	409	14.35	5.25	4.25	2.15	.10						
409 -	411	14.55	5.40	4.40	2.35	.30						
411 -	413	14.70	5.55	4.55	2.50	.45						
413 -	415	14.85	5.75	4.70	2.65	.60						
415 -	417	15.00	5.90	4.85	2.80	.75						
417 -	419	15.15	6.05	5.00	2.95	.90						
419 -	421	15.30	6.20	5.20	3.10	1.05						
421 -	423	15.50	6.35	5.35	3.30	1.25						
423 -	425	15.65	6.50	5.50	3.45	1.40						
425 -	427	15.80	6.70	5.65	3.60	1.55						
427 -	429	15.95	6.85	5.80	3.75	1.70						
429 -	431	16.10	7.00	5.95	3.90	1.85						
431 -	433	16.25	7.15	6.15	4.10	2.00						
433 -	435	16.45	7.30	6.30	4.25	2.20	.15					
435 -	437	16.60	7.45	6.45	4.40	2.35	.30					
437 -	439	16.75	7.65	6.60	4.55	2.50	.45					
439 -	441	16.90	7.80	6.75	4.70	2.65	.60					
441 -	443	17.05	7.95	6.90	4.85	2.80	.75					
443 -	445	17.20	8.10	7.10	5.05	2.95	.90					
445 -	447	17.40	8.25	7.25	5.20	3.15	1.10					
447 -	449	17.55	8.40	7.40	5.35	3.30	1.25					
449 -	451	17.70	8.60	7.55	5.50	3.45	1.40					

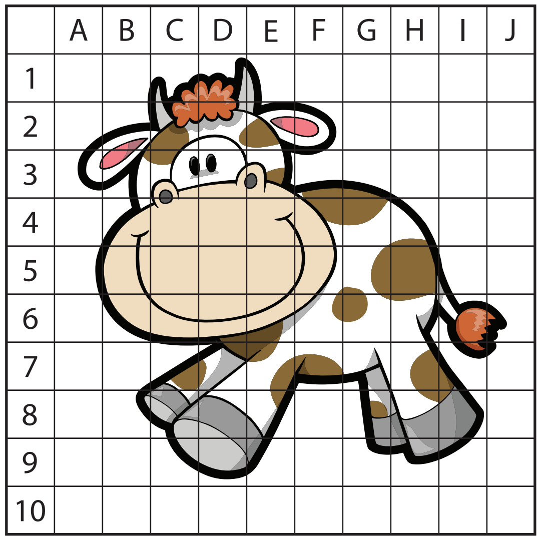
*You normally use claim code "0" only for non-resident employees. However, if you have non-resident employees who earn less than the minimum amount shown in the "Pay" column, you may not be able to use these tables. Instead, refer to the "Step-by-step calculation of tax deductions" in Section "A" of this publication.

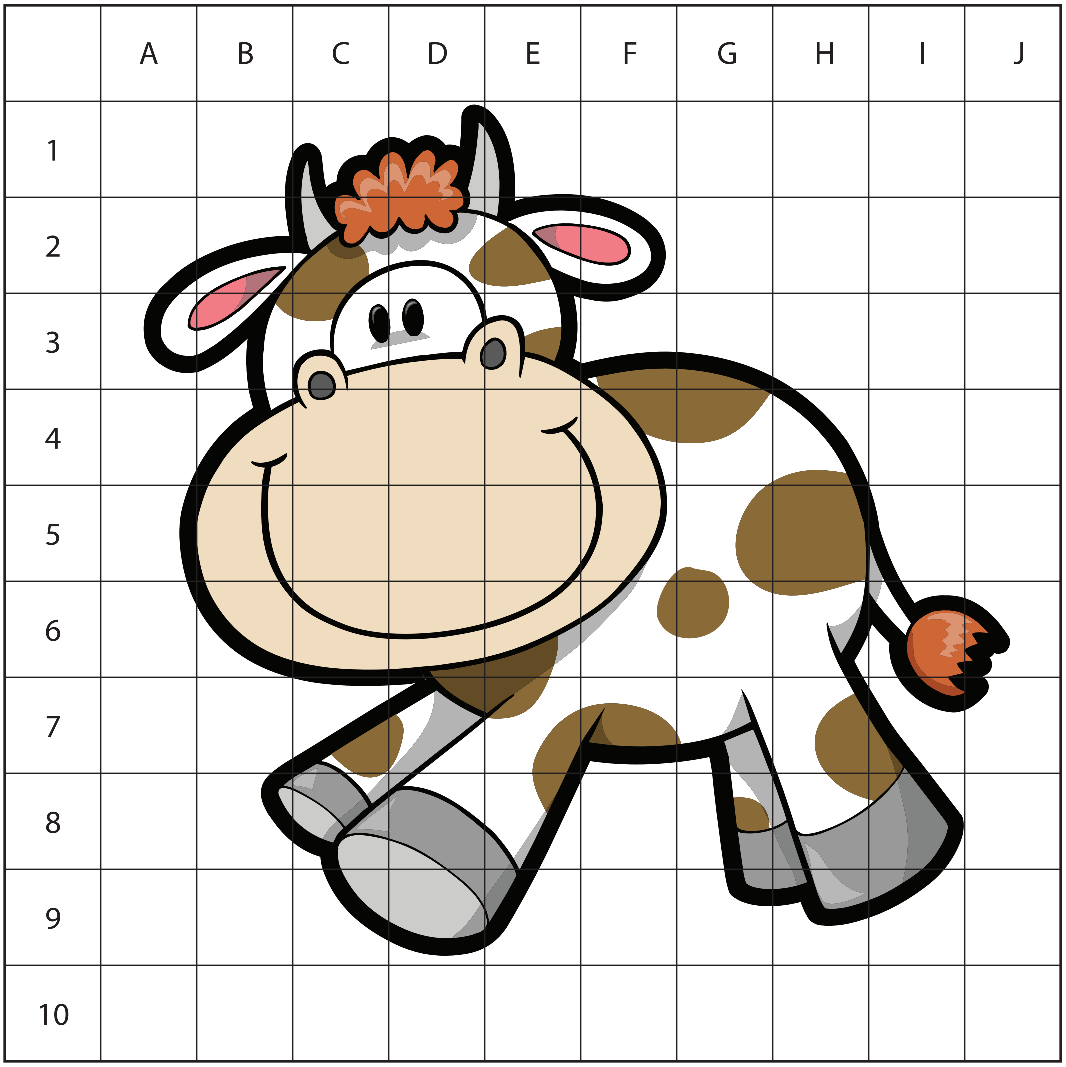
*Le code de demande «0» est normalement utilisé seulement pour les non-résidents. Cependant, si la rémunération de votre employé non résidant est inférieure au montant minimum indiqué dans la colonne «Rémunération», vous ne pourrez peut-être pas utiliser ces tables. Reportez-vous alors au «Calcul des retenues d'impôt, étape par étape» dans la section «A» de cette publication.

Solutions

Lesson A: Similar Polygons

Lesson A: Activity 1: Try This





1. The scale factor is two. This makes sense because the squares on the quarter-inch grid are twice the size of the squares on the eighth-inch grid.

$$
\begin{aligned}
2 \times \frac{1}{8} &= \frac{2}{1} \times \frac{1}{8} \\
&= \frac{2}{8} \\
&= \frac{1}{4}
\end{aligned}
$$

2. The area increases as the square of the scale factor. (Recall what you learned in Module 2, Section 2, Lesson F about scale factors and areas.)

   The area increases (scale factor)^2 = 2^2 = 4 times.

3. The scale factor is four. This makes sense because the squares on the half-inch grid are four times the size of the squares on the eighth-inch grid.

$$\begin{aligned} 4 \times \frac{1}{8} &= \frac{4}{1} \times \frac{1}{8} \\ &= \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$

4. The area increases as the square of the scale factor.

   The area increases (scale factor)^2 = 4^2 = 16 times.

5. If the original were drawn on the quarter-inch grid, draw its image on the eighth-inch grid.

   If the original were drawn on the half-inch grid, draw its image on the quarter-inch grid.

Lesson A: Activity 2: Try This

Answers will vary. Sample answers are given. Where measurements are listed, they are the measurements of the example polygon.

1. The corresponding angles are congruent. Approximate values are:
   $\angle A \cong \angle A' \cong \angle A'' = 113°$
   $\angle B \cong \angle B' \cong \angle B'' = 98°$
   $\angle C \cong \angle C' \cong \angle C'' = 138°$
   $\angle D \cong \angle D' \cong \angle D'' = 99°$
   $\angle E \cong \angle E' \cong \angle E'' = 91°$

2.

$\frac{\text{length of } \overline{A'B'}}{\text{length of } \overline{AB}}$	$\frac{\text{length of } \overline{A'B'}}{\text{length of } \overline{AB}} = \frac{16 \text{ mm}}{32 \text{ mm}} \approx 0.5$
$\frac{\text{length of } \overline{B'C'}}{\text{length of } \overline{BC}}$	$\frac{\text{length of } \overline{B'C'}}{\text{length of } \overline{BC}} = \frac{14 \text{ mm}}{27 \text{ mm}} \approx 0.5$
$\frac{\text{length of } \overline{C'D'}}{\text{length of } \overline{CD}}$	$\frac{\text{length of } \overline{C'D'}}{\text{length of } \overline{CD}} = \frac{13 \text{ mm}}{26 \text{ mm}} \approx 0.5$
$\frac{\text{length of } \overline{D'E'}}{\text{length of } \overline{DE}}$	$\frac{\text{length of } \overline{D'E'}}{\text{length of } \overline{DE}} = \frac{20 \text{ mm}}{39 \text{ mm}} \approx 0.5$
$\frac{\text{length of } \overline{E'A'}}{\text{length of } \overline{EA}}$	$\frac{\text{length of } \overline{E'A'}}{\text{length of } \overline{EA}} = \frac{17 \text{ mm}}{33 \text{ mm}} \approx 0.5$

3.

$\frac{\text{length of } \overline{A''B''}}{\text{length of } \overline{AB}}$	$\frac{\text{length of } \overline{A''B''}}{\text{length of } \overline{AB}} = \frac{64 \text{ mm}}{32 \text{ mm}} \approx 2.0$
$\frac{\text{length of } \overline{B''C''}}{\text{length of } \overline{BC}}$	$\frac{\text{length of } \overline{B''C''}}{\text{length of } \overline{BC}} = \frac{54 \text{ mm}}{27 \text{ mm}} \approx 2.0$
$\frac{\text{length of } \overline{C''D''}}{\text{length of } \overline{CD}}$	$\frac{\text{length of } \overline{C''D''}}{\text{length of } \overline{CD}} = \frac{51 \text{ mm}}{26 \text{ mm}} \approx 2.0$
$\frac{\text{length of } \overline{D''E''}}{\text{length of } \overline{DE}}$	$\frac{\text{length of } \overline{D''E''}}{\text{length of } \overline{DE}} = \frac{77 \text{ mm}}{39 \text{ mm}} \approx 2.0$
$\frac{\text{length of } \overline{E''A''}}{\text{length of } \overline{EA}}$	$\frac{\text{length of } \overline{E''A''}}{\text{length of } \overline{EA}} = \frac{65 \text{ mm}}{33 \text{ mm}} \approx 2.0$

4. The ratios of the corresponding side lengths were equal—each was 0.5. This makes sense because the squares on the eighth-inch grid are half the size of the squares on the quarter-inch grid.

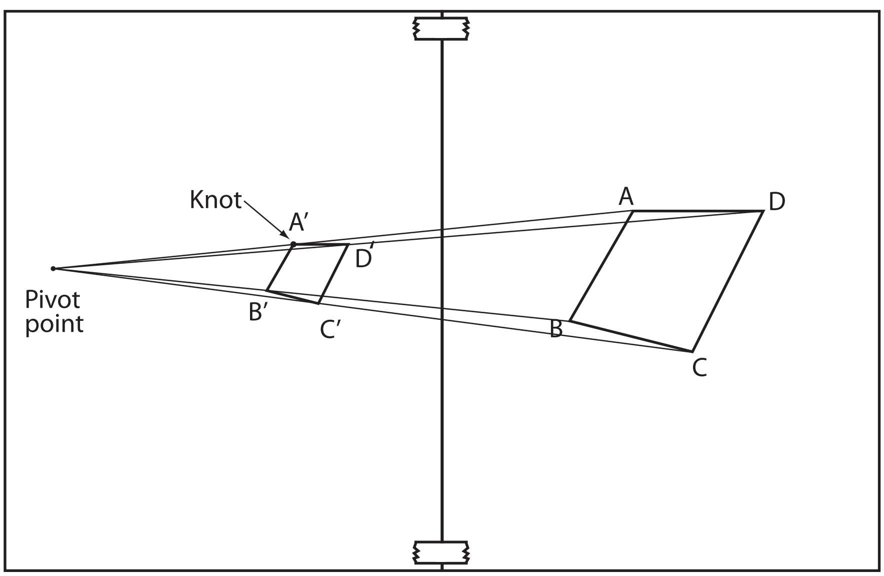
5. The ratios of the corresponding side lengths were equal—each was 2. This makes sense because the squares on the half-inch grid are twice the size of the squares on the quarter-inch grid.

Lesson A: Activity 3: Try This

Check to see that corresponding angles are equal in measure. Also, check that the corresponding side lengths are proportional. If they are, then A'B'C'D' is similar to ABCD.

Lesson A: Activity 4: Mastering Concepts

With the end of one elastic fixed at the pivot point, stretch the elastics so that the end of the second elastic is at point A. The knot will lie between the pivot and A. Mark the location of the knot as A'. Repeat this process to locate B', C', and D'. Join the points to form A'B'C'D'. This figure will be similar to, but smaller than, ABCD.



Lesson B: Ratios and Similar Polygons

Lesson B: Activity 1: Self-Check

1.
$$\frac{x}{12}=\frac{16}{18}$$
$$\frac{x}{12}=\frac{8}{9}$$
$$12\times\frac{x}{12}=12\times\frac{8}{9}$$
$$x=\frac{96}{9}$$
$$x=\frac{32}{3}\text{ or }10\frac{2}{3}$$

2.
$$\frac{x}{13}=\frac{26}{39}$$
$$\frac{x}{13}=\frac{2}{3}$$
$$13\times\frac{x}{13}=13\times\frac{2}{3}$$
$$x=\frac{26}{3}\text{ or }8\frac{2}{3}$$

3.
$$\frac{x}{7}=\frac{18}{15}$$
$$\frac{x}{7}=\frac{6}{5}$$
$$7\times\frac{x}{7}=7\times\frac{6}{5}$$
$$x=\frac{42}{5}\text{ or }8\frac{2}{5}$$

4.
$$\frac{7}{x}=\frac{14}{31}$$
$$\frac{x}{7}=\frac{31}{14}$$

Flip each ratio to put x on top.

$$7\times\frac{x}{7}=7\times\frac{31}{14}$$
$$x=\frac{31}{2}\text{ or }15\frac{1}{2}$$

Lesson B: Activity 2: Try This

1. The corresponding angles are congruent and corresponding side lengths are proportional, that is, the ratios are equal.

2. To prove that the two triangles you just drew are similar triangles, you would have to show the following:
   - corresponding angles are congruent
   - corresponding sides are proportional in length

Lesson B: Activity 3: Self-Check

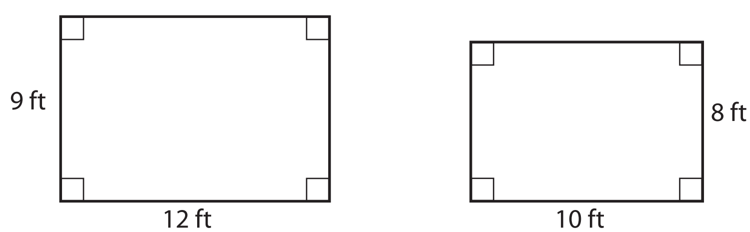
1. The ratio of each side of ΔA'B'C' to each side of ΔABC $= \frac{5 \text{ cm}}{10 \text{ cm}}$

   $= \frac{1}{2}$

   The scale factor is $\frac{1}{2}$.

   To form ΔA'B'C' from ΔABC, simply multiply each side of ΔABC by $\frac{1}{2}$.

   10 cm × $\frac{1}{2}$ = 5 cm.

2. 

   The corresponding angles are equal in measure. They are all right angles. The rectangles might be similar. Compare the ratios of the corresponding sides.

   Compare the widths.

   $\frac{9 \text{ ft}}{8 \text{ ft}} = \frac{9}{8}$

   Compare the lengths.

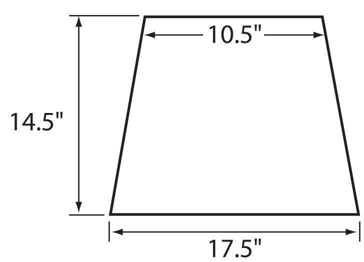
   $\frac{12 \text{ ft}}{10 \text{ ft}} = \frac{6}{5}$

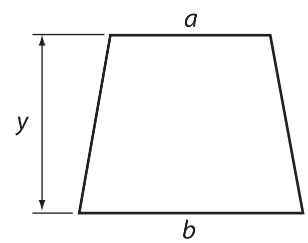
   The ratios of the corresponding sides are not equal. So, the two rectangles are not similar polygons. So, unlike squares, not all rectangles are similar.

Lesson B: Activity 4: Self-Check

1. The dimensions of the reduced piece will be 10% smaller. The dimensions of the reduced piece will be 100% – 10%, or 90% of those of the original pattern.

   The scale factor is 90% $= \frac{90}{100}$ or $\frac{9}{10}$.





   Let the width across the top be a.

$$\frac{a}{10.5} = \frac{9}{10}$$
$$10.5 \times \frac{a}{10.5} = 10.5 \times \frac{9}{10}$$
$$a = 9.45$$

   The width across the top should be approximately $9\frac{1}{2}$ inches.

   Let the width across the bottom be b.

$$\frac{b}{17.5} = \frac{9}{10}$$
$$17.5 \times \frac{b}{17.5} = 17.5 \times \frac{9}{10}$$
$$b = 15.75$$

   The width across the bottom should be approximately $15\frac{3}{4}$ inches.

   Let the height be y.

$$\frac{y}{14.5} = \frac{9}{10}$$
$$14.5 \times \frac{y}{14.5} = 14.5 \times \frac{9}{10}$$
$$y = 13.05$$

   The height should be approximately 13 inches.

2. Let the length be x.

$$\begin{aligned}\frac{x}{12} &= \frac{5}{6}\\ 12\times\frac{x}{12} &= 12\times\frac{5}{6}\\ x &= 10\end{aligned}$$

The length is 10 cm.

3. The scale factor is $\frac{1}{32}$.

Let the model's wingspan be x. The wingspan of the *Silver Dart* = 15 m.

$$\begin{aligned}\frac{x}{15} &= \frac{1}{32}\\ 15\times\frac{x}{15} &= 15\times\frac{1}{32}\\ x &= 0.46875\end{aligned}$$

The model's wingspan will be 0.47 m or 47 cm.

4. Use proportions.

$$\begin{aligned}\frac{x}{5} &= \frac{4}{3}\\ 5\times\frac{x}{5} &= 5\times\frac{4}{3}\\ x &= 6.666\ldots\\ x &\approx 6.7\text{ cm}\end{aligned}$$

$$\begin{aligned}\frac{y}{2.5} &= \frac{4}{3}\\ 2.5\times\frac{y}{2.5} &= 2.5\times\frac{4}{3}\\ y &= 3.333\ldots\\ y &\approx 3.3\text{ cm}\end{aligned}$$

$$\begin{aligned}\frac{z}{6} &= \frac{3}{4}\\ 6\times\frac{z}{6} &= 6\times\frac{3}{4}\\ z &\approx 4.5\text{ cm}\end{aligned}$$

The measurements are as follows: 6.7 cm, 3.3 cm, and 4.5 cm.

Lesson B: Activity 5: Mastering Concepts

$$\frac{\text{Width of small rectangle}}{\text{Width of large rectangle}} = \frac{\text{Length of small rectangle}}{\text{Length of large rectangle}}$$

$$\frac{x}{1} = \frac{1}{1.618}$$
$$x = 0.618046$$
$$x \approx 0.618$$

This makes sense from the diagram, because 1 m + 0.618 m = 1.618 m.

Lesson C: Similar Triangles

Lesson C: Activity 1: Try This

1. The ratios of the corresponding sides are equal; that is, the sides are proportional. For the two triangles in the diagram,

   $$\frac{\text{AB}}{\text{DE}} = \frac{6}{3} = 2,\ \frac{\text{BC}}{\text{EF}} = \frac{12}{6} = 2, \text{ and } \frac{\text{AC}}{\text{DF}} = \frac{9}{4.5} = 2.$$

   These equal ratios tell you that the sides of the larger triangle are twice as long as the corresponding sides of the smaller triangle.

2. The ratios flipped over are:

   $$\frac{\text{DE}}{\text{AB}} = \frac{3}{6} = 0.5,\ \frac{\text{EF}}{\text{BC}} = \frac{6}{12} = 0.5, \text{ and } \frac{\text{DF}}{\text{AC}} = \frac{4.5}{9} = 0.5.$$

   Notice that these ratios are also equal. These equal ratios tell you that the sides of the smaller triangle are one-half as long as the corresponding sides of the larger triangle.

Lesson C: Activity 2: Self-Check

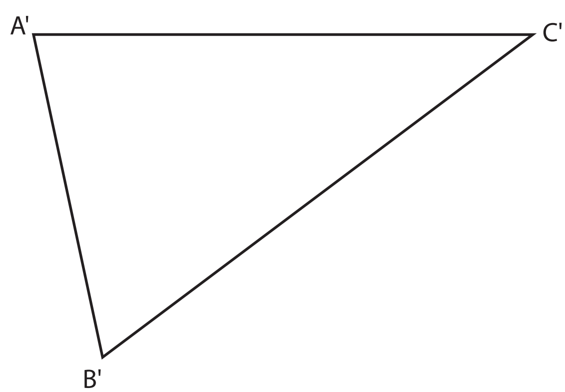
1. $$\frac{7}{x} = \frac{21}{11}$$
   $$\frac{x}{7} = \frac{11}{21}$$
   $$(7)\frac{x}{7} = \frac{11}{21}(7)$$
   $$x = \frac{11}{3}$$
   $$x = 3\frac{2}{3}$$

2. $$\begin{aligned} \frac{4}{x} &= \frac{8}{3} \\ \frac{x}{4} &= \frac{3}{8} \\ (4)\frac{x}{4} &= \frac{3}{8}(4) \\ x &= \frac{3}{2} \\ x &= 1\frac{1}{2} \end{aligned}$$

3. $$\begin{aligned} \frac{16}{x} &= \frac{8}{7} \\ \frac{x}{16} &= \frac{7}{8} \\ (16)\frac{x}{16} &= \frac{7}{8}(16) \\ x &= 14 \end{aligned}$$

Lesson C: Activity 3: Try This

1. a. Answers will vary. One example of a similar triangle is given below.



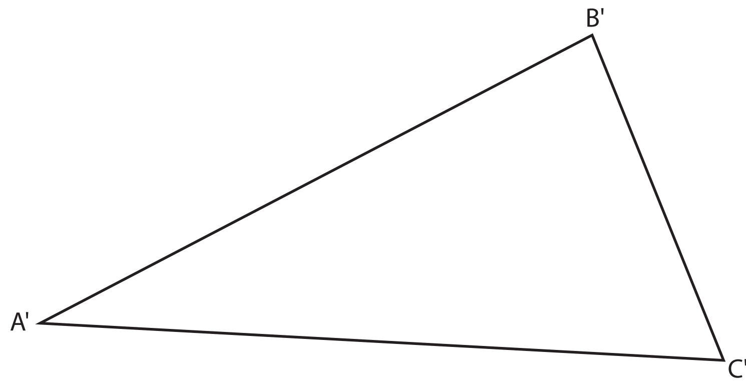
b. Since the angle sum of a triangle is180°, if two angles are known, the third angle can be found by subtracting the sum of the first two from 180º.

For example:

$\angle A + \angle B + \angle C = 180º$

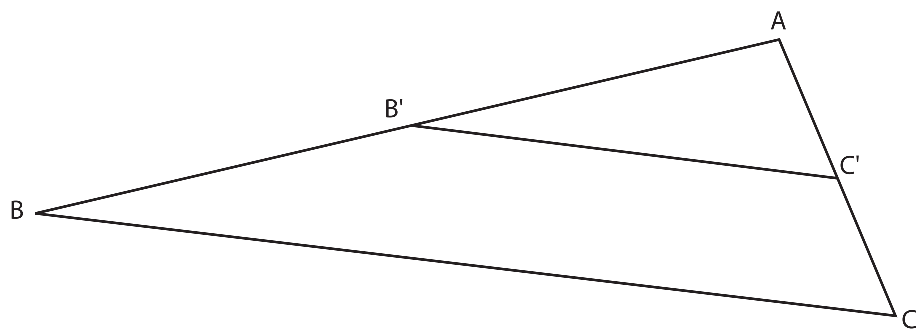
$\angle A = 180º – (\angle B + \angle C)$

2. a. Answers will vary. One example of a similar triangle is given below.



   b. The corresponding angles should be within a degree or two of being equal. The triangles are similar, but once again, differences may arise through construction or measurement errors.

3. a. Answers will vary. One example of a similar triangle is given below.



   b. $\overline{B'C'}$ and $\overline{BC}$ are parallel because $\angle AB'C' = \angle B$ and $\angle AC'B' = \angle C$. These angle pairs are corresponding angles formed when $\overline{B'C'}$ and $\overline{BC}$ are cut by the transversals $\overline{AB}$ and $\overline{AC}$ respectively.

4. Answers will vary. You should present a logical case for the method chosen.

   For example: "I prefer the three-angle method because measuring angles using a protractor is simpler than using a compass or calculating how long sides must be."

Lesson C: Activity 4: Self-Check

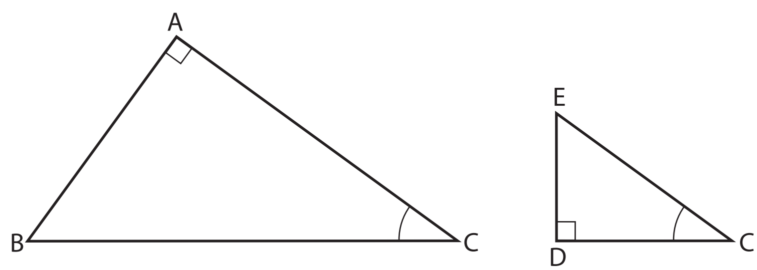
1. a. Jill is correct. Her statement follows from Condition 1 (AAA Similarity).

   The reasoning is—if triangles PQR and ABC have 2 pairs of angles that are equal, then the other pair of angles must also be equal, because the angles in each triangle must add up to 180°.

   b. In two right triangles, you already know there is at least one pair of congruent angles, namely the right angles. Also, if you are told a pair of acute angles are congruent, that is a second pair. It follows from Jill's statement that the triangles are similar.

2. Yes, $\Delta ABC \sim \Delta DEC$, because they are both right triangles and share the acute angle, C.

3. It often helps to separate the triangles and orient them the same way. Since $\Delta ABC \sim \Delta DEC$, the ratios of their corresponding sides are equal. Remember the corresponding sides lie opposite the congruent angles.



$$\frac{AB}{DE} = \frac{AC}{DC} = \frac{BC}{EC} \text{ or } \frac{DE}{AB} = \frac{DC}{AC} = \frac{EC}{BC}$$

Remember, the ratios in a proportion may be flipped over (inverted).

4. You'll have to think back to Section 1, Lessons E and F to solve this question. You have a transversal crossing a set of parallel lines—recall the angle relationships.

   In ΔACX and ΔBDX,

   $\angle AXC \cong \angle BXD$ — vertically opposite angles

   $\angle A \cong \angle B$ — alternate interior angles formed by a transversal crossing parallel lines

   $\angle C \cong \angle D$ — alternate interior angles formed by a transversal crossing parallel lines

   The corresponding angles in ΔACX and ΔBDX are congruent, so the two triangles are, therefore, similar (by Condition 1, AAA Similarity).

5. $\angle B \cong \angle E$. However, the triangles are not similar because the ratios of the corresponding sides are not equal.

   $\frac{6}{3} = 2$, but $\frac{12.6}{6.7} = 1.8805\ldots$

6. In ΔADE and ΔABC,

   $\angle A$ is common to both triangles.

   $\angle ADE \cong \angle B$ — corresponding angles formed by a transversal $\overline{AB}$ cutting the two parallel lines

   $\angle AED \cong \angle C$ — corresponding angles formed by a transversal $\overline{AC}$ cutting the two parallel lines

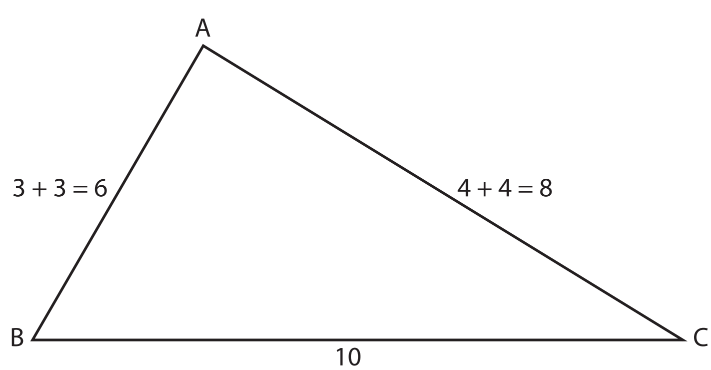
   ΔADE and ΔABC are similar, because the corresponding angles of the two triangles are congruent.

7. Your answer may be different, but the diagram in choice (c) makes the corresponding parts very easy see. With the triangles oriented this way, the corresponding angles and sides are in the same position.

   If you favour another diagram, make sure that you have provided an explanation that is as valid to you as the explanation provided.

Lesson C: Activity 5: Mastering Concepts

1. Separate the triangles.



In ΔABC and ΔADE, $\angle A$ is common.

The ratios of the sides forming $\angle A$ are equal.

$$\frac{AB}{AD} = \frac{6}{3} = 2$$
$$\frac{AC}{AE} = \frac{8}{4} = 2$$

So, $\Delta ABC \sim \Delta ADE$.

Since the triangles are similar, the corresponding angles are congruent.

So, $\angle B \cong \angle ADE$.

Therefore, $\overline{DE} \parallel \overline{BC}$.

corresponding angles formed by a transversal ($\overline{AB}$) are congruent

2. Since $\Delta ABC \sim \Delta ADE$,

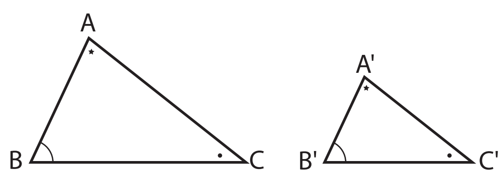
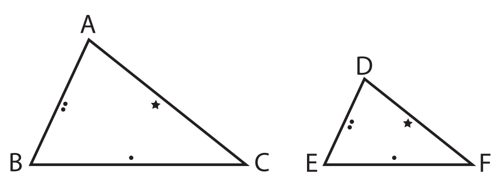
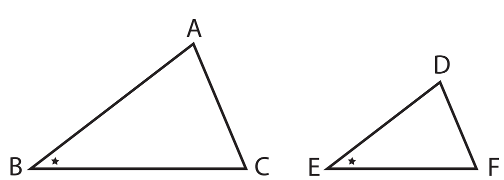
$$\begin{aligned}
\frac{AB}{AD} &= \frac{BC}{DE} \\
\frac{6}{3} &= \frac{10}{DE} \\
\frac{2}{1} &= \frac{10}{DE} \\
\frac{DE}{10} &= \frac{1}{2} \\
(10)\frac{DE}{10} &= \frac{1}{2}(10) \\
DE &= 5
\end{aligned}$$

Lesson D: Applying Similar Triangles

Lesson D: Activity 1: Self-Check

1.

Conditions for Triangle Similarity

Graphic Representation	Description in Words	Symbolic Description	Nickname
Condition 1 	If the corresponding angles of two triangles are equal in measure, then the triangles are similar.	In the diagram, if $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, and $\angle C \cong \angle C'$, then $\Delta ABC \sim \Delta A'B'C'$.	AAA Similarity
Condition 2 	If the corresponding sides of two triangles are proportional, the triangles are similar.	In the diagram, if $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{EF}$, then $\Delta ABC \sim \Delta DEF$.	SSS Similarity
Condition 3 	If two pairs of sides of two triangles are proportional, and the angles between those pairs of sides are congruent, the triangles are similar.	If, in the diagram, $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A \cong \angle D$, then $\Delta ABC \sim \Delta DEF$.	SAS Similarity

2. a. $\angle A \cong \angle F$
   $\angle B \cong \angle E$
   $\angle C \cong \angle D$

   b. a corresponds to f
   b corresponds to e
   c corresponds to d

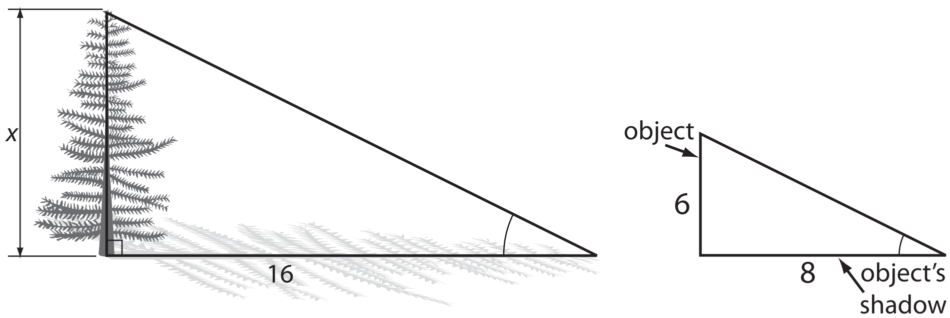
   c. Answers will vary. A sample answer is provided.

   $$\frac{a}{f} = \frac{b}{e}$$

Lesson D: Activity 2: Try This

Answers will vary. This is a sample series of answers based on a simple scenario in which measurements are in whole numbers.

1.



2. The right triangles are similar, because there is a pair of corresponding acute angles that are congruent. That angle is created by the angle of the sunlight.

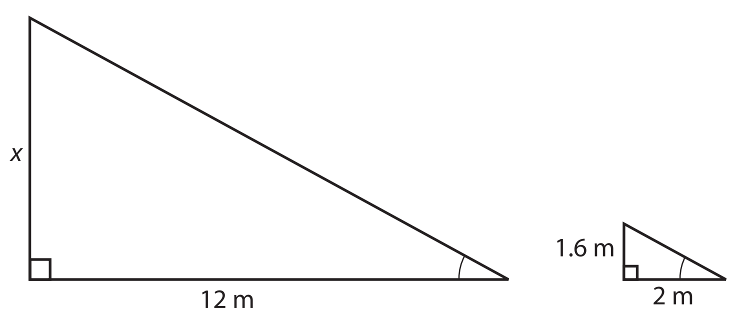
3. $$\frac{x}{6} = \frac{16}{8}$$
   $$(6)\frac{x}{6} = \frac{16}{8}(6)$$
   $$x = 12$$

   The height of the tree is 12 ft.

4. Yes, a tree height of 12 ft is reasonable. The tree appeared to be only about 2 × my height based on a rough estimate.

Lesson D: Activity 3: Self-Check

1. Separate the triangles.



   Are the triangles similar?

   The two right triangles share an acute angle. Therefore, the triangles are similar.

Set up a proportion.

$$\begin{aligned}\frac{x}{1.6} &= \frac{12}{2}\\(1.6)\frac{x}{1.6} &= \frac{12}{2}(1.6)\\x &= 9.6\end{aligned}$$

The height of the school is about 9.6 m.

2. Are the right triangles similar?

   The two right triangles have congruent acute angles at S (vertically opposite angles).

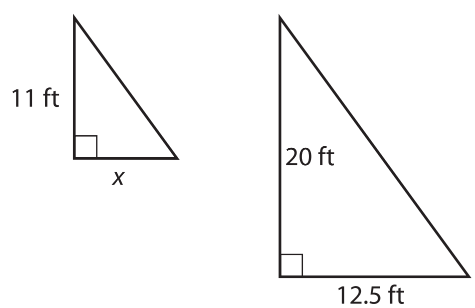
   Therefore, the triangles are similar.

   Set up a proportion.

$$\begin{aligned}\frac{x}{15} &= \frac{50}{10}\\(15)\frac{x}{15} &= \frac{50}{10}(15)\\x &= 75\end{aligned}$$

   The fisherman is 75 m away from Dace.

3. There are four similar right triangles in the diagram. Let $x = \frac{1}{2}$ the width of the balcony.



   Are the right triangles similar?

   The two right triangles share an acute angle. Therefore, the triangles are similar.

Set up a proportion.

$$\frac{x}{12.5} = \frac{11}{20}$$
$$(12.5)\frac{x}{12.5} = \frac{11}{20}(12.5)$$
$$x = 6.875$$

The chalet's balcony width is 2 × 6.875 ft = 13.75 ft.

4. a. In ΔABE and ΔDCE, ∠AEB ≅ ∠DEC. vertically opposite angles

   Since $\overline{AB} \parallel \overline{CD}$ and $\overline{AD}$ is a transversal, ∠BAE ≅ ∠CDE. alternate interior angles

   Therefore, ΔABE ~ ΔDCE.

   b. Set up a proportion.

   Compare corresponding sides. Remember, corresponding sides lie opposite congruent angles.

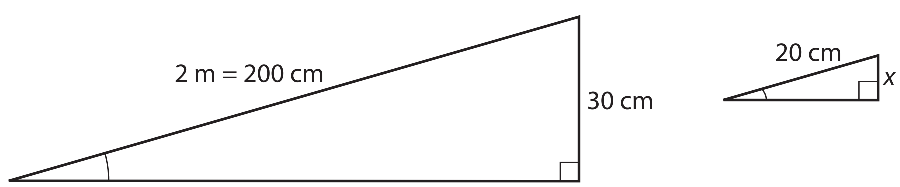
   Now, x and 32 in are the measures opposite ∠ABE and ∠DCE which are congruent.

   So,

$$\frac{x}{32} = \frac{12}{28}$$
$$(32)\frac{x}{32} = \frac{12}{28}(32)$$
$$x = 13.7142\ldots$$
$$x = 13.7$$

   The missing dimension of the ironing board is approximately 13.7 in.

5. Separate the triangles.



The two right triangles share an acute angle. Therefore, the triangles are similar.

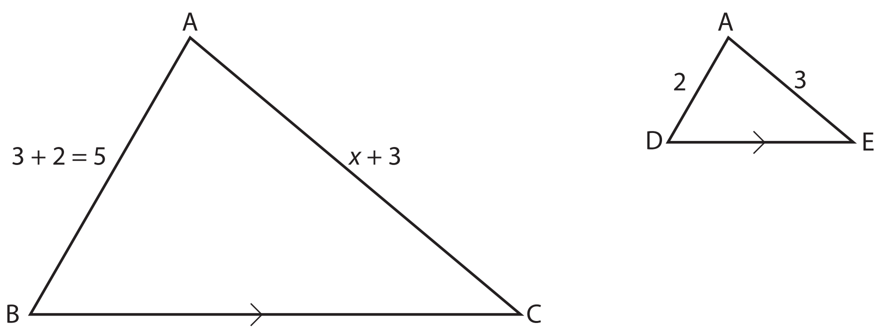
Set up a proportion.

$$\frac{x}{30} = \frac{20}{200}$$
$$(30)\frac{x}{30} = \frac{20}{200}(30)$$
$$x = 3$$

The timber is raised 3 cm off of the ground.

Lesson D: Activity 4: Mastering Concepts

1. Separate the triangles.



Since $\overline{DE} \parallel \overline{BC}$ and $\overline{AB}$ is a transversal, $\angle ADE \cong \angle ABC$. corresponding angles

$\angle A \cong \angle A$ common angle

Therefore, $\Delta ABC \sim \Delta ADE$. Two pairs of corresponding angles are congruent.

2. Set up a proportion.

$$\frac{x+3}{3} = \frac{5}{2}$$
$$(3)\frac{x+3}{3} = \frac{5}{2}(3)$$
$$x + 3 = 7.5$$
$$x = 7.5 - 3$$
$$x = 4.5$$

Lesson E: Pythagorean Theorem

Lesson E: Activity 1: Try This

1. It should look pretty much the same.

2. It should have been close to 90º. If your measurement was a bit off, it could be due to a small error when marking off the "knots" on the string or to the difficulty of measuring the angle while your partner's fingers were holding the vertices.

3. No. There is only one triangle with sides measuring 3 units, 4 units, and 5 units.

Lesson E: Activity 2: Try This

1. Yes, the squares of the legs add up to the square of the hypotenuse (when the triangle is a right triangle).

2. a. The hand-drawn triangle should not be a right triangle.

   b. You were asked to draw a triangle for which the Pythagorean Theorem does not apply. Therefore, no, $f^2 + g^2 = h^2$ is NOT true for this triangle.

   Your data in the two columns on the right of the table "Investigating Right Triangles" should be consistent with the answers to Questions 1 and 2. That is, $f^2 + g^2 = h^2$ if, and only if, the triangle (with sides f, g, and h) is a right triangle.

Lesson E: Activity 3: Self-Check

1. a. 16
   b. 234.09
   c. 7
   d. 29.7610...

2. $$\begin{aligned} x^2 &= 144 \\ \sqrt{x^2} &= \sqrt{144} \\ x &= 12 \end{aligned}$$

To isolate x, take the square root of both sides of the equation.

Lesson E: Activity 4: Try This

1. The two acute angles of a right triangle are complementary—they add to 90º. At each vertex there are three angles, which add up to 180°. If two are complementary, the third angle must be a right angle.

   Each of the four sides of the square is the length of the hypotenuse of one of the right triangles.

   So, the figure has four right angles and four sides that are equal in length. It is a square.

2. Each side of the square is c-units. Its area is $c \times c = c^2$.

3. None of the triangles has been removed; they have just been repositioned. Therefore, the black showing will be the same in area.

4. Again, sides are equal and the four angles are right angles. Acute angles of a right triangle are complementary.

5. Its area is b^2. The larger square is b-units on a side, $b \times b = b^2$.

6. The larger square is a-units on a side, $a \times a = a^2$.

7. The total area = $a^2 + b^2$.

8. The areas are equal, $c^2 = a^2 + b^2$. This is the Pythagorean Theorem.

Lesson E: Activity 5: Self-Check

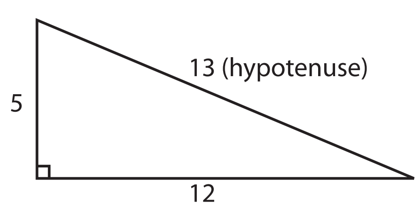
1. a. $13^2 = 169$

   $$12^2 + 5^2 = 144 + 25$$
   $$= 169$$

   So, $12^2 + 5^2 = 13^2$.

   Therefore 5, 12, 13 is a Pythagorean triple.

   b.

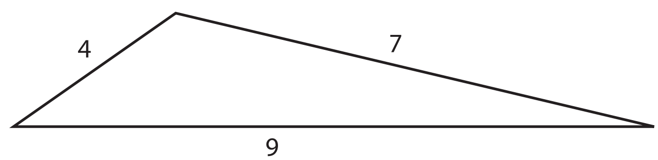


c. 5, 12, 13
   2(5, 12, 13) = 10, 24, 26
   3(5, 12, 13) = 15, 36, 39
   4(5, 12, 13) = 20, 48, 52

2. a. $9^2 = 81$

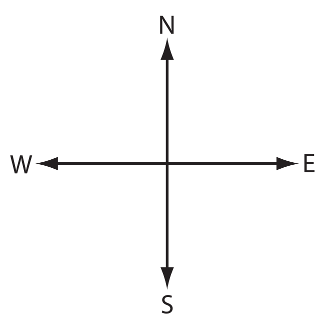
   $$4^2 + 7^2 = 16 + 49$$
   $$= 65$$

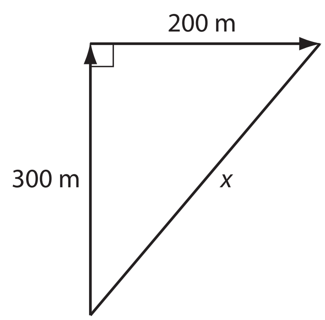
   Since $9^2 \neq 4^2 + 7^2$, 4, 7, 9 is not a Pythagorean triple.

   b. 

   c. This triangle is not a right triangle because 4, 7, 9 is not a Pythagorean triple. The Pythagorean Theorem only *works* for right triangles.

3. Draw a diagram.





   Let Kale's distance from his starting point be x.

   $(\text{hypotenuse})^2 = (\text{leg } 1)^2 + (\text{leg } 2)^2$

$$x^2 = 300^2 + 200^2$$
$$x^2 = 90\ 000 + 40\ 000$$
$$x^2 = 130\ 000$$
$$x = \sqrt{130\ 000}$$
$$x = 360.5551\ldots$$
$$x \approx 361$$

Kale is approximately 361 m from his starting point.

Lesson E: Activity 6: Mastering Concepts

$$\begin{aligned}(3n)^2 + (4n)^2 &= 9n^2 + 16n^2 \\ &= 25n^2 \\ (5n)^2 &= 25n^2\end{aligned}$$

Because $(3n)^2 + (4n)^2 = (5n)^2$; $3n$, $4n$, $5n$ is a Pythagorean triple.

Lesson F: Applying the Pythagorean Theorem

Lesson F: Activity 1: Try This

1. $\angle C = 90°$

2. If ΔABC satisfies the Pythagorean Theorem equation $c^2 = a^2 + b^2$ then ΔABC is a right triangle.

   $$\begin{aligned}c^2 &= (13)^2 \\ &= 169\end{aligned}$$

   $$\begin{aligned}a^2 + b^2 &= 12^2 + 5^2 \\ &= 144 + 25 \\ &= 169\end{aligned}$$

   Since $c^2 = a^2 + b^2$, ΔABC is a right triangle.

3. The numbers 5, 12, 13 are a Pythagorean triple.

Lesson F: Activity 2: Self-Check

Triangle	*a*	*b*	*c*	a^2	b^2	c^2	$a^2 + b^2$	Measured $\angle C$
#1	3	4	5	9	16	25	25	90°
#2	4	5	5	16	25	36	41	83°
#3	5	6	7	25	36	49	64	78°
#4	6	8	10	36	64	100	100	90°
#5	8	15	17	64	225	289	289	90°
#6	6	16	17	36	256	289	292	99°

1. The hypotenuse is always the longest side. So, if a triangle is a right triangle, the right angle will be opposite the longest side. The square on the longest side is equal to the sum of the squares on the two other sides.

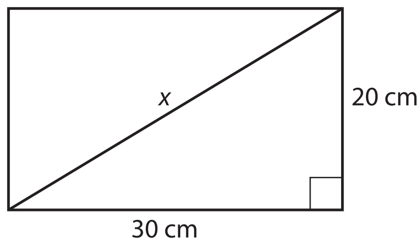
2. The triangles with sides 3, 4, 5, sides 6, 8, 10, and sides 8, 15, 17. For these triangles the table cells in column c^2 and columns $a^2 + b^2$ are equal. That equality shows that the side lengths of these triangles satisfy the Pythagorean Theorem. So, these triangles are right triangles.

3. 3, 4, 5, and 6, 8, 10, and 8, 15, 17 are Pythagorean triples.

4. Multiply by 2 (16, 30, 34)
   Multiply by 3 (24, 45, 51)
   Multiply by 4 (32, 60, 68)
   Multiply by 5 (40, 75, 85)

Lesson F: Activity 3: Self-Check

1.



Let the length of the diagonal be x.

$x^2 = 20^2 + 30^2$
$x^2 = 400 + 900$
$x^2 = 1300$
$x = \sqrt{1300}$
$x = 36.0555\ldots$
$x \approx 36$

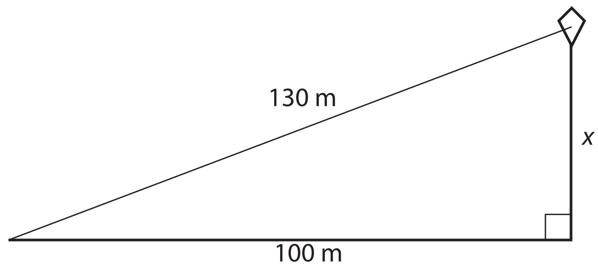
The length of the longest rod is approximately 36 cm.

2. Let the distance be x.

$x^2 = 1^2 + 2^2$
$x^2 = 1 + 4$
$x^2 = 5$
$x = \sqrt{5}$
$x = 2.2360\ldots$
$x \approx 2.2$

Maxim is approximately 2.2 miles from his starting point.

3.

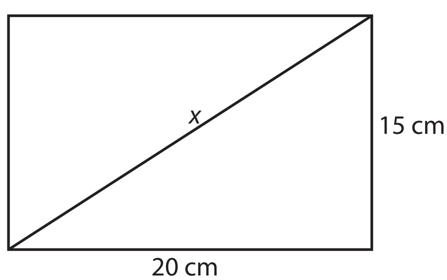


Let the height be x metres.

$$
\begin{aligned}
x^2 + 100^2 &= 130^2 \\
x^2 + 10\ 000 &= 16\ 900 \\
x^2 &= 16\ 900 - 10\ 000 \\
x^2 &= 6900 \\
x &= \sqrt{6900} \\
x &= 83.0662\ldots \\
x &\approx 83
\end{aligned}
$$

The height of the kite is approximately 83 m.

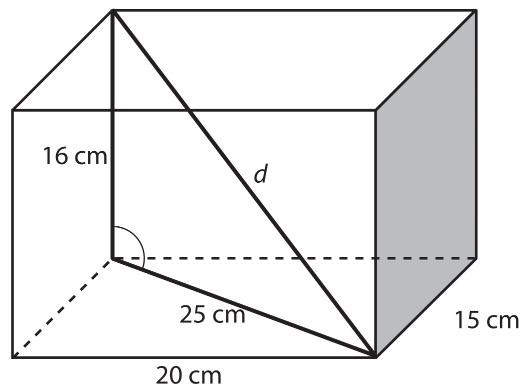
4. First determine if the knitting needle can fit flat along the diagonal on the bottom of the box.



Let the diagonal distance across the bottom of the box be x.

$$
\begin{aligned}
x^2 &= 20^2 + 15^2 \\
x^2 &= 400 + 225 \\
x^2 &= 625 \\
x &= \sqrt{625} \\
x &= 25
\end{aligned}
$$

The diagonal along the bottom is only 25 cm long. So, the 28 cm knitting needle cannot lie flat. However, can it lie diagonally from one corner on the bottom, across the centre of the box, to the opposite corner at the top?

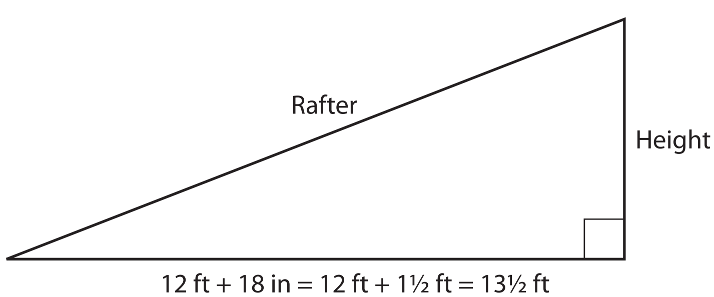


To calculate this diagonal distance d, apply the Pythagorean Theorem a second time.

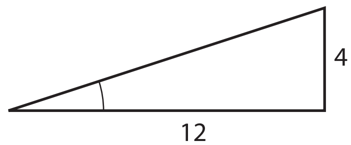
$$
\begin{aligned}
d^2 &= 25^2 + 16^2 \\
d^2 &= 625 + 256 \\
d^2 &= 881 \\
d &= \sqrt{881} \\
d &= 29.6816\ldots \\
d &\approx 29.7
\end{aligned}
$$

The diagonal distance from one bottom corner across to the opposite top corner is approximately 29.7 cm. So, a 28 cm knitting needle can fit in the box with the lid closed.

Lesson F: Activity 4: Mastering Concepts



From the middle of the truss to the wall, and then 18 in beyond, is a horizontal distance of 13.5 ft (span ÷ 2 + 18 in).



The slope of the roof is a rise of 4 in and a run of 12 in. The rise and run can be in feet or inches.

Because the two triangles shown above are similar, the actual roof height can be found using proportional reasoning.

$$\begin{aligned}\frac{\text{height}}{4} &= \frac{13.5}{12}\\ (4)\frac{\text{height}}{4} &= \frac{13.5}{12}(4)\\ \text{height} &= 4.5\end{aligned}$$

The height is 4½ feet.

Apply the Pythagorean Theorem to find the rafter length using the actual horizontal distance and height of the roof support.

$$\begin{aligned}(\text{rafter length})^2 &= (13.5)^2 + (4.5)^2\\ (\text{rafter length})^2 &= 182.25 + 20.25\\ (\text{rafter length})^2 &= 202.5\\ \text{rafter length} &= \sqrt{202.5}\\ \text{rafter length} &= 14.2302\ldots\\ \text{rafter length} &= 14 \text{ ft} + 0.2302\ldots \text{ ft}\\ \text{rafter length} &= 14 \text{ ft} + (0.2302\ldots)(12) \text{ in}\\ \text{rafter length} &= 14 \text{ ft } 2 \text{ in} + (0.7629\ldots)\frac{16}{16} \text{ in}\\ \text{rafter length} &= 14 \text{ ft } 2 \text{ in} + \frac{12.2078\ldots}{16} \text{ in}\\ \text{rafter length} &\approx 14 \text{ ft } 2\frac{12}{16} \text{ in}\end{aligned}$$

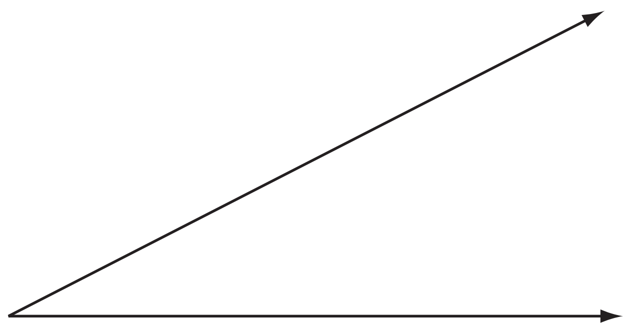
The rafter length, to the nearest sixteenth of an inch, is 14 ft 2 $\frac{12}{16}$ in, or 14 ft 2 $\frac{3}{4}$ in.

Glossary

acute angle

an angle greater than 0° but less than 90°

For example, this is an acute angle.



adjacent angles

angles which share a common vertex and lie on opposite sides of a common arm

adjacent side

the side next to the reference angle in a right triangle. (The adjacent side cannot be the hypotenuse.)

alternate exterior angles

exterior angles lying on opposite sides of the transversal

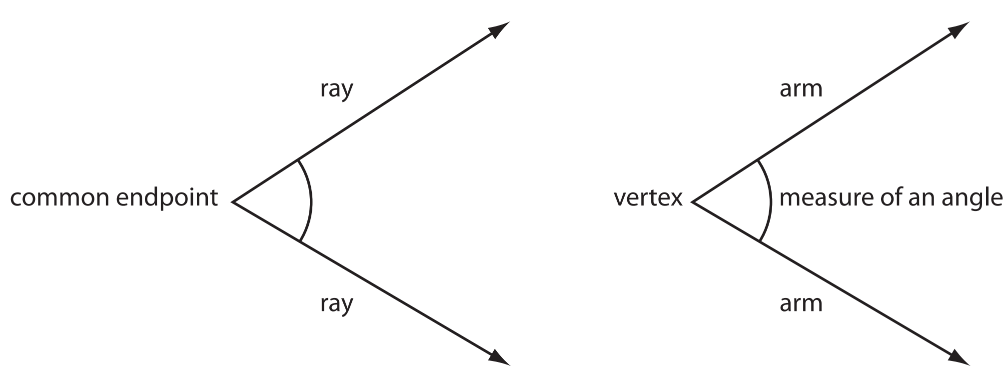
alternate interior angles

interior angles lying on opposite sides of the transversal

angle

a geometric shape formed by two rays with a common endpoint

Each ray is called an *arm of the angle*. The common endpoint of the arms of the angle is the vertex of the angle.



angle of depression

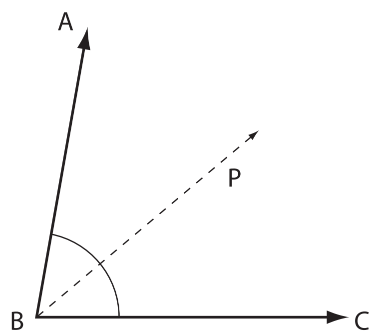
an angle below the horizontal that an observer must look down to see an object that is below the observer

angle of elevation

the angle above the horizontal that an observer must look to see an object that is higher than the observer

bisect

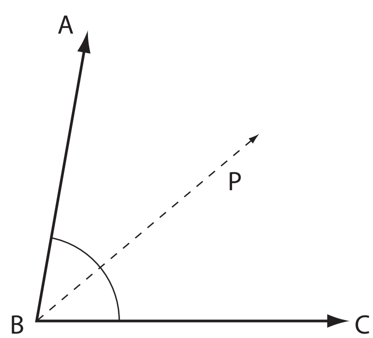
divide into two congruent (equal in measure) halves



bisector

a line or ray which divides a geometric shape into congruent halves

Ray BP is a bisector of ∠ABC, since it bisects ∠ABC into two congruent halves.



$\angle ABP \cong \angle PBC$

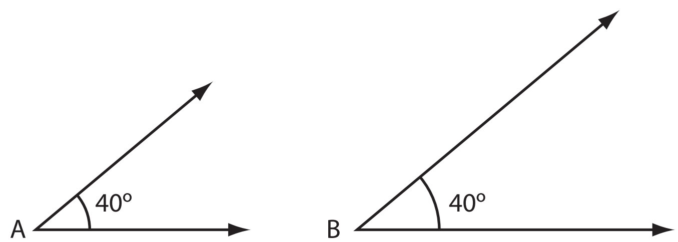
clinometer

a device for measuring angles to distant objects that are higher or lower than your position

complementary angles
two angles with measures that add up to 90°

One angle is called the *complement* to the other.

congruent angles
angles with the same measure



In the diagram ∠A = 40° and ∠B = 40°. So, ∠A and ∠B are congruent.

There is a special symbol for "is congruent to." The congruence symbol is ≅.

So, you can write ∠A ≅ ∠B.

corresponding angles
angles in the same relative positions when two lines are intersected by a transversal

cosine ratio
the ratio of the length of the side adjacent to the reference angle, to the length of the hypotenuse of the right triangle

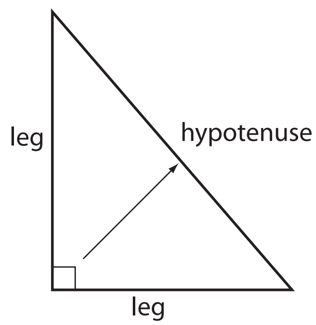
exterior angles
angles lying outside two lines cut by a transversal

full rotation
an angle having a measure of 360°

This is a full rotation angle.

hypotenuse

in a right triangle, the side opposite the right angle; the longest side in a right triangle



indirect measurement

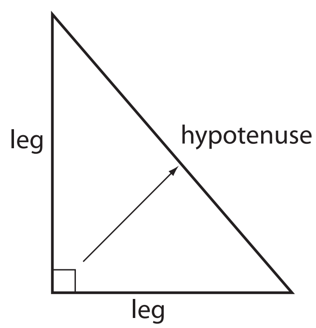
taking one measurement in order to calculate another measurement

interior angles

angles lying between two lines cut by a transversal

leg

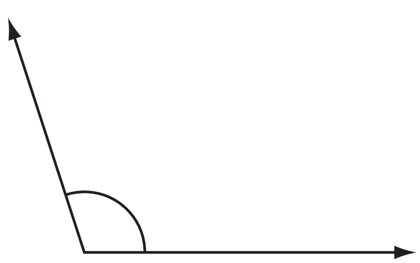
one of the two sides of a right triangle that forms the right angle



obtuse angle

an angle greater than 90° but less than 180°

For example, this is an obtuse angle.



opposite side
the side across from the reference angle in a right triangle

parallel
lines that are the same distance apart everywhere: they never meet

perpendicular
lines that meet at right angles

polygon
a many-sided figure

A triangle is a polygon with three sides, a quadrilateral is a polygon with four sides, and so on.

proportion
the statement showing two ratios are equal

Pythagorean Theorem
for any right triangle, the square of the hypotenuse is equal to the sum of the squares of the two legs

Pythagorean triple
three whole numbers, which represent the lengths of the sides of a right triangle

There are an infinite number of such triples.

reference angle
an acute angle that is specified (example, shaded) in a right triangle

referent
an object or part of the human body you can refer to when estimating length or distance

reflex angle
an angle having a measure greater than 180° but less than 360°

This is an example of a reflex angle.

regular polygon
a polygon with all its angles equal in measure and all its sides equal in measure

right angle
one quarter of a complete rotation. It is 90° in measure.

scale factor
the number by which the length and the width of a figure is multiplied to form a larger or smaller similar figure

similar figures
figures with the same shape but not necessarily the same size

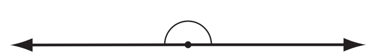
A figure similar to another may be larger or smaller

sine ratio
the ratio of the length of the side opposite to the reference angle, over the hypotenuse of the right triangle

solve a right triangle
to find all the missing sides and angles in a right triangle

straight angle
one half a rotation; an angle 180°

This is a straight angle.



straightedge
a rigid strip of wood, metal, or plastic having a straightedge used for drawing lines

When a ruler is used without reference to its measuring scale, it is considered to be a straightedge.

supplementary angles
two angles, which add up to 180°

In a pair of supplementary angles, one angle is the supplement to the other.

symmetry
the property of being the same in size and shape on both sides of a central dividing line

tangent ratio
the ratio of the length of the side opposite to the selected acute angle, to the length of the side adjacent to the selected acute angle in a right triangle

transversal
a line that cuts across two or more lines

trigonometry
the branch of mathematics based originally on determining sides and angles of triangles, particularly right triangles

vertically opposite angles
angles lying across from each other at the point where two lines intersect

Vertically opposite angles are also referred to as *opposite angles*.

Pythagorean Theorem Proof Template